Examining nonextensive statistics in relativistic heavy-ion collisions

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Simon, A., & Wolschin, G. (2018). Examining nonextensive statistics in relativistic heavy-ion collisions. *Physical Review C*, *97*(4), 044913.

Setting and Problem

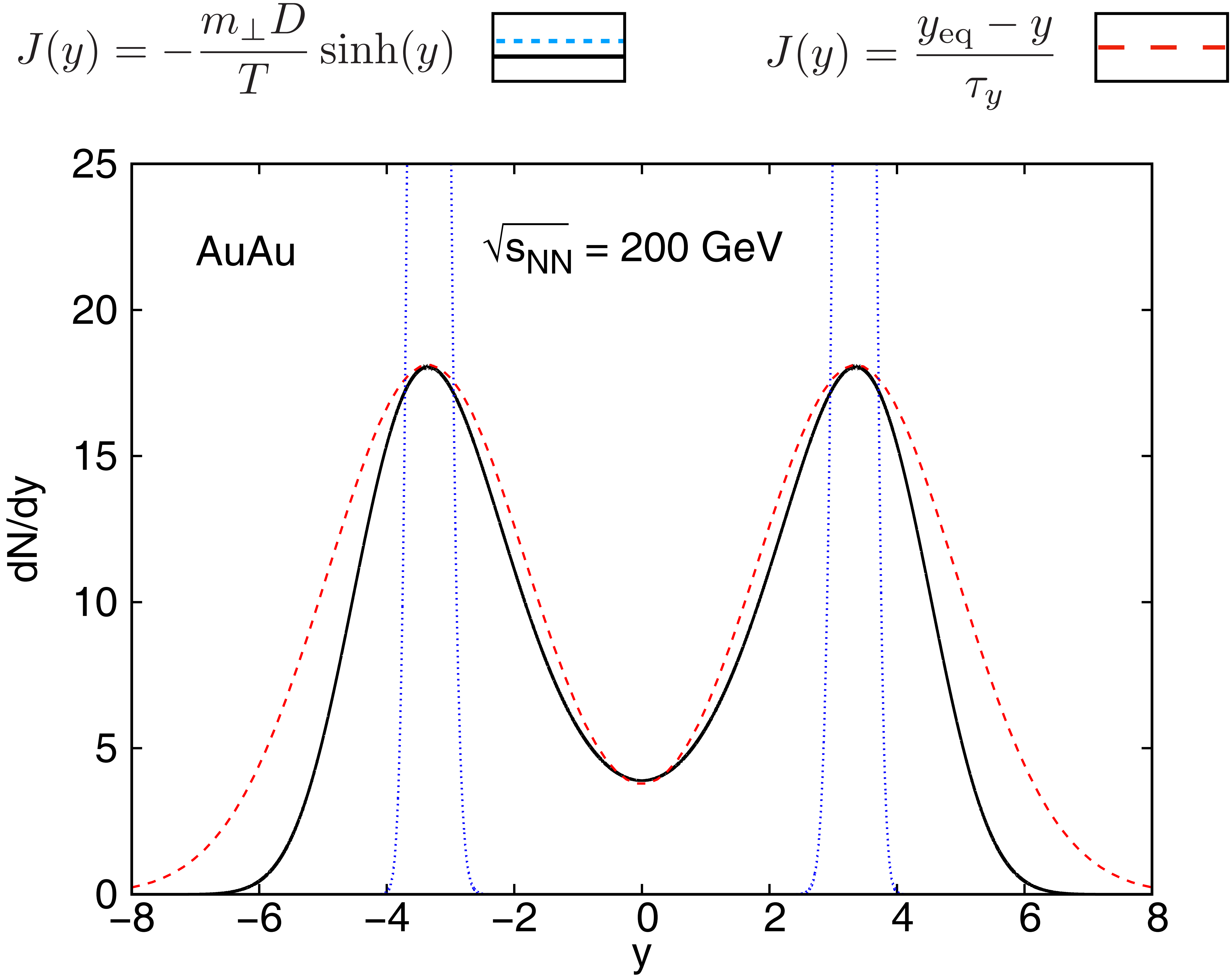
- Model stopping of (proton - antiproton) @ heavy-ion collisions (RHIC, PbPb)
- Use relativistic diffusion model

$$\frac{\partial}{\partial t} W(y,t) = -\frac{\partial}{\partial y}[J(y,t)W(y,t)] + \frac{\partial^2}{\partial y^2}[D(y,t)W(y,t)]$$

= drift **= diffusion**

- Know drift coefficient J(y,t) by:
  — interaction time and peak position: <u>amplitude</u>
  — stationary solution: <u>functional form</u>

- Determine diffusion by fluctuation-dissipation theorem



Forndran, F., & Wolschin, G. (2017). Relativistic diffusion model with nonlinear drift. *The European Physical Journal A*, *53*(2), 37.

