RG flow of the Higgs potential

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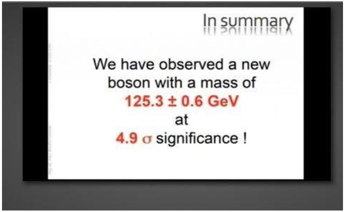
& C. Gneiting, R. Sondenheimer, PRD 89 (2014) 045012, [arXiv:1308.5075],
& S. Rechenberger, M.M. Scherer, L. Zambelli, EPJC 73 (2013) 2652 [arXiv:1306.6508]
& R. Sondenheimer, arXiv:1407.XXXX; L. Zambelli, arxiv:1408.YYYY

21.07.2014

Search for the Higgs boson

▷ 4 Jul. 2012
ATLAS & CMS
@CERN



▷ 14 Mar 2013, CERN press release:

"...the new particle is looking more and more like a Higgs boson..."

$$\text{CMS'12}: \quad 125.3 \pm 0.4(stat) \pm 0.5(sys) GeV,$$

$$\text{ATLAS'12}: \quad 126.0 \pm 0.4(stat) \pm 0.4(sys) GeV$$

▷ Success of Experiment & Theory

(ANDERSON'62; BROUT,ENGLERT'64; HIGGS'64; GURALNIK,HAGEN,KIBBLE'64)


JOGHURT MILD
Ehrmann
Bioghurt
PUR
JOGHURT MILD
Ehrmann
Bioghurt
PUR

Numbers matter

standard model

best before:

$\Lambda = M_{\text{Planck}}$?

Validity range of the standard model

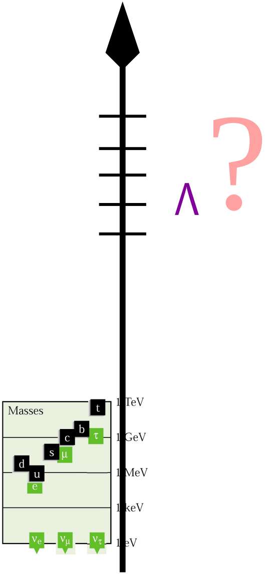
▷ Λ:

- UV cutoff

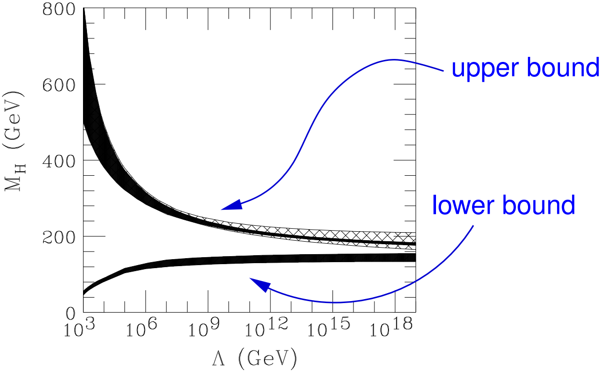
  SM as effective theory

- scale of maximum UV extension

- scale of new physics:
  $\Lambda_{NP} \leq \Lambda$



Higgs boson mass and maximum validity scale

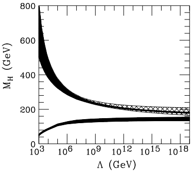


(HAMBYE,RIESSELMANN'97)

SM: e.g. (KRIVE,LINDE'76; MAIANI,PARISI,PETRONZIO'78; KRASNIKOV'78; POLITZER, WOLFRAM'78; HUNG'79; LINDNER'85; WETTERICH'87; SHER'88; FORT,JONES,STEPHENSON, EINHORN'93; ALTARELLI,ISIDORI'94; SCHREMPP,WIMMER'96; . . .)

BSM: e.g., (CABBIBO, MAIANI,PARISI,PETRONZIO'79; ESPINOSA,QUIROS'91; . . .)

Lower bound of Higgs boson mass



▷ vacuum stability / meta-stability bound

▷ effective potential á la Coleman Weinberg:

$$U_{\text{eff}}(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{2}\lambda(\phi)\phi^4$$

▷ e.g., $\lambda(\phi)$ from “RG-improved” perturbation theory:

$$\partial_t \lambda = \frac{3}{4\pi^2}\left(-h_t^4 + h_t^2\lambda + \frac{1}{16}\left[2g^4 + (g^2 + g'^2)^2\right] - \frac{1}{4}\lambda(3g^2 + g'^2) + \lambda^2\right)$$

Lower bound of Higgs boson mass

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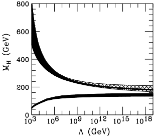

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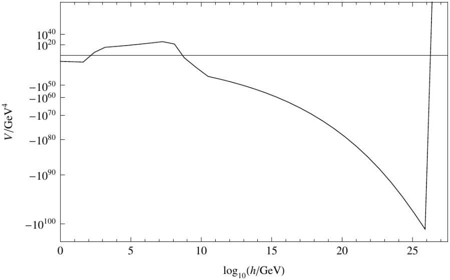
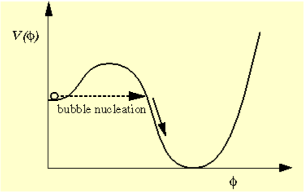


(Gabrielli et al.'13)

