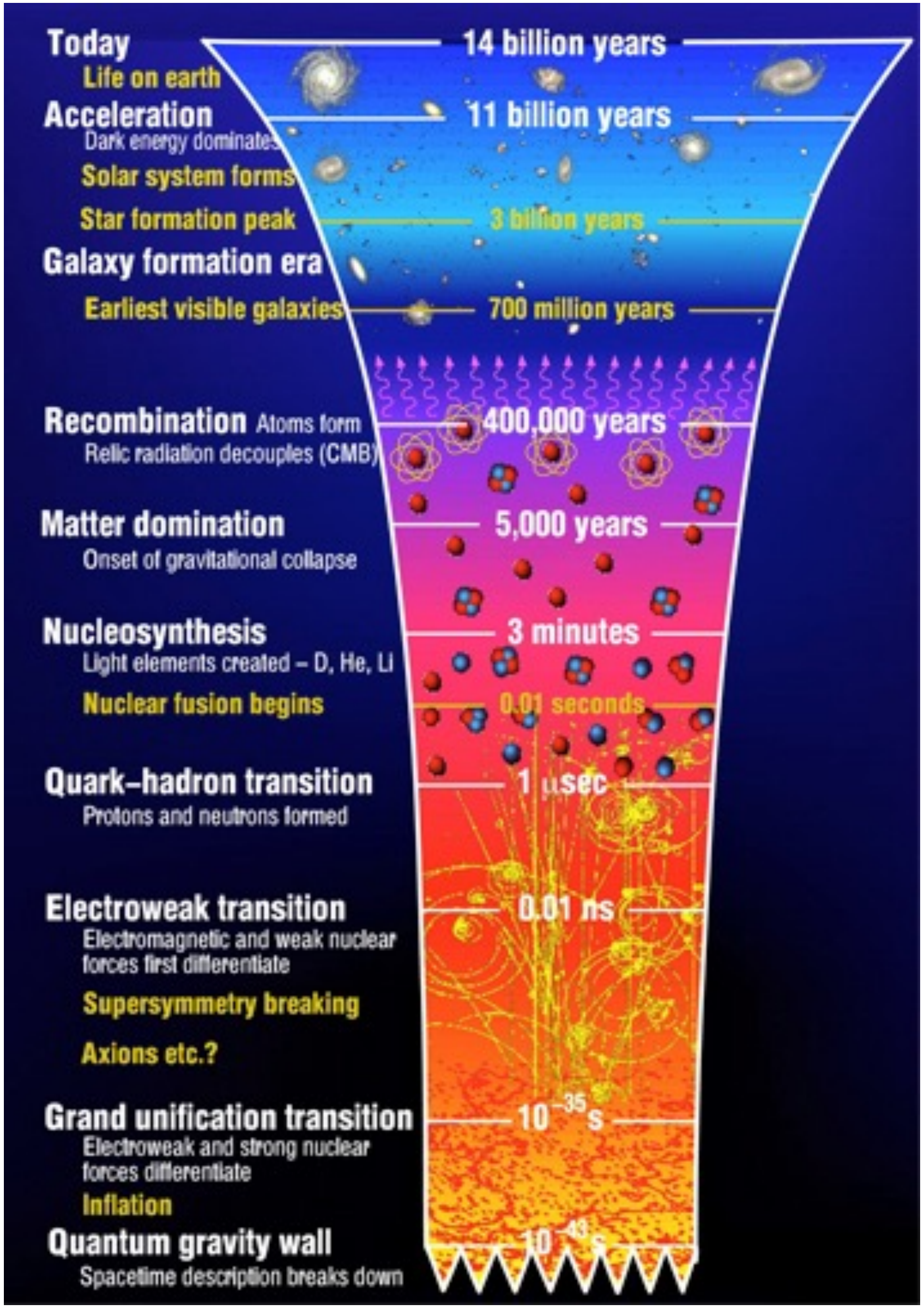
Thermal processes

Cosmology WS22/23



10^{-4} eV

1 eV

1 MeV

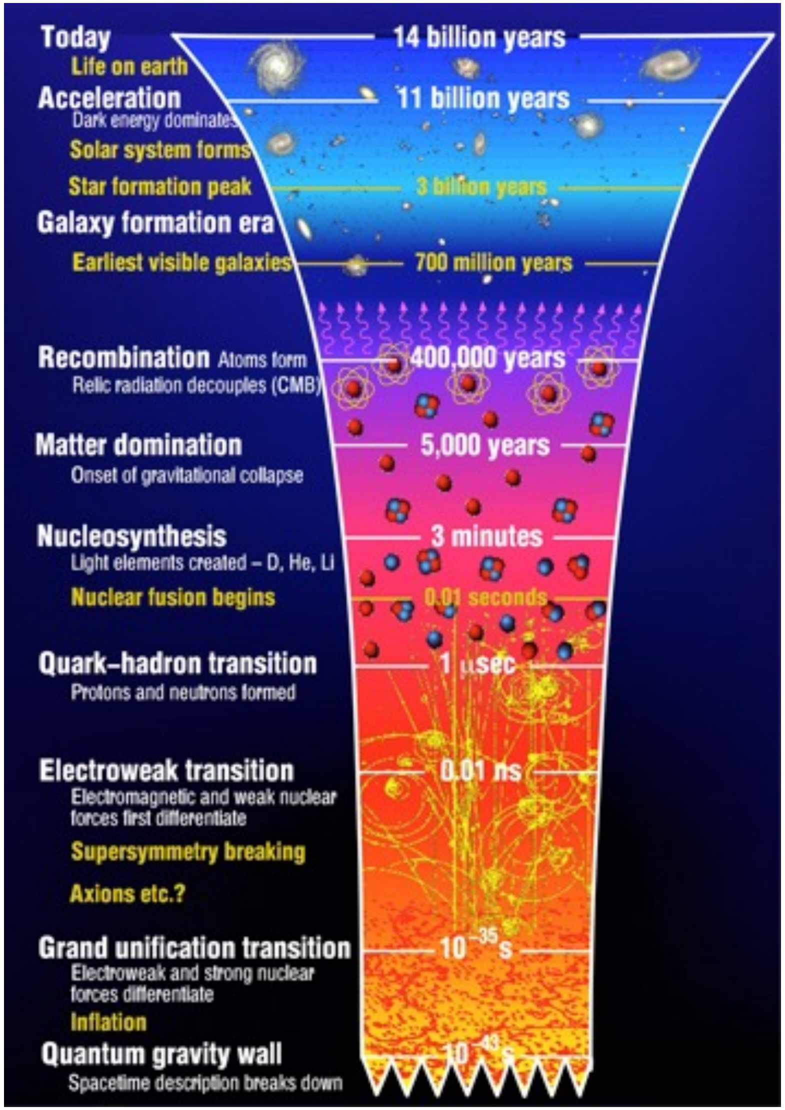
1 GeV

100 GeV

10^{15} GeV

10^{19} GeV

Temperature



1 Abundance of cosmic neutrinos

2 Big bang nucleosynthesis

3 Matter-radiation decoupling

Cosmological time vs temperature

$$T(z) = \frac{T_0}{a} = \frac{2.7K}{a} = 2.7K(1+z)$$

$$(2.7K = 2.3 \bullet 10^{-4}\, eV)$$

$$t(z) = \frac{1}{H_0}\int_z^{\infty} \frac{dz}{(1+z)E(z)} \quad \rightarrow \quad t(T)$$

so e.g. T=1 MeV (nucleosynthesis) occurs at

$$z(T = 1MeV) \approx 10^{10}$$

$$t(T = 1MeV) \approx 10 \quad \text{seconds}$$

Distribution functions

number density of particles with momentum k in d^3k

$$f(\mathbf{k},t)d^3k = \frac{g_A}{(2\pi)^3}[e^{\frac{E-\mu_A}{T(t)}} \pm 1]^{-1} d^3k$$

+1 fermions
-1 bosons

$$E^2 = k^2 + m^2$$

