

Loop Quantum Gravity: Four Recent Advances and a Dozen Frequently Asked Questions

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As per organizers' request, my talk at the 11th Marcel Grossmann Conference consisted of two parts. In the first, I illustrated recent advances in loop quantum gravity through examples. In the second, I presented an overall assessment of the status of the program by addressing some frequently asked questions. This account is addressed primarily to researchers outside the loop quantum gravity community.

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I. EXAMPLES OF RECENT ADVANCES

In this section I will attempt to provide a flavor of the progress that is being made on various fronts of Loop Quantum Gravity (LQG). I have chosen examples from four areas: i) Mathematical foundation of the theory [1]; ii) Planck scale physics near the big bang singularity in the FRW cosmologies [2]; iii) Effective matter theories obtained by integrating out gravitational degrees of freedom [3]; and iv) Recovery of the graviton propagator starting from a non-perturbative, background independent theory [4]. The first two of these developments arose in the canonical formulation of the theory while the last two refer to spin foams —the path integral framework.

I want to emphasize that these are only illustrations. Because of time limitation, I could not include other recent advances, in particular the mathematically interesting extensions of gauge theories using quantum groups [5]; the master constraint program [6] and algebraic quantum gravity [7]; explorations of the Planck scale geometry in symmetry reduced midi-superspaces [8]; ramifications of quantum geometry for dynamics of matter [9]; ideas on black hole evaporation and information loss [10] and numerous phenomenological developments in quantum cosmology [12]. For detailed reviews, see, e.g., [13, 14, 15].

A. Power of background independence: uniqueness of LQG kinematics

In the Hamiltonian framework of any background independent theory —such as general relativity— one encounters first class constraints which tell us that diffeomorphisms generate gauge transformations.¹ In the fifties and sixties Bergmann and Dirac analyzed such classical

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¹ For simplicity of presentation, in this discussion I will restrict myself to spatially compact space-times. In the asymptotically flat (or AdS) context, constraints generate only those diffeomorphisms which are asymptotically identity; only these are treated as gauge. Although for definiteness I have discussed the Dirac quantization program below, considerations remain unaltered in the BRST scheme.

systems and Dirac introduced a systematic quantization program. Here, one first ignores constraints and introduces a kinematic framework consisting of an algebra \mathfrak{a} of quantum operators and a representation thereof on a Hilbert space \mathcal{H}_{kin} . This provides the arena for defining and solving the quantum constraints. When equipped with a suitable inner product, the space of solutions defines the physical Hilbert space \mathcal{H}_{phy} .

In particle mechanics one often begins with the Heisenberg-algebra generated by operators q and p , satisfying the canonical commutation relations, or the Weyl algebra generated by $U(\lambda) := \exp i\lambda q$ and $V(\mu) := \exp i\mu p$. The von Neumann theorem ensures us that, under suitable physically motivated assumptions, the algebra admits a unique irreducible representation, namely the standard Schrödinger one. Therefore for constrained mechanical systems one generally uses this representation for quantum kinematics. However for field theories —i.e. for systems with an infinite number of freedom— the uniqueness theorem fails and there is an infinite number of inequivalent representations. To select a preferred one, additional physical inputs are necessary.

A systematic approach to finding these representations is provided by the celebrated Gel'fand-Naimark-Segal (GNS) construction which may be summarized as follows. A *state* on a \star -algebra \mathfrak{a} is a positive linear functional, i.e., a linear map F from \mathfrak{a} to \mathbb{C} such that $F(a^*a) \geq 0$ for all a in \mathfrak{a} and $F(I)=1$ (where I is the identity element of \mathfrak{a}). Given a state F , GNS provided a constructive procedure to obtain a \star -representation of \mathfrak{a} by operators on a Hilbert space \mathcal{H} such that: i) there is a normalized cyclic vector Ψ_F in \mathcal{H} (i.e., the action \mathfrak{a} on Ψ_F yields a dense sub-space of \mathcal{H}); and, ii) $F(a) = \langle \Psi_F | a | \Psi_F \rangle$. Thus, in this GNS representation the original positive linear functional yields just expectation values of operators in the cyclic state.² To summarize, the task of finding a representation is *neatly reduced to that of finding a state F on the algebra*. If the state F is invariant under an automorphism of \mathfrak{a} , that automorphism is represented by an unitary transformation on \mathcal{H} . For free field theories in Minkowski space-time, the requirement of Poincaré invariance (together with certain technical conditions) selects a unique state F on the Weyl algebra of operators —the corresponding Ψ_F is the Fock vacuum. Since Ψ_F is Poincaré invariant, the Poincaré group is unitarily implemented on the Fock space \mathcal{H} .

In LQG, the basic variables are a gravitational spin connection A_a^i on a 3-dimensional manifold M and its conjugate momentum E_i^a which enjoys the geometric interpretation of an orthonormal triad on M (with density weight 1). The \star -algebra \mathfrak{a} is generated by holonomies h_e of A_a^i along edges e in M and fluxes $E_{f,S} := \int_S E_i^a f^i dS_a$ of triads (smeared by test fields f^i) across 2-surfaces S in M . (It is thus analogous to the algebra generated by functions $\exp i\lambda q$ and p on the phase space of a non-relativistic particle.) However, since the system has an infinite number of degrees of freedom, \mathfrak{a} admits infinitely many inequivalent representations. The GNS construction provides a convenient avenue to arrive at a preferred representation.

What condition should we use to select a state F or a class of such states? The construction of the algebra does not require a background geometry or indeed any background field. As a consequence, each diffeomorphism on M gives rise to an automorphism of this algebra. To promote background independence in the quantum theory it is natural to ask that the state be invariant under these automorphisms. A surprisingly powerful recent theorem is that the algebra \mathfrak{a} admits *exactly one* diffeomorphism invariant state [1]! In this precise

² In the mathematical literature, Ψ_F is often referred to as the ‘vacuum’ although it may not be directly related to the Hamiltonian of the theory.

sense, *quantum kinematics of LQG is uniquely determined by the requirement of background independence*. Note that in contrast to the situation with Poincaré invariance, there is no assumption about dynamics here; the requirement of diffeomorphism invariance is much more powerful than that of Poincaré invariance.