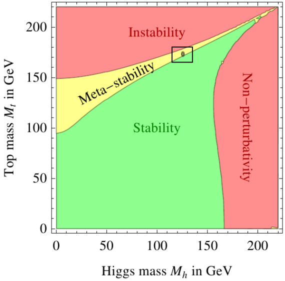
Lower Bound of Higgs boson mass

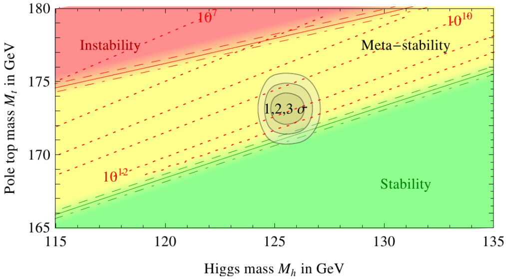
▷ meta-stability:

tunneling time > age of universe



▷ “Near critical” standard model: (BUTTAZZO ET AL.’13)

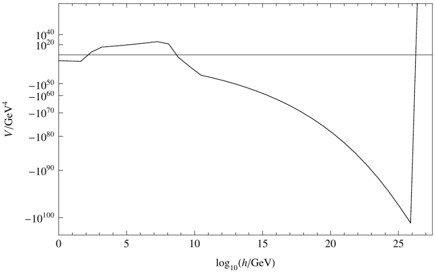




NNLO calculation (DEGRASSI ET AL.’12)

earlier calculations, e.g., (ISIDORI,RIDOLFI,STRUMIA’01)

2nd thoughts on the lower bound

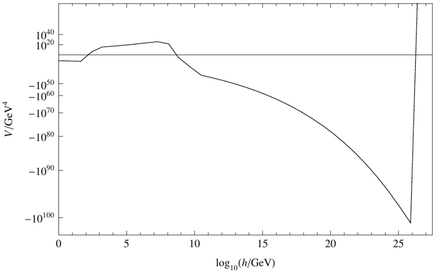


- ▷ True minimum of $U_{\mathrm{eff}}(\phi)$ at

  $\phi \sim 10^{25}\mathrm{GeV} > M_{\mathrm{Pl}}$?

- ▷ UV$\rightarrow$IR RG flow?

2nd thoughts on the lower bound



▷ True minimum of $U_{\text{eff}}(\phi)$ at

$\phi \sim 10^{25}\text{GeV} > M_{\text{Pl}}$?

▷ UV→IR RG flow?

▷ simple top-Higgs Yukawa model:

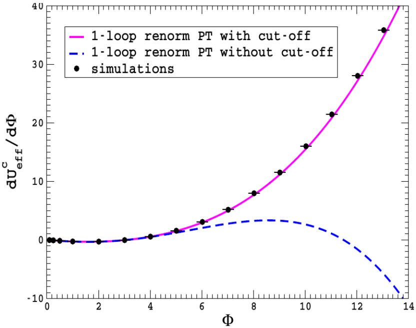
lattice simulation

vs. 1-loop PT with cutoff

vs. 1-loop "Λ-removed" PT

(HOLLAND,KUTI'03; HOLLAND'04)

Criticism: to few scales? (EINHORN,JONES'07)



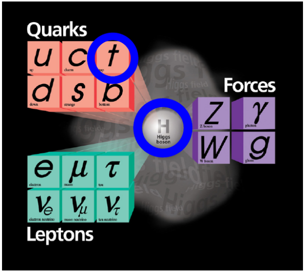
Top-Higgs Yukawa models

Z_2 symmetric model

$$S_{int} = \int ih\phi\bar{\psi}\psi$$

chiral model

$$S_{int} = \int i\bar{h}_t(\bar{\psi}_L\phi_C t_R + \bar{t}_R\phi_C^\dagger\psi_L)$$



- includes relevant top quark + Higgs field (+ largest Yukawa coupling)
- discrete model → no Goldstone bosons (as in SM)
- avoids intricate questions arising from gauge symmetry

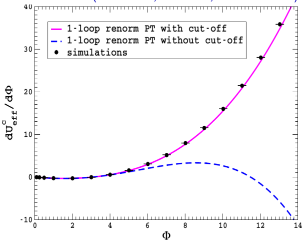
(FROHLICH,MORCHIO,STROCCHI'81; MAAS'12)

Top-Higgs Yukawa models

▷ generating functional:

$$Z[J] = \int_\Lambda \mathcal{D}\phi\mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-S[\phi,\bar{\psi},\psi]+\int J\phi}$$
$$= \int_\Lambda \mathcal{D}\phi\ e^{-S[\phi,\bar{\psi},\psi]-S_{F,\Lambda}[\phi]+\int J\phi}$$

(HOLLAND, KUTI'03; HOLLAND'04)



▷ top-induced effective potential

$$U_F(\phi) = -\frac{1}{2\Omega} \ln \frac{\det_\Lambda(-\partial^2 + h_t^2\phi^\dagger\phi)}{\det_\Lambda(-\partial^2)},$$

▷ CAVE: cutoff Λ / regularization dependent

Top-induced effective potential

▷ exact results for fermion determinants for homogeneous ϕ

▷ e.g., sharp cutoff:

$$U_{\mathrm{F},t}(\phi) = \underbrace{-\frac{\Lambda^2}{8\pi^2} h_t^2 |\phi|^2}_{<0 \text{ (mass-like term)}} + \frac{1}{16\pi^2} \underbrace{\left[h_t^4 |\phi|^4 \ln\left(1 + \frac{\Lambda^2}{h_t^2|\phi|^2}\right) + h_t^2|\phi|^2\Lambda^2 - \Lambda^4 \ln\left(1 + \frac{h_t^2|\phi|^2}{\Lambda^2}\right)\right]}_{>0 \text{ (interaction part)}}$$