$$d^3k = 4\pi k^2 dk = 4\pi(E^2 - m^2)dk$$

$$n = \int f(k)d^3k = \frac{g}{2\pi^2}\int \frac{(E^2-m^2)^{1/2}EdE}{e^{\frac{E-\mu_A}{T}} \pm 1}$$

number density

$$\rho = \int Ef(k)d^3k = \frac{g}{2\pi^2}\int \frac{(E^2-m^2)^{1/2}E^2dE}{e^{\frac{E-\mu_A}{T}} \pm 1}$$

energy density

$$p = \int \frac{k^2}{3E}f(k)d^3k = \frac{g}{6\pi^2}\int \frac{(E^2-m^2)^{3/2}dE}{e^{\frac{E-\mu_A}{T}} \pm 1}$$

pressure

Pressure in SR

$$k^{\alpha} = \{\gamma m, \gamma m v^{i}\}$$

$$v^{i} = \frac{k^{i}}{k^{0}}; \quad v = \frac{k}{E}$$

$$p = \frac{1}{3} n(mv)v \rightarrow \frac{1}{3} nkv = \frac{k^{2}}{3E} n$$

$$n = f(k) d^{3}k$$

Relativistic components

Only two particles in the standard model are massless (or almost massless):

photons and neutrinos

We can “easily” measure the abundance of photons from their present CMB temperature