Account for collective expansion

- ‘Artificially’ through stronger diffusion coefficient, or
- Change underlying equation: non-linear Fokker-Planck equation

$$\frac{\partial}{\partial t} W(y,t)^{\mu} = -\frac{\partial}{\partial y}\left[J(y,t) W(y,t)^{\mu}\right] + \frac{\partial^2}{\partial y^2}\left[D(y,t) W(y,t)^{\nu}\right]$$

Non-extensive entropy

Boltzmann-Gibbs entropy

$$S_{BG} = -k \sum_i p_i \ln p_i$$

$$S_B(\mathbf{A} + \mathbf{B}) = S_B(\mathbf{A}) + S_B(\mathbf{B})$$

q entropy

$$S_q = \frac{k}{q-1}(1 - \sum_i p_i^q)$$

$$S_q(\mathbf{A} + \mathbf{B}) = S_q(\mathbf{A}) + S_q(\mathbf{B}) + (1-q)S_q(\mathbf{A})S_q(\mathbf{B})$$

$$S_q = \left\langle \ln_q \frac{1}{p_i} \right\rangle$$

Tsallis, C. (1988). Possible generalization of Boltzmann-Gibbs statistics. *Journal of statistical physics*, *52*(1), 479-487.

Connection to NLFPE?

- Maximize entropy $$S_q[p] = \frac{1 - \int du[p(u)]^q}{q - 1}$$

$$p_q(y,t) = \frac{\{1 - \beta(t)(1-q)[y - y_m(t)]^2\}^{1/(1-q)}}{Z_q(t)}$$

- Insert as ansatz into NLFPE and chose linear drift
- Leads to: $q = 1 + \mu - \nu$

Tsallis, C., & Bukman, D. J. (1996). Anomalous diffusion in the presence of external forces: Exact time-dependent solutions and their thermostatistical basis. *Physical Review E*, *54*(3), R2197.

