Limiting Fragmentation at LHC Energies

Benjamin Kellers

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Agenda

- **Introduction**
- Preparation
- LF in the RDM with three sources
- Refining the Model
- Summary

Important Variables and Concepts

- Rapidity

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$$

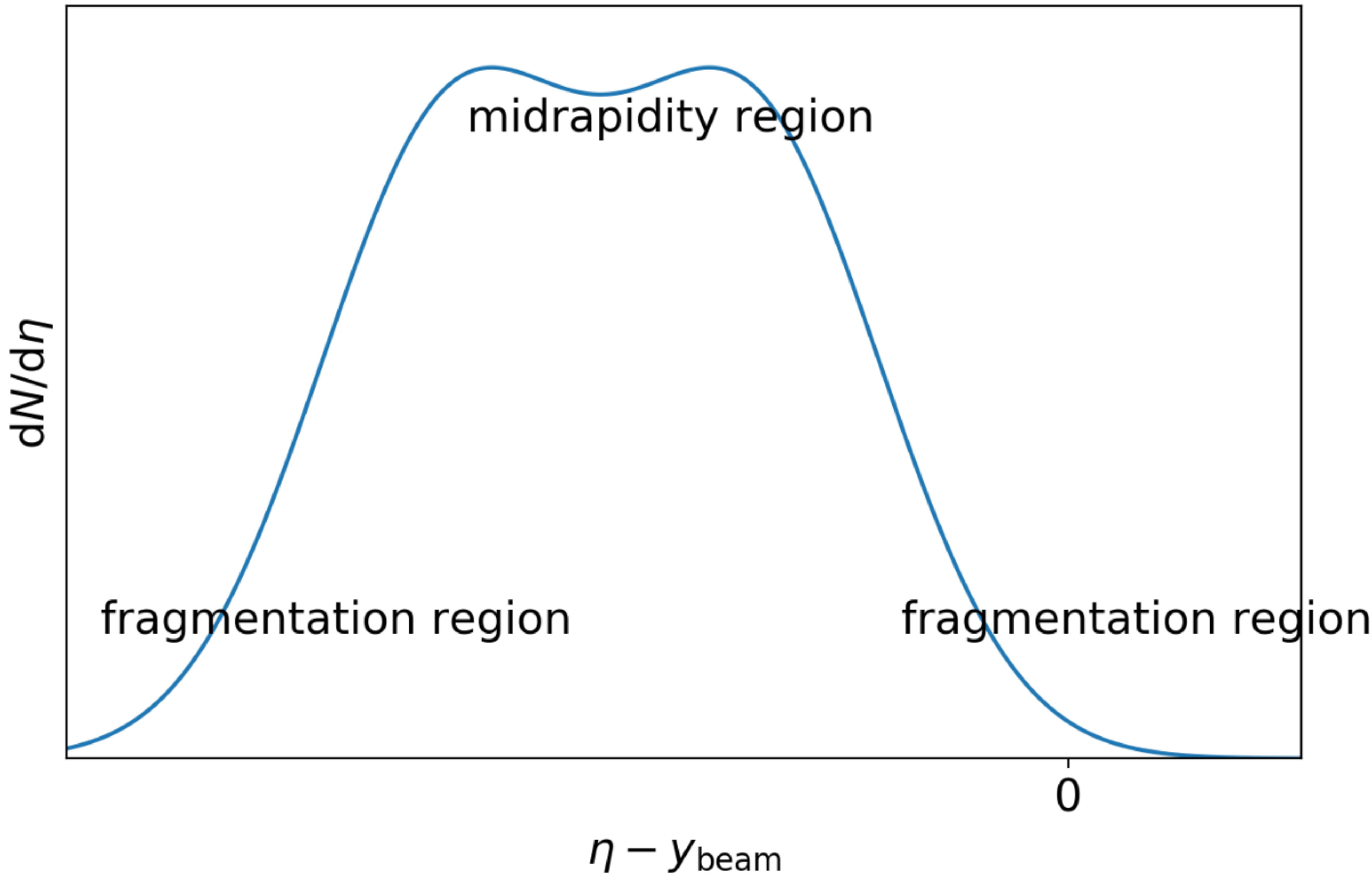
- Pseudorapidity

$$\eta = \tfrac{1}{2} \ln \tfrac{|\vec{p}| + p_z}{|\vec{p}| - p_z} = -\ln\left(\tan\left(\tfrac{\theta}{2}\right)\right)$$

- Beam Rapidity

$$\mathrm{y}_{\mathrm{beam}} = \mp \ln\left(\frac{\sqrt{s_{\mathrm{NN}}}}{m_{\mathrm{p}}}\right)$$

