

Distances and acceleration

Cosmology WS22/23

Luminosity distance...

$$f = \frac{L}{4\pi r^2(z)(1+z)^2}$$

$$d_r = r(z)(1+z)$$

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 + \Omega_{k0} H_0^2 r^2} + r^2 d\Omega^2 \right]$$

$$d_L(z) = \frac{1+z}{H_0 \sqrt{\Omega_{k0}}} \sinh \left(H_0 \sqrt{\Omega_{k0}} \int \frac{dz}{H(z)} \right)$$

$$H(z) = H_0 (\Omega_{m0} a^{-3} + \Omega_{\Lambda} + \Omega_{k0} a^{-2})^{1/2}$$

...and magnitudes

$$f = \frac{L}{4\pi d_L^2}$$

Apparent magnitude

$$m = -2.5 \log f + c_1$$

Absolute magnitude

$$M = -2.5 \log L + c_2$$

$$m = M + 25 + 5 \log d_L(\text{Mpc})$$

Mapping the expansion rate

$$f = \frac{L}{4\pi d_L^2}$$

$$d_L \sim \int \frac{dz}{E(z; \Omega_{i,0}, w_i)}$$

Flux ---> distance ---> cosmology

Distance indicators: parallax

1836
Friedrich Bessel

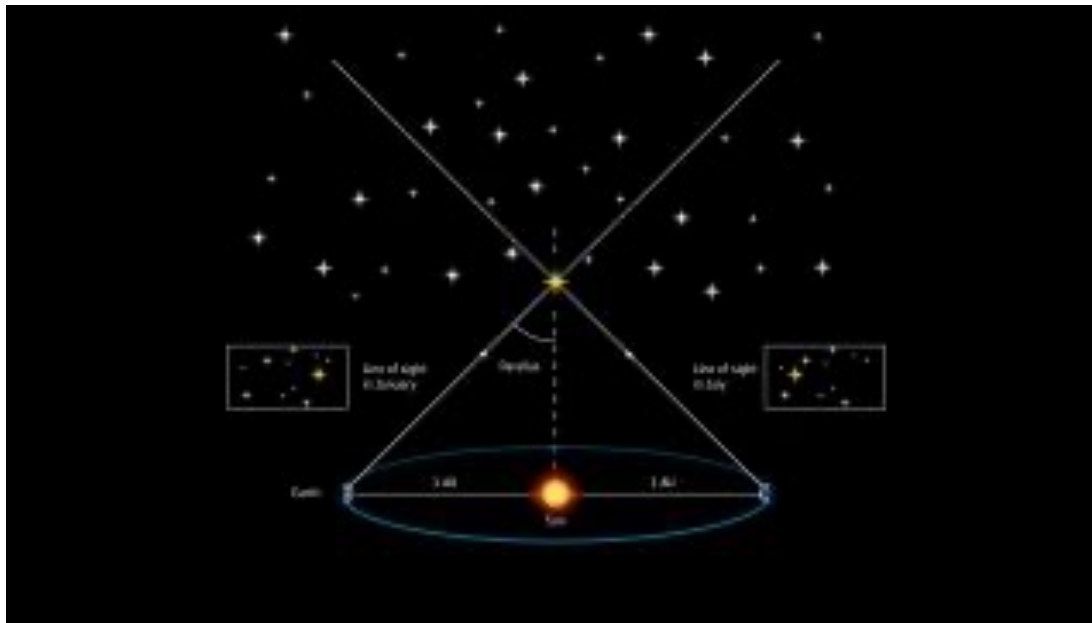


Image credit: ESA

$$d = \frac{1\text{pc}}{\theta[\text{arcsec}]} \quad 1\text{pc} = 2.3 \cdot 10^{13}\text{km}$$

GAIA satellite 10^{-5}arcsec ---> 50-100kpc

Distance indicators: parallax

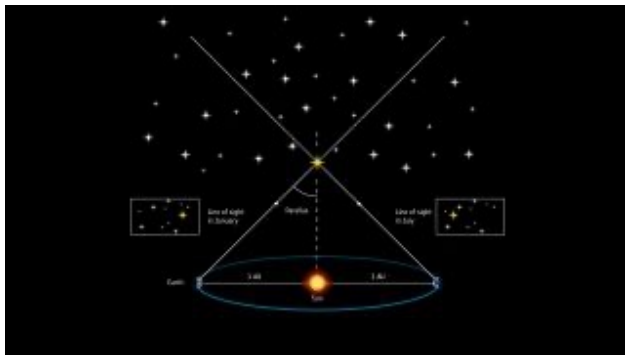
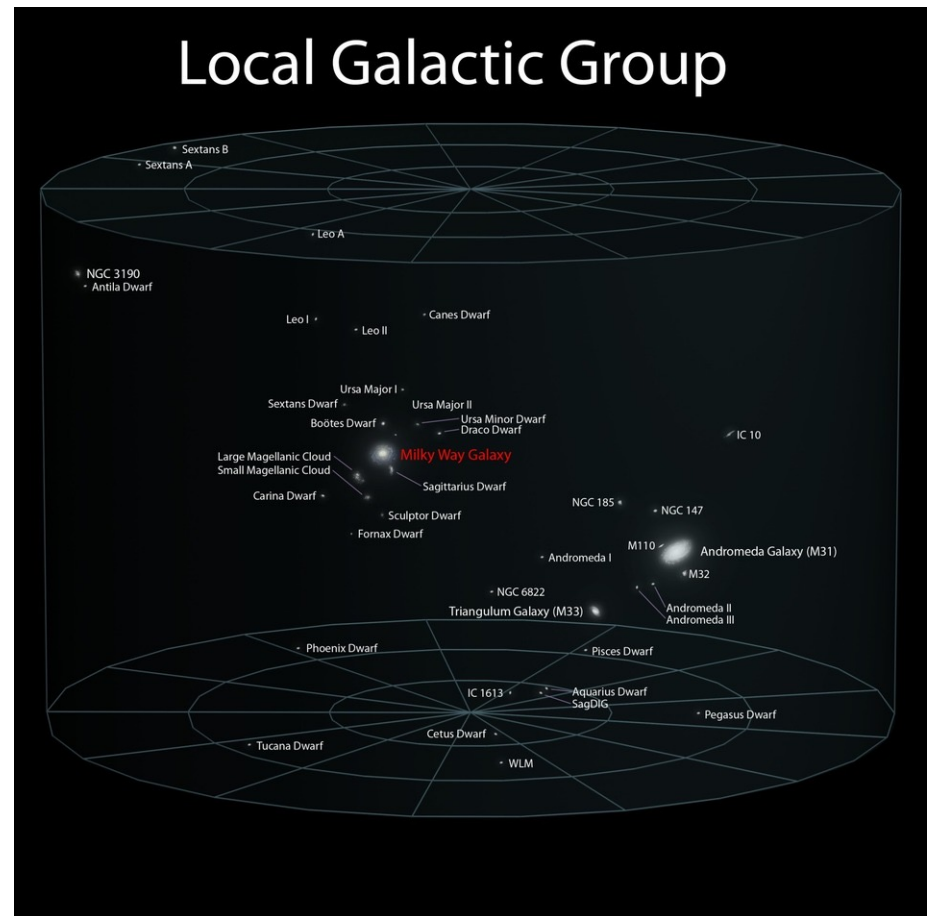


