INSTITUT FÜR THEORETISCHE PHYSIK HEIDELBERG
PROF. DR. LUCA AMENDOLA
COMPUTATIONAL STATISTICS – SUMMER SEMESTER 2022 –

Tutors:

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Please, send (in pdf) or hand in the solution to this exercise sheet to your tutor, following their instructions, and carefully respecting the delivery date shown below. All exercise sheets will be graded. You can solve them individually or in pairs (with another student of the same tutorial group). In the latter case deliver please only one document with the solutions. Write the name of the file in the following format: X.Name1.Surname1 or X.Name1.Surname1-Name2.Surname2.pdf, with X being the number of the corresponding exercise sheet.

Exercise sheet 9

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9.1 Python exercise: Linear fitting (5pt)

Write a python script which generates artificial data (e.g. 10 data points) following the model $y(x) = ax + b$ including uncorrelated Gaussian noise with variance σ^2 .
(a) Fit the model to the artificial data using the analytical formula, repeat for a large number of times, and derive the distributions $p(a)$ and $p(b)$ of the model parameters (2pt),
(b) as well as their correlation coefficient r_{ab} . Plot as well the cloud of points (a, b)(1pt).
(c) What happens to the distributions $p(a)$ and $p(b)$ if you decrease the noise σ^2 (1pt)?
(d) What happens to the width of the distributions for a and b if you change the x-range(1pt)?

9.2 Fisher matrix for a nonlinear model (4pt)

The model $y(x) = \exp(-\alpha x)$ is fitted to data $y_i = y(x_i) + \Delta y_i$, where Δy_i is an uncorrelated Gaussian noise with constant variance σ^2.
a) Please compute the Fisher matrix $F_{\alpha\alpha}$ where α is the only parameter of our model (2pt).
b) Show that the errors depend on α but if the linearization $\exp(-\alpha x) \approx 1 - \alpha x$ is used, the errors become independent of α (2pt).

9.3 Combination of independent measurements (5pt)

If you combine independent measurements by multiplying their likelihoods, $\mathcal{L} = \mathcal{L}_1 \mathcal{L}_2$,
a) why would that imply that the Fisher matrices add, i.e. $F_{\mu\nu} = F_{\mu\nu}^{(1)} + F_{\mu\nu}^{(2)}$ (1pt)?
b) assuming for simplicity that we have one parameter and that the error for each new set of data is the same, show that the total error decreases with the addition of more data sets (1.5pt)
c) What happens then to the Shannon entropy presented in a previous sheet (Hint: start by writing the entropy action assuming no priors). Was the behavior expected, why ? (2.5pt)?