The GNS representation that results from this F has been known for sometime now. It was introduced ab-initio some ten years ago [16] and underlies the quantum geometry of LQG kinematics. Specifically, it is in this representation that one can introduce a spin network basis [17] and show that geometric operators such as areas of surfaces and volumes of regions have purely discrete eigenvalues [13, 14, 15]. The uniqueness result shows that the origin of these features lies in background independence and makes these results compelling.

B. Quantum nature of the big bang in FRW cosmologies

In the standard cosmology based on classical general relativity, space-time and matter are both born at the big bang and it is meaningless to ask what was there before. However, one expects quantum effects to dominate when the curvature enters the Planck regime. Thus, big-bang is a prediction of general relativity in a domain where it is inapplicable! Key questions to any quantum gravity theory are then: What is the quantum nature of the big bang? Is the classical singularity resolved by quantum effects? Is there a quantum extension of the classical space-time?

These questions were raised already in the late sixties. To analyze them, DeWitt [18] and Misner [19] observed that in classical general relativity one can restrict oneself just to spatially homogeneous and isotropic situations to an excellent degree of approximation and suggested that one begin with the same strategy also in quantum theory. This led to quantum cosmology. Now, imposition of these symmetries freezes all but a finite number of degrees of freedom. Therefore, field theoretical difficulties are by-passed and one is led to a quantum mechanical system. Recall however that background independence of general relativity leads to constraints. Because of strong spatial symmetries most of them are automatically satisfied. However, one Hamiltonian constraint survives. Thus we are led to a quantum mechanical system with a single constraint. Since there are only a finite number of degrees of freedom, it was customary to represent kinematical quantum states by wave functions $\Psi(a, \phi_i)$ of the scale factor a and matter fields ϕ_i . Imposition of the quantum constraint then reduces to solving a second order differential equation, called the Wheeler-DeWitt (WDW) equation, which governs quantum dynamics. However, because of background independence, extracting physics is not trivial: a priori, there is no time, no obvious inner product on solutions to this equation nor candidate Dirac observables. Therefore, much of the early work was confined to a WKB approximations.

However, in the case when matter fields include a massless scalar field ϕ_o , it is possible to complete the Misner-Wheeler-DeWitt (MWDW) program [2]. The massless scalar field serves as an intrinsic or emergent time variable both in classical and quantum theories. The WDW equation takes the form

$$\partial_{\phi_o}^2 \Psi(a, \phi_i) = \Theta \Psi(\phi_i, a) \quad (1.1)$$

where Θ is a second order differential operator involving only the scale factor and matter fields other than ϕ_o . one can construct the Hilbert space of physical states by a group averaging method used in full LQG and find a convenient, complete set of Dirac observables.

One can now return to the key questions posed at the beginning of this sub-section. Thus, one can start with a quantum state which is peaked, in a well-defined sense, at a chosen FRW solution at late times and evolve it backward. The good news is that it remains peaked at the classical trajectory showing that general relativity is recovered from this quantum framework. The bad news is that the agreement continues all the way into the big bang singularity. Thus, in the MWDW theory, the singularity is *not* resolved by quantum effects.

Because there are only a finite number of degrees of freedom, it appeared for about two decades that the big bang singularity cannot be resolved in quantum cosmology without an external input such as matter fields violating energy conditions, or new boundary conditions, e.g., a la Hartle and Hawking. However, six years ago, Bojowald [11] showed that this status-quo changes dramatically in loop quantum cosmology. The primary reason is that if one closely mimics full LQG in the symmetry reduced models, one is led to a representation which is inequivalent to Schrödinger's. Thus loop quantum cosmology (LQC) is *inequivalent to the* MWDW theory already at the kinematical level.

Last year, the program was completed by first constructing the physical Hilbert and Dirac observables and then analyzing quantum dynamics in detail [2]. Now, the second order differential operator $\underline{\Theta}$ of the WDW equation (1.1) is replaced by a second order *difference* operator Θ . In the low curvature region, Θ is well approximated by $\underline{\Theta}$ but near the Planck regime there are major differences. In particular, if we start with a state which is semi-classical at late times and evolve backwards, it follows the classical trajectory only until scalar curvature R —the only independent curvature invariant for this model— reaches a critical value $R_{\text{crit}} \approx 13\pi/\ell_{\text{Pl}}^2$. Then quantum geometry gives rise to an effective repulsive force so that the wave function bounces, giving rise to a pre-big-bang branch which again obeys the classical Einstein equations once the curvature becomes significantly less than R_{crit} .

This quantum evolution has been analyzed in detail using numerical methods as well as effective equation techniques for the $k = 0$ and $k = 1$ FRW models with and without cosmological constant (and with less rigor and completeness for the $k = -1$ and the Bianchi I anisotropic models). I will conclude with a few illustrative examples from the $k = 1$ case. Here, Einstein's equations imply that the universe expands after the big bang to a maximum radius a_{max} and then recollapses. First, one can ask for the volume V_{min} of the universe at the quantum bounce. This depends on the classical trajectory about which the wave function is peaked at a late time. If the trajectory is such that a_{max} is a megaparsec, then $V_{\text{min}} \approx 6 \times 10^{16} \text{cm}^3$, some 10^{115} times larger than the Planck volume! Furthermore V_{min} scales as a_{max}^2 , whence more 'macroscopic' the universe, larger the value of V_{min} . Quantum effects dominate when the space-time curvature or matter density —and not of volume— enters the Planck regime. Second, as a result of the recollapse, in the $k = 1$ case the classical universe exhibits a big bang as well as a big crunch singularity. Both are replaced by bounces in the quantum theory. Therefore, we are led to a cyclic scenario. Thus, quantum space-time is *enormously larger* than what classical general relativity would have us believe. Finally, one of the outstanding questions in full LQG is whether the theory admits a rich semi-classical sector. Even though the LQC departures from general relativity are small when $\rho \ll \rho_{\text{Pl}}$, the expansion lasts for a very long time. Can the quantum effects accumulate so that the universe does not recollapse or recollapses at a value significantly different from a_{max} ? Indeed, using an early form of the LQC Hamiltonian constraint, Green and Unruh [20] argued that there would be no recollapse. However the recent, much more complete and detailed analysis [2] has shown that the universe does recollapse in LQC and agreement with

classical general relativity on a_{\max} is excellent. Even for universes which are so small that $a_{\max} \approx 30\ell_{\text{Pl}}$, the classical Friedmann formula $\rho_{\min} = 3/(8\pi G a_{\max}^2)$ holds to one part in 10^{-5} and the agreement improves greatly for ‘macroscopic’ universes, i.e., ones with macroscopic values of a_{\max} . This detailed agreement suggests that LQG has the potential for curing the ultraviolet divergences of general relativity while admitting a rich semi-classical sector.