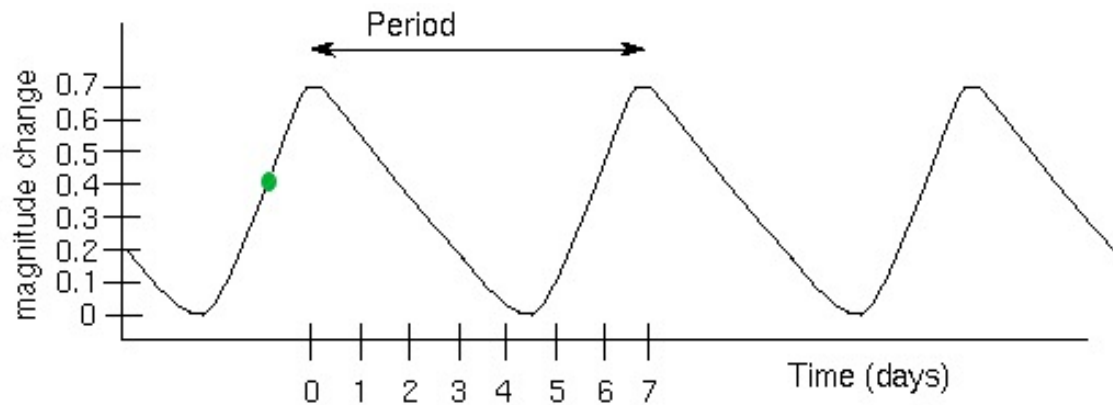
Image credit: ESA

GAIA satellite 10^{-5} arcsec
---> 50-100kpc



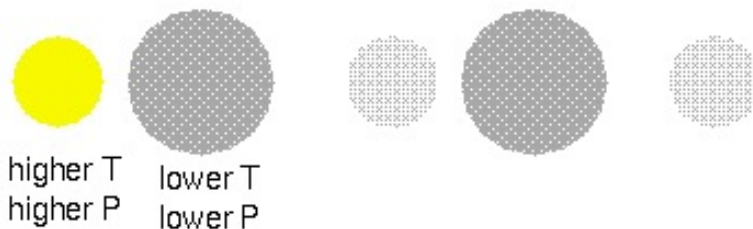
Credit: By Azcolvin429, via Wikimedia Commons

Distance indicators: Cepheids



Henrietta Leavitt
1910's

$$L \sim P^{1.3}$$

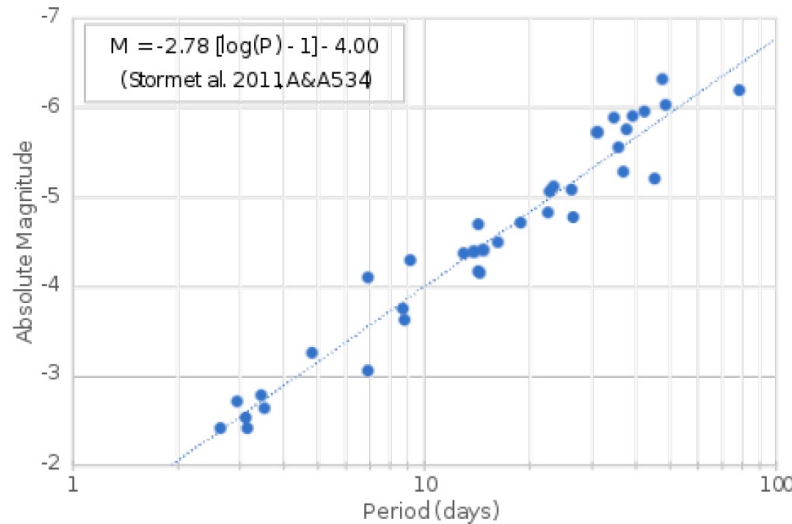


Variability phase lasts for
some thousand years

Cepheid variables: outward pressure (P) and inward gravity compression are out of sync, so star changes size and temperature: it **pulsates**.

RR-Lyrae variables are smaller and have pulsation periods of less than 24 hours. Also, their light curve looks different from the Cepheid light curve.

Distance indicators: Cepheids



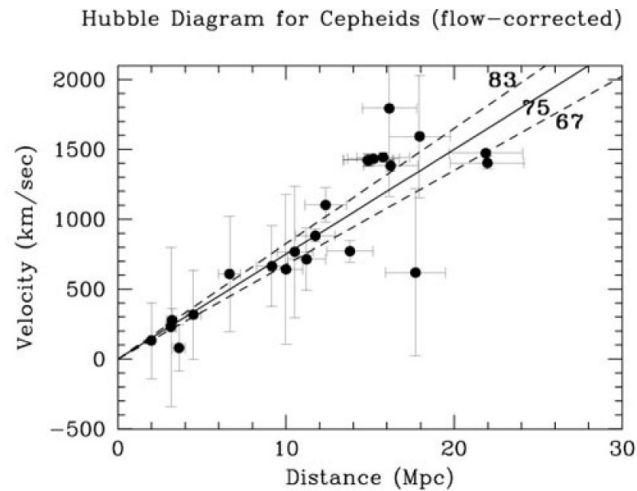
Period-luminosity-color
relation

$$M_V = 3.32(V-I) - 3.93 \log P - 2.92,$$

Sandage et al. 2008

Figure 5.2.1: Period-Luminosity relation for Magellanic Cloud Cepheid variable stars; data from Storm et al. 2011, A&A, 534. (Wikicommons, author: Dbenford)

Distance indicators: Cepheids



Distances up to 30-50 Mpc

Figure 5.2.2: Hubble constant from the Cepheid method (Freedman et al., *Ap. J.*, Volume 553, Issue 1, pp. 47-72, 2001. © AAS. *Reproduced with permission*).

Distance indicators: planetary nebulae



Late phase
of red giants

Image credits: B. Balick, R. Sahai, A. Hajian, M. Meixner, P. Harrington, K. Borowski et al. (ST/SCI/AURA/NASA/ESA/NOAO), G. Heiligman et al. (HST, ESA, NASA), A. Block (NOAO/AURA/NSF), G. Jacoby (Wfyn/NOAO/NSF), D. Malin (AAO) and A. Zijlstra (IPHAS).