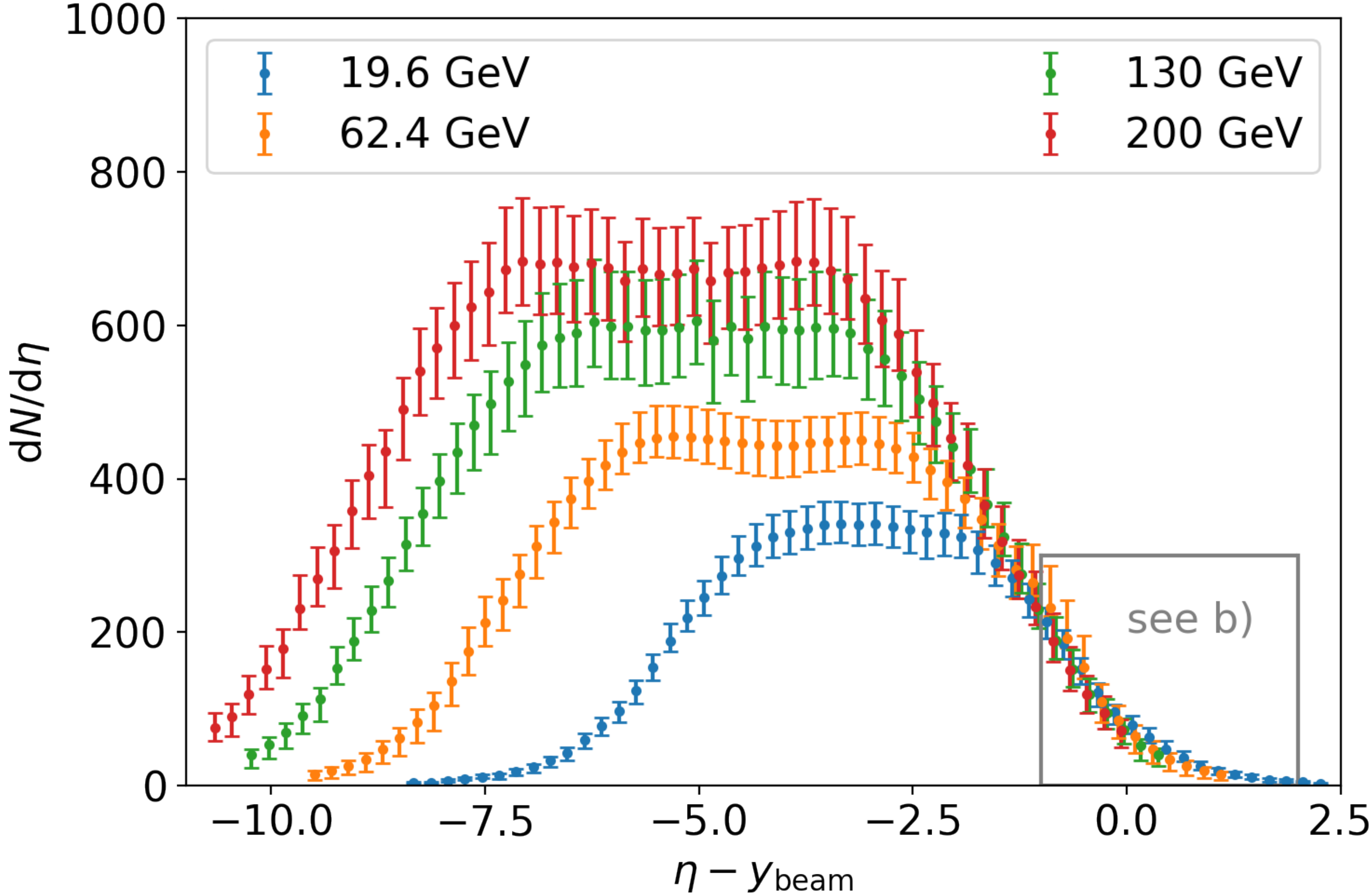
Limiting Fragmentation (LF) Hypothesis



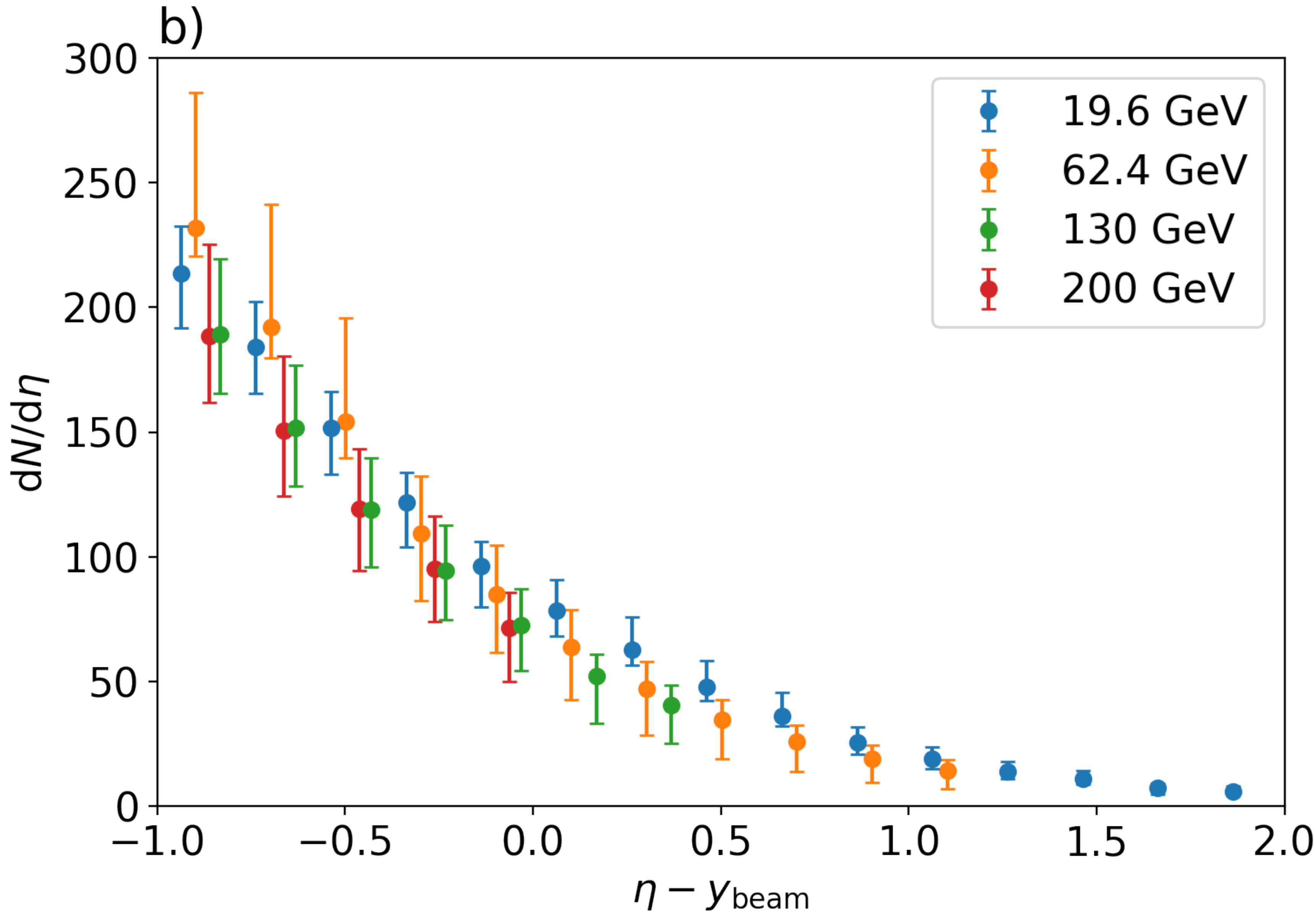
Main Questions

- Limiting fragmentation at LHC energies?
- Three sources RDM: a good model?
- Why do we ask these questions? …