▷ mass-like term: contributes to χSB $\Longrightarrow$ $v \simeq 246\text{GeV}$

▷ interaction part: strictly positive

$\Longrightarrow$ cannot induce instability for any finite Λ

(HG,Sondenheimer'14ip)

“Rederiving” the instability

▷ try to send $\Lambda \to \infty$:

$$U_{\mathrm{F},t}(\phi) = -\frac{\Lambda^2}{8\pi^2} h_t^2 |\phi|^2 + \frac{1}{16\pi^2} h_t^4 |\phi|^4 \left[\ln \frac{\Lambda^2}{h_t^2 |\phi|^2} + \text{const.} + \mathcal{O}\left(\frac{h_t^2 |\phi|^2}{\Lambda^2} \right) \right]$$

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▷ renormalization: trade $(\Lambda, m_\Lambda, \lambda_\Lambda)$ for (μ, v, λ_v)

$$U_{F,t}(\phi) \overset{?}{\to} -\frac{1}{16\pi^2} h_t^4 |\phi|^4 \left(\ln \frac{h_t^2 |\phi|^2}{\mu^2} + \text{const.} \right)$$

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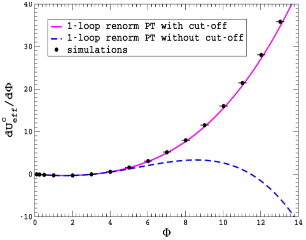

(HOLLAND, KUTI'03; HOLLAND'04)

▷ "instability" occurs beyond $\frac{h_t^2|\phi|^2}{\Lambda^2} > 1$

(HG,SONDENHEIMER'14IP)

▷ implicit renormalization conditions would violate unitarity

(BRANCHINA,FAIVRE'05; GNEITING'05)

Gauge-invariant regulator

▷ ζ function regularization (interpolating reg.: propertime $\leftrightarrow$ dim.reg.)

(HG,SONDENHEIMER'14IP)

$$U_{\mathrm{F},t}(\phi) = \underbrace{-\frac{4\mu^{4-d}\Lambda^{d-2}}{(d-2)(4\pi)^{d/2}} h_t^2 \, |\phi|^2}_{<0 \text{ (mass-like term)}} + \underbrace{\frac{2\mu^{4-d}}{(4\pi)^{d/2}} \int_{1/\Lambda^2}^{\infty} \frac{dT}{T^{1+(d/2)}} \left(e^{-h_t^2 |\phi|^2 T} + h_t^2 |\phi|^2 T - 1 \right)}_{>0 \text{ (interaction part)}}$$

▷ interaction part: strictly positive

$\Longrightarrow$ cannot induce instability for any finite Λ, μ, d

Gauge-invariant regulator

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(HG,Sondenheimer'14ip)

$$U_{\mathrm{F},t}(\phi) = \underbrace{-\frac{4\mu^{4-d}\Lambda^{d-2}}{(d-2)(4\pi)^{d/2}} h_t^2\,|\phi|^2}_{<0 \text{ (mass-like term)}} + \underbrace{\frac{2\mu^{4-d}}{(4\pi)^{d/2}}\int_{1/\Lambda^2}^{\infty}\frac{dT}{T^{1+(d/2)}}\left(e^{-h_t^2|\phi|^2 T} + h_t^2|\phi|^2 T - 1\right)}_{>0 \text{ (interaction part)}}$$

▷ BUT: Limit $\Lambda \to \infty$ and expansion about $d = 4 - \epsilon$

$$U_{\mathrm{F},t}(\phi) \overset{?}{\to} \frac{\#}{\epsilon}\, h_t^4|\phi|^4 - \frac{1}{16\pi^2} h_t^4|\phi|^4 \left(\ln\frac{h_t^2|\phi|^2}{\mu^2} + \text{const.}\right)$$

$\Longrightarrow$ "instability": artifact of dim.reg

use dim.reg. in the presence of large fields: (Brown'76,Luscher'82)

Summary, Part I

- no in-/meta-stability from top (fermion) fluctuations

  …if cutoff Λ is kept finite but arbitrary

- no in-/meta-stability at all ?

$$U_{\mathrm{eff}}(\phi) = U_{\Lambda}(\phi) + U_{\mathrm{B}}(\phi) + U_{\mathrm{F}}(\phi)$$

Summary, Part I

- no in-/meta-stability from top (fermion) fluctuations

  . . . if cutoff Λ is kept finite but arbitrary

- no in-/meta-stability at all ?

$$U_{\text{eff}}(\phi) = \underbrace{U_\Lambda(\phi)}_{\text{arbitrary}} + \underbrace{U_{\text{B}}(\phi) + U_{\text{F}}(\phi)}_{\text{generically stable}}$$

. . . in-/meta-stabilities from the bare action/UV completion

- lower Higgs mass bounds?