How can we predict the abundance of neutrinos?

Since neutrinos are actually massive (but very light), they could even represent dark matter….

Temperature and abundance of cosmic neutrinos

Relativistic case: m<< T

Bosons (photons)

$$n_\gamma = \frac{2\zeta(3)}{\pi^2} T^3$$

$(\varsigma(3) \approx 1.2)$

$$\rho_\gamma = g_\gamma \frac{\pi^2}{30} T^4$$

Fermions (neutrinos)

$$\frac{3}{4} n_\gamma$$

$$\frac{7}{8} \rho_\gamma$$

$$p = \frac{1}{3} \rho$$

Energy conservation

$$TdS = pdV + dU$$

Putting $U = \rho V$ and constant entropy, $dS = 0$, this becomes

$$TdS = (\rho + p)dV + V\,d\rho = 0$$

This is the conservation equation: V=$V_0 a^3$ → $\dot{\rho} + 3H(\rho + p) = 0$

We see that

$$dS = \frac{(\rho + p)}{T}dV + \frac{V}{T}\frac{d\rho}{dT}dT$$

Energy conservation

since entropy is a state function (i.e., a total differential)

$$\frac{\partial S}{\partial V} = \frac{\rho + p}{T}, \quad \frac{\partial S}{\partial T} = \frac{V}{T}\frac{d\rho}{dT}$$

and since

$$\frac{\partial^2 S}{\partial V \partial T} = \frac{\partial^2 S}{\partial T \partial V}$$

we obtain

$$\frac{d(\rho + p)}{TdT} - \frac{\rho + p}{T^2} = \frac{1}{T}\frac{d\rho}{dT}$$

from which

$$\frac{dp}{dT} = \frac{\rho + p}{T}$$

Energy conservation

Now we can insert this expression in

$$TdS = (\rho + p)dV + V\,d\rho = d[(\rho + p)V] - V\,dp = d[(\rho + p)V] - V\,\frac{\rho + p}{T}dT$$

to obtain

$$dS = d\left[\frac{(\rho + p)V}{T}\right]$$

and finally

$$s = \frac{S}{V} = \frac{\rho + p}{T}$$

entropy density

Since $S=sa^3$ is conserved, s decreases as a^{-3}

Energy conservation

Now we apply this to relativistic particles: one has then

$$s = \frac{\rho + p}{T} = \frac{4\rho}{3T} = A\frac{4}{3} g \frac{\pi^2}{30} T^3 \quad (sa^3 \quad \text{constant})$$

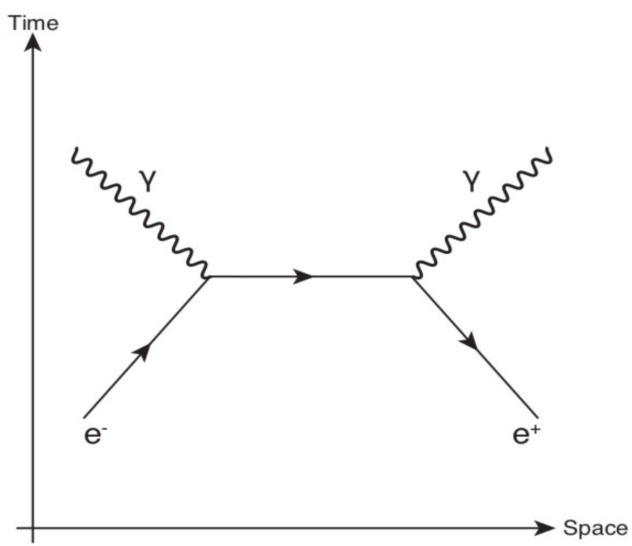
$$A = 1 \ \ bosons \quad (photons); \quad g = 2 \quad (photons)$$

$$A = \frac{7}{8} \quad fermions\,(neutrinos); \quad g = N_\nu \quad (neutrinos); \quad g = 2 \quad (electrons)$$