These attractive results have been obtained for the simplest models —FRW space-times with a massless scalar field— by making a symmetry reduction at the classical level and then going to the quantum theory. This LQC is yet to be derived from full LQG. Within this key limitation, it is nonetheless interesting that one can probe Planck scale physics near the most interesting curvature singularities, the results are strikingly different from the MWDW theory, and the difference originates in the quantum nature of geometry underlying LQG.

C. Emergence of non-commutative quantum field theory

An outstanding question facing any quantum gravity theory is: What are the ramifications of the quantum nature of gravity on dynamics of ordinary matter fields? In the canonical framework, analysis involves two steps: i) construction of quantum field theories on quantum geometries; and ii) dynamical selection of suitable quantum geometries appropriate for low energy physics. There has been considerable work on the first part which has clarified in particular how the fundamental discreteness inherent to quantum geometry could tame the ultra-violet divergences that plague quantum field theories in the continuum [9, 15, 21]. However, investigation of the second step is still at a preliminary stage. Consequently, reliable predictions on quantum gravity ramifications for low energy physics are yet to emerge from the canonical theory.³

Path integral methods are better suited to address this issue. For, the program can be stated rather simply: Given a quantum theory with matter, what is the effective theory that results after gravitational degrees of freedom have been integrated out? The technical execution of the program is however not as simple because of the task of integrating out the gravitational degrees of freedom. What measure should one use? Since Einstein gravity is not perturbatively renormalizable, one cannot use methods that are readily available in the more familiar Minkowskian field theories.

Spin foams provide a non-perturbative and rigorous framework to evaluate path integrals for quantum gravity [22]. Their strength lies in the fact that they supply a definition of a quantum space-time in algebraic and combinatorial terms. In essence, one can think of the quantum space-time as a generalized, 2-dimensional Feynman diagram with degrees of freedom propagating along surfaces. These considerations can be made explicit in 2+1 dimensions. Through a series of steps, it has recently become possible to go further and integrate out the gravitational degrees of freedom to obtain effective theories for matter fields [3]. Furthermore, the process seems to naturally lead to non-commutative field theories where particles can be envisaged as living in a *non-commutative space-time*.

Let us start with the partition function $Z = \int Dg \int D\phi \exp i(S[\phi, g] + S[g])$ where $S[\phi, g]$ is the action for a scalar field ϕ (with non-derivative interactions) on a 3-dimensional space-

³ See, however, the work of the Madrid group [8] based on Hamiltonian methods. Although it does not lie in the strict confines of canonical LQG, it is based on quantum geometry considerations.

time with metric g_{ab} and $S[g]$ is the general relativity action. The goal is to integrate out the gravitational degrees of freedom and obtain an effective action $S_{\text{eff}}[\phi]$ for the matter field ϕ . The idea is to achieve this by first expanding out the ϕ -integral in terms of Feynman diagrams which depend on a background metric g_{ab} : $Z = \sum_{\Gamma} C_{\Gamma} \left(\int Dg I_{\Gamma}[g] \exp iS[g] \right) \equiv \sum_{\Gamma} C_{\Gamma} \hat{I}_{\Gamma}$. The next and even more non-trivial step is to re-sum the Feynman diagrams and obtain an effective action: $Z = \int D\phi \exp iS_{\text{eff}}[\phi, g_o]$, where g_o is the flat metric.

This procedure has been carried out by making an ansatz for coupling gravity to the Feynman diagrams of particles [3]. A rigorous derivation of the ansatz starting from the action $S[\phi, g]$ is still lacking. But the final result is rather simple and therefore attractive: The standard Abelian products of fields in the original action $S[\phi, g]$ are just replaced by a non-Abelian \star -product in $S_{\text{eff}}[\phi, g_o]$. If metrics g_{ab} have signature $+,+,+$, the resulting theory is called Riemannian while if the signature is $+,-,-$, (or $-,+,-$) it is the Lorentzian theory of direct physical interest. Detailed analysis has been carried out in both cases. (Note that the partition function involves $\exp iS$ in both cases; thus the traditional Euclidean quantum theory is not considered.)

While the action $S[\phi, g_o]$ is invariant under the six dimensional Poincaré group, $S_{\text{eff}}[\phi, g_o]$ is invariant under the so-called κ deformation of the Poincaré group—which is again six dimensional— where κ now has the value $4\pi G$. Ordinarily, deformed Poincaré theories suffer from huge ambiguities because we do not a priori know which elements of the Hopf algebra are to be identified with physical energy and momentum. However, in the present case these are resolved because the deformation is tied directly to quantum gravity effects stemming from the Ponzano-Regge model. The effect of quantum gravity on matter is two folds. First, the mass gets renormalized via $m \rightarrow \sin \kappa m / \kappa$. Second, the momentum space is no longer a flat space but the homogeneous space $\text{SO}(3)$ in the Riemannian case and $\text{SO}(2,1)$ in the Lorentzian. The underlying group structure enables one to ‘add’ momenta of particles but this operation is non-Abelian. These particles can be regarded as moving in the ‘dual’, non-commutative position space co-ordinatized by X_i satisfying the relations:

$$[X_i, X_j] = i\kappa\hbar\epsilon_{ijk}X_k. \quad (1.2)$$

The \star -algebra generated by the three operators X_i subject to these commutation relations is referred to as the κ -Minkowski space. Detailed expressions show that energy of matter particles is bounded above by $1/\kappa$ in any rest frame (because $p_0^2 - p_1^2 - p_2^2 < 1/\kappa^2$). Similarly, there is a minimal length scale $\sim \kappa\hbar$ accessible to the theory.