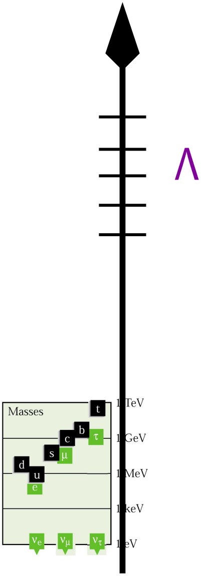
$\Longrightarrow$ nonperturbative methods recommended if not needed

▷ extensive lattice simulations: (FODOR,HOLLAND,KUTI,NOGRADI,SCHROEDER'07)

(GERHOLD,JANSEN'07'09'10)

▷ constraining 4th generations: (GERHOLD,JANSEN,KALLARACKAL'10; BULAVA,JANSEN,NAGY'13)

Higgs boson mass bounds as a UV to IR mapping



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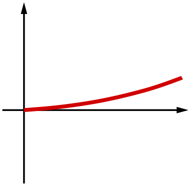

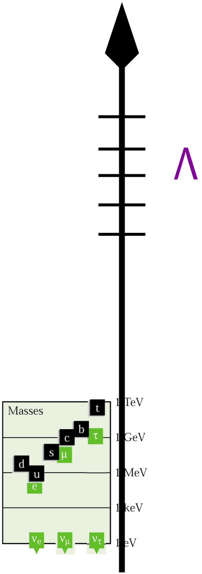

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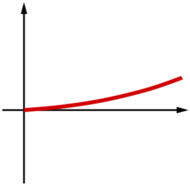

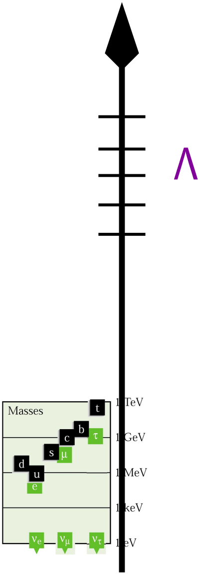

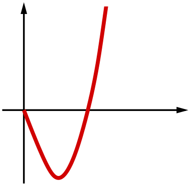

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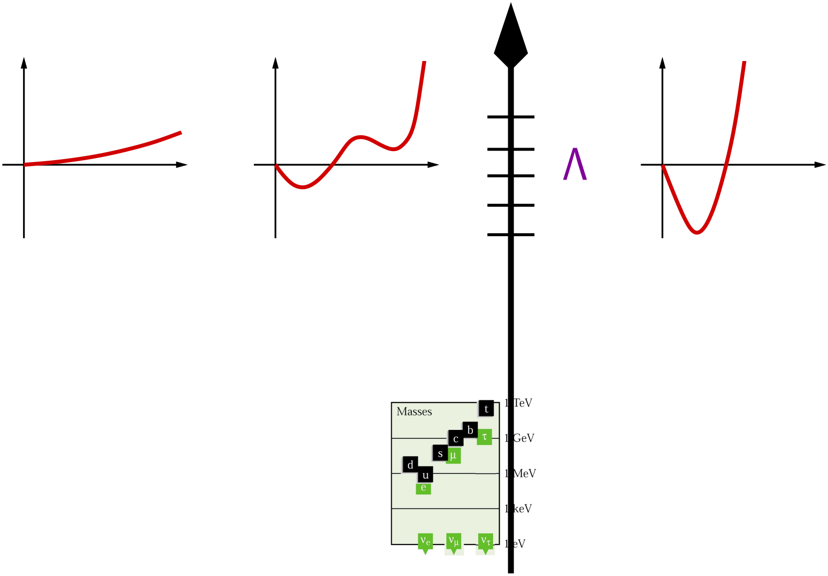

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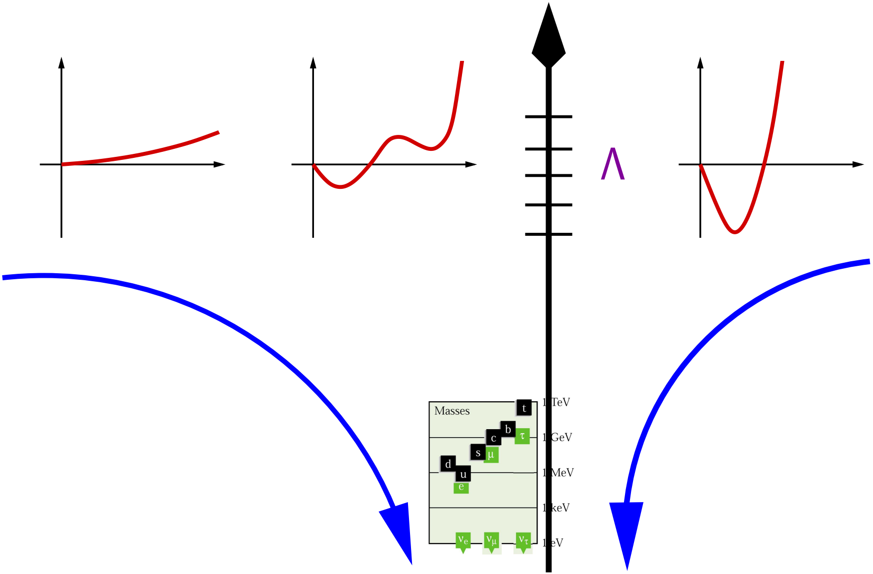