Final equation

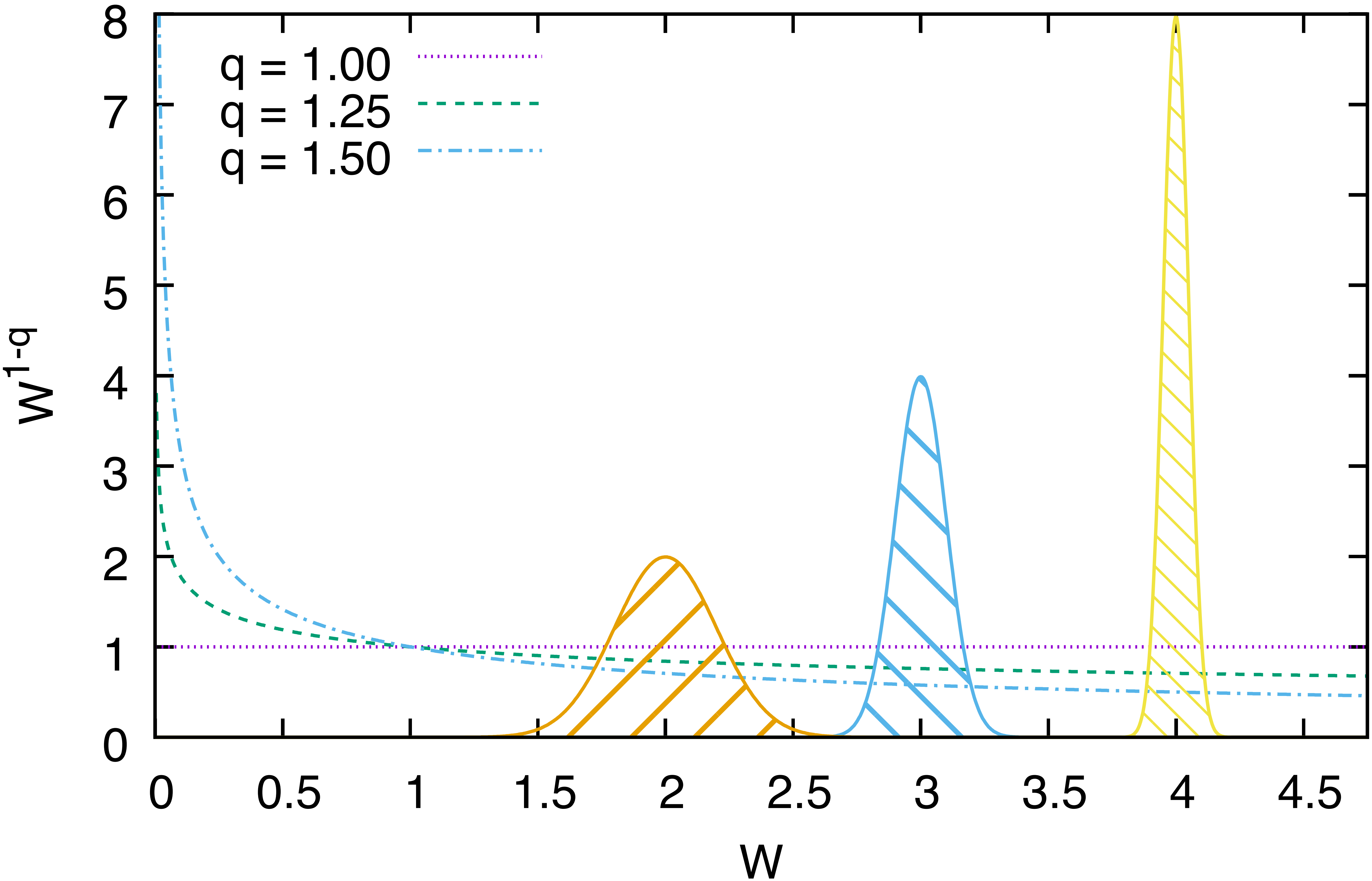
- Rescale time and get

$$\frac{\partial f}{\partial \tau} = \frac{\partial}{\partial y}[\sinh(y)\ f(y,t)] + \gamma\ \frac{\partial^2}{\partial y^2}[f(y,t)^{2-q}],\ \gamma = T/m_\perp$$

- Non-linear diffusion coefficient

$$\frac{\partial^2}{\partial y^2}[DW^{2-q}] = \frac{\partial^2}{\partial y^2}[(DW^{1-q})\ W]$$

