Thermal processes: Big bang nucleosynthesis

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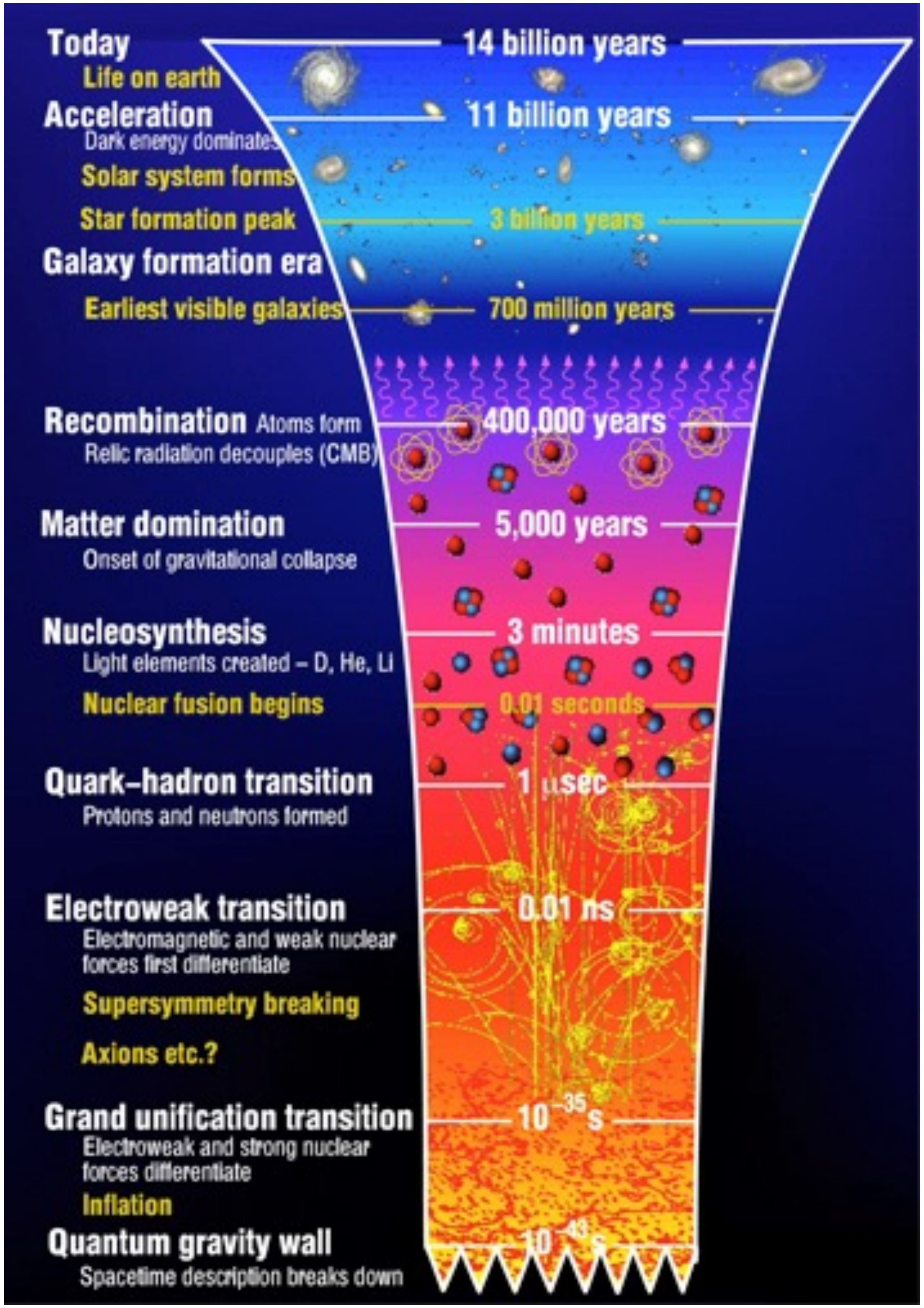
Cosmology WS22/23