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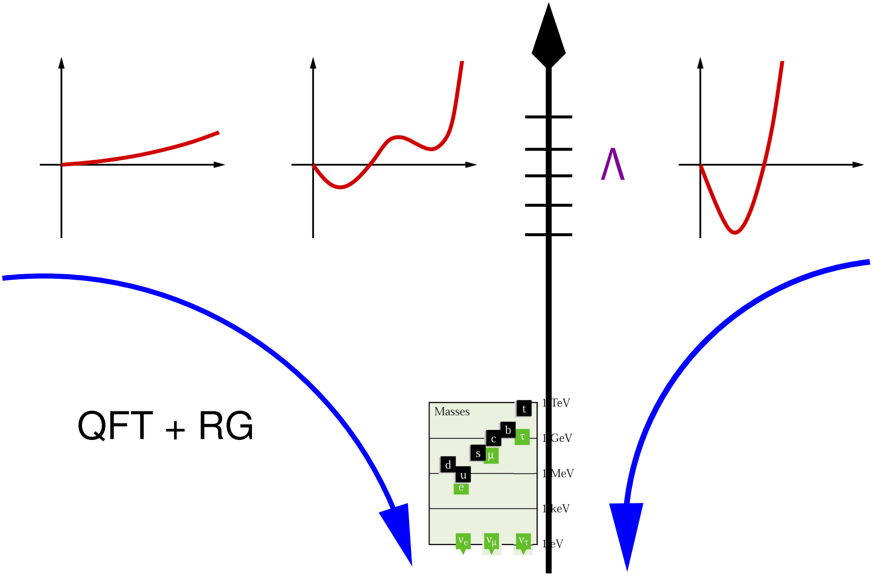

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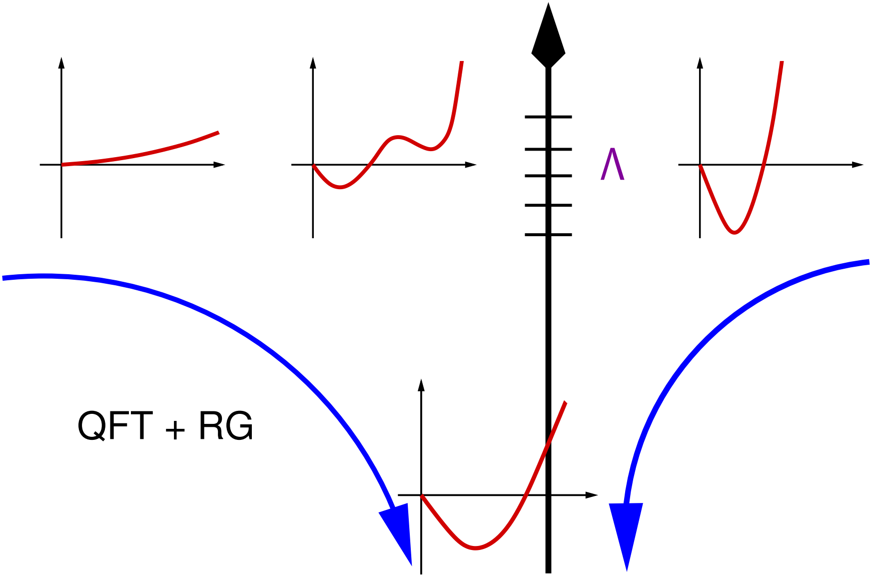


Higgs boson mass bounds as a UV to IR mapping

▷ microscopic action at cutoff Λ:

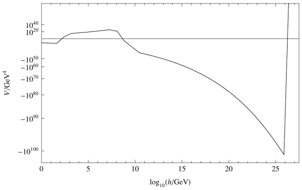
$$S_\Lambda = S_\Lambda(m_\Lambda^2, \lambda_\Lambda, \lambda_{6,\Lambda}, \dots, h_\Lambda, \dots)$$

$\Longrightarrow$ RG: mapping to IR observables

$$\stackrel{\text{RG}}{\Longrightarrow} \quad v \simeq 246\text{GeV}, \; m_{\text{top}} \simeq 173\text{GeV}, \; m_{\text{H}} = m_{\text{H}}[S_\Lambda]$$

▷ By contrast: "RG-improved" PT

$$\text{fix: } v, m_{\text{top}} \; m_{\text{H}} \quad \stackrel{\text{upwards RG}}{\Longrightarrow} \quad U_{\text{eff}}$$

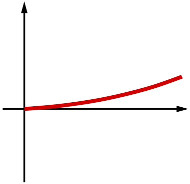


Simple Example: mean-field theory

(HG,GNEITING,SONDENHEIMER'13)

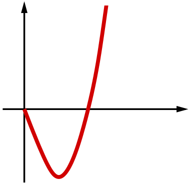
- ▷ MFT ≙ large-N_f limit ≙ fermion det only:

$$U_{\mathrm{MF}}(\phi) = U_{\Lambda}(\phi) + U_{\mathrm{F},t}(\phi)$$



- ▷ UV bare potential, e.g,

$$U_{\Lambda}(\phi) = \frac{1}{2} m_{\Lambda}^2 \phi^2 + \frac{1}{8} \lambda_{\Lambda} \phi^4, \quad \lambda_{\Lambda} \geq 0$$



- ▷ trade: $m_{\Lambda}, h_{\Lambda} \quad \Longleftrightarrow \quad v, m_{\mathrm{top}}$