Temperature of cosmic neutrinos

Photons and neutrinos are both relativistic, but there are two differences:
1) photons are bosons, while neutrinos are fermions
2) photons are heated by electron/positron annihilation, neutrinos are not

Electrons and photons get out of equilibrium at T<0.5 MeV (freeze out); from that moment on, photons are heated by electron annihilations



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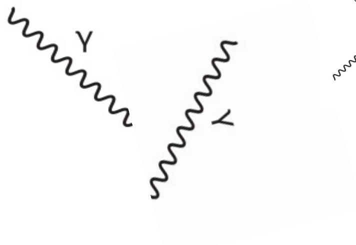
Freeze out

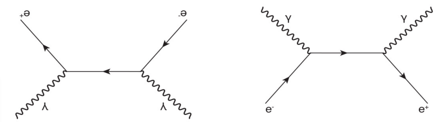
pair annihilation
and pair creation

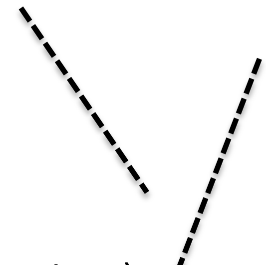
photons

neutrinos

T > 0.5 MeV







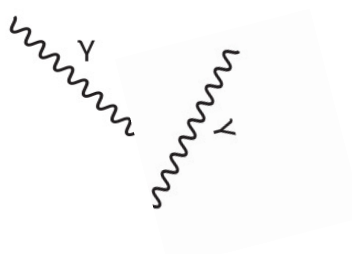
(same temperature for photons and neutrinos)

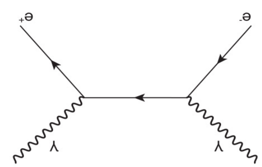
photons

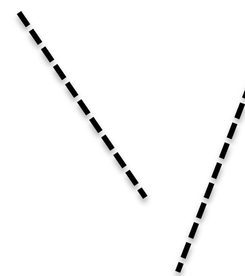
pair annihilation

neutrinos

T < 0.5 MeV







(photons are heated wrt neutrinos)

Temperature of cosmic neutrinos

$$s_\nu = \frac{4\rho}{3T} = \frac{7}{8} g_\nu \frac{4\pi^2}{3\cdot 30} T^3 = \frac{7}{8} g_\nu \frac{2\pi^2}{45} T^3$$

$$s_\gamma = g_\gamma \frac{2\pi^2}{45} T^3$$

before e^- e^+ freeze-out: electrons, positrons, neutrinos, photons

$$s_b = \frac{2\pi^2}{45} T_b^3 \left(2 + \frac{7}{8}\cdot 4 + \frac{7}{8}\cdot 2N_\nu g_\nu\right)$$

(same temperature!)

after e^- e^+ freeze-out: photons and neutrinos

$$s_a = \frac{2\pi^2}{45}\left(2T_\gamma^3 + \frac{7}{8}\cdot 2 g_\nu N_\nu T_\nu^3\right)$$

Since sa^3 is constant, we have

$$(a_b T_b)^3 [2 + \frac{7}{8} \cdot 4 + \frac{7}{8} \cdot 2 g_\nu N_\nu] = (a_a T_\nu)^3 [2 \left(\frac{T_\gamma}{T_\nu}\right)^3 + \frac{7}{8} \cdot 2 g_\nu N_\nu]$$

$$a_b T_b = a_a T_\nu$$

because neutrinos have T=1/a at all times

Therefore finally:

$$\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3} \approx 0.714$$

Since today $T_\gamma \approx 2.7$K, the unobservable T_ν should be around 1.9 K.

Relativistic degrees of freedom

$$\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3} \approx 0.714$$

$$\rho_\gamma(T) = AT^4$$

$$\rho_v(T_v) = \frac{7}{8} N_v g_v \rho_\gamma(T_v) = \frac{7}{8} N_v g_v A \left(\frac{T_v}{T_\gamma}\right)^4 T_\gamma^{\ 4} = \frac{7}{8} N_v g_v \left(\frac{4}{11}\right)^{4/3} \rho_\gamma(T_\gamma)$$

$$\approx 0.68 \rho_\gamma$$

$$\rho_v + \rho_\gamma \approx 1.68 \rho_\gamma$$

$g_\gamma \rightarrow 2 \bullet 1.68 = 3.36$ effective relativistic deg. of freedom

$$\Omega_{\gamma+\upsilon} = \left(\frac{\rho_v + \rho_\gamma}{\rho_{crit}}\right)_0 = 8 \cdot 10^{-5}$$

relativistic density fraction today

Massive neutrinos?