Distance indicators: planetary nebulae

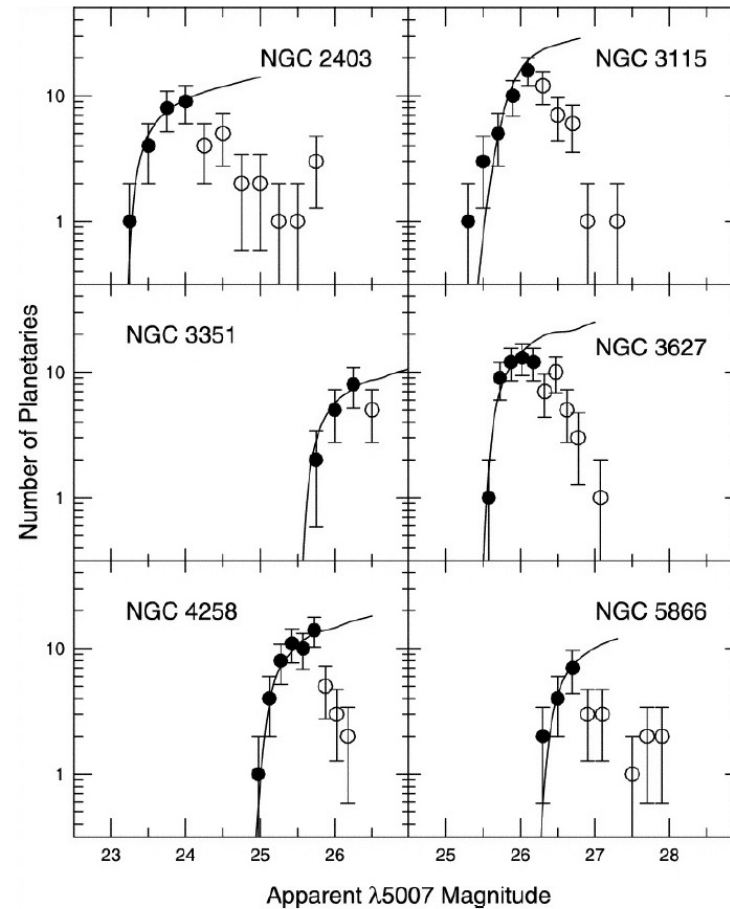
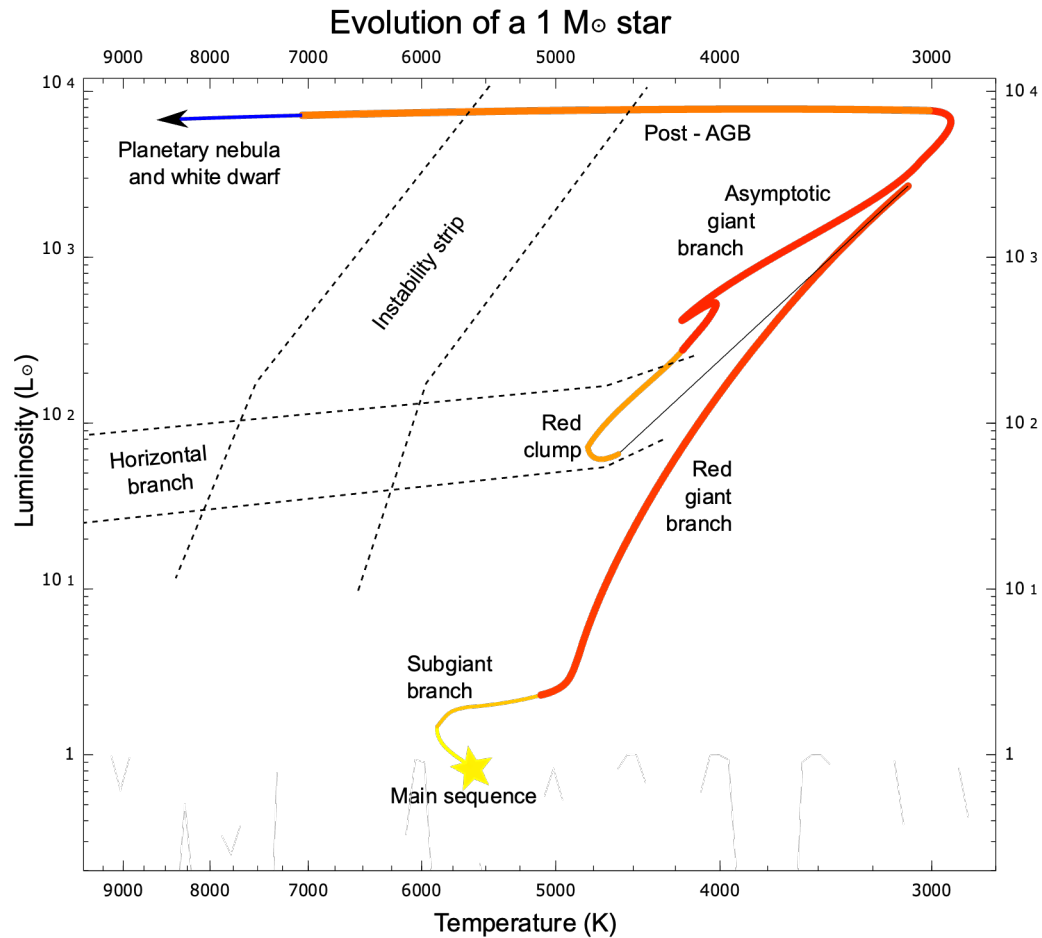


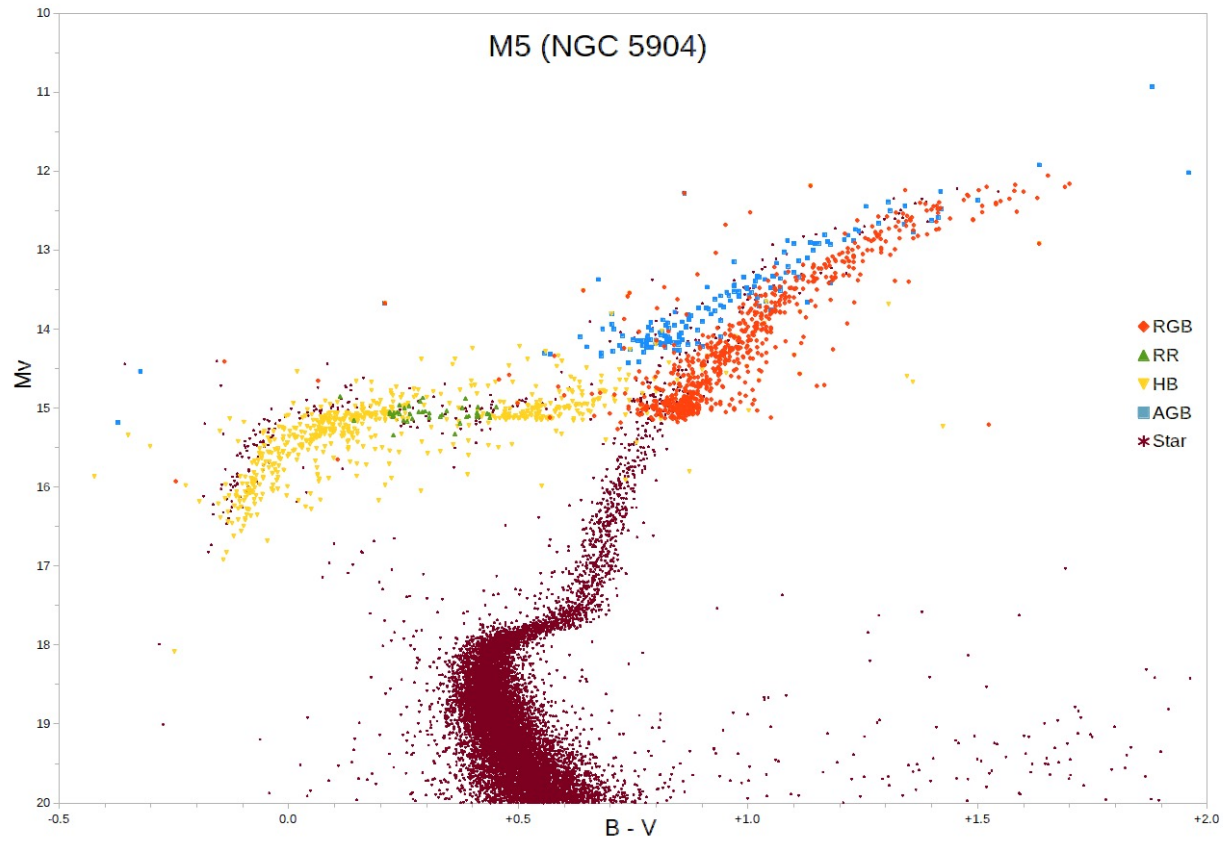
Figure 5.3.1: Luminosity functions of planetary nebulae in a sample of galaxies. Notice the sharp cut-off at high luminosity. (From Ciardullo et al. 2002, astro-ph/0206177, *Astrophys.J.* 577 (2002) 31-50. © AAS. *Reproduced with permission.*)

Distance indicators: Tip of Red Giant Branch



By Lithopsian - Own work, CC BY-SA 4.0, <https://commons.wikimedia.org/w/index.php?curid=48486177>

Distance indicators: Tip of Red Giant Branch



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Distance indicators: Tully-Fisher

Spiral galaxies

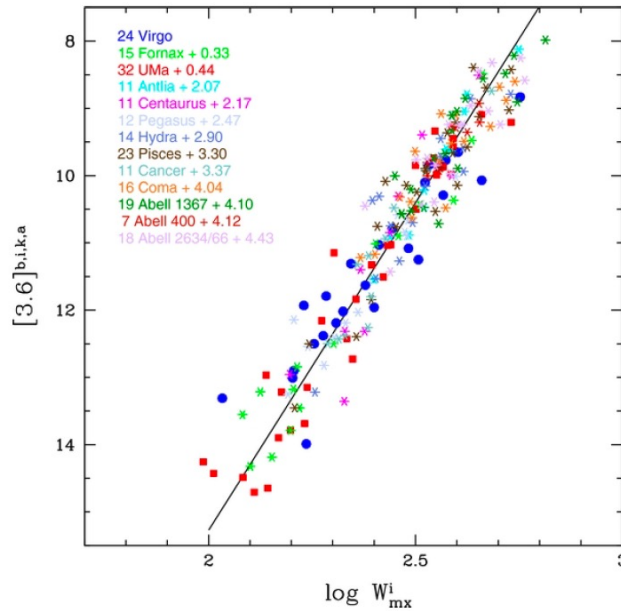


Figure 5.5.2: Tully-Fisher relation in the IR band around $3.6\mu\text{m}$ (Sorice et al., 2013ApJ...765...94S, © AAS. *Reproduced with permission*) for galaxies up to roughly 200 Mpc.