- ▷ Higgs boson mass:

$$m_{\mathrm{H}}^2(\Lambda, \lambda_{\Lambda}) \quad = \quad \frac{m_{\mathrm{top}}^4}{4\pi^2 v^2} \left[2 \ln \left(1 + \frac{\Lambda^2}{m_{\mathrm{top}}^2} \right) - \frac{3\Lambda^4 + 2 m_{\mathrm{top}}^2 \Lambda^2}{(\Lambda^2 + m_{\mathrm{top}}^2)^2} \right] + v^2 \lambda_{\Lambda}$$

Simple Example: mean-field theory

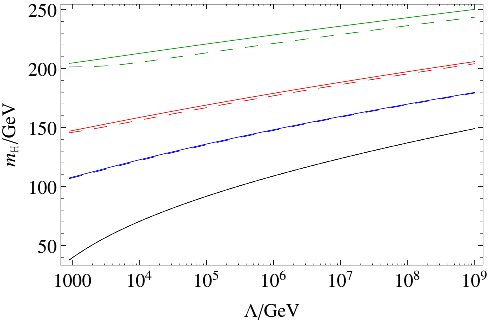
▷ Higgs boson mass bound (HG,GNEITING,SONDENHEIMER'13)

$$m_H^2(\Lambda,\lambda_\Lambda) = \frac{m_{top}^4}{4\pi^2 v^2}\left[2\ln\left(1+\frac{\Lambda^2}{m_{top}^2}\right) - \frac{3\Lambda^4 + 2m_{top}^2\Lambda^2}{(\Lambda^2+m_{top}^2)^2}\right] + v^2\lambda_\Lambda$$

requirement: well defined path integral: $\lambda_\Lambda \geq 0$ for ϕ^4 theory

cf. lattice (HOLLAND'04; FODOR,HOLLAND,KUTI,NOGRADI,SCHROEDER'07; GERHOLD,JANSEN'07'09'10)

▷ extended mean field $\hat{=}$ NLO $1/N_f$ expansion:



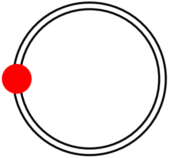
$$\lambda_\Lambda = \begin{cases} 0 \\ 1/6 \\ 1/3 \\ 2/3 \end{cases}$$

Nonperturbative tool: functional RG

▷ RG flow equation:

$$\partial_t \Gamma_k \equiv k\partial_k \Gamma_k \; = \; \frac{1}{2}\,\mathrm{Tr}\; \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1}$$

$$=$$

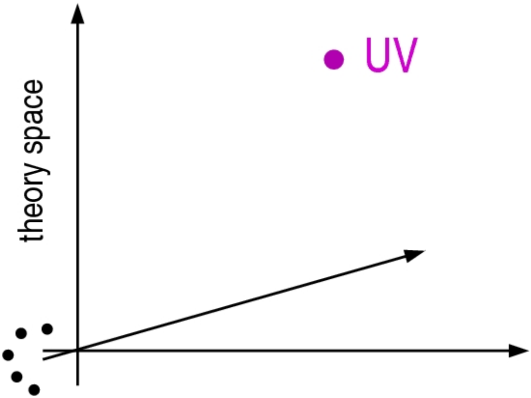


(WETTERICH'93)

RG Flow Equation

$$\partial_t \Gamma_k = \frac{1}{2} \operatorname{Tr} \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1}$$

▷ RG trajectory: $\Gamma_{k=\Lambda} = S_\Lambda = \int \frac{1}{2}(\partial\phi)^2 + U_\Lambda(\phi) + \ldots$



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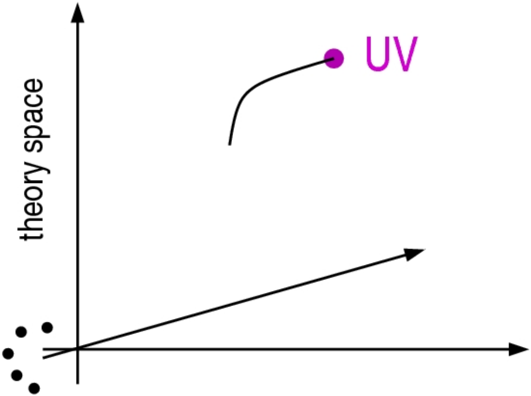

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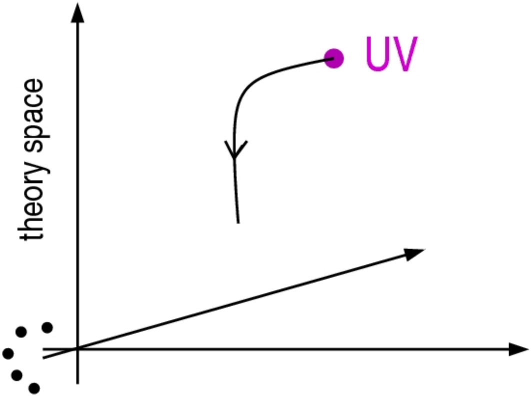
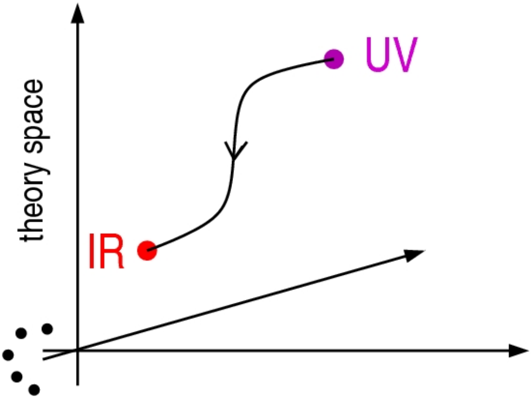