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10^{-4} eV

1 eV

1 MeV

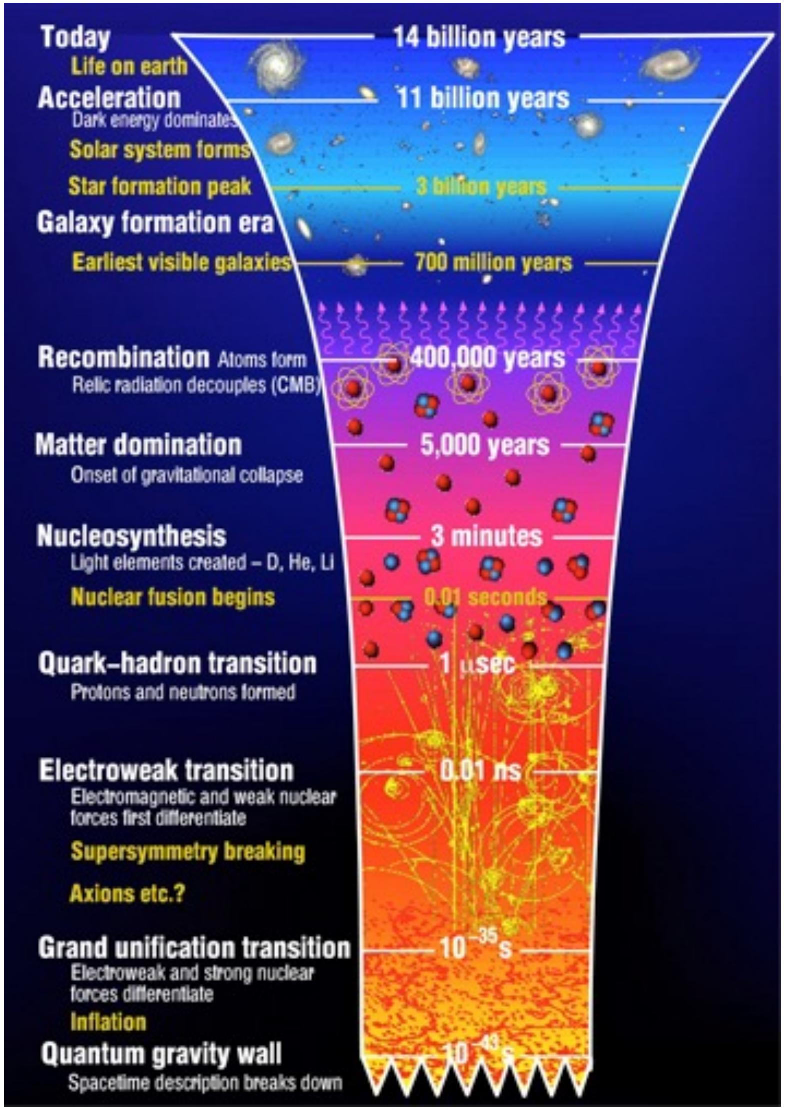
1 GeV

100 GeV

10^{15} GeV

10^{19} GeV

Temperature



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1 Abundance of cosmic neutrinos

2 Big bang nucleosynthesis

3 Matter-radiation decoupling

Cosmological time vs temperature

$$T(z) = \frac{T_0}{a} = \frac{2.7K}{a} = 2.7K(1+z)$$

$$(2.7K = 2.3 \bullet 10^{-4}\, eV)$$

$$t(z) = \frac{1}{H_0}\int_z^{\infty} \frac{dz}{(1+z)E(z)} \quad \rightarrow \quad t(T)$$

so e.g. T=1 MeV (nucleosynthesis) occurs at

$$z(T = 1MeV) \approx 10^{10}$$

$$t(T = 1MeV) \approx 10 \quad \text{seconds}$$

Distribution functions

number density of particles with momentum k in d^3k

$$f(\mathbf{k},t)d^3k = \frac{g_A}{(2\pi)^3}[e^{\frac{E-\mu_A}{T(t)}} \pm 1]^{-1} d^3k$$

+1 fermions
-1 bosons

$$E^2 = k^2 + m^2$$

$$n = \int f(k)d^3k = \frac{g}{2\pi^2}\int \frac{(E^2-m^2)^{1/2}EdE}{e^{\frac{E-\mu_A}{T}} \pm 1}$$

number density

$$\rho = \int Ef(k)d^3k = \frac{g}{2\pi^2}\int \frac{(E^2-m^2)^{1/2}E^2dE}{e^{\frac{E-\mu_A}{T}} \pm 1}$$

energy density

$$p = \int \frac{k^2}{3E}f(k)d^3k = \frac{g}{6\pi^2}\int \frac{(E^2-m^2)^{3/2}dE}{e^{\frac{E-\mu_A}{T}} \pm 1}$$

pressure

Distribution functions

$$n = \int f(k) d^3k = \frac{g}{2\pi^2} \int \frac{(E^2 - m^2)^{1/2} E dE}{e^{\frac{E-\mu_A}{T}} \pm 1}$$

number density

$$\rho = \int E f(k) d^3k = \frac{g}{2\pi^2} \int \frac{(E^2 - m^2)^{1/2} E^2 dE}{e^{\frac{E-\mu_A}{T}} \pm 1}$$

energy density

$$p = \int \frac{k^2}{3E} f(k) d^3k = \frac{g}{6\pi^2} \int \frac{(E^2 - m^2)^{3/2} dE}{e^{\frac{E-\mu_A}{T}} \pm 1}$$

pressure

$E^2 = k^2 + m^2$ $\longrightarrow$ Non-relativistic limit $\begin{cases} m \gg T \\ E \approx m + \frac{1}{2}\frac{k^2}{m^2} \end{cases}$

