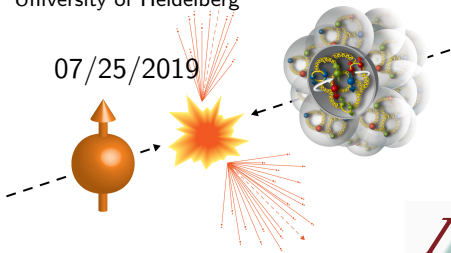


# Microscopic Sources of Particle Production in Relativistic Heavy-Ion Collisions

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Multiparticle Dynamics Seminar  
University of Heidelberg

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### 1 Motivation

- QCD and Hadrons
- Relativistic Heavy-Ion Collisions

### 2 The Parton Model

- QCD Factorization at High Energies
- Parton Distribution Function
- kT-Factorization
- Gluon Saturation

### 3 Results for Charged Hadron Production

- Relativistic Diffusion Model
- Hybrid-Factorization

### 4 Summary

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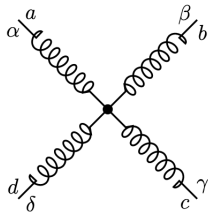
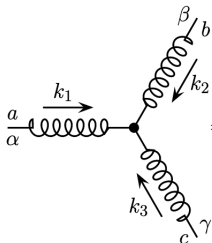
### 4 Summary

## Motivation: QCD and Hadrons

- QCD Lagrangian, color charge  $a$

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{q}(i\not{D} - m_q)q - \frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$$

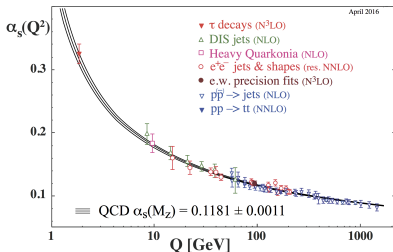


## Motivation: QCD and Hadrons

- First order running coupling:

$$\alpha_s(Q^2) = \frac{12 \cdot \pi}{(33 - 2n_f) \cdot \ln(Q^2/\Lambda_{\text{QCD}}^2)}$$

- Non-perturbative regime:  $\Lambda_{\text{QCD}} \approx 0.22 \text{ GeV}$



[Bethke2017]

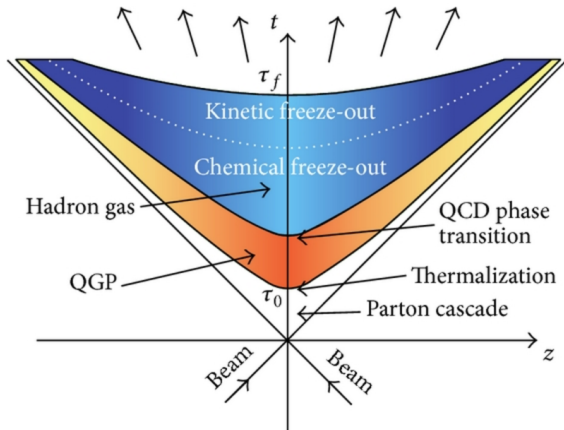
- $q - \bar{q}$  Potential:  $V(r) = \frac{4}{3} \frac{\alpha_s(r)}{r} + k \cdot r$
- Confinement:  $q, g$

⇒ Hadrons (non point-like particles)

## Relativistic Heavy-Ion Collisions

- Observable: produced charged hadrons  $d\sigma^{ch}$
- High Energy at LHC:

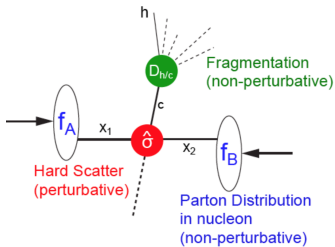
$$pp \rightarrow H, pPb \rightarrow H, PbPb \rightarrow H, \text{ with } p = uud$$



[Fig:Tawfik2014]

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## QCD Factorization at High Energies



[fig:Reygers2017]

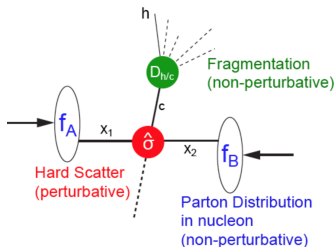
- Factorization: hard- and soft-processes

$$d\sigma_{in} = \sum_{A,B,C} f_A \otimes f_B \otimes d\sigma(AB \rightarrow C) \otimes D_{h/C}$$

$$\sigma_{in} = \sum_{A,B,C} \int dx_1 dx_2 f_A(x_1) f_B(x_2) \sigma(AB \rightarrow C) \otimes D_{h/C}$$



## QCD Factorization at High Energies



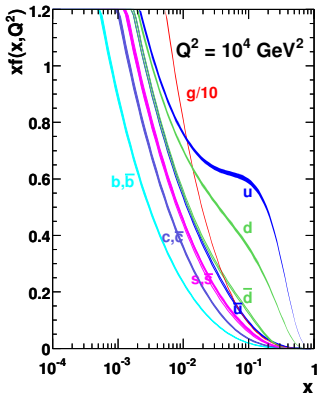
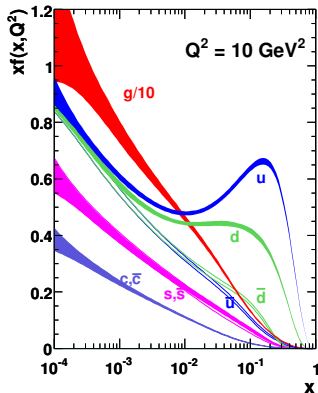
[fig:Reygers2017]

### Relativistic Heavy-Ion Collisions

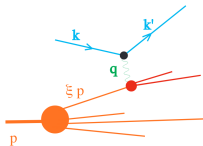
- Parton Distribution Functions: PDFs, nPDFs (nuclear PDFs)
- Collinear Factorization,  $k_T$ -Factorization
- Color Glass Condensate, Initial State Models
- Hard Scattering  $\Rightarrow$  Dipol Color Cross section
- Fragmentation  $\Rightarrow$  Parton-Hadron Duality

# Parton Distribution Function

MSTW 2008 NLO PDFs (68% C.L.)

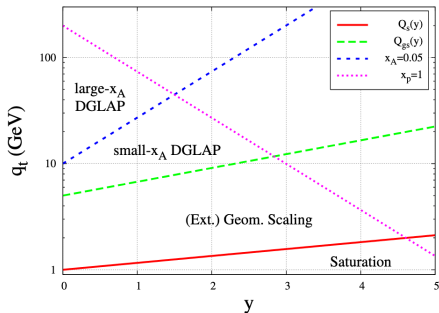


- infinite-momentum frame:  $f(x, Q^2)$



[fig: hep-ph/9702203]

## Collinear Factorization



$$y = \ln(x_0/x)$$

[fig: Dumitru2006]

- Infrared divergencies  $\Rightarrow$  DGLAP for  $f_i(x)$

$$\frac{\partial g(x, Q^2)}{\partial \log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} \left( \sum_i P_{gq} \otimes (q_i + \bar{q}_i) + P_{gg} \otimes g \right)$$

- Splitting function for quarks  $P_{AB} = P(A \rightarrow B) \sim \alpha_s(Q^2)^n$
- Splitting function for gluons  $P_{gg}(x) \sim \alpha_s(Q^2)^n \log^{n-1}(1/x)$

## $k_T$ -Factorization

- $1/x$  divergencies  $\Rightarrow$  B-JIMWLK equations
- large  $N_c$ -limit B-JIMWKL

Balitsky-Kovchegov (BK) equation

- Definition: unintegrated gluon distribution  $\varphi(x, k_T)$

$$xg(x, Q^2) = \int^{Q^2} \frac{d^2 k_T}{(2\pi)^2} \varphi(x, k_T)$$

- Cross section in  $k_T$ -Factorization:

$$\frac{d^2 \sigma_g^h}{dx_1 dx_2} = \int_{k_T, q_T} \frac{1}{k_T^2 q_T^2} \varphi(x_1, k_T) \varphi(x_2, q_T) \delta(|k_T + q_T - p_T|)$$

## Balitsky-Kovchegov equation

- Color-dipole cross section:  $\mathcal{N}(\mathbf{r}, Y)$  with  $\mathbf{x} = (x_{T,0}, x_{T,1})$

$$\varphi(\mathbf{k}, Y) \sim \int d^2r e^{-i\mathbf{k}\mathbf{r}} \nabla_r^2 \mathcal{N}(\mathbf{r}, Y)$$

- BK equation with  $Y = \log(x_0/x)$  (tree level)

$$\frac{\partial}{\partial Y} \mathcal{N}(\mathbf{x}, \mathbf{y}; Y) \sim \int_{\mathbf{z}} \mathcal{K}_{\mathbf{x}, \mathbf{y}, \mathbf{z}} [\mathcal{N}(\mathbf{x}, \mathbf{z}, Y) \mathcal{N}(\mathbf{z}, \mathbf{y}, Y) - \mathcal{N}(\mathbf{x}, \mathbf{y}, Y)]$$

- Kernel:

$$\mathcal{K}^{LO}(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \frac{N_c \alpha_s^{\text{fixed}}}{2\pi^2} \frac{(\mathbf{x} - \mathbf{y})^2}{(\mathbf{x} - \mathbf{z})^2 (\mathbf{z} - \mathbf{y})^2}$$

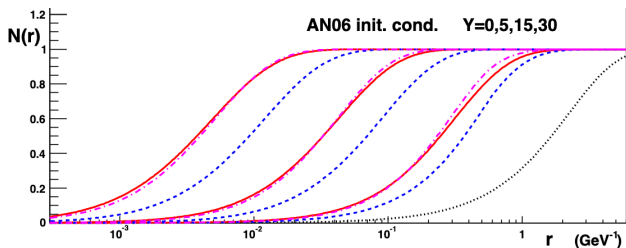
## Balitsky-Kovchegov equation

- Running coupling BK (rcBK):  $\frac{\partial}{\partial Y} \mathcal{N}(r, Y) \sim$

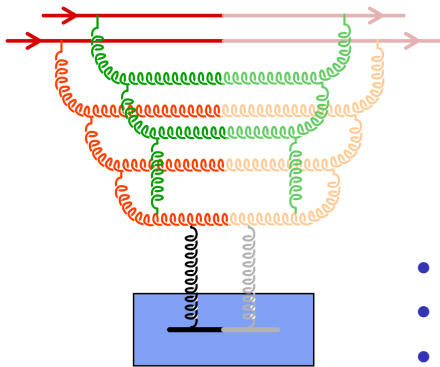
$$\int \mathcal{K}_{r,r_1,r_2}^{\text{run}} [\mathcal{N}(r_1, Y) + \mathcal{N}(r_2, Y) - \mathcal{N}(r, Y) - \mathcal{N}(r_1, Y)\mathcal{N}(r_2, Y)]$$

- kernel in transverse position space:  $\mathcal{K}_{r,r_1,r_2}^{\text{run}} =$

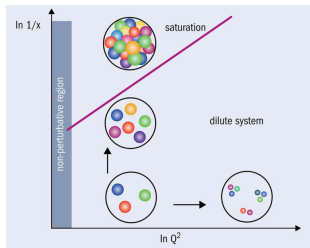
$$\frac{N_c \alpha_s(r^2)}{2\pi^2} \left[ \frac{1}{r_1^2} \left( \frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_2^2} \left( \frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) \right]$$



## Gluon Saturation



[fig: Gelis2007]



- $xg(x, Q^2) \sim 1/x$
- Gluon Saturation via recombination
- Unitarity of gluon distribution
- Dynamical Saturation Scale

$$Q_s^2(x) = x^{-\lambda} Q_0^2 A^{1/3}$$

[fig2: cerncourier.com]

## Analytical Solutions of BK

- Solution of fixed BK with Feynman- $x$

$$\mathcal{N}(r_T, x) = 1 - \exp\left(-\frac{1}{4} (r_T^2 Q_s^2(x))\right)^{\gamma(y, k_T)}$$

$$\varphi(x, k_T) = - \int d^2 r_T e^{ik_T \cdot r_T} \mathcal{N}(r_T, x)$$

→ Hankel transformation

- Simplest Model:  $\gamma = 1 \Rightarrow$  Golec-Biernat, Wusthoff (GBW)

$$\varphi^{\gamma=1}(k_T^2) = \frac{4\pi k_T^2}{Q_s^2} \exp(-k_T^2/Q_s^2)$$

- Model:  $\gamma = 1/2 \Rightarrow$  Boer, Utermann, Wessels (BUW)

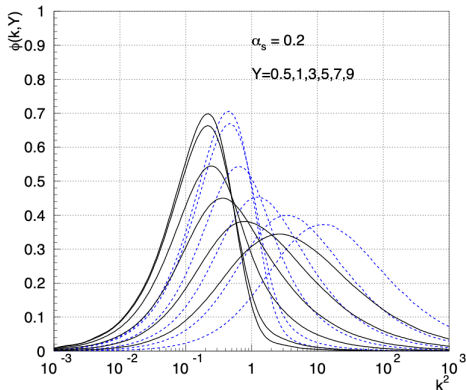
$$\varphi^{\gamma=1/2}(k_T^2) = \frac{32\pi k_T^2}{Q_s^2} \frac{1}{(1 + 16k_T^2/Q_s^2)^{3/2}}$$



## Color Glass Condensate

- McLerran-Venugopalan (MV) Model

$$N(r, Y = 0) = 1 - \exp \left( - \left( \frac{r^2 Q_{s0}^2}{4} \right)^\gamma \ln \left( \frac{1}{\Lambda_{\text{QCD}} \cdot r} + e \right) \right)$$



[fig: Venugopalan2008]

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## Relativistic Diffusion Model

- Fokker-Planck DGL for  $f(p_0, p_1)$  with  $t \equiv p_0$ ,  $y \equiv p_1$

$$\frac{\partial}{\partial t} f(y, t) = -\frac{1}{\tau_y} \frac{\partial}{\partial y} [(y_{\text{eq}} - y) f(y, t)] + D_y \frac{\partial^2}{\partial y^2} f(y, t)$$

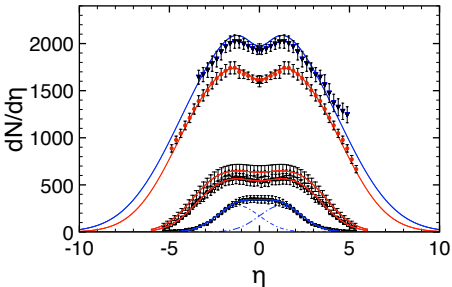
- Observable: Produced charged hadrons:

$$\frac{d\sigma_{ch}}{Ad\eta} = \frac{dN^{ch}}{d\eta} = J(\eta, m_\pi / \langle p_T \rangle) \frac{dN^{ch}}{dy}$$

$$y(\eta, m, p_T) = \frac{1}{2} \log \left( \frac{\sqrt{m^2 + p_T^2} \cosh^2(\eta) + p_T \sinh(\eta)}{\sqrt{m^2 + p_T^2} \cosh^2(\eta) - p_T \sinh(\eta)} \right)$$

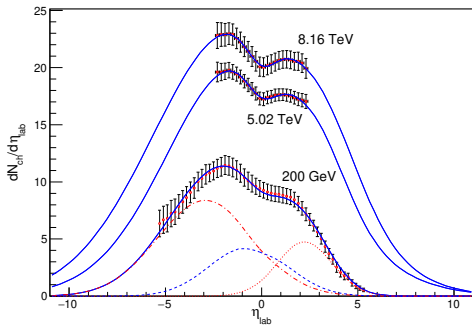
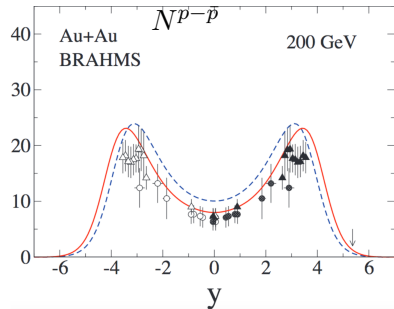
- For  $\langle p_T \rangle \gg m \Rightarrow y \approx \eta$
- Non-equilibrium Relativistic Diffusion Model (RDM)

$$\frac{dN^{ch}}{d\eta} = J \int dp_0 N^{ch} f(y, p_0) \delta(p_0 - \tau_{int}) \equiv J \sum_{i=1}^3 N_i R_i$$



## RDM

- (up) RDM for PbPb [Schulz,Wolschin2018]
- (right) RDM for pPb [Schulz,Wolschin2018]
- (left) GBW for Stopping [Wolschin2016]



## Hybrid-Factorization

- Fragmentation region  $\Rightarrow$  large  $y \Rightarrow$  Dilute-dense regime
- Single Inclusive Hadron Production with GBW:

$$\frac{dN_A^h}{dy} = \frac{C}{2\pi} \int_{z_0/z}^1 \frac{dx}{x} x f_A(x, p_T^2) \varphi(x^{2+\lambda} e^\tau)$$

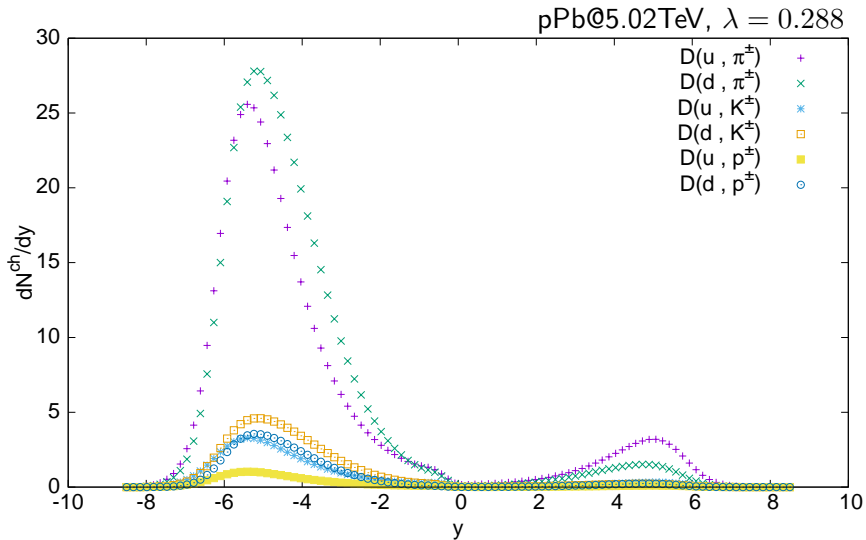
- $\tau \equiv \ln(s_{NN}/Q^2) - \ln A^{1/3} - 2(1 + \lambda)y$
- Geometric scaling:  $z_0 \rightarrow 0$

$$C \equiv \int_{z_0}^1 dz D_{h/g}(z, p_T^2)$$

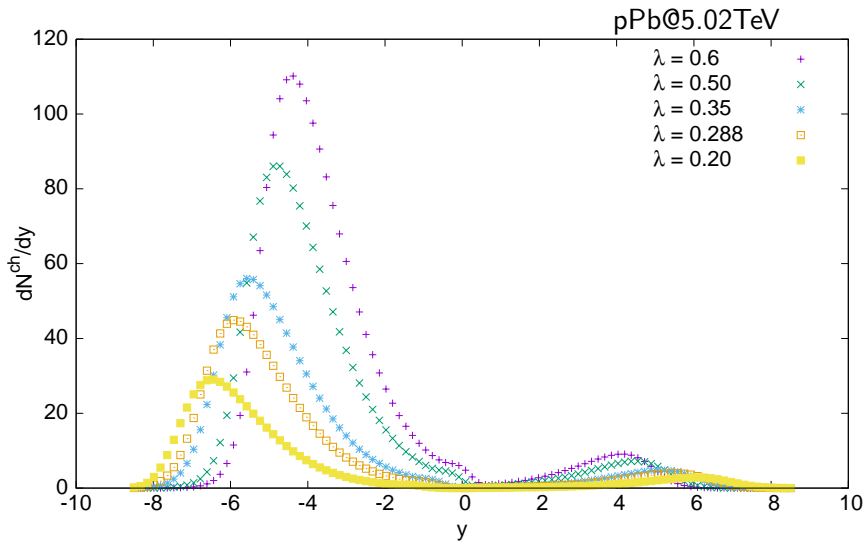
- Fragmentation function  $D \leftrightarrow$  parton-hadron duality
- Parametrisation:

$$D_i^{h^\pm}(z, M_0^2) = N_i^{h^\pm} z^{a_i^{h^\pm}} (1-z)^{b_i^{h^\pm}} \left( 1 + c_i^{h^\pm} (1-z)^{d_i^{h^\pm}} \right)$$

# Charged Hadron Production



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## Summary

- Relativistic Diffusion Model
  - Rapidity distribution of charged hadrons
  
- Midrapidity source
  - Nearly thermalized, small- $x$  small- $x$  gluon scattering
  
- Fragmentation sources
  - small- $x$  gluon high- $x$  quark scattering

## References



[[phys.org20150211](#)]

<https://phys.org/news/2015-02-polarized-protons-uncover-secrets.html>, 07/10/2018



[[Bethke2017](#)]

S. Bethke,  $\alpha_s$  2016, Nucl. Part. Phys. Proc. 282-284, 149 (2017)



[[Schulz,Wolschin2018](#)]

P. Schulz, G. Wolschin, Diffusion-model analysis of pPb and PbPb collisions at LHC energies, Mod. Phys. Lett. A33, 1850098 (2018)



[[Wolschin2016](#)]

G. Wolschin, Beyond the thermal model in relativistic heavy ion collisions, Phys. Rev. C 94, 024911 (2016)



[[Tawfik2014](#)]

A. Tawfik, A. G. Shalaby, Balance Function in High-Energy Collisions, Advances in High Energy Physics Volume 2015, 186812 (2015)



[[Reygers2017](#)]

J. Stachel, K. Reygers, Lecture Notes: Quark-Gluon Plasma Physics, Heidelberg University (2015)



[[MSTW2008](#)]

A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt, Parton distributions for the LHC, Eur. Phys. J. C63, 189-285 (2009)



[[Gelis2007](#)]

F. Gelis, Lecture Notes: low- $x$  physics, saturation and diffraction, School on QCD, Copanello (2007)