LF at RHIC Energies



LF at RHIC Energies



Relativistic Diffusion Model (RDM)

- RDM with 3 sources
- Each source fulfills the linear FPE

$$\frac{\partial}{\partial t} R_k = -\frac{1}{\tau_y} \frac{\partial}{\partial y} \left[(y_{\mathrm{eq}} - y) \cdot R_k(y,t) \right] + D_y^k \frac{\partial^2}{\partial y^2} R_k(y,t)$$

$$\text{for } k = 1,\ 2,\ \mathrm{gg}.$$

- Symmetric Case (e.g. Au-Au, Pb-Pb)

$$\mathrm{y_{eq}} = 0$$

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Solving the FPE (symmetric case)

$$\frac{\partial}{\partial t} R_k = \frac{1}{\tau_y} \frac{\partial}{\partial y} \left[y \cdot R_k(y,t) \right] + D_y^k \frac{\partial^2}{\partial y^2} R_k(y,t)$$

$$\text{for } k = 1,\ 2,\ \text{gg}.$$

$$R_{1,2}(y, t = 0) = \delta(y \pm y_{\max})$$
$$R_{\text{gg}}(y, t = 0) = \delta(y).$$

Solving the FPE (symmetric case)

$$R_k(y,t) = \frac{1}{\sqrt{2\pi\sigma_k^2(t)}} \exp\left(-\frac{(y \pm y_{\max} e^{-t/\tau_y})^2}{2\sigma_k^2(t)}\right)$$

$$\text{with} \quad \sigma_k^2(t) = D_y^k \tau_y (1 - e^{-2t/\tau_y}).$$

$$\Rightarrow \quad \langle y_k \rangle = \mp y_{\max} e^{-t/\tau_y}$$

Problem

We still have one problem:

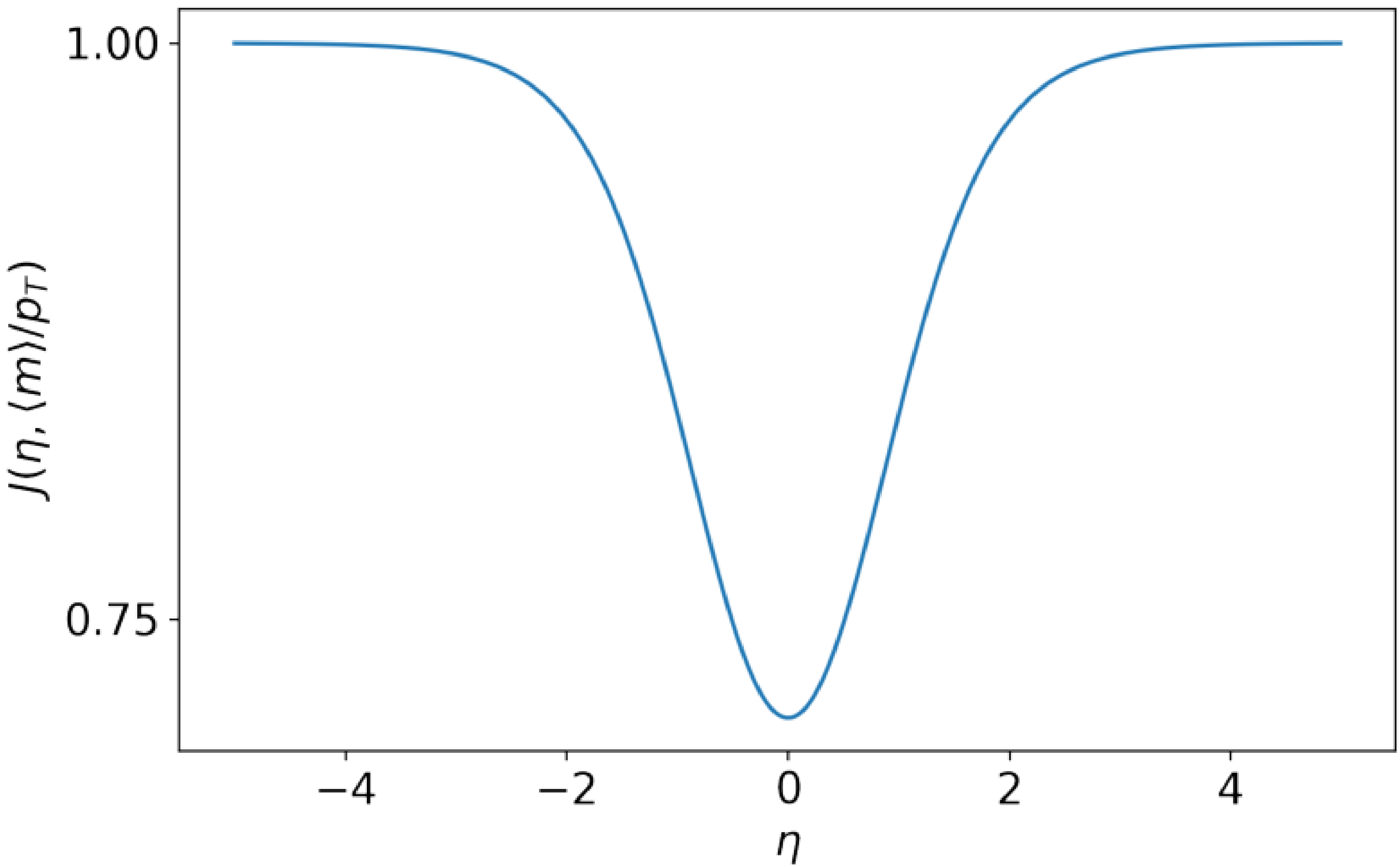
Our model is based on the **rapidity space** but measurements are taken in **pseudorapidity space**.