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-\frac{m-\mu}{T}}$$

$$\rho = nm$$

$$p = nT \ll \rho$$

Neutron-proton freeze out

$$m_{protons} = 938.3MeV$$

$$m_{neutrons} = 939.5MeV$$

Freeze-out temperature for neutrons and protons: $T_D \approx 0.7\ \mathrm{MeV}$

$$n + \nu_e \leftrightarrow p + e^-$$

freezing-out ratio $$\frac{n_n}{n_p} = e^{-\frac{\Delta m}{T_D}} \approx \frac{1}{6}$$

$$t_D = \frac{1}{H_0\sqrt{\Omega_\gamma}} \int_{z_D}^{\infty} \frac{dz}{(1+z)^3} = \frac{1}{2H_0\sqrt{\Omega_\gamma}(1+z_D)^2} \approx 10s$$

neutrons life-time 900 sec so

$$\frac{n}{p} \approx \frac{1}{7}$$

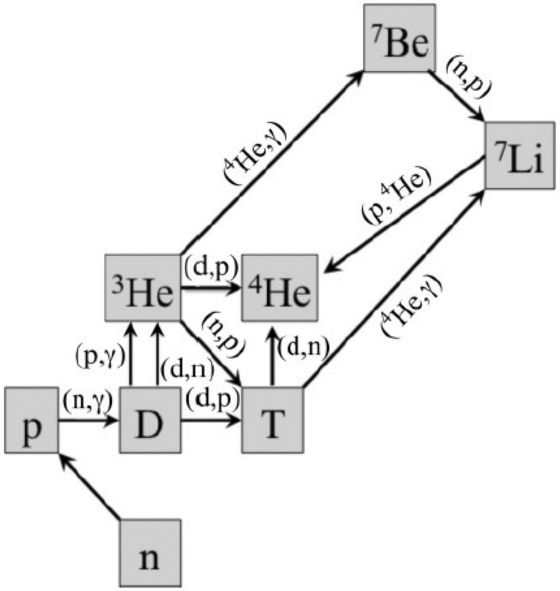
Abundance of Helium-4

so there are seven protons for each neutrons. Suppose all neutrons end up in He-4 (the most stable Helium isotope). Then out of 14p and 2n, one produces one He-4 nuclei and 12 H nuclei (the left-over protons). Then we end up with 4 nucleons in He-4 out of the initial 16, i.e. a quarter of the mass in He4. We define the mass ratio Y=He4/(He4+H)

$$Y = \frac{4(n_n/2)}{n_n + n_p} \approx 0.25$$

Network of interactions

$$\begin{aligned}
p + n &\rightarrow D + \gamma \\
D + p &\rightarrow {}^3He + \gamma \\
D + D &\rightarrow {}^3He + n \\
D + D &\rightarrow T + p \\
{}^3He + n &\rightarrow T + p \\
{}^3He + D &\rightarrow {}^4He + p \\
T + D &\rightarrow {}^4He + n
\end{aligned}$$



re 4.3.1: Big bang nucleosynthesis main reactions (from WikiCommons, author Pamputt).

(D=deuterium; T=tritium)

primordial nuclei: hydrogen, deuterium, tritium, helium3, helium4, lithium, beryllium