- What changes?



q = 1.00
q = 1.25
q = 1.50
W^{1-q}
W
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
0
1
2
3
4
5
6
7
8

FEM

- Weak formulation (integral equation)

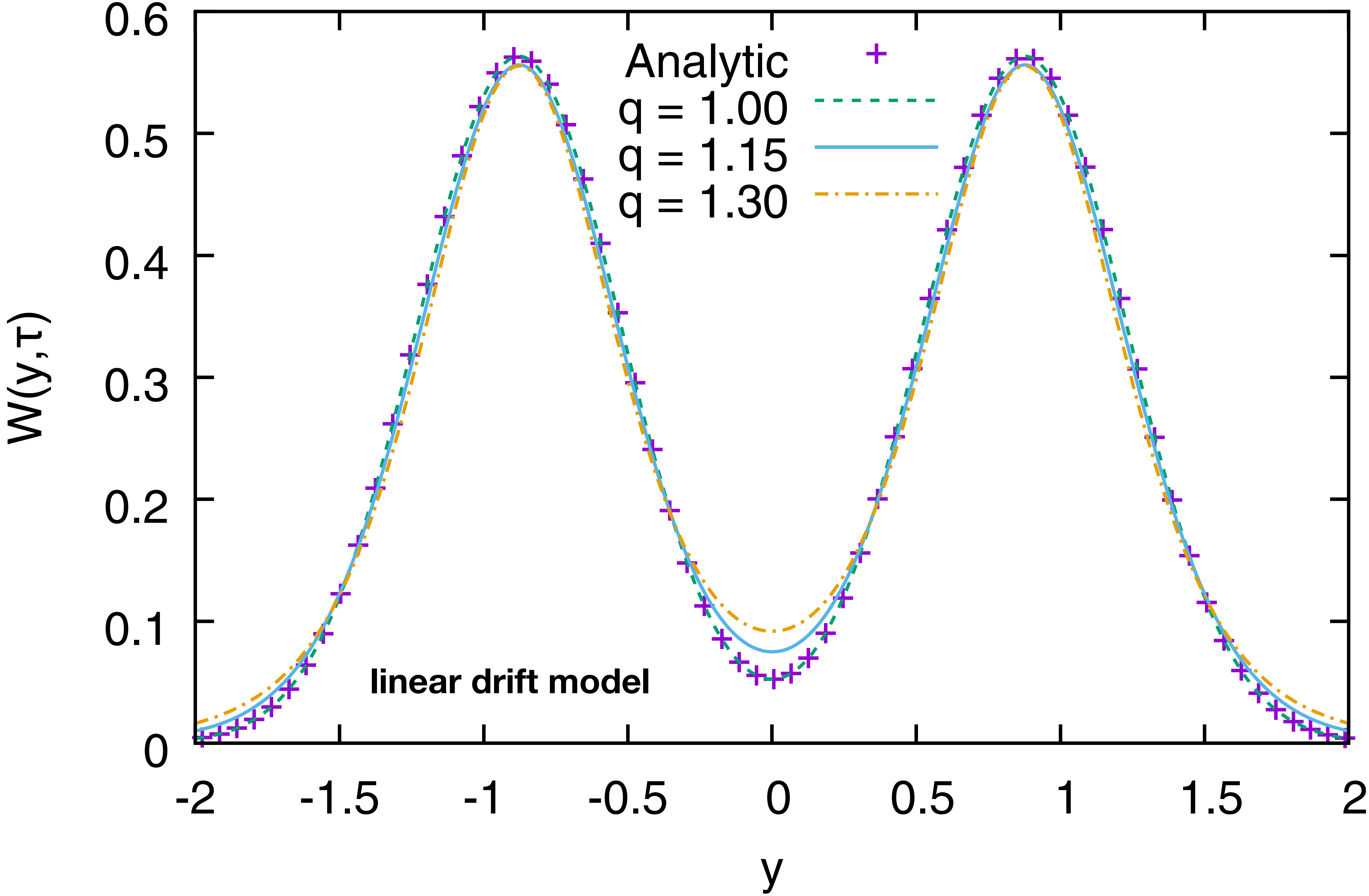
$$\int_{\Omega} dy \left\{ g(y) \frac{\partial}{\partial y} \left[\sinh(y)\ f(y,t) + \gamma\ \frac{\partial}{\partial y} f(y,t)^{2-q} \right] \right\}$$