Thus, thanks to the spin foam technology that has been systematically developed over the last decade, a new approach to analyze the quantum gravity effects on matter fields has now emerged. Although some issues still remain, it is encouraging that the analysis provides a concrete realization of several features that have been heuristically expected in the literature on physical grounds: a quantum gravity induced cut-off, non-commutative fields, effective non-commutative space-times, and the idea proposed by Snyder [23] (sixty years ago!) that a curved momentum space could regularize field theories. However, the detailed work to date *is* tied to features of 3d gravity. Although extensions to 4 dimensions have already been suggested, it is not yet clear which of these features will filter down in 4-dimensional effective theories.

D. The graviton propagator

Background dependence of LQG has powerful consequences. As we saw in section IA, it selects a unique kinematical framework and leads one to a specific quantum theory of geometry with an in-built discreteness [1, 16]. This in turn has implications on the quantum nature of singularities [2, 11] and the ultraviolet behavior of matter fields [9, 15]. However, precisely because of this emphasis on Planck scale discreteness, familiar properties of gravity and matter that can be easily derived using space-time continuum are now difficult to obtain. For instance in the traditional perturbative framework, the gravitational attraction between two point masses arises from an exchange of virtual gravitons, described by the Feynman propagator. Loop quantum gravity, on the other hand, has no background metric. Therefore one cannot even begin to calculate the propagator along these traditional lines.

The tension can be illustrated by the following simple argument. In Minkowskian field theories, the propagator is a 2-point function $W(x, y)$ which can be expressed as a path integral $W(x, x') = \int D\phi \phi(x)\phi(x') \exp iS[\phi]$. However, this expression cannot be taken over naively to background independent theories. For, in such a theory the measure and the action are diffeomorphism invariant whence we would have $W(x, x') = W(y, y')$ for any y, y' which are images of (x, x') under some diffeomorphism. A distribution $W(x, x')$ with this property would not look anything like the familiar graviton propagator which falls off as $|x - x'|^{-2}$. Thus, the naive calculation sketched above is inadequate. To obtain the standard propagator, at the very least we must inject into the calculation enough structure that lets us speak of distance between x and x' . It turns out that an appropriate strategy is to consider a space-time region with a boundary on which x and x' lie, and fix a state on the boundary which has enough information to speak of the bulk-distance between these points. To obtain the propagator, one can then integrate over fields in the bulk (i.e. interior) which are compatible with the given boundary state.⁴ First steps in the implementation of this idea have been carried out over the last year. So far, calculations have been done only in the Riemannian (rather than Lorentzian) framework [4].

Consider then \mathbb{R}^4 and introduce in it a topological 3-sphere S . Introduce on S Cauchy data (q_{ab}, k^{ab}) induced by a flat Euclidean 4-metric. By ‘evolving’ this data via Einstein’s equation one would recover an Euclidean 4-metric g_{ab} in the interior of S . However, this 4-metric will be unique only up to diffeomorphisms (which are identity on the boundary S). But the geodesic distance between x and x' is invariant with respect to this diffeomorphism freedom. The idea is that the desired propagator should be obtained by fixing points x, x' on the boundary S and summing over all configurations in the interior bulk which agree with the boundary data (q_{ab}, k^{ab}) . In quantum theory then one fixes a LQG boundary state $\Psi_{q,k}(s)$ peaked at some flat space initial data (q_{ab}, k^{ab}) , where s is a (diffeomorphism equivalence class of) spin network(s) on the 3-sphere S . Formally the gravitational propagator is given by

$$W^{aba'b'}(\{S, q, k; x, x'\}) = N \sum_{s, s'} W[s'] \langle s' | h^{ab}(x) h^{a'b'}(x') | s \rangle \Psi_{q,k}[s] \quad (1.3)$$

where $\{S, q, k; x, x'\}$ stands for a diffeomorphism class of $(S, q_{ab}, k^{ab}, x, x')$ and $W(s')$ the amplitude obtained by summing over all spin-foams in the interior which are compatible

⁴ For details on why this is a natural generalization of the standard procedure in Minkowski space-time, see [4] and references therein.

with the spin network s on the boundary.⁵ Note that while the construction does require a state peaked at some initial data for Euclidean space, the answer depends only on the diffeomorphism equivalence class $\{S, q, k; x, x'\}$ of the quintet inside the curly brackets.

Technically, the non-trivial parts of the calculation are of course the evaluation of the transition amplitude $W(s)$ and the computation of the sum. The well-developed spin-foam technology enables one to perform the first task rigorously. The result is manifestly finite. Computation of the sum turns out to be more delicate. Here, there is a conceptually important but mathematically delicate cancellation between phase factors that arise from $W(s)$ and the semi-classical state $\Psi_{q,k}(s)$ which makes the sum convergent. This subtle interplay provides considerable support for the way the problem has been set up. The final result can be expressed as a power series in ℓ_{Pl}/R , where R is the geodesic distance between x and x' with respect to the Euclidean metric. The leading term is independent of ℓ_{Pl} and proportional to $1/R^2$ just as one would expect from perturbative treatments. The full result includes corrections of the order ℓ_{Pl}/R and higher, which encode the ultraviolet non-trivialities of LQG.

I have sketched only the main idea. There are several unresolved issues, both conceptual and technical. Conceptually, so far all calculations are incomplete because the required boundary state is introduced by hand. While states used *are* well motivated the choice is far from being unique and higher order corrections depend on the choice. Ultimately this state should represent the ‘Minkowskian vacuum’, determined dynamically. Technically, the most important question is whether the construction can be extended to the physical, Lorentzian sector. The non-trivial achievement is that a conceptual framework has been introduced to calculate n-point functions within a background independent setting, thereby bridging the Planck scale quantum geometry to the familiar continuum physics.

This concludes the discussion of recent advances. The kinematic results of section IA are rigorous and apply to full LQG. The subsequent three examples refer to dynamics: The LQC analysis summarized in section IB resolves singularities of direct physical interest; the effective non-commutative field theory of section IC provides new a approach to quantum gravity phenomenology; and results of section ID on the propagator open up the possibility of relating LQG to the more familiar Minkowskian perturbation theory and pin-pointing the limitations of the latter. However, since these three examples refer to non-perturbative *dynamics*, important issues remain: relation of LQC with LQG in section IB; a systematic derivation of the ansatz used in section IC; and determination of the boundary state in the Lorentzian sector in ID. Together, these examples illustrate that progress is being made on several different fronts.