Primordial abundances of light elements

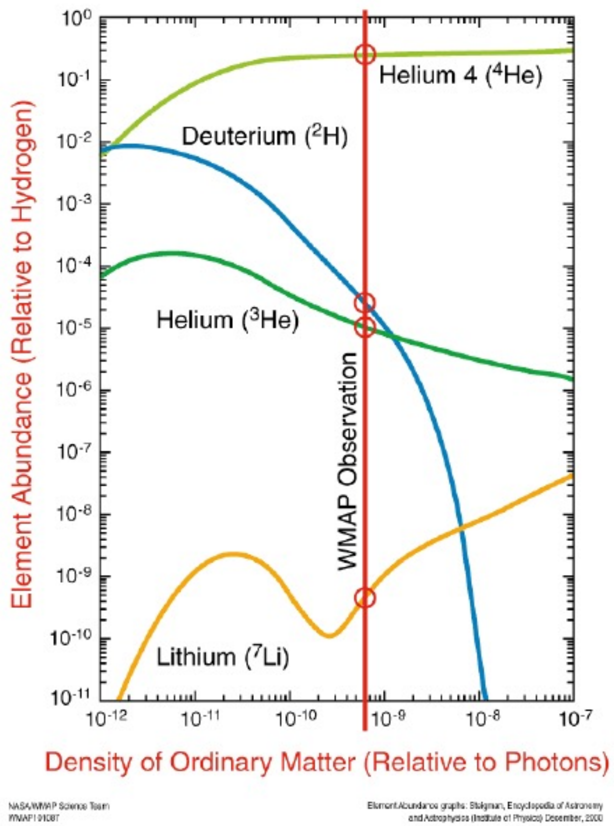


Figure 4.3.2: Big bang nucleosynthesis yields (NASA/WMAP Science team)

key parameter:

baryon/photon ratio

$$\eta \approx 10^{-8}\Omega_b h^2$$

independent of time!

(baryons: proton+neutrons)

Primordial abundances of light elements

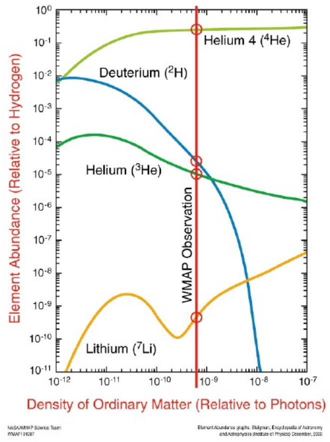


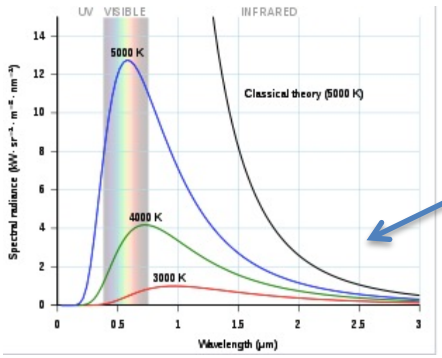
Figure 4.3.2: Big bang nucleosynthesis yields (NASA/WMAP Science team)

key parameter:

baryon/photon ratio

$$\eta \approx 10^{-8}\Omega_b h^2$$

since there are so many photons per particle, one has to reach temperatures much lower than the binding energy to prevent the high-energy tail of the photon distribution to ionize the nuclei



tail of
high-energy photons

More in detail

$$n_B = \frac{\rho_B}{m_B c^2} = \Omega_b \frac{\rho_c}{m_B c^2} = (\Omega_b h^2) 10^{-47} GeV^3$$

baryons today

$$\eta = \frac{n_B}{n_\gamma} = 2.68 \cdot 10^{-8} (\Omega_b h^2)$$

constant baryon/photon ratio

$$\begin{aligned} n &= g(\frac{mT}{2\pi})^{3/2} e^{-\frac{m-\mu}{T}} \\ \rho &= nm \\ p &= nT \ll \rho \end{aligned}$$

non-relativistic densities/pressure

so for neutrons and protons:

$$\begin{aligned} n_n &= 2(\frac{m_B T}{2\pi})^{3/2} e^{-\frac{m_n - \mu_n}{T}} \\ n_p &= 2(\frac{m_B T}{2\pi})^{3/2} e^{-\frac{m_p - \mu_p}{T}} \end{aligned}$$

More in detail

consider this reaction to form a nucleus N with A nucleons and Z protons:

$$Z \text{ protons} + (A-Z) \text{ neutrons} \rightarrow {}^{A}N_{Z} \text{ nucleus}$$

$$n_n = 2\left(\frac{m_B T}{2\pi}\right)^{3/2} e^{-\frac{m_n - \mu_n}{T}}$$

$$n_p = 2\left(\frac{m_B T}{2\pi}\right)^{3/2} e^{-\frac{m_p - \mu_p}{T}}$$

densities for neutrons, protons, generic nuclei N

$$n_A = g_A\left(\frac{m_A T}{2\pi}\right)^{3/2} e^{-\frac{m_A - \mu_A}{T}}$$

Conservation of energy

In every reaction, the total chemical potential is conserved

$$Z \text{ protons} + (A - Z) \text{ neutrons} \rightarrow {}^{A}N_{Z} \text{ nuclei}$$