RG Flow Equation

$$\partial_t \Gamma_k = \frac{1}{2} \,\mathrm{Tr}\, \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1}$$

▷ RG trajectory: $\Gamma_{k\to 0} = \Gamma = \int U_{\text{eff}} + \dots$

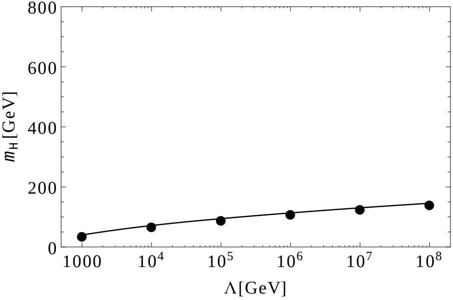


Higgs boson mass bounds from functional RG

▷ Z_2 Yukawa model (HG,Gneiting,Sondenheimer'13)

▷ Systematic derivative expansion:

$$\Gamma_k = \int d^dx \left(\frac{Z_{\phi k}}{2} \partial_\mu \phi \partial^\mu \phi + U_k(\phi^2) + Z_{\psi k} \bar{\psi} i \partial\!\!\!/ \psi + i h_k \phi \bar{\psi} \psi \right)$$



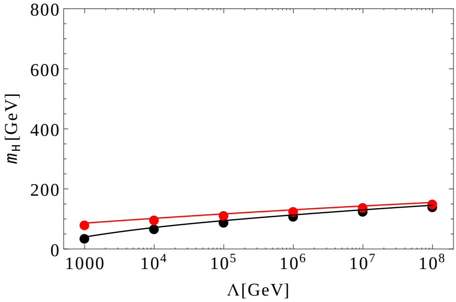
$\lambda_\Lambda = 0$

Higgs boson mass bounds from functional RG

▷ Z_2 Yukawa model (HG,GNEITING,SONDENHEIMER'13)

▷ Systematic derivative expansion:

$$\Gamma_k = \int d^d x \left(\frac{Z_{\phi k}}{2} \partial_\mu \phi \partial^\mu \phi + U_k(\phi^2) + Z_{\psi k} \bar{\psi} i \partial\!\!\!/ \psi + i h_k \phi \bar{\psi} \psi \right)$$



$\lambda_\Lambda = 0$

$\lambda_\Lambda = 0.1$

Higgs boson mass bounds from functional RG

▷ Z_2 Yukawa model (HG, GNEITING, SONDENHEIMER'13)

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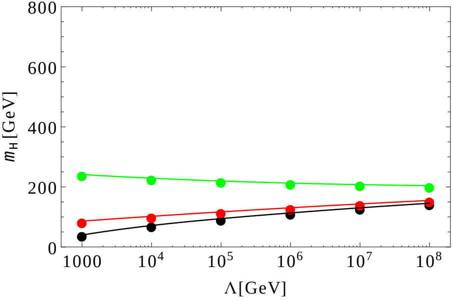


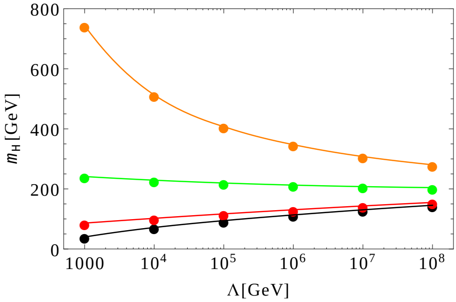
$\lambda_\Lambda = 0$
$\lambda_\Lambda = 0.1$
$\lambda_\Lambda = 1$

Higgs boson mass bounds from functional RG

▷ Z_2 Yukawa model (HG,GNEITING,SONDENHEIMER'13)

▷ Systematic derivative expansion:

$$\Gamma_k = \int d^d x \left(\frac{Z_{\phi k}}{2} \partial_\mu \phi \partial^\mu \phi + U_k(\phi^2) + Z_{\psi k} \bar{\psi} i \partial\!\!\!/ \psi + i h_k \phi \bar{\psi} \psi \right)$$

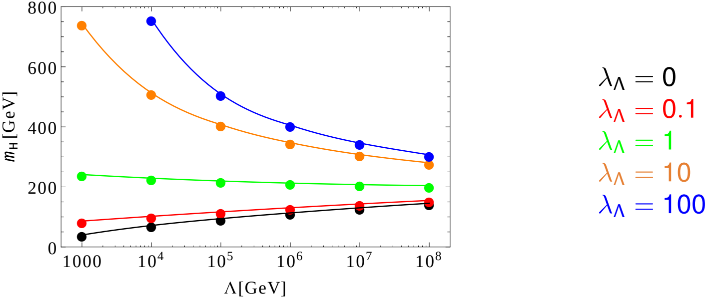


$\lambda_\Lambda = 0$
$\lambda_\Lambda = 0.1$
$\lambda_\Lambda = 1$
$\lambda_\Lambda = 10$

Higgs boson mass bounds from functional RG

- ▷ Z_2 Yukawa model (HG,GNEITING,SONDENHEIMER'13)
- ▷ Systematic derivative expansion:

$$\Gamma_k = \int d^d x \left(\frac{Z_{\phi k}}{2} \partial_\mu \phi \partial^\mu \phi + U_k(\phi^2) + Z_{\psi k} \bar{\psi} i \partial\!\!\!/ \psi + i h_k \phi \bar{\psi} \psi \right)$$