Jacobian

$$\frac{\mathrm{d}N}{\mathrm{d}\eta} = \frac{\mathrm{d}N}{\mathrm{d}y}\frac{\mathrm{d}y}{\mathrm{d}\eta} =: J\left(\eta, \frac{\langle m \rangle}{p_\mathrm{T}}\right)\frac{\mathrm{d}y}{\mathrm{d}\eta}$$

$$J\left(\eta, \frac{\langle m \rangle}{p_\mathrm{T}}\right) = \frac{\cosh(\eta)}{\sqrt{1 + \left(\frac{\langle m \rangle}{p_\mathrm{T}}\right)^2 + \sinh^2(\eta)}}$$

Jacobian



→ not very important for LF

Jacobian

- *J* is a function of η
- Parameters are not known for most of η-space
- Trick: *J* is known at midrapidity

$$J\left(\eta, \frac{\langle m \rangle}{p_{\mathrm{T}}}\right)\Bigg|_{y=0} = \frac{1}{\sqrt{1+\left(\left.\frac{\langle m \rangle}{p_{\mathrm{T}}}\right|_{y=0}\right)^2}} =: J_0$$

Jacobian

- Compute a p_T that reproduces *J* best at midrapidity

$$\langle p_{\mathrm{T,eff}} \rangle = \frac{m_\pi J_0}{\sqrt{1 - J_0^2}}$$

- *J* at midrapidity is given by

$$J_0 = J\big|_{y=0} = \frac{\left.\frac{\mathrm{d}N}{\mathrm{d}\eta}\right|_{\eta=0}}{\left.\frac{\mathrm{d}N}{\mathrm{d}y}\right|_{y=0}},$$

Jacobian

$$\left.\frac{\mathrm{d}N}{\mathrm{d}\eta}\right|_{\eta=0}$$ is directly measured

$$\left.\frac{\mathrm{d}N}{\mathrm{d}y}\right|_{y=0}$$ can be derived from p_T-diagrams

$$\left.\frac{\mathrm{d}N}{\mathrm{d}y}\right|_{y=0} = \int \left.\frac{\mathrm{d}N^2}{\mathrm{d}y\mathrm{d}p_\mathrm{T}}\right|_{y=0} \mathrm{d}p_\mathrm{T}.$$

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Three Sources RDM

- Fitfunction

$$\frac{\mathrm{d}N}{\mathrm{d}\eta}(\eta) = J(\eta)\,\left\{A_{\mathrm{f}}\left[\exp\left(\frac{(\eta-\mu_{\mathrm{f}})^2}{2\sigma_{\mathrm{f}}^2}\right)+\exp\left(\frac{(\eta+\mu_{\mathrm{f}})^2}{2\sigma_{\mathrm{f}}^2}\right)\right]+A_{\mathrm{gg}}\exp\left(\frac{\eta^2}{2\sigma_{\mathrm{gg}}^2}\right)\right\}$$

- Jacobian

$$J(\eta) = \frac{\cosh(\eta)}{\sqrt{3.2639+\sinh^2(\eta)}}$$

because

$$\langle m\rangle = m_\pi = 139.57\,\mathrm{MeV}$$

$$p_{\mathrm{T}} = 0.21\,\mathrm{GeV}$$

[D. M. Röhrscheid and G. Wolschin, PhysRevC 86, 024902]

Three Sources RDM

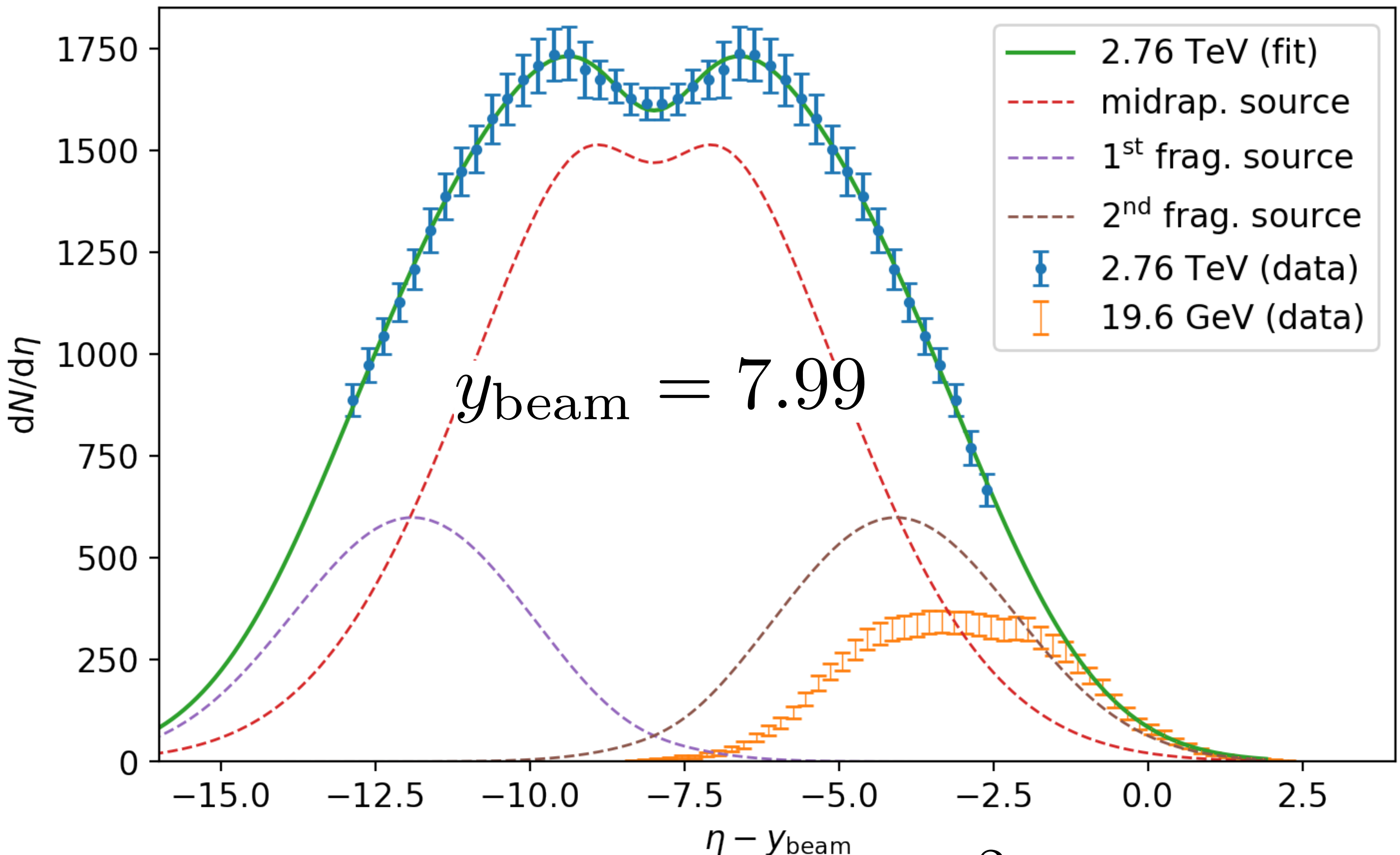
- Number of produced charged particles:

$$N_{1,2} = A_{\mathrm{f}}\sqrt{2\pi\sigma_{\mathrm{f}}^2} = 2900,$$

$$N_{gg} = A_{\mathrm{gg}}\sqrt{2\pi\sigma_{\mathrm{gg}}^2} = 11895.$$

$$N_{\mathrm{total}} = 17695$$

Pb-Pb at 2.76 TeV



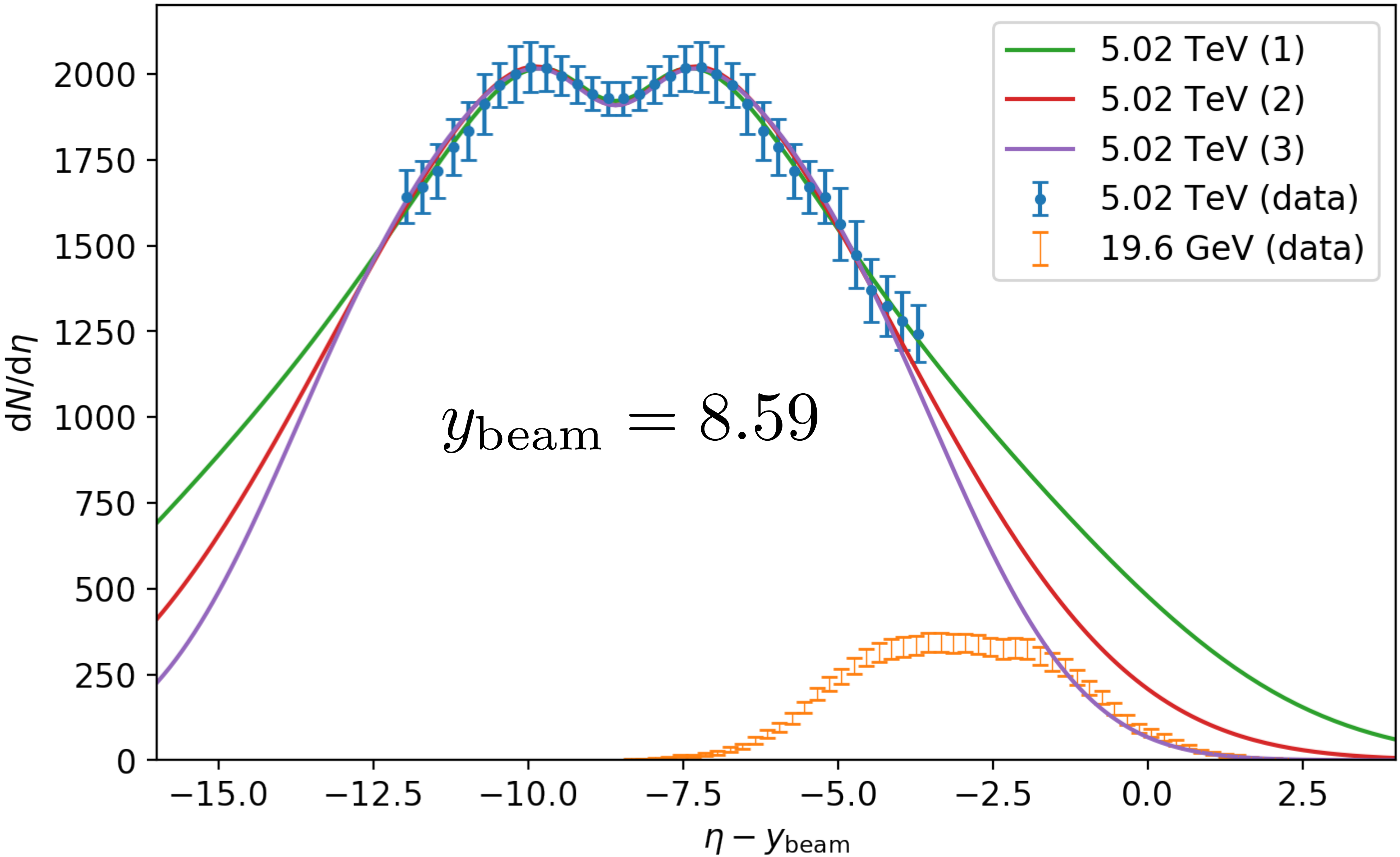
$$\chi^2_{\mathrm{red}} = 1.10,\ 31\,\%$$

Pb-Pb at 5.02 TeV

- Important information about the fragmentation region is not in the data
- Regular fit leads to unphysical result
  → boundaries for the fit parameters

Parameter Boundaries	A_{f} −	A_{f} +	A_{gg} −	A_{gg} +	μ_{f} −	μ_{f} +	σ_{f} −	σ_{f} +	σ_{gg} −	σ_{gg} +
(1)	599	∞	1765	∞	3.9	∞	1.9	∞	2.6	∞
(2)	599	∞	1765	∞	3.9	4	1.9	∞	2.6	∞
(3)	599	∞	1765	∞	3.9	4	1.9	2	2.6	∞