$$\mu_A = Z\mu_p + (A - Z)\mu_n$$

$$\begin{aligned} n_n &= 2\left(\frac{m_B T}{2\pi}\right)^{3/2} e^{-\frac{m_n - \mu_n}{T}} \\ n_p &= 2\left(\frac{m_B T}{2\pi}\right)^{3/2} e^{-\frac{m_p - \mu_p}{T}} \end{aligned}$$

$$n_A = g_A\left(\frac{m_A T}{2\pi}\right)^{3/2} e^{-\frac{m_A - \mu_A}{T}}$$

$$\Rightarrow$$

$$\begin{aligned} n_A &= g_A 2^{-A} A^{3/2} \left(\frac{m_B T}{2\pi}\right)^{3(1-A)/2} n_p^Z n_n^{A-Z} e^{\frac{B_A}{T}} \\ B_A &\equiv Z m_p + (A - Z) m_n - m_A \end{aligned}$$

($B_A > 0$ binding energy)

Abundances of nuclei

$$
\begin{aligned}
n_A &= g_A 2^{-A} A^{3/2} \left(\frac{m_B T}{2\pi}\right)^{3(1-A)/2} n_p^Z n_n^{A-Z} e^{\frac{B_A}{T}} \\
B_A &\equiv Z m_p + (A-Z) m_n - m_A
\end{aligned}
$$

$$X_A = \frac{A n_A}{n_B}$$

defines the mass fraction in nucleus N
(n_B = total density of baryons)

then finally we obtain a relation between abundance of N and temperature

$$X_A = F(A)\left(\frac{T}{m_B}\right)^{3(A-1)/2} \eta^{A-1} X_p^Z X_n^{A-Z} e^{B_A/T}$$

$$\eta = \frac{n_B}{n_\gamma} = 2.68 \cdot 10^{-8} (\Omega_b h^2)$$

$$F(A) = g_A A^{5/2} \zeta(3)^{A-1} \pi^{(1-A)/2} 2^{(3A-5)/2}$$

numerical order
unity factor

when T decreases, X_A increases: nuclei form

Conservation of energy

then finally we obtain a relation between abundance of N and temperature

$$X_A = F(A)(\frac{T}{m_B})^{3(A-1)/2}\eta^{A-1}X_p^Z X_n^{A-Z}e^{B_A/T}$$

X_A = 1 means production of that nucleus stops

Since η is very small, X reaches unity at temperatures quite lower than the binding energy, so much less than 1MeV

useful numerical approximation:

$$T_A \approx \frac{B_A/(A-1)}{\log \eta^{-1} + 1.5\log(m_B/T)}$$

$$T_A \approx 0.07, 0.11, 0.28\text{MeV for } {}^2H, {}^3He, {}^4He,$$

Primordial abundances of light elements

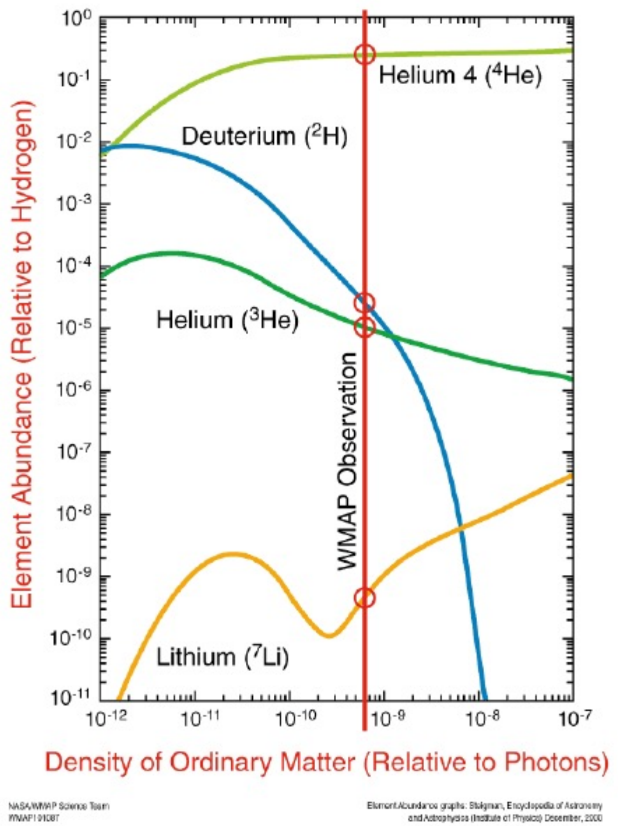


Figure 4.3.2: Big bang nucleosynthesis yields (NASA/WMAP Science team)

key parameter:

baryon/photon ratio

$$\eta \approx 10^{-8}\Omega_b h^2$$

(baryons: proton+neutrons)