II. FREQUENTLY ASKED QUESTIONS

The organizers of the conference noted that there has been growing interest in loop quantum gravity, illustrated, e.g., by some semi-popular articles and reviews written by authors who do not work in this field, and requested that I clarify issues that puzzle ‘outsiders’ and respond to questions string theorists often raise. These outside perspectives are extremely

⁵ $W(s')$ is the spin-foam analog of the sum over geometries $W({}^3g) := \int D[g] \exp iS_{\text{EH}}[g]$ where the integral is performed over all 4-geometries g which agree with the pre-specified 3-geometry 3g on the boundary.

helpful because they can bring freshness.⁶ The LQG community very much appreciates these efforts. However, such reviews can also put the program in a pre-conceived, conceptual straightjacket inherited from author's own expertise, thereby missing the spirit of the endeavor. This is not surprising because we all know the difference between working in a field and learning about it by reading; reading alone cannot give a deep and encompassing perspective that arises after years of thinking about problems and working out technical details. My assigned task was then to try to create a bridge from LQG to the broader quantum gravity community, particularly string theorists.

This is not straightforward particularly because, as in any healthy, developing field, the LQG community has diverse viewpoints on some of the key open issues. Therefore, to prepare my talk I consulted a number of leading researchers. But my answers typically represent only a broad consensus rather than a sharp, unanimous view. Also, at some points I have taken the liberty to express my personal take on the issue.

A. Structure of LQG

• 1. *Uniqueness of the LQG kinematic framework rests on the result [1] that there is a unique diffeomorphism invariant state. But how can this be? Shouldn't there be an infinite number of diffeomorphism invariant states in quantum gravity?*

As explained in section IA, a state is a positive linear functional on a \star -algebra. The uniqueness result refers to the basic holonomy-flux algebra \mathfrak{a} of LQG. The *kinematical* Hilbert space \mathcal{H}_{kin} of LQG that results from the GNS construction therefore admits a unique diffeomorphism invariant ray. All vectors in the infinite dimensional *physical* Hilbert space of LQG are indeed diffeomorphism invariant, but they are positive linear functionals on a *different* algebra, consisting of diffeomorphism invariant operators.

This point may seem confusing. Let me therefore consider the more familiar example of quantum geometrodynamics (QGD). Now the \star algebra \mathfrak{a} is usually taken to be generated by the metric q_{ab} and its conjugate momentum p_{ab} (both smeared with test fields), subject to the standard canonical commutation relations $[q_{ab}(x), p^{cd}(y)] = -i\hbar \delta_{(a}^c \delta_{b)}^d \delta(x, y) I$. One's first inclination is to expect that \mathfrak{a} admits infinitely many diffeomorphism invariant states (e.g. $\Psi(q) = \int_M R_{ab} R^{ab} dV_q$, where R_{ab} is the Ricci tensor of q_{ab}). If so, the uniqueness results in LQG would seem strange, perhaps an artifact of some hidden assumption. Let us therefore explore this issue. Suppose we find a diffeomorphism invariant state F on this \mathfrak{a} . As in section IA, let us denote by Ψ_F the cyclic vector in the Hilbert space \mathcal{H} that results from the GNS construction. Then it follows that the expectation values of $q_{ab}(x)$, $q_{ab}(x)q_{a'b'}(x')$ etc. in the state Ψ_F must be diffeomorphism invariant distributions on M . But the only such distribution is the zero distribution. It follows immediately that $q_{ab}(x)\Psi_f = 0$. An identical argument tells us that $p^{cd}(y)\Psi_f = 0$. But this is impossible as it would contradict the canonical commutation relations. Thus, contrary to one's initial expectation, the canonical algebra of quantum geometrodynamics does not admit even a single diffeomorphism invariant state.⁷

⁶ For example, in his recent book 'Road to Reality' Roger Penrose has given such fresh perspectives on string theory.

⁷ The detailed version of this argument is rigorous. There are no subtleties concerning domains of operators

Following the footsteps of LQG, let us modify the kinematical algebra appropriately. Let \mathfrak{a} be the algebra generated by smeared operators $q(f) := \int_M q_{ab} f^{ab}(x) d^3x$ and exponentiated operators $\exp i \int p^{ab}(x) g_{ab}(x) d^3x$ (where f^{ab} is a test tensor density of weight 1 and g_{ab} a test tensor field with density weight zero). Then one can indeed find a diffeomorphism invariant state F which is completely analogous to that of LQG. In the resulting GNS representation the operators $q(f)$ are well defined but there is no operator corresponding to $p(g)$ because the operators $\exp i \int p^{ab}(x) g_{ab}(x) d^3x$ fail to be weakly continuous in g . This is completely analogous to the fact that in LQG operators h_e and $E_{S,f}$ are well-defined but there is no operator corresponding to the connection because holonomy operators fail to be continuous in the edge e . Next, let us consider the cyclic state Ψ_F . Reasoning of the last paragraph implies that the state is sharply peaked at the zero metric! (The same is true in LQG). Thus Ψ_F is the analog of Witten's ‘non-perturbative, diffeomorphism invariant vacuum’ in 2+1 dimensions. The problem with the canonical commutation relations is bypassed because there is no operator $p(g)$.

I hope this close similarity between LQG and the more familiar quantum geometrodynamics provides some intuition for the uniqueness result of [1]. These surprisingly strong restrictions bring out the fact that background independence is a largely unfamiliar territory that can hold many surprises. The situation in quantum geometrodynamics may also help in making the nature of the LQG cyclic state and the non-existence of a connection operator in the theory less mysterious.

• 2. *LQG seems to start with general relativity or supergravity. Shouldn't Einstein's equations get quantum corrections?*

Yes. And they *do* receive quantum corrections in LQG.

Let us first consider QED. There one begins with the classical Maxwell-Dirac action and then proceeds with quantization. One does not argue that the classical action must be modified because of quantum corrections. Yet, the effective action—which is meant to incorporate all quantum effects—has all sorts of additional terms representing quantum corrections.

The viewpoint is similar with Einstein-matter actions in LQG. The classical theory one ‘quantizes’ is general relativity (or supergravity). But Einstein equations do receive quantum corrections. In fact, one would expect the ‘effective action’ to exhibit *non-localities* at the Planck scale (in addition to the common non-local terms one encounters in Minkowskian quantum field theories).

