INSTITUT FÜR THEORETISCHE PHYSIK HEIDELBERG
PROF. DR. LUCA AMENDOLA
COMPUTATIONAL STATISTICS – SUMMER SEMESTER 2022 –

Tutors:

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Please, send (in pdf) or hand in the solution to this exercise sheet to your tutor, following their instructions, and carefully respecting the delivery date shown below. All exercise sheets will be graded. You can solve them individually or in pairs (with another student of the same tutorial group). In the latter case deliver please only one document with the solutions. Write the name of the file in the following format: X.Name1.Surname1 or X.Name1.Surname1-Name2.Surname2.pdf, with X being the number of the corresponding exercise sheet.

Exercise sheet 6

RECEIVED: May.30 - DELIVERY: June.07

Exercise 6.1: Wick‘s theorem (4pt)

Assume that x, y and z are Gaussian random variables with zero mean and variances $\sigma_x^2, \sigma_y^2, \sigma_z^2$. Furthermore, x and y as well as y and z are correlated with the correlation coefficient ρ, while x and z are uncorrelated. Use Wick‘s theorem to calculate a) $E[x^2y^2]$ (2pt) and b) $E[xy^3z^2]$ (2pt).

Exercise 6.2: Difference, product and quotient of Gaussian distributed random variables (6pt)

a) Find the PDF of the difference $d = x - y$ of two uncorrelated Gaussian random variables x, y with zero mean and the same variance σ^2. (2pt)

b) Find the PDF of the product $p = xy$ of two uncorrelated Gaussian random variables x, y with zero mean and the same variance σ^2. (2pt)

c) Find the PDF of the quotient $m = x/y$ of two uncorrelated Gaussian random variables x, y with zero mean and the same variance σ^2. (2pt)

Exercise 6.3: Python exercise: Inversion sampling (5pt)

Consider the Cauchy distribution

$$p(x) = \frac{1}{\pi(1+x^2)}. \tag{1}$$

a) Write a script to perform rejection sampling using the method you saw in Exercise 5.3. (2 pt)

b) Write a script to perform inversion sampling which consists in the following:

- Generate a random number y from the uniform unit interval.
- Map y onto $x = P^{-1}(y)$, where $P^{-1}(y)$ is the inverse of the cumulative distribution function of the Cauchy PDF defined as

$$P(x) = \int_{-\infty}^{x} p(x')\mathrm{d}x'. \tag{2}$$

Show that x is distributed according to $p(x)$. (2pt)

c) Which of the two algorithms is faster for the same convergence criteria (e.g. reaching the theoretically expected median to within 1%) (1pt) ?

For that matter please use `import time`. With this library you can set the command `start-time` = `time.time()` at the beginning of your algorithm and the command `print(time.time()-start-time)` at the end of your algorithm. This will then give you the time your code takes to run on your computer.