Abundance of baryons

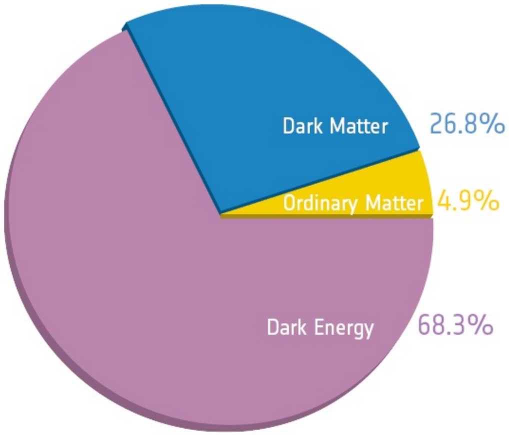
the baryon/photon ratio is fixed
by comparing the predicted abundances of light
nuclei with observations

$$\eta = \frac{n_B}{n_\gamma} = 2.68 \cdot 10^{-8} (\Omega_b h^2)$$

since we know the abundance of photons from CMB, we get

$$\Omega_b h^2 = 0.022 \pm 0.002$$

$$\Omega_b \approx 0.05$$



ESA

so we need non-baryonic dark matter to reach
a matter density fraction 0.3!

Thermal processes: matter-radiation decoupling

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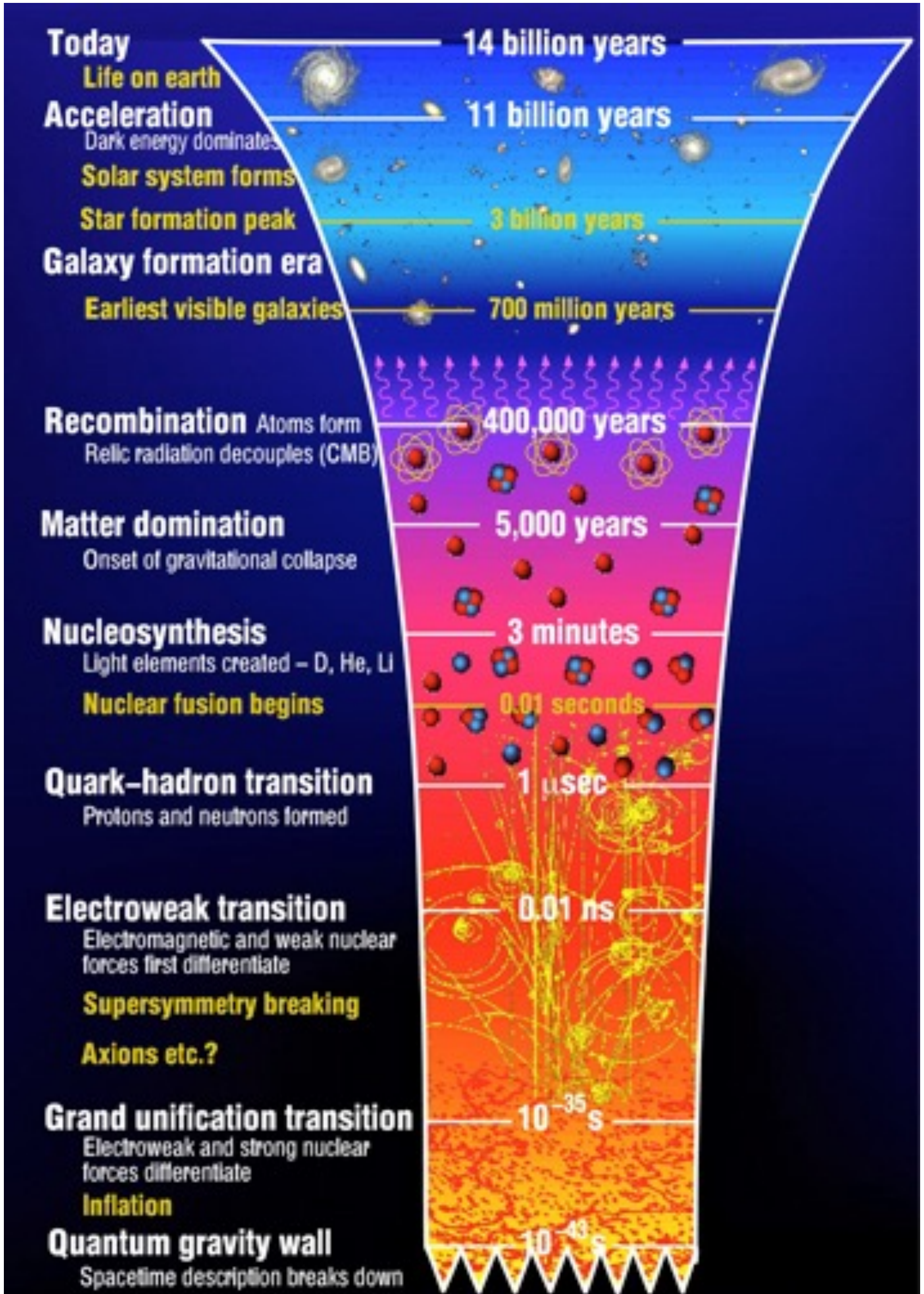