We already know the leading corrections in quantum cosmology. The $k=0$ Friedmann equation $(\dot{a}/a)^2 = 8\pi G/3$ is replaced by $(\dot{a}/a)^2 = (8\pi G/3)[1 - \rho_{\text{matter}}/\rho_{\text{crit}}]$ where $\rho_{\text{crit}} \approx 0.8\rho_{\text{Pl}}$. The precise form of this quantum correction leads to profound departures from general relativity in the Planck regime.

because the GNS construction guarantees that the products $q(f)p(g)$ and $p(g)q(f)$ of smeared operators have a well-defined action on Ψ_F . The main result holds also for the affine algebra of Klauder's [24].

B. Quantum dynamics

- 3. *Isn't there a large number of ambiguities in the dynamics of LQG?*

Yes. Indeed, these have been pointed out in many reviews (see, e.g. [13]) and have constituted a focal point of concern in the LQG community. This is precisely the current incompleteness of the mathematical framework of LQG. Ambiguities can be broadly divided into three classes of increasing importance.

The first type corresponds to factor ordering choices which are present also in ordinary quantum mechanics. They are partially reduced by the requirement that the constraint should be self-adjoint so one can pass to the physical Hilbert space by group averaging. These ambiguities persist in symmetry reduced models including quantum cosmology. In simple models, explicit calculations have shown that this freedom does not change the qualitative predictions of the theory. In particular in the FRW models with scalar fields, the resolution of the big bang singularity and the main features of quantum physics near the big bang are robust [2, 11]. Therefore there is reason to hope that the situation would be similar in the full theory.

The second and more significant source of ambiguities is associated with the choice of representation j associated with the new edge added by the action of the Hamiltonian constraint on \mathcal{H}_{kin} . This does change the constraint qualitatively. In quantum cosmology the choice $j = 1/2$ leads to a second order difference equation which reduces to the Wheeler DeWitt equation when the curvature is low compared to the Planck scale. Vandersloot has shown that the use of $j > 1/2$ leads to a higher order difference equation which has many more solutions [25]. One can argue that the extra solutions are spurious. Starting with LQG in 2+1 dimensions, Perez has given arguments to the effect that we should only use the fundamental representation $j = 1/2$ also in 3+1 dimensions [25]. Thus, while the issue is not definitively settled, there are several pointers indicating that this ambiguity could be naturally resolved.

Since there is a certain similarity with lattice gauge theories, it is instructive to compare the two situations. In lattice gauge theories ambiguities arise in the intermediate stage (because of irrelevant operators) but disappear in the continuum limit. In LQG we are already in the continuum but the underlying diffeomorphism invariance enables one to get rid of a host of ambiguities. However, a large number of them still persist, in particular, those associated with the choice of an initial triangulation of the 3-manifold M and treatment of the so called 'non-Euclidean part of the constraint'. This third set of ambiguities is much more severe. In my view, the most unsatisfactory aspect of the current status of the program is that the physical meaning and ramifications of these ambiguities are still poorly understood. One can invoke 'naturalness' and 'simplicity' criterion to remove them but without a deeper understanding of their physical meaning, these criteria can be subjective and therefore not compelling.

- 4. *In the current approach, the constraint algebra can not be verified in quantum theory. Is this not a fatal drawback?*

No. *Logically* there is nothing wrong in: i) first imposing the Gauss constraint; ii) defining the Diff constraint only on the Hilbert space of gauge invariant states and solving it; and iii) then defining the Hamiltonian constraint on the Hilbert space of gauge and Diff invariant

states and then solving it. If this procedure leads to a theory with a rich semi-classical sector, it would be a viable physical theory.

However, as I just discussed, there is still a large number of poorly controlled ambiguities in the definition of the Hamiltonian constraint. Requiring the satisfaction of the constraint algebra could be very helpful in reducing them and to have a better chance at arriving at a viable theory. Progress along these lines has been made recently by Giesel and Thiemann in that they are able to show that for a certain class of constructions of the quantum constraint, the expected algebra is indeed realized on semi-classical states [7].

• 5. *Aren't these ambiguities just a reflection of the ambiguities associated with perturbative non-renormalizability of Einstein gravity?*

No! The two have entirely different origins: The ambiguities in LQG arise from an incompleteness of our current understanding while the infinite ambiguities of perturbative quantum general relativity are an expression of the inadequacy of the Gaussian fixed point.

2+1-dimensional general relativity is also power counting non-renormalizable but exactly soluble in LQG and spin foam calculations have also established this elegantly in the path integral framework [26]. In another direction, the Madrid group has compared and contrasted the perturbative and non-perturbative treatments for 2+1 gravity coupled to a scalar field [8]. The precise and important differences in geometry, causal structure and correlation functions that emerge at the Planck scale appear to bear out the qualitative expectations and hopes researchers have expressed for some time. Thus, a priori there is no relation between perturbative and non-perturbative ambiguities. Finally, this view is re-enforced also by results on ‘causal dynamical triangulations’ [27].

Indeed, there exist perturbatively non-renormalizable but exactly soluble models, e.g., the Gross-Neveu model in three dimensions. It admits a non-Gaussian fixed point (NGFP). Initially it was thought that the correlation functions will not be tempered distributions, i.e., will be worse behaved than those in renormalizable field theories, reflecting the perturbative non-renormalizability. This turned out not to be the case! By now there is significant evidence that Einstein gravity also admits a NGFP [28]. Furthermore there are some qualitative similarities on the nature of the Planck scale geometry in these ‘asymptotically safe’ scenarios and in spin-foams/LQG. In particular, the effective space-time dimension at the Planck scale turns out to be two in both theories.

• 6. *Will Lorentz invariance be violated in the low energy limit of LQG dynamics?*

Let me answer in steps in part because the question has several connotations and inequivalent precise formulations.

LQG is based on a Hamiltonian theory and sometimes field theorists implicitly assume that this feature would automatically lead to a Lorentz violation. This is not the case. For example, in the canonical description of Minkowskian field theories the full Poincaré group acts unitarily. Similarly, in the asymptotically flat context the canonical phase space of general relativity does carry a symplectic representation of the asymptotic Poincaré group and the Hamiltonian generating these transformations are the total energy-momentum and angular-momentum. Thus, by itself the 3+1 split is not an obstruction to Lorentz invariance. Finally, sometimes quantization of area and volume in quantum kinematics is taken to indicate Lorentz violations. This is not the case: recall that quantization of eigenvalues of

angular momentum operators J_i does not break spherical symmetry.⁸

In full non-perturbative quantum gravity there is no background metric whence some care is needed to speak of Lorentz invariance. The question can only refer either to asymptotic symmetries in the asymptotically flat context or effective low energy descriptions. I would expect LQG will have the first type of Lorentz invariance generated by global charges corresponding to asymptotic symmetries. But unfortunately so far global issues related to asymptotic flatness have received very little attention.