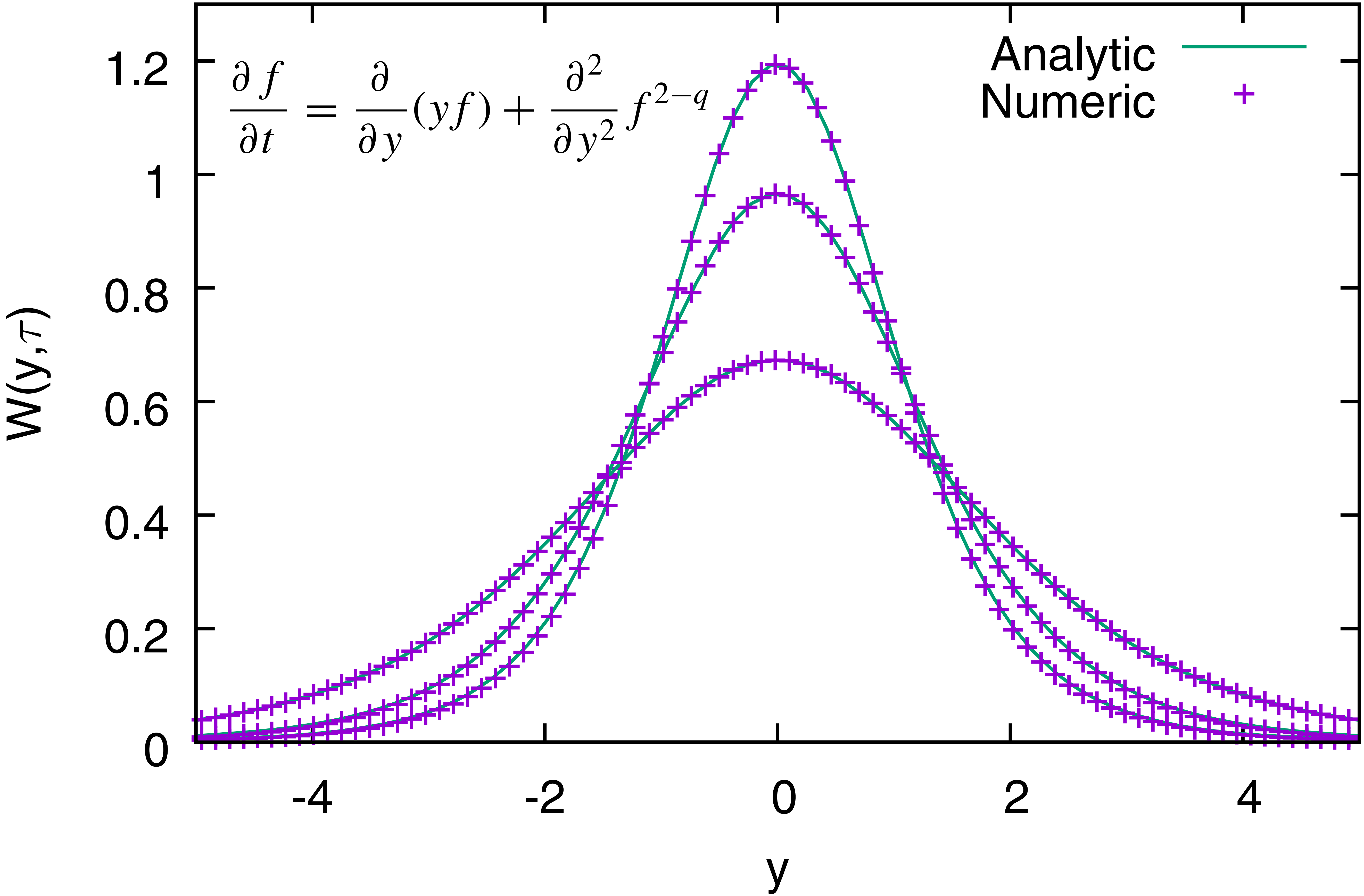
$$\left[g(y) \left\{ \sinh(y) f(y,t) + \gamma \frac{\partial}{\partial y} f(y,t)^{2-q} \right\} \right] \Bigg|_{\partial\Omega}$$
$$- \int_{\Omega} dy \left\{ \frac{\partial g}{\partial y} \left[\sinh(y) f(y,t) + \gamma\ \frac{\partial}{\partial y} f(y,t)^{2-q} \right] \right\}$$