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10^{-4} eV

1 eV

1 MeV

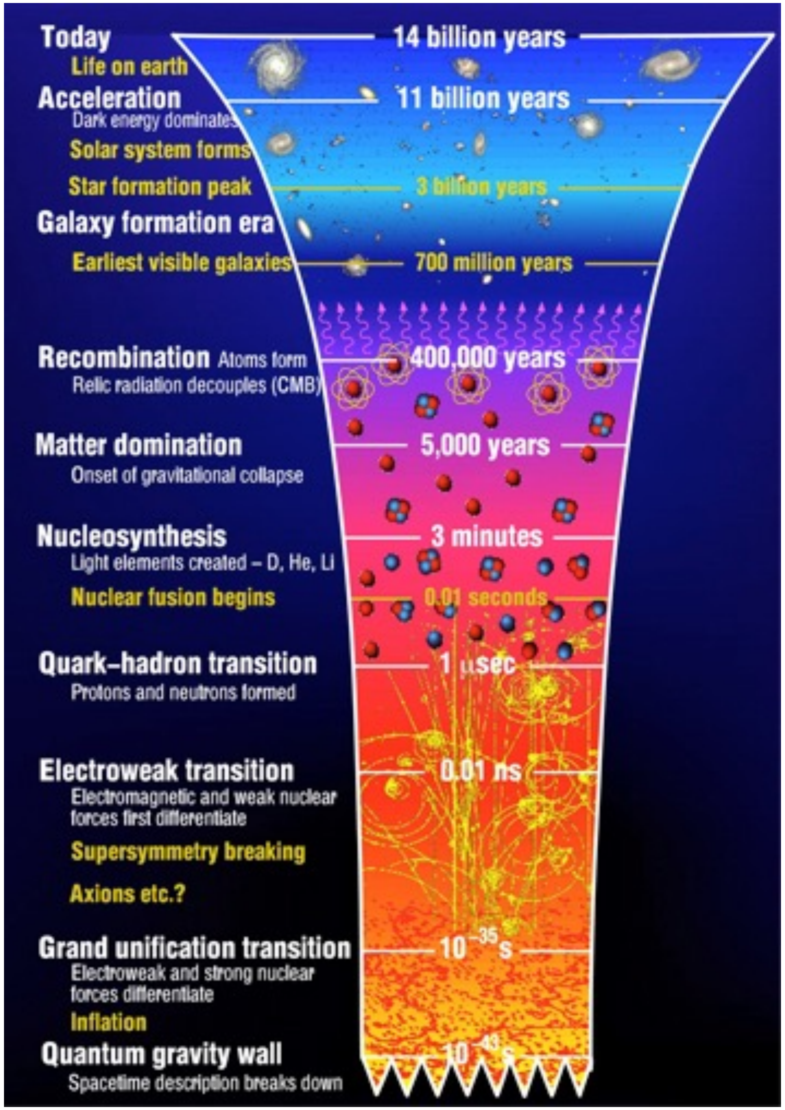
1 GeV

100 GeV

10^{15} GeV

10^{19} GeV

Temperature



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Formation of atoms

ionization energy of hydrogen atom: 13.6 eV

$$p + e \to n_H + \gamma$$ reaction

non-relativistic regime: m>>T

$$n_i = 2(\frac{m_i T}{2\pi})^{3/2} e^{-\frac{m_i - \mu_i}{T}}$$ density of non-relativistic particles

$$\mu_p + \mu_e = \mu_H$$ conservation of energy

Formation of atoms

same calculation as for BBN

$$n_H = \frac{g_H}{g_p g_e} n_p n_e \left(\frac{m_e T}{2\pi}\right)^{-3/2} e^{B/T}$$