Virial theorem

$$\langle v^2 \rangle = \langle \Phi \rangle = \frac{GM}{R_{vir}}$$

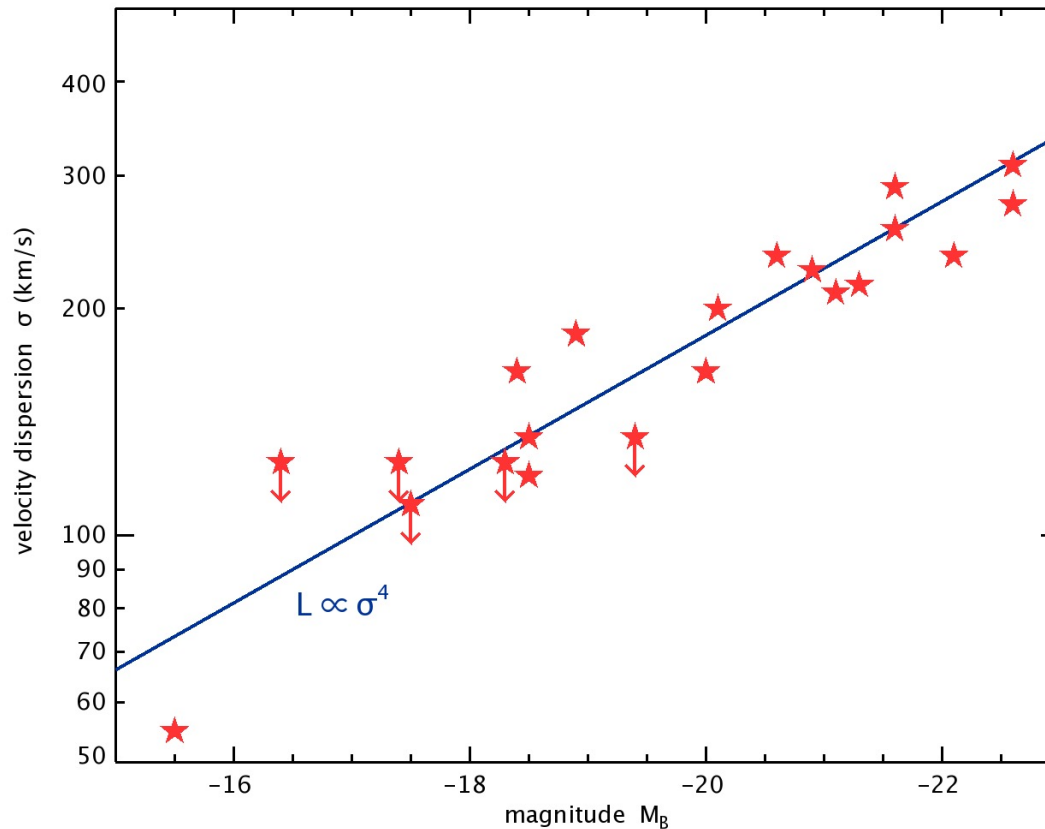
$$M \sim L \sim R^2$$

$$L \sim \sigma_{rot}^\alpha$$

$$\alpha \approx 3.5 - 4$$

Distance indicators: Faber-Jackson

Elliptical galaxies



Virial theorem

$$\langle v^2 \rangle = \langle \Phi \rangle = \frac{GM}{R_{vir}}$$

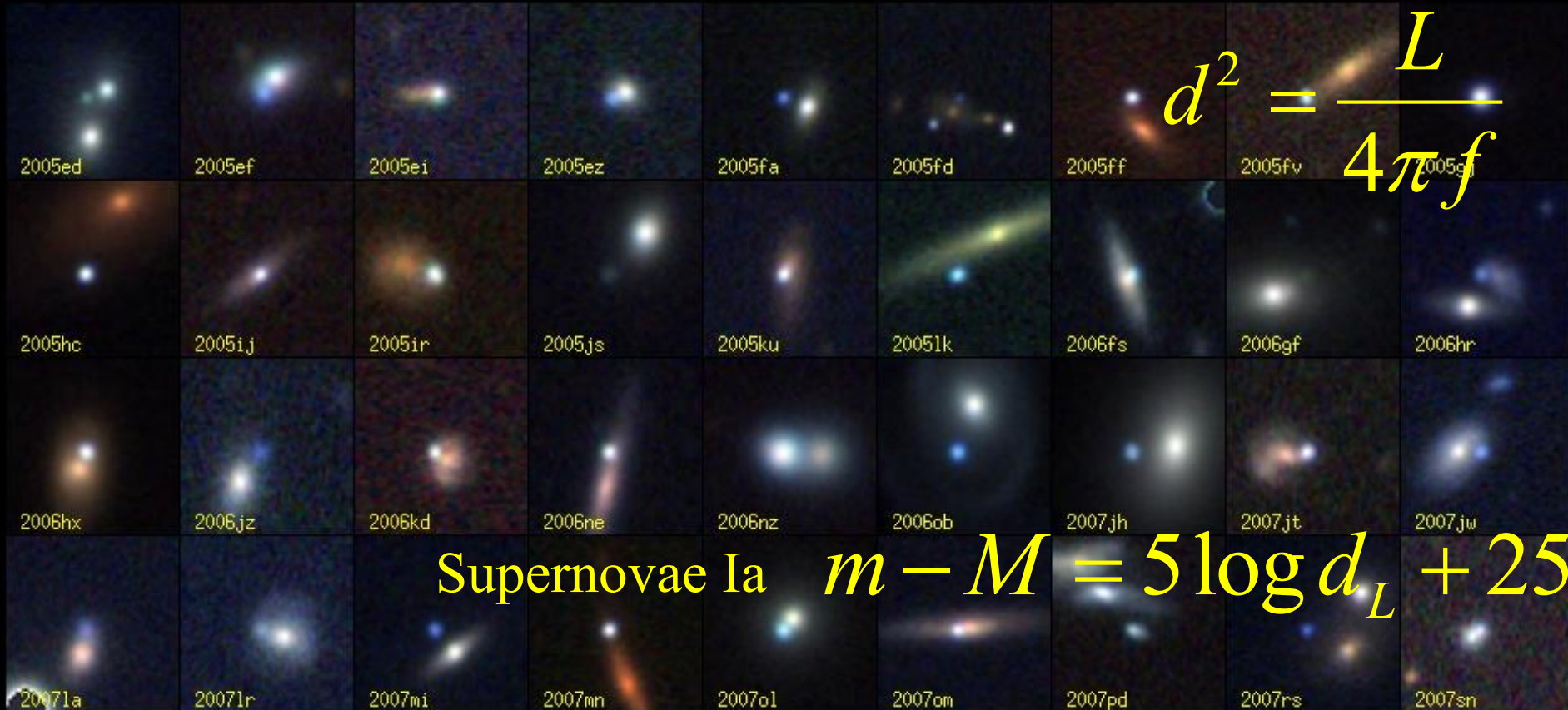
$$L \approx \sigma^\gamma$$

$$\gamma \approx 3 - 4$$