- Time integration by Backward-Euler

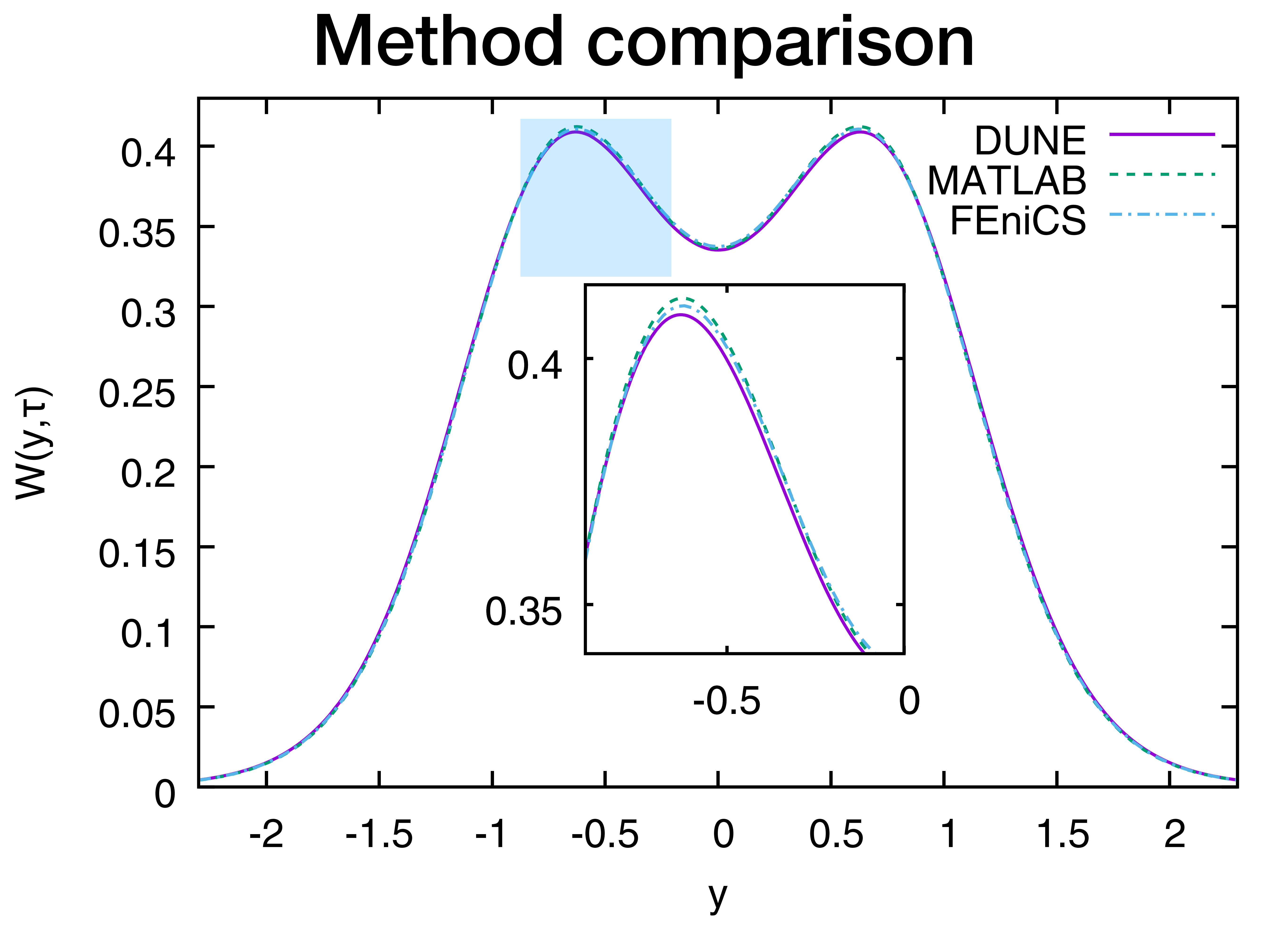
$$\frac{\partial f(t_n)}{\partial t} = \frac{f(t_n) - f(t_{n-1})}{\Delta t} + O(\| \Delta t^2 \|)$$