Pb-Pb at 5.02 TeV



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Nonlinear FPE

- Boltzmann-Gibbs statistics leads to a nonlinear drift in the FPE

$$\frac{\partial}{\partial t} R_k(y,t) = A \frac{\partial}{\partial y}[\sinh(y) R_k(y,t)] + D_k \frac{\partial^2}{\partial y^2} R_k(y,t)$$

- Requires numerical approach (Alessandros talk)

Nonlinear FPE

- Dimensionless version of FPE

$$t \to \tau = \frac{t}{t_c} \quad \Rightarrow \quad \frac{\partial}{\partial t} = \frac{1}{t_c}\frac{\partial}{\partial \tau}$$

$$\frac{\partial}{\partial \tau}R(y,\tau) = t_c A \frac{\partial}{\partial y}[\sinh(y)R(y,\tau)] + t_c D \frac{\partial^2}{\partial y^2}R(y,\tau)$$

$$t_c \equiv 1/A$$

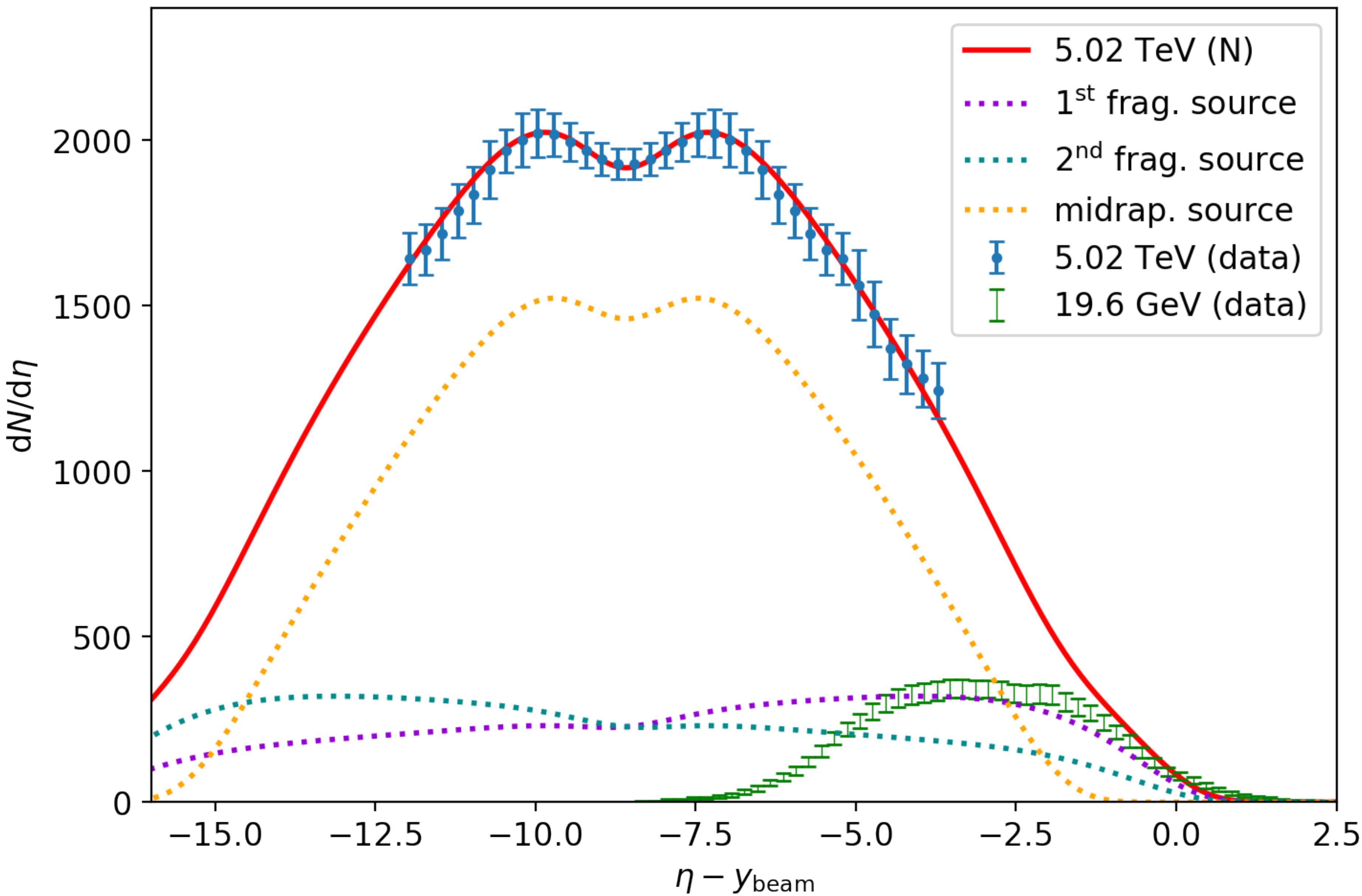
$$\boxed{\frac{\partial}{\partial \tau}R(y,\tau) = \frac{\partial}{\partial y}[\sinh(y)R(y,\tau)] + \gamma \frac{\partial^2}{\partial y^2}R(y,\tau)}$$