For less massive galaxies

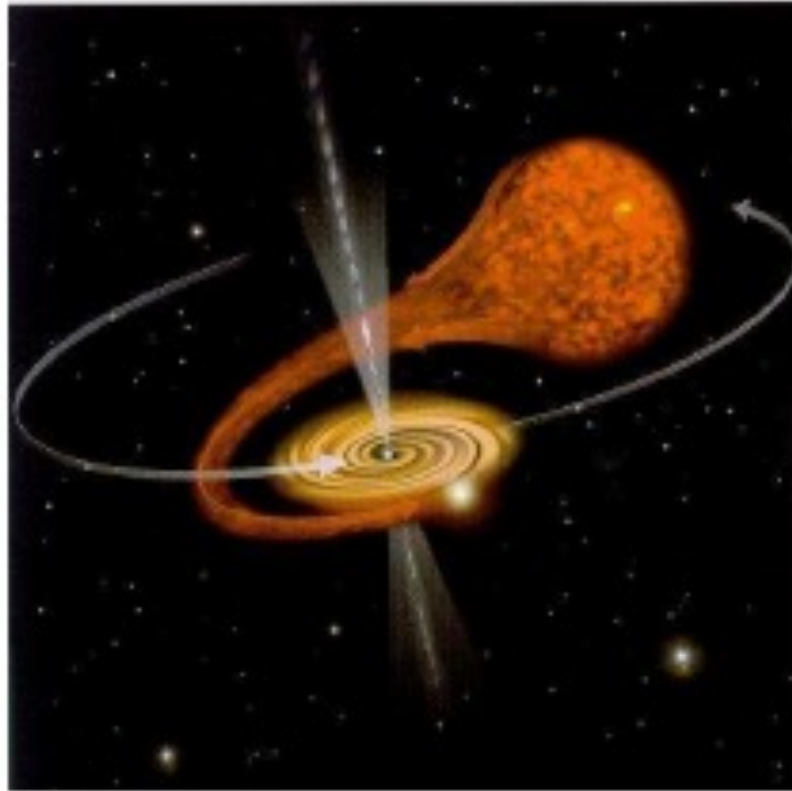
Public Domain, <https://en.wikipedia.org/w/index.php?curid=13405907>

Lighthouses in the dark



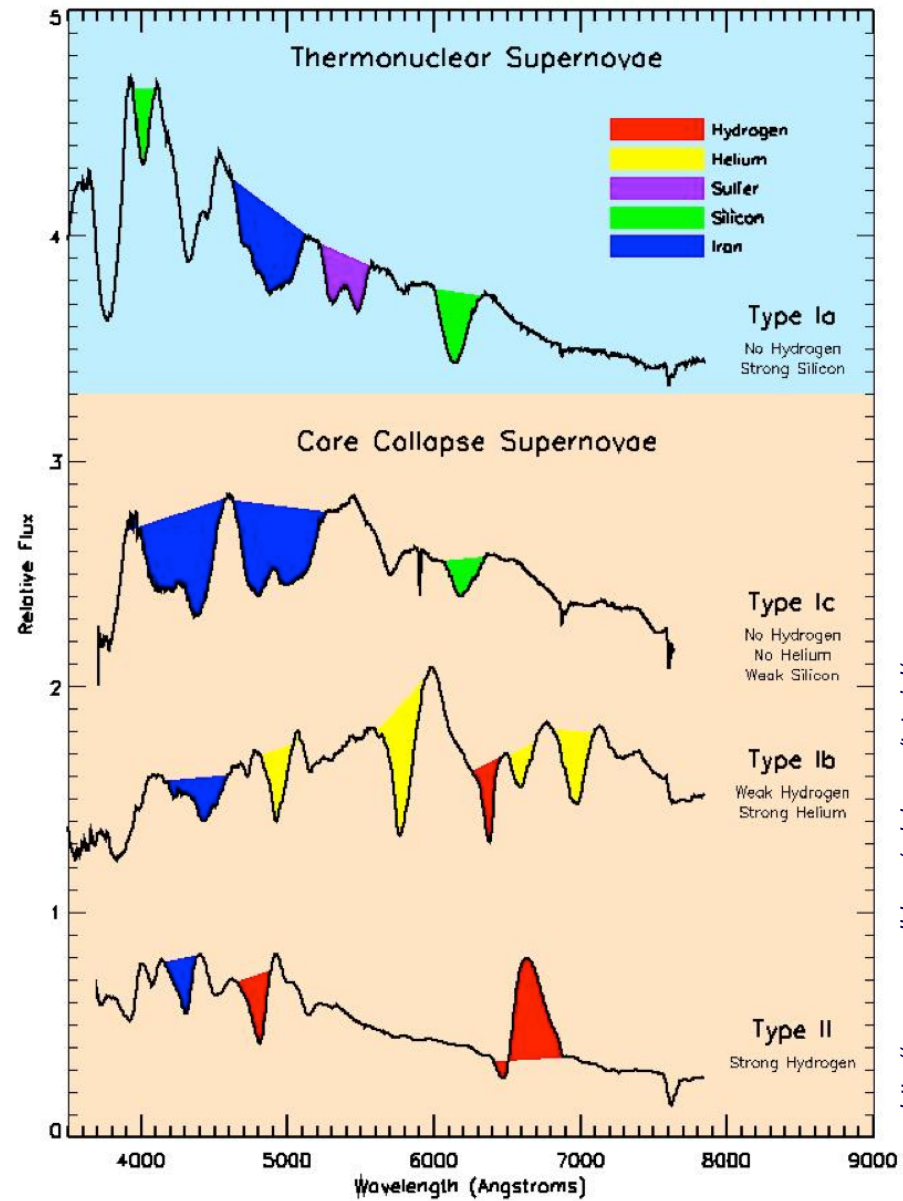
$$d^2 = \frac{L}{4\pi f}$$

Supernovae Ia $m - M = 5 \log d_L + 25$

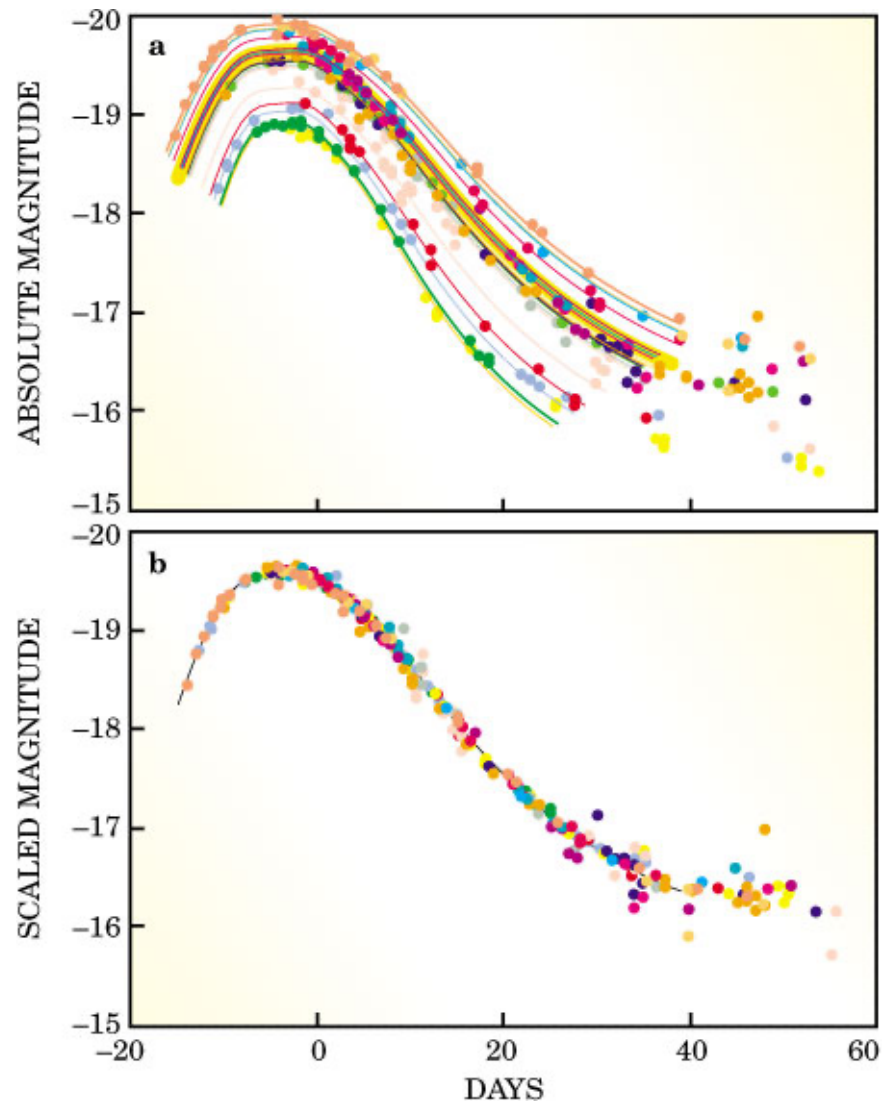


- ▶ This hypothesis can be tested and calibrated through a local sample whose distance we know by other means.