Numerical checks

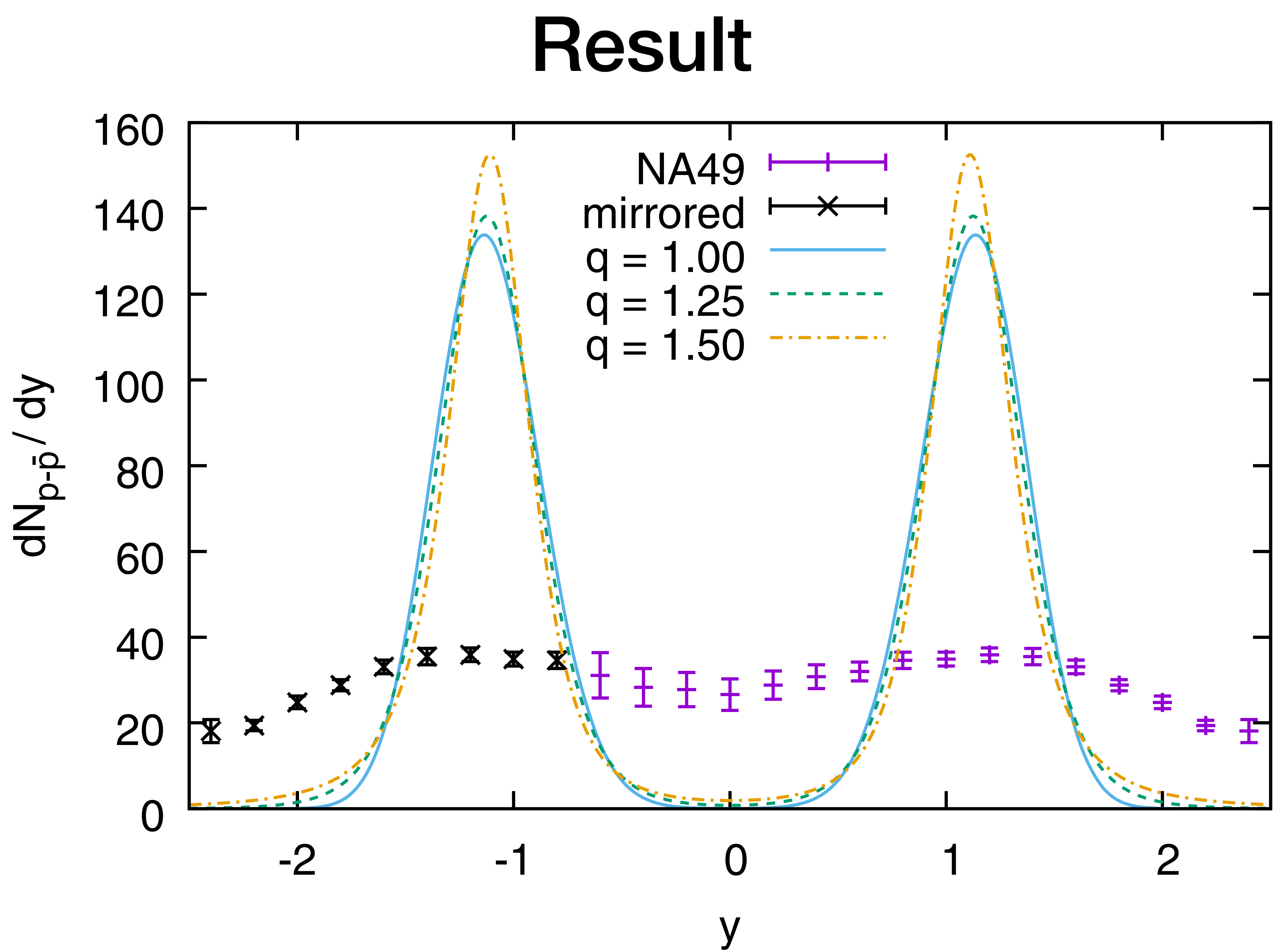




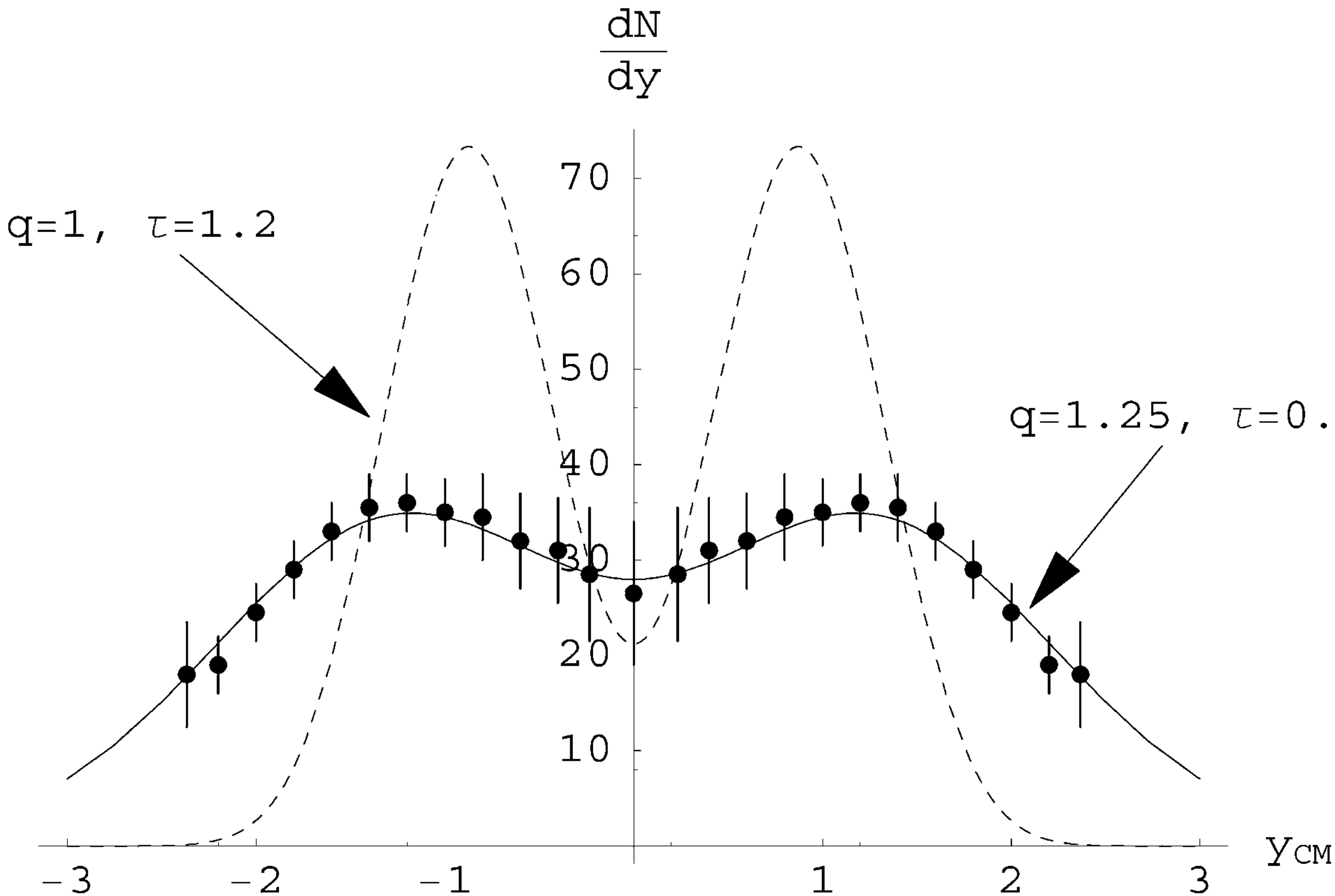
Borland, L., Pennini, F., Plastino, A. R., & Plastino, A. (1999). The nonlinear Fokker-Planck equation with state-dependent diffusion-a nonextensive maximum entropy approach. *The European Physical Journal B-Condensed Matter and Complex Systems*, *12*(2), 285-297.