For effective low energy theories, the main issue is whether the effective actions will have terms which will violate *local* Lorentz invariance. For instance the manner in which quantization ambiguities are resolved may require a background structure and this in turn may give rise to background fields in the effective low energy theory, violating Lorentz invariance. Such a violation could rule out the theory because there are very strict experimental constraints and also theoretical arguments which say that the Lorentz violations at very high energies would trickle down to low energy physics because of closed loop effects in the Feynman expansion. However, these constraints arise by assuming that the violation is due to a background vector field (a rest frame) or a background tensor field. While quantum geometry of LQG probably undergoes violent fluctuations at the Planck scale, it seems highly unlikely that their coarse graining will give rise to such background fields. Their effect is likely to be analogous to the 2+1 effective field theory [3] I discussed in section IC. That description is free of all the usual experimental or theoretical difficulties [30] normally discussed under ‘Lorentz violations’. Indeed, on one particle states, the action of the Lorentz group is the standard one. However, in the effective theory, spin statistics and addition of momenta is non-standard. These modifications are not what is normally called ‘Lorentz violations’. Experimental constraints on them appear not to have been discussed in the literature.

However, it *is* true that some choices of quantization of operators in the Hamiltonian constraints may lead to Lorentz violations in the effective theory in the standard sense, leading to conflicts with experiments. So, the requirement that there be no such violations will serve as an important criterion in narrowing down the ambiguities.⁹ Such external, physical criteria are essential since a theory with a large number of free parameters will not have predictive power in practice.

C. Spin foams, black hole entropy and quantum cosmology

• 7. *Much of the work in spin foams is carried out in the Riemannian context which has no simple relation to the physical Lorentzian sector. Why is it then interesting?*

Because it addresses some of the long standing problems in a background independent fashion, with all due care of mathematical physics. Faced simultaneously with a number of difficult technical and conceptual issues, it is not uncommon in mathematical physics to ignore one or two important features in order to gain insight into the remaining aspects

⁸ For further discussion, see [29] and [13, 14]. For an early discussion on why there is no inherent conflict between Lorentz invariance and discreteness, see [23].

⁹ This strategy was explicitly realized in a recent analysis of the CGHS model from a canonical quantization perspective in the recent work of Tavares, Varadarajan and the author.

of the problem. An outstanding example is the AdS/CFT correspondence in which one makes certain aspects of quantum gravity mathematically tractable at the cost of using unphysical boundary conditions in which not only there is a negative cosmological constant but the extra compact dimensions have radius of a cosmological—rather than Planck—size. Furthermore, unlike the AdS/CFT correspondence, the Riemannian spin-foams can directly suggest strategies for the physically interesting Lorentzian sector. As we saw in section IC in 3 dimensions these strategies can be generally implemented in detail. In 4 dimensions, arguments generally go through at a level of rigor that is often considered acceptable in particle physics.¹⁰

• 8. *In the black hole entropy calculation, what is the justification of assuming the Boltzmann statistics for punctures?*

This is a misconception! No such assumption was made. To compute entropy, one just counts the number of Chern Simons states on the horizon—i.e., the number of states of the quantum horizon geometry—paying due respect to the subtleties of diffeomorphism invariance. In this calculation, the punctures are not treated as particles, whence the issue of their statistics does not even arise. At the end of the calculation, one may try to reinterpret the result by constructing a heuristic picture of the black hole horizon as a gas of punctures. One can then conclude that one would reproduce the result of the calculation by using Boltzmann statistics for the hypothetical puncture particles.

Perhaps an analogy would help. From the Schwarzschild solution we know that the radius R_H of the horizon of a black hole of M is given by $R_H = 2GM/c^2$. There are also celebrated, 18th century calculations by Mitchell and Laplace which use the formula for escape velocity from Newtonian gravity, set the escape velocity to the speed of light and conclude that the radius R of a black hole of mass M is given by $R = 2GM/c^2$. The frequently asked question I began with is somewhat analogous to asking to Schwarzschild: “what is the justification of setting the escape velocity to be c when the speed of light is not absolute in Newtonian gravity?” Concepts used explicitly and implicitly in the question are irrelevant to the systematic derivation.

• 9a. *A puzzle about black hole entropy is that it scales with the area of the black hole, not the volume. While LQG calculations directly use the physical, black hole space-time in contrast to stringy discussions, isn't it the case that they do not explain why entropy does not scale with the volume?*

The intuition about scaling with volume comes from ordinary thermodynamical systems. But because of the singularity, the notion of ‘volume of a black hole’ does not have well-defined meaning. Let us first consider a static star. Its volume at an instant of time can be calculated by measuring the volume of the 3-surface which is orthogonal to the Killing field and whose boundary is the surface of the star at that instant. In the case of a static black hole, the Killing field is *space-like* in the interior of the horizon whence the 3-surface

¹⁰ In this discussion, I restricted myself to more conceptual issues. A number of more technical points were addressed by Laurent Friedel in <http://www.math.columbia.edu/~woit/wordpress/?p=330> January 23rd, two entries under *L says* towards the end.

orthogonal to it is *time-like*. If one just takes a cross-section of the horizon and asks how much volume it contains, one can get any answer between zero and infinity, depending on the choice of the 3-surface whose boundary is the given cross-section (and the singularity). So it is meaningless to look for an expression of entropy that scales with volume.

- *9b. If one is not considering black-hole interior, what is then one counting?*

The viewpoint in LQG is that entropy is not an intrinsic attribute of space-time but depends on its division into exterior and interior regions. Operationally, it is tied to the class of observers who live in the exterior region, for whom the isolated horizon is a *physical* boundary that separates the part of the space-time they can access from the part they can not. (This point is especially transparent for cosmological horizons which are also encompassed by the LQG calculation and for which there is no intrinsic analog of the ‘black hole region’.)