Similarly, we can derive their number density:

$$n_\nu = \frac{3}{2}\frac{4}{11}N_\nu g_\nu n_\gamma = \frac{6}{11}N_\nu g_\nu n_\gamma$$

340 neutrinos per cubic centimeter today!

Neutrinos have actually a very small mass, less than 1 eV.
So they are non-relativistic now (2.7k=$10^{-4}eV$).

Their energy density today is then *mass x number density*:

$$\rho_{\nu,0} = \frac{6}{11}g_\nu n_\gamma \sum m_\nu$$

and finally we have a current fraction

$$\Omega_{\nu,0} = \frac{8\pi G}{3H_0^2}\frac{6}{11}g_\nu n_\gamma \sum m_\nu \approx \frac{\sum m_\nu}{93h^2 eV}$$

i.e. around 0.01, insufficient to explain dark matter !

Relativistic degrees of freedom

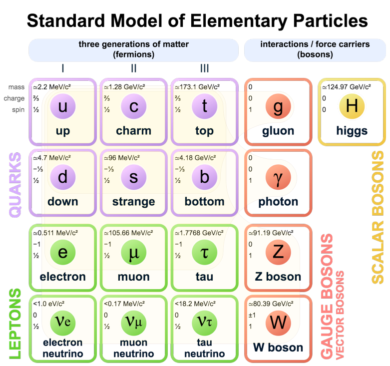
Main message:

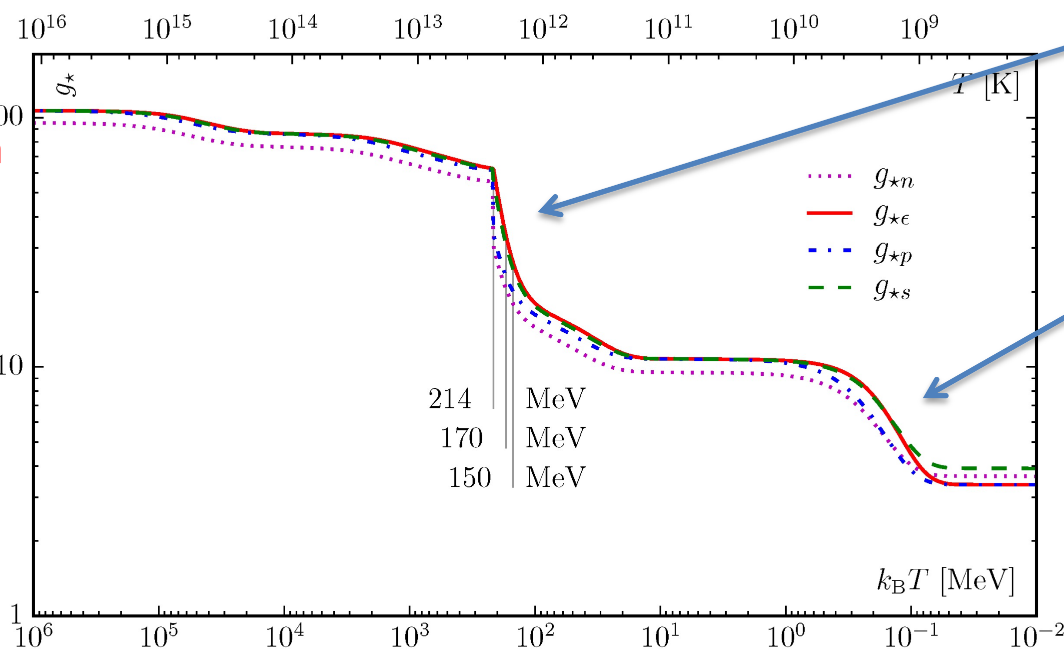
$$T \sim \frac{1}{a}$$

only in thermodynamic equilibrium!

If there are processes of freeze-out, T undergoes rapid jumps:

Relativistic degrees of freedom





Quark-Hadron transition

Electron pair annihilation

3.36

temperature

Husdal, Galaxies 2016, 4, 78.