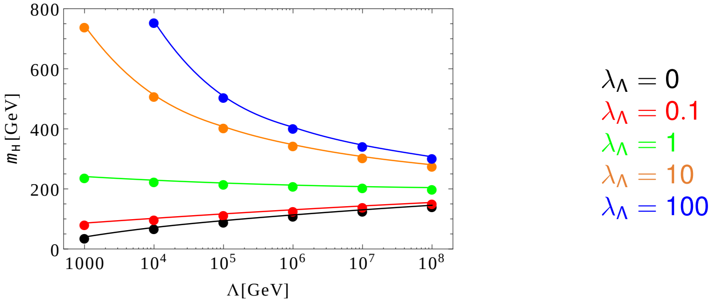


Higgs boson mass bounds from functional RG

▷ Z_2 Yukawa model (HG,GNEITING,SONDENHEIMER'13)

▷ Systematic derivative expansion:

$$\Gamma_k = \int d^d x \left(\frac{Z_{\phi k}}{2} \partial_\mu \phi \partial^\mu \phi + U_k(\phi^2) + Z_{\psi k} \bar{\psi} i \partial\!\!\!/ \psi + i h_k \phi \bar{\psi} \psi \right)$$



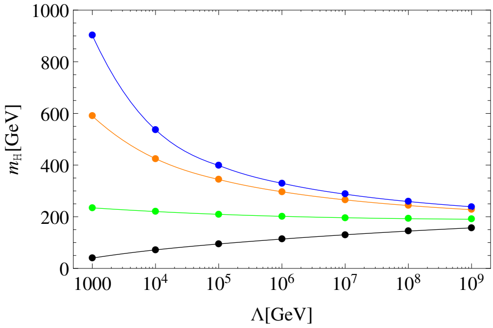
$\Longrightarrow$ lower bound for $\lambda_\Lambda = 0$ ($\simeq$ mean-field result)

agreement with lattice (HOLLAND'04; FODOR,HOLLAND,KUTI,NOGRADI,SCHROEDER'07; GERHOLD,JANSEN'07'09'10)

Conventional lower Higgs boson mass bound

▷ chiral top-bottom-Higgs Yukawa model (+Goldstone decoupling):

(HG,Sondenheimer'14ip)



FRG:
NLO derivative expansion

$\lambda_{2\Lambda} = 0$
$\lambda_{2\Lambda} = 1$
$\lambda_{2\Lambda} = 10$
$\lambda_{2\Lambda} = 100$

$\Longrightarrow$ lower bound close to Z_2 model:

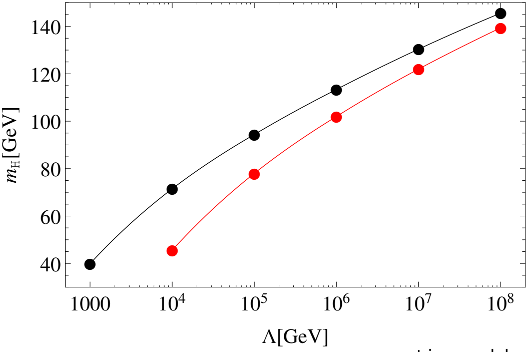
. . . bottom quark has little quantitative influence

General microscopic actions

▷ S_Λ is a priori unconstrained. Consider, e.g.,

$$U_\Lambda = \frac{\lambda_{1\Lambda}}{2}\phi^2 + \frac{\lambda_{2\Lambda}}{8}\phi^4 + \frac{\lambda_{3\Lambda}}{24}\phi^6$$

▷ for $\lambda_{3\Lambda} > 0$ we can choose $\lambda_{2\Lambda} < 0$:



$\lambda_{3\Lambda} = 0,\ \lambda_{2\Lambda} = 0$

$\lambda_{3\Lambda} = 3,\ \lambda_{2\Lambda} = -0.08$

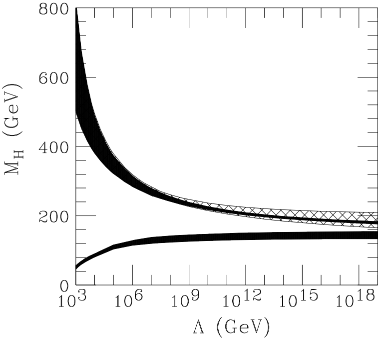
▷ lower bound relaxed

string models with $\lambda < 0$: (Hebecker,Knochel,Weigand'13)

Renormalizable field theories

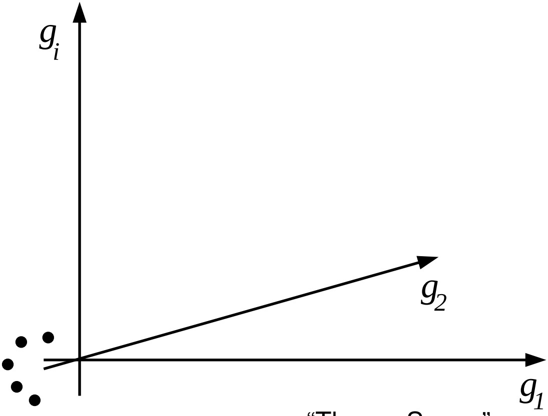
▷ seeming contradiction with common wisdom . . . ?

". . . observables are determined by renormalizable operators . . . "