While there is an ‘observer dependence’ in this sense, entropy cannot refer to all the interior degrees of freedom that are inaccessible to the observers under consideration: Since inside the horizon one can join-on entire universes which do not communicate to the exterior region, the number of potential interior states compatible with the data accessible to the exterior observers is uncontrollably large. Instead, as in the membrane paradigm, in LQG one counts those black hole states which can interact with the outside world, whence the entropy refers to the micro-states of the boundary itself, i.e. of the quantum geometry of the horizon. (For further discussion, see sections VI.C and VII of [31].)

To summarize the goal of LQG calculations has been to answer the following question: Given that there is an isolated horizon, what is the entropy associated with it? Because of the conditional nature of this question, one begins with a suitably restricted sector of general relativity and *then* carries out quantization. However, as a result, the description is only an effective one. Fortunately, for thermodynamic considerations involving large black holes, effective descriptions are adequate.

- *10. Isn't Loop Quantum Cosmology too restrictive because of the huge symmetry reduction?*

Absolutely! Inclusion of inhomogeneities is crucial. This is a focal point of significant current research.

However, it is quite possible that the qualitative results are more robust than one has any right to expect a priori. This possibility is suggested by two considerations. First, the Belinskii-Lifshitz-Khalatnikov conjecture—which seemed too sweeping to be true when it was first proposed some 30 years ago—has now received considerable support in classical general relativity. As we heard at this conference from Claes Uggla, the conjecture implies that near space-like singularities (such as the big bang) the homogeneity approximation becomes successively better (because ‘space derivative terms’ can be neglected compared to ‘time derivative terms’). Therefore, lessons from quantum theory in the homogeneous context may be more potent than one might a priori imagine. The second point is illustrated by the hydrogen atom. Suppose, hypothetically, we had no experimental data on atomic spectra to assist us, nor a theory of hydrogen atom but knew that the charged-particle-photon systems should be treated using quantum electrodynamics. Suppose Dirac then came up with his solution of the bound state problem for an electron in an external Coulomb field. Since all but a finite number of the field degrees of freedom are frozen in this model,

a priori one might have thought that the solution would be a poor representation of the physical hydrogen atom in which true quantum fluctuations can excite any and all of the frozen degrees of freedom. A priori this is a perfectly reasonable concern but we know that it is misplaced in the real world. Neither of these two points is compelling but together they suggest that the qualitative features of the symmetry reduced analysis could well turn out to be robust.

Finally, the symmetry reduced models are generally useful in providing intuition and guidance. For example, detailed analytical/numerical studies in loop quantum cosmology [2] have led to new insights for constructing the physical sector of the theory, forced us to abandon naive dynamics and revealed interesting physics in the Planck domain. In this respect these calculations are somewhat analogous to checking AdS/CFT conjecture in symmetric situations, e.g. the Penrose limit. Furthermore, FRW cosmologies are not tailored just to make the problem tractable, but are of direct physical interest.

III. ASSESSMENT AND COMPARISON

• 11. *If a definite Hamiltonian constraint has not yet emerged in LQG, could one not say that there has been little progress since the Wheeler-DeWitt geometrodynamics?*

It is perhaps clarifying to compare the situation with that of string theory. There, the perturbation series is known to diverge—furthermore it does so uncontrollably because it is not even Borel summable. So, to any physical question—such as the value graviton-graviton scattering amplitude—the full answer in perturbative string theory is *infinite*. Now, one may say that this also happens in QED. But there we know the theory is incomplete and we should not trust its predictions beyond a certain number of terms in perturbation theory. But a theory which claims to be complete can not take such a refuge. Therefore, the standard belief in string theory is that the infinite answer of the perturbation theory is incorrect because non-perturbative effects are crucial. This was realized almost two decades ago but we still only have a skeleton of the candidate, non-perturbative M theory. So, the analogous question in string theory would be: could one not say that there has been little progress since supergravity?

I think that in both cases there is considerable incompleteness and diversity of ideas on how to address it. *But there has also been considerable progress: In both cases, internally consistent scenarios have emerged and special cases have been well-understood.*

Here are a few examples from LQG:

- i) There is a strong uniqueness theorem for the kinematical framework;
- ii) We have a fairly good understanding of the geometry of quantum horizons in equilibrium and a statistical mechanical derivation of entropy of astro-physically realistic black holes;
- iii) Spin-foam models have led to a fertile approach to obtaining the graviton propagator, effective low energy theories *and* probing non-perturbative aspects of Yang-Mills theories [32]; and,
- iv) Dynamical ideas have been implemented *in detail* in mini-superspaces and have led to physically desirable results, including singularity resolutions.

• 12. *Are LQG and string theory two versions of the same theory or are they mutually incompatible?*

My personal answer is: Neither! Certainly, the two are not equivalent in the sense that the Pauli's algebraic treatment of the hydrogen atom is equivalent to Schrödinger's treatment based on wave functions. Both LQG and string theory are very incomplete and they start out with very different basic assumptions. These differences are important but, in my view, they make the two attempts complementary rather than incompatible. In particular, LQG has taught lessons on implementation of background independence and the use of quantum geometry in the resolution of realistic singularities. String theory has provided a qualitatively new strategy for unification, well developed perturbative treatments and valuable experience with effective low energy actions. All these are likely to be useful in our search for a viable quantum gravity theory.

Interchange of ideas has been minimal so far because not only do the two approaches have different starting points but they use very different mathematical frameworks, making translations non-trivial. For example, Hilbert spaces with 'polymer-like' spin network states and operators with discrete eigenvalues feature prominently in LQG. String theory, on the other hand, tends to use path integrals and continuum conformal field theories. Such differences in mathematical frameworks naturally lead each community to address a set of questions not easily accessible to the other. For example, because of background independence, there are no classical fields at all in the fundamental description of LQG. Therefore, effective low energy actions which play an important role in string theory are generally difficult to construct. Reciprocally, the resolution of the FRW big-bang singularity of loop quantum cosmology is difficult to recover in string theory. Unfortunately, because of these deep differences of emphasis and language, misconceptions can arise rather easily. But this variety is also very good. For, as Feynman emphasized during a lecture at CERN on his way back from Stockholm:

"It is very important that we do not all follow the same fashion... It's necessary to increase the amount of variety the only way to do it is to implore you few guys to take a risk ..."

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