Types of supernovae



<http://supernova.lbl.gov/~dnkasen/tutorial/>

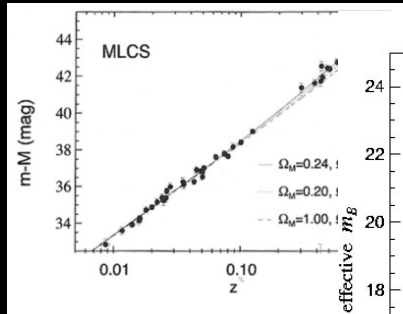


Phillips, Hamuy, et al.
1993, 1995

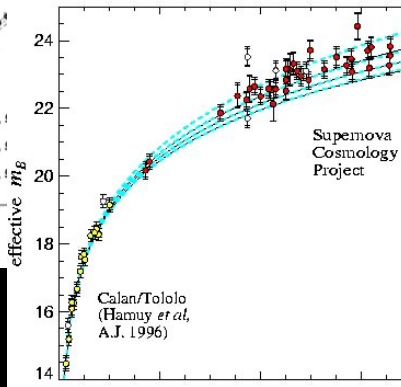
- ▶ Then, we compare $m_{obs}(z)$ with

$$m_{theor}(z) = M + 25 + 5 \log d(z; \Omega_M, \Omega_\Lambda, \dots)$$

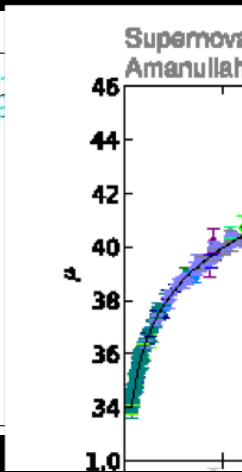
Hubble diagram



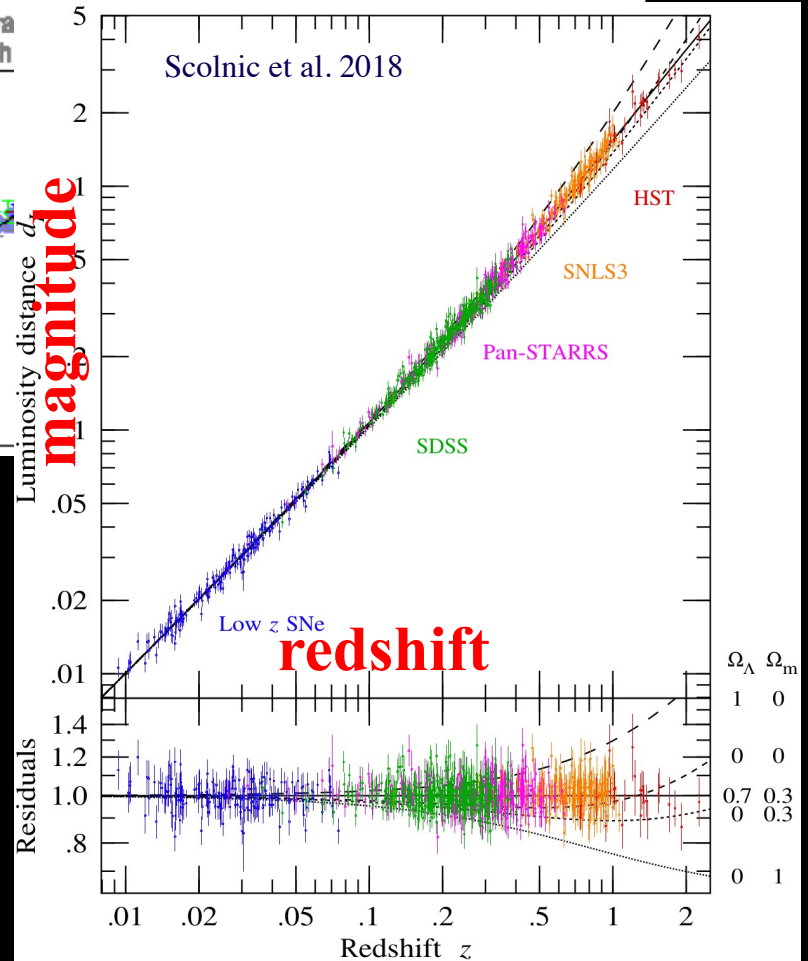
1997



1998

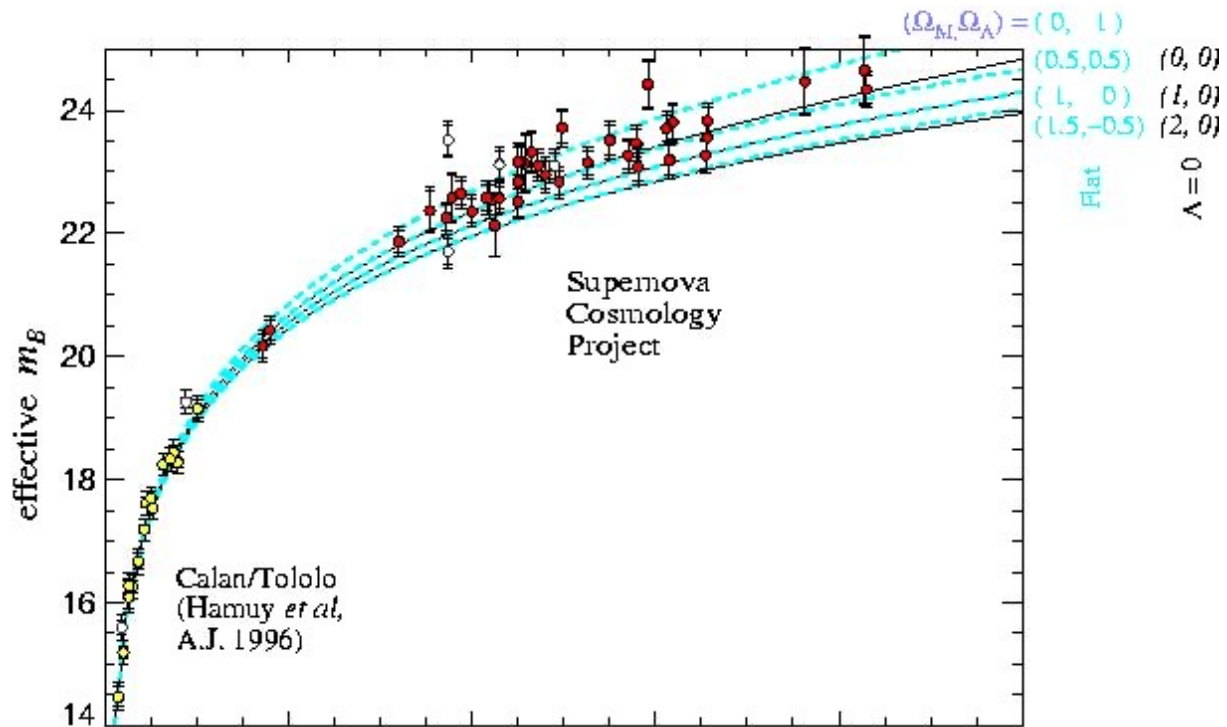


2010



2018

SNIa are dimmer than expected!