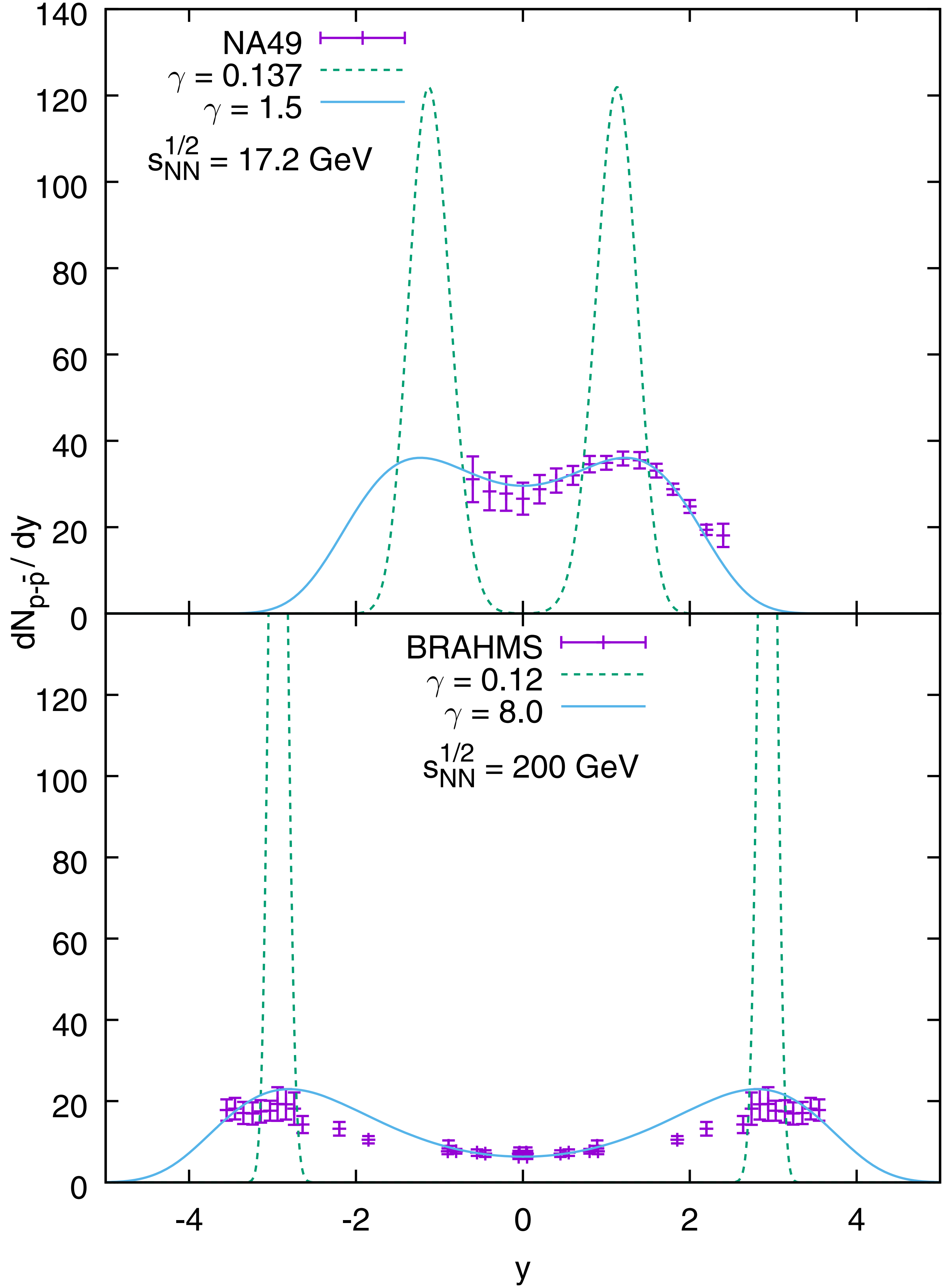
Method comparison
DUNE
MATLAB
FEniCS
W(y,τ)
y
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
0.4
0.35
-0.5
0


Result
NA49
mirrored
q = 1.00
q = 1.25
q = 1.50
160
140
120
100
80
60
40
20
0
-2
-1
0
1
2
y
dN_{p-p̄} / dy

≠ Lavagno's result



Lavagno, A. (2002). Anomalous diffusion in non-equilibrium relativistic heavy-ion rapidity spectra. *Physica A: Statistical Mechanics and its Applications*, *305*(1), 238-241.



- Return to linear diffusion
- Adjust diffusion coefficient manually

Summary

- Measured rapidity spectra broader than anticipated
- Nonextensive statistics supposed to produce collective expansion
- Numerically solved the resulting nonlinear Fokker-Planck equation
- Effect is too weak to account for experimental data