density of hydrogen atoms

$$g_{p,e} = 2 \text{ and } g_H = 4,$$

$$B = m_p + m_e - m_H = 13.6 \text{ eV}.$$

ionization energy

Formation of atoms

$$n_H = \frac{g_H}{g_p g_e} n_p n_e \left(\frac{m_e T}{2\pi}\right)^{-3/2} e^{B/T}$$

$$B = m_p + m_e - m_H = 13.6 \text{ eV}.$$

define ionization fraction $x_i = n_i/n_B.$

Since $n_p = n_e$, and $n_p + n_H = n_B$

we have $x_p = x_e$ and $x_H = (n_H/n_B) = 1 - x_e.$

when x_e goes to zero, recombination is complete

In practice, we say recombination is complete when x_e =0.1 << 1

Formation of atoms

Then from here

$$n_H = \frac{g_H}{g_p g_e} n_p n_e \left(\frac{m_e T}{2\pi}\right)^{-3/2} e^{B/T}$$

We obtain this

$$\frac{1-x_e}{x_e^2} = \frac{4\sqrt{2}\zeta(3)}{\sqrt{\pi}} \eta \left(\frac{T}{m_e}\right)^{3/2} e^{B/T} \approx 3.84\eta \left(\frac{T}{m_e}\right)^{3/2} e^{B/T}$$

Saha equation: ionization as function of temperature
when T decreases, x_e decreases

Formation of atoms

Saha equation

very small number!

$$\frac{1-x_e}{x_e^2} = \frac{4\sqrt{2}\zeta(3)}{\sqrt{\pi}}\eta\left(\frac{T}{m_e}\right)^{3/2} e^{B/T} \approx 3.84\eta\left(\frac{T}{m_e}\right)^{3/2} e^{B/T}$$

useful analytical approximation:

$$\tau \equiv T/1eV$$

$$\tau^{-1} = 3.084 - 0.00735\log(\Omega_b h^2)$$

$$T \approx 0.3eV$$ temperature of recombination

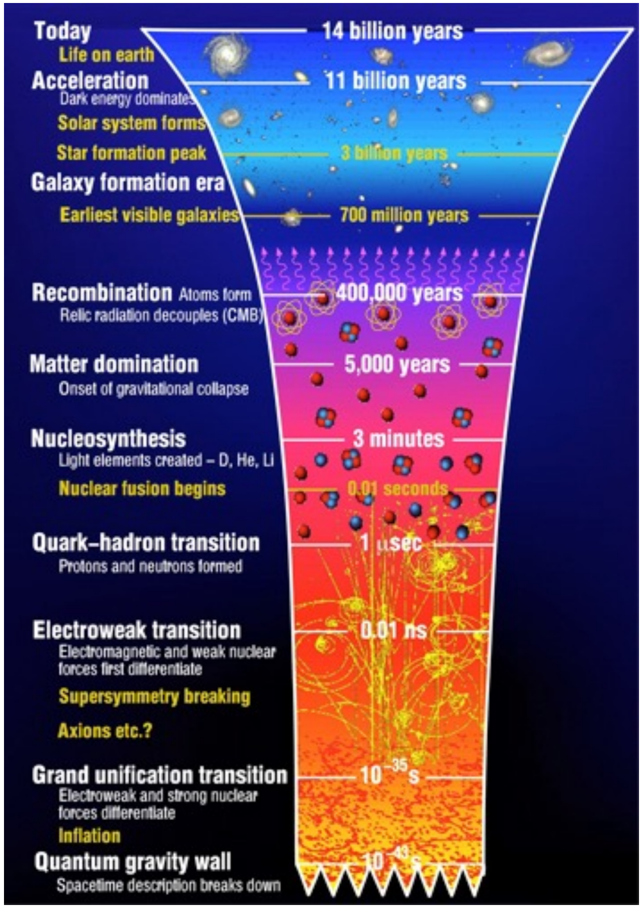
$$z = (T/T_{2.7K}) = \tau(1eV/T_{2.7K}) \approx 4200\tau$$

redshift of recombination

$$z \approx 1367(1 - 0.024\log\Omega_b h^2)^{-1}$$

Decoupling

Decoupling occurs when photons can travel through the Universe without being absorbed by matter, or when:



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mean free path > 1/H = Hubble radius

This occurs just after recombination, because atoms are neutral:

$$z_{dec} \approx 1100$$

The Universe becomes transparent!