Basic property 1

Local
Hubble
law

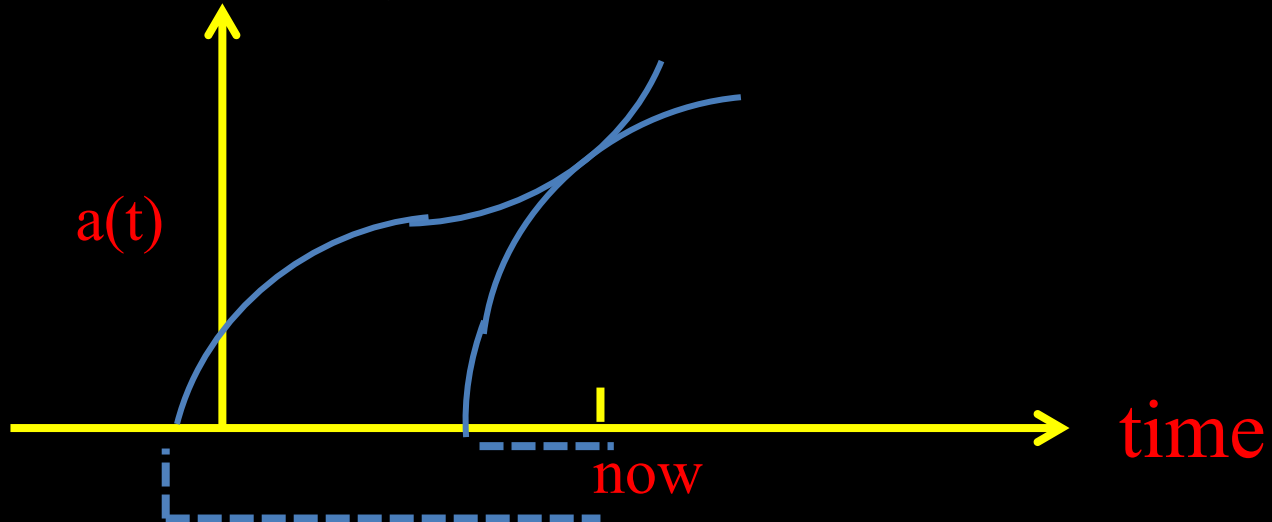
$$r(z) = \frac{z}{H_0}$$



$$r(z) = \int \frac{dz}{H(z)}$$

Global
Hubble
law

If $H(z)$ in the past is smaller (i.e. **acceleration**), then $r(z)$ is larger: larger distances (for a given redshift) make dimmer supernovae



LambdaCDM

$$H^2 = H_0^2 [\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0} + (1 - \Omega_{m,0} - \Omega_{\Lambda,0})(1+z)^2]$$

Only two parameters!

$$d_L \sim \int \frac{dz}{E(z; \Omega_{i,0}, w_i)}$$

$$d_L(z; \Omega_{m,0}, \Omega_{\Lambda,0}) \sim \int \frac{dz}{[\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0} + (1 - \Omega_{m,0} - \Omega_{\Lambda,0})(1+z)^2]^{1/2}}$$

Statistics in three steps

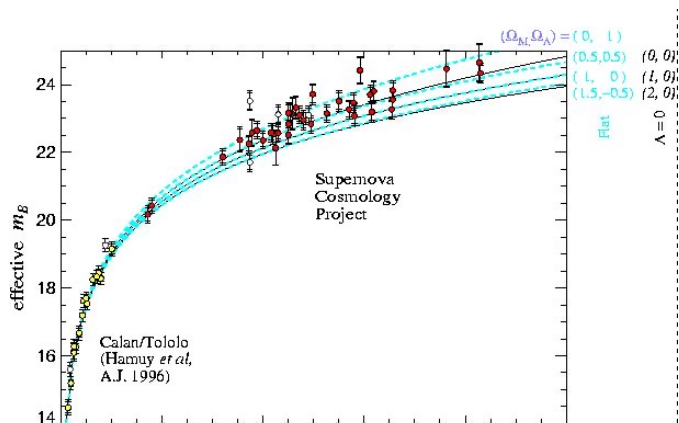
1: take a model

$$H^2 = H_0^2 E^2(z; \text{params})$$

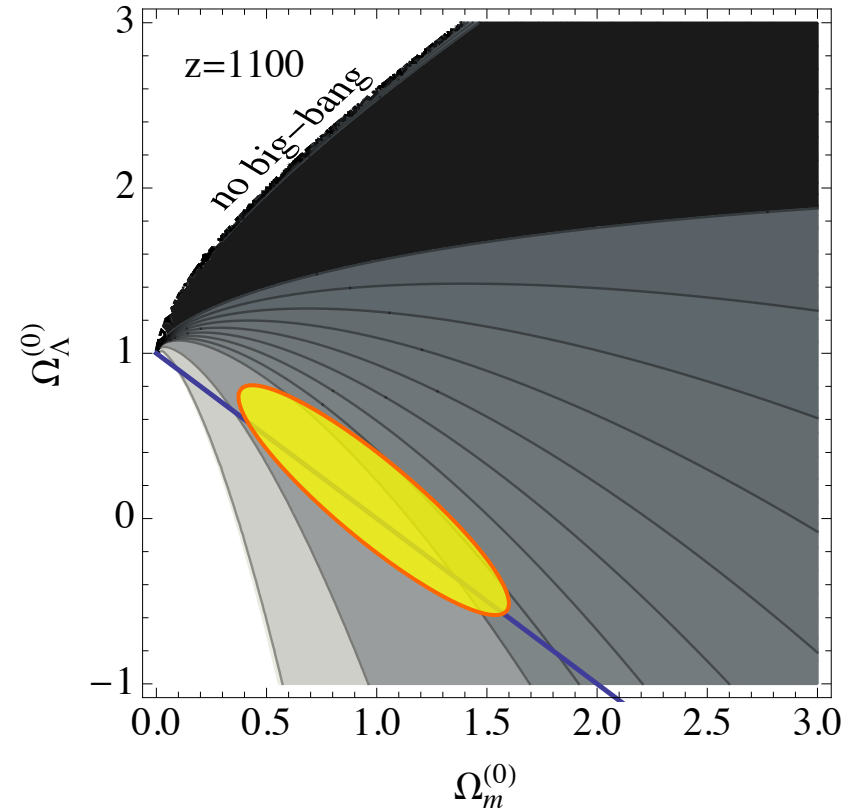
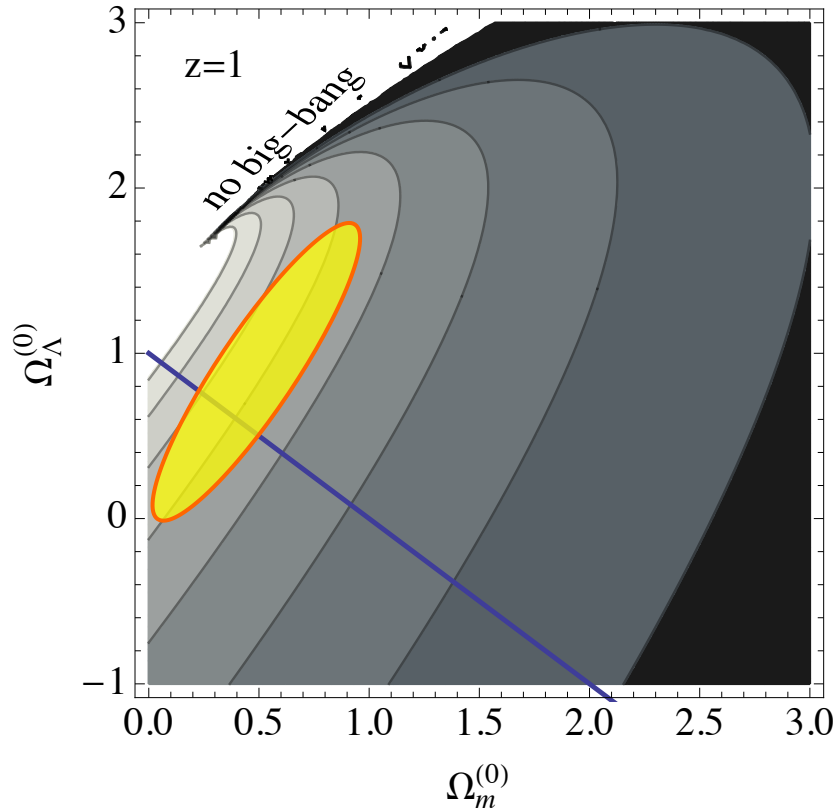
2: find the distance

$$d_L(z; \text{params}) \sim \int \frac{dz}{E(z)}$$

3: vary the parameters and minimize the chi-squared with data

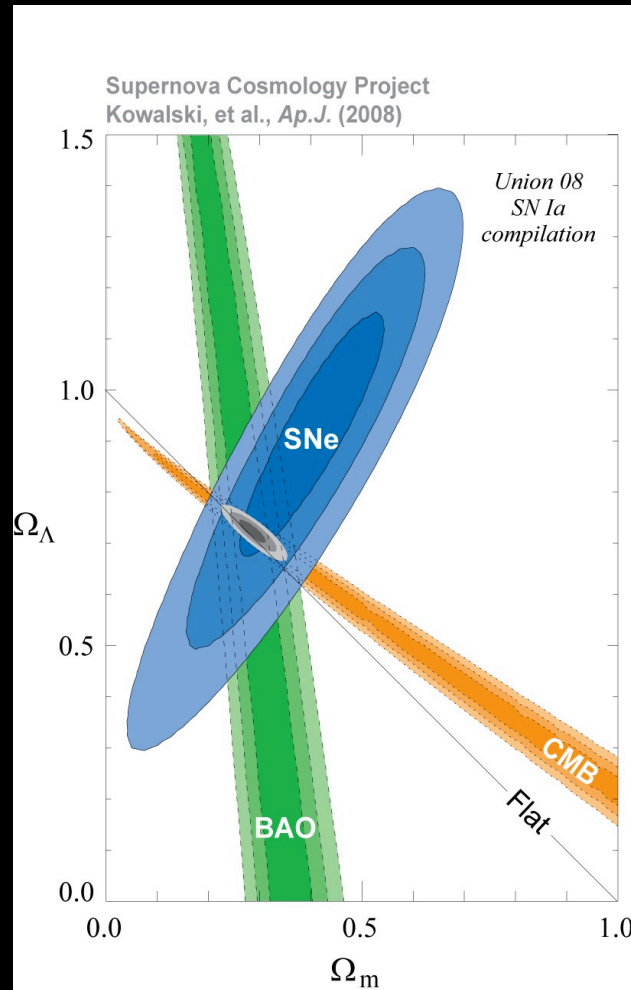


Basic property 2

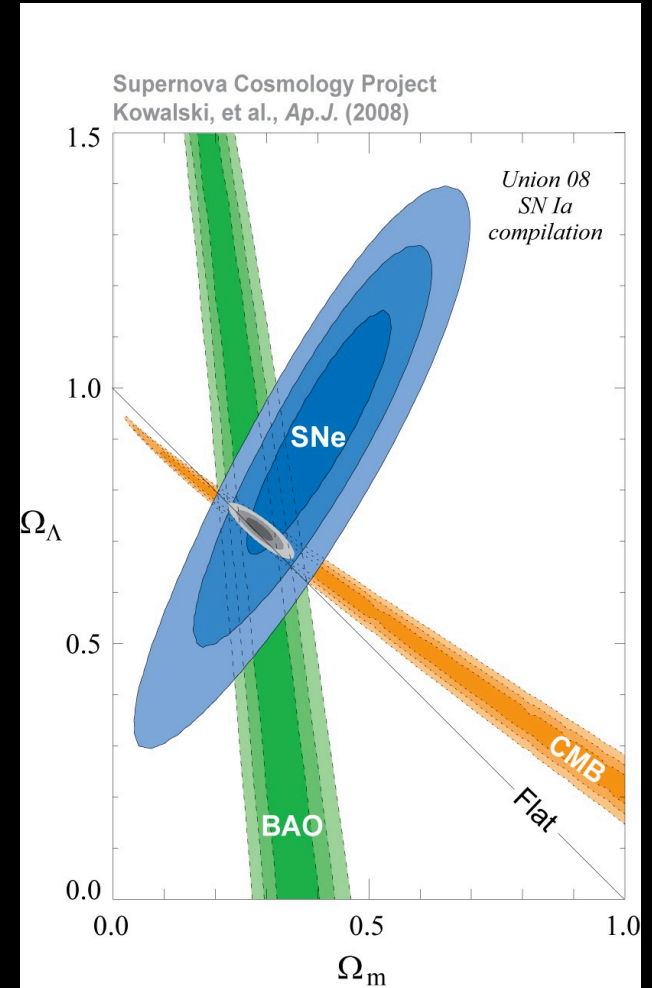
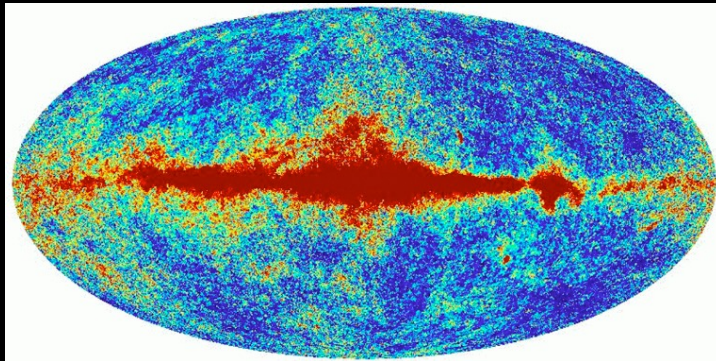
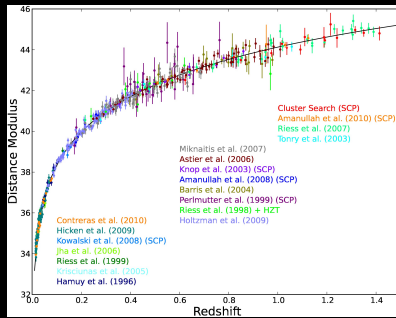


Curves of constant luminosity distance

Properties of Dark Energy



Properties of Dark Energy



Constant EoS

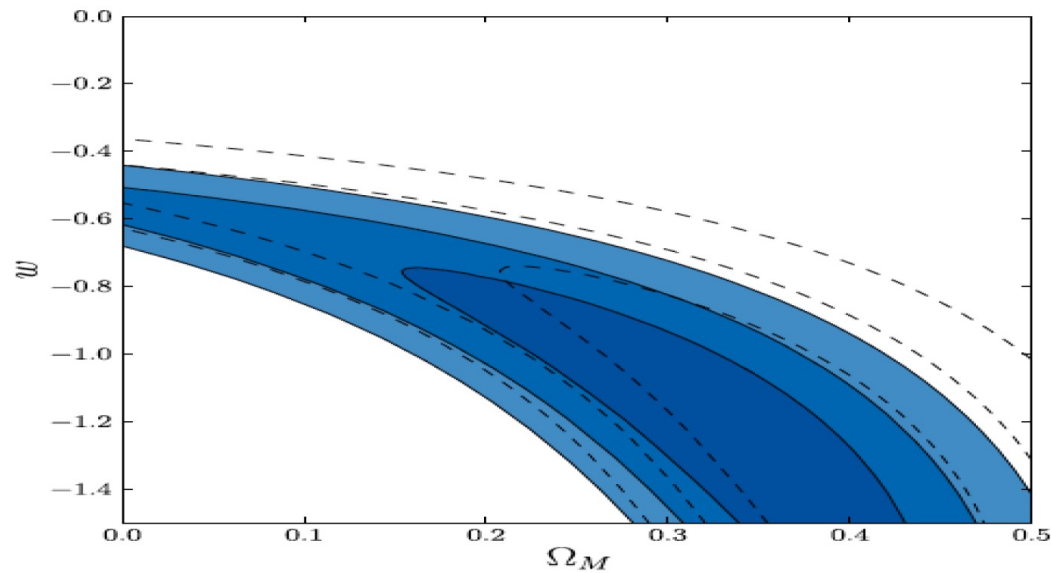


Figure 6.1.3: 68.3 %, 95.4 % and 99.7 % confidence level contours on $(\Omega_m^{(0)}, w_{\text{DE}})$ from the SN Ia observations (denoted as Ω_M and w in the figure) compiled in Amanullah et al. 2010ApJ...716..712A (© AAS. *Reproduced with permission*).