Nonlinear FPE

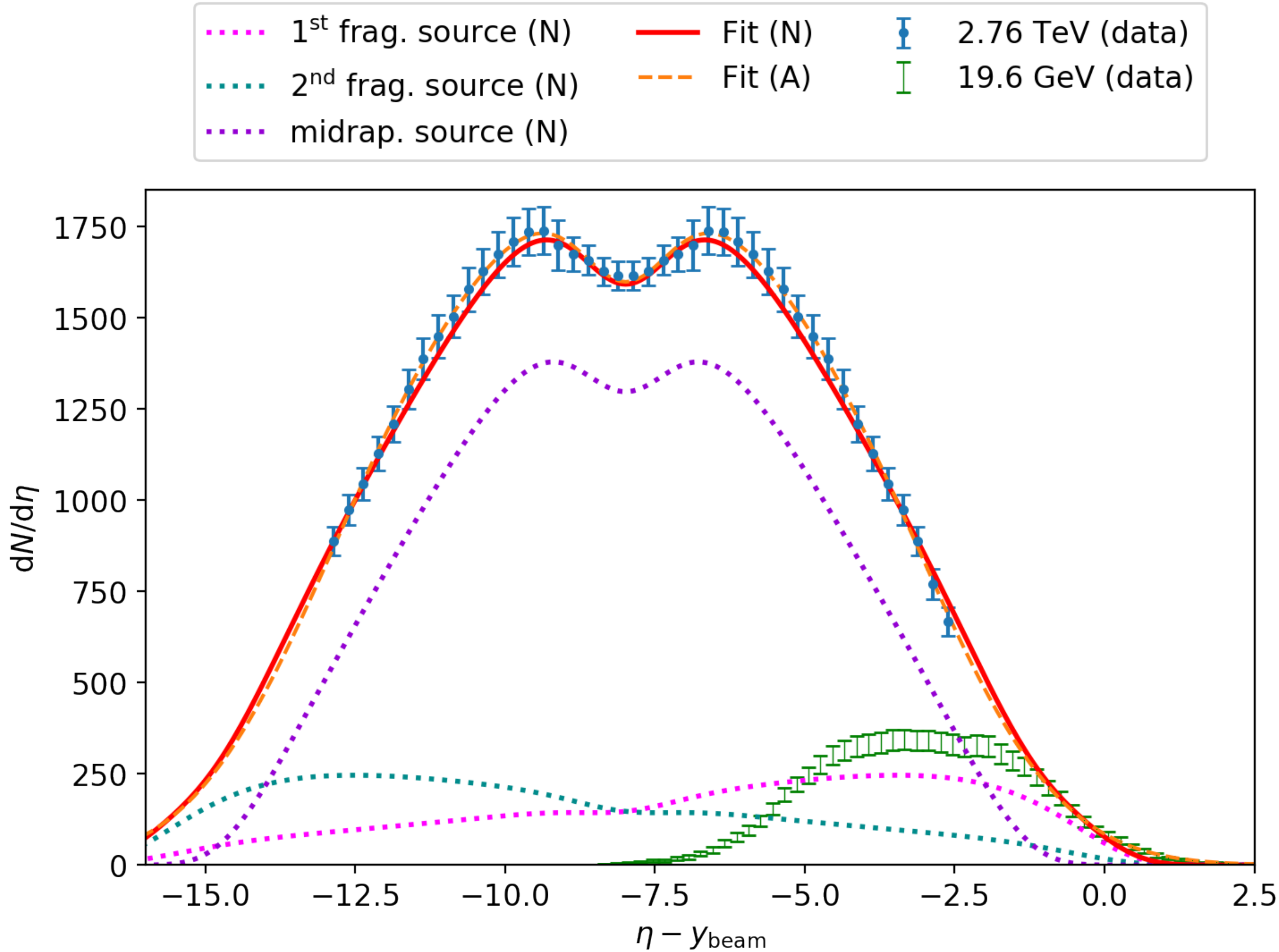
- Initial conditions
  $\rightarrow$ Gaussians at $0,\ \pm y_{\text{beam}}$ with $\sigma = 0.1$

- Dirichlet boundary conditions
  $\rightarrow R(y = \pm 15, t) = 0$

Pb-Pb at 5.02 TeV



Pb-Pb at 2.76 TeV



Produced Charged Particles

$\sqrt{s_{\mathrm{NN}}} = 5.02\,\mathrm{TeV}$	$\sqrt{s_{\mathrm{NN}}} = 2.76\,\mathrm{TeV}$
$N_{1,2} = 3900,$	$N_{1,2} = 2500,$
$N_{gg} = 14000$	$N_{\mathrm{gg}} = 12500$
$N_{\mathrm{total}} = 21800$	$N_{\mathrm{total}} = 17500$

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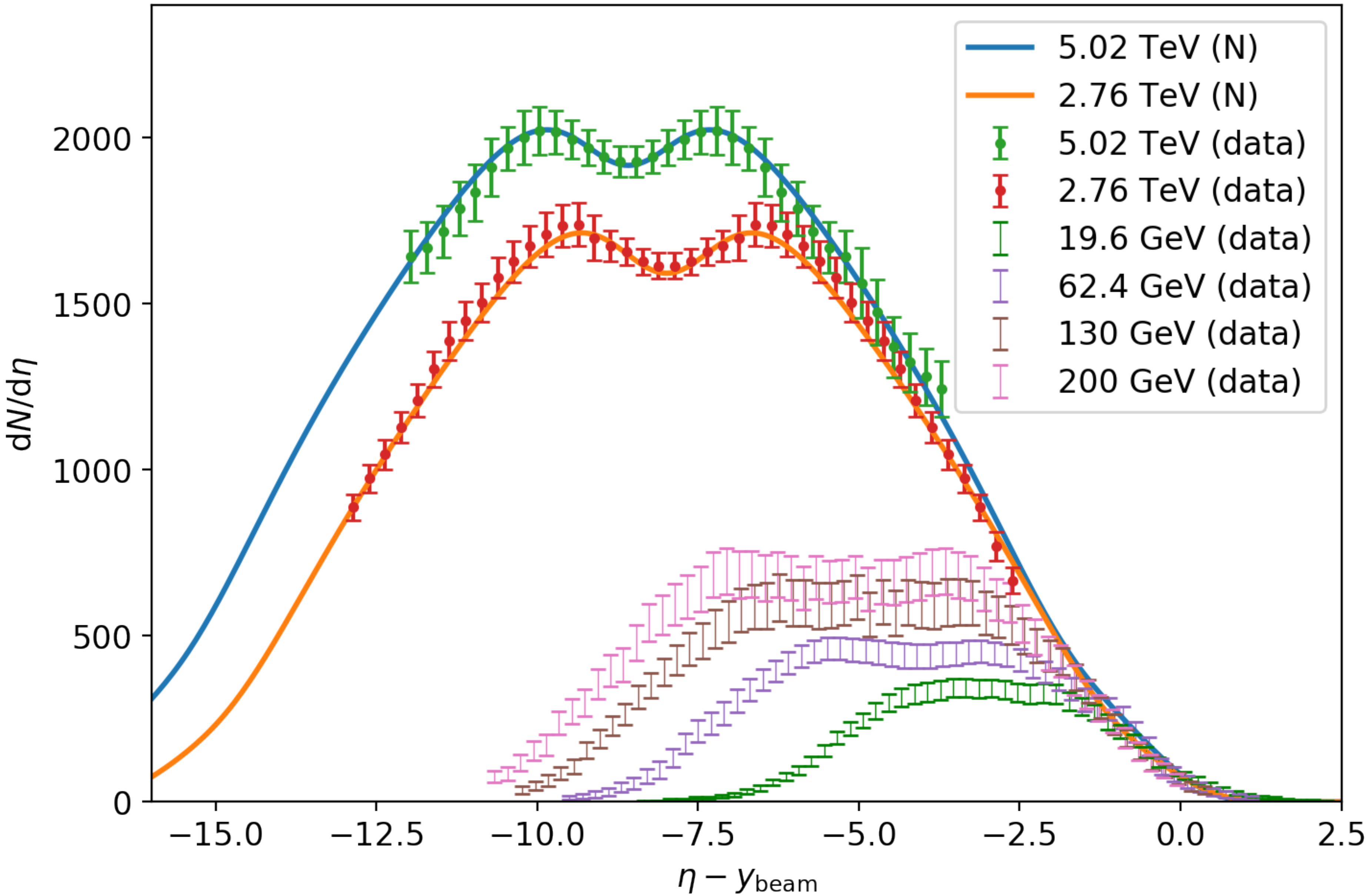
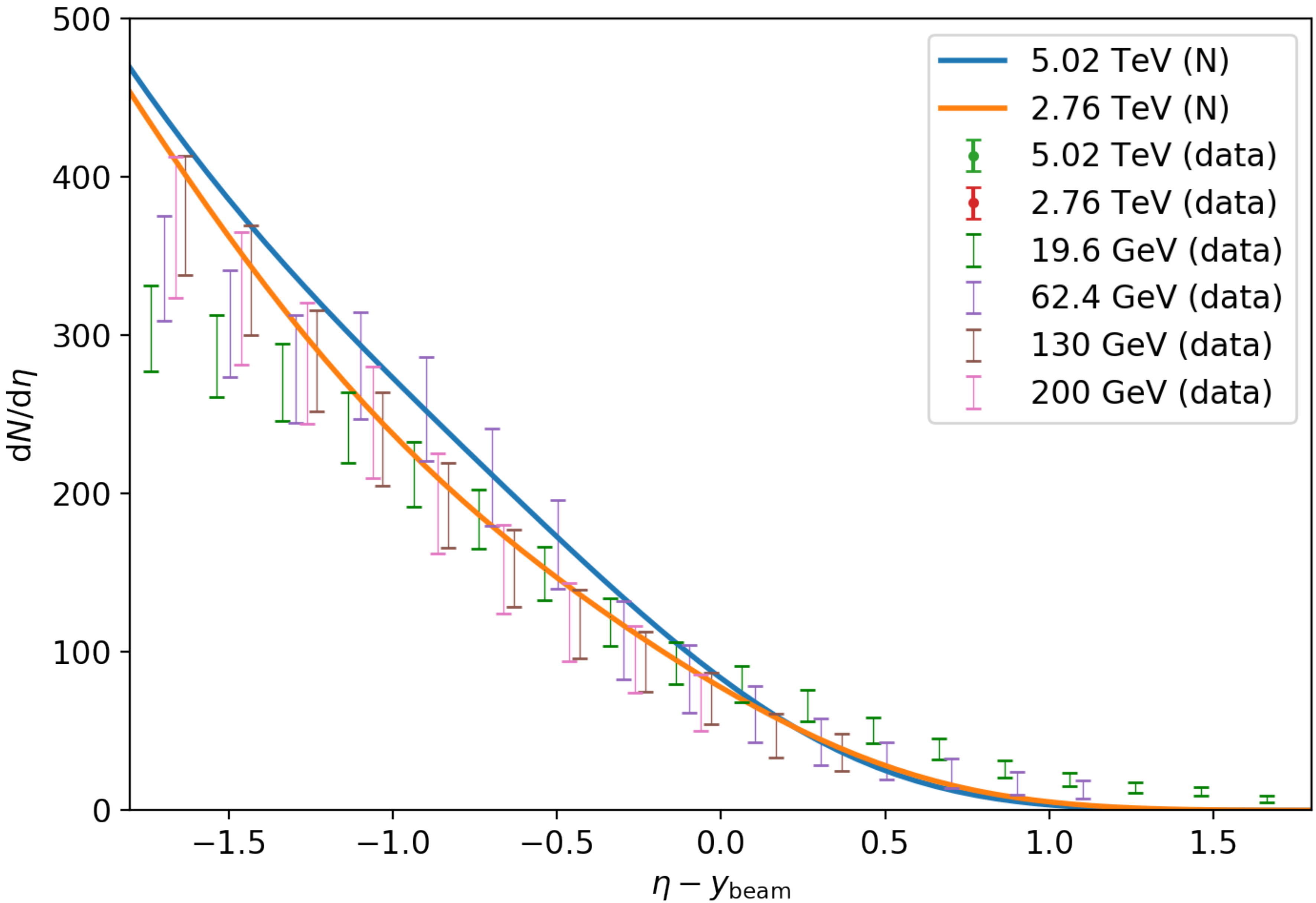


5.02 TeV (N)
2.76 TeV (N)
5.02 TeV (data)
2.76 TeV (data)
19.6 GeV (data)
62.4 GeV (data)
130 GeV (data)
200 GeV (data)
dN/dη
0
500
1000
1500
2000
−15.0
−12.5
−10.0
−7.5
−5.0
−2.5
0.0
2.5
$\eta - y_{\text{beam}}$

Summary



500
400
300
200
100
0
dN/dη
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
η − y_beam
5.02 TeV (N)
2.76 TeV (N)
5.02 TeV (data)
2.76 TeV (data)
19.6 GeV (data)
62.4 GeV (data)
130 GeV (data)
200 GeV (data)

Summary

- Linear FPE is a good approximation
- LF is possible, even for the linear FPE
- Nonlinear FPE allows for LF in the 3sRMD at LHC energies
- LF might exist at even larger energy scales
- Final answer lies in future data (maybe after the next Pb-run)

Literature

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