INSTITUT FÜR THEORETISCHE PHYSIK HEIDELBERG
PROF. DR. LUCA AMENDOLA
COMPUTATIONAL STATISTICS – SUMMER SEMESTER 2022 –

Tutors:

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Please, send (in pdf) or hand in the solution to this exercise sheet to your tutor, following their instructions, and carefully respecting the delivery date shown below. All exercise sheets will be graded. You can solve them individually or in pairs (with another student of the same tutorial group). In the latter case deliver please only one document with the solutions. Write the name of the file in the following format: X.Name1.Surname1 or X.Name1.Surname1-Name2.Surname2.pdf, with X being the number of the corresponding exercise sheet. The sheets are to be submitted always on Mondays by 11:00 am.

Exercise Sheet 2

RECEIVED: May. 02 - DELIVERY: May. 09

Exercise 2.1: Full Width Half Maximum (2pt)

A definition for the width of a PDF is given by the 'Full width at half maximum' (FWHM), i.e. the width of a PDF at half of its maximum. Since σ is usually reserved for Gaussian PDF's, the FWHM is mostly employed for non-Gaussian PDF's. Show that the FWHM of a Gaussian PDF is equal to 2.35σ. (2pt)

Hint : at what value the PDF reaches its maximum ?

Exercise 2.2: Kolmogorov's Axioms (3pt)

In a random experiment the set of all possible outcomes, called Ω, consists of all the individual outcomes ω. P is called a probability for $P : \mathcal{P}(\omega) \to [0, 1]$ if P obeys the so-called Kolmogorov's axioms:

$$P(\Omega) = 1 \tag{1}$$

$$P(A) \geq 0\,, \qquad \forall A \subset \Omega \tag{2}$$

$$P(A \cup B) = P(A) + P(B)\,, \qquad \forall A \cap B = \{\} \tag{3}$$

Denoting "the complement of A", i.e. "not-A", with $\bar{A}$, prove the following :

1. $P(A) + P(\bar{A}) = 1$ and $P(\{\}) = 0$ (1pt)
2. If $A \subset B \rightarrow P(A) \leq P(B)$ (1pt)
3. $P(A \cup B) + P(A \cap B) = P(A) + P(B)$ (1pt)

Hints: you need some identities from set theory, e.g.

$$(\bar{A} \cap B) \cup (A \cap B) = B \tag{4}$$

Exercise 2.3: Conditional probabilities (3pt)

Show that

$$P(A|B) + P(A|\bar{B}) = 2P(A) \tag{5}$$

a) if A and B are independent (1pt); and 2) if $P(B) = P(\bar{B})$. (2pt)

Exercise 2.4: Bayes Theorem (4pt)

Use Bayes Theorem to calculate the probability of you having Covid, if you get a positive test result after doing 1) a PCR test or 2) a quick test (2pt). Also, what is the probability of having Covid if you get a negative results after doing 3) a PCR test or 4) a quick test?) (2pt)

First calculate the probability $p(covid)$ by just using the information for Heidelberg, by dividing the people currently being infected with Covid by the total number of citizens in Heidelberg (please get this info on the internet). For simplicity just assume that the 7 day incidence contains all the infected people and nobody is infected longer than 7 days. Assume that the quick test detects 90% of all Covid cases with a 2% false positive rate and the PCR test 99% with a 0.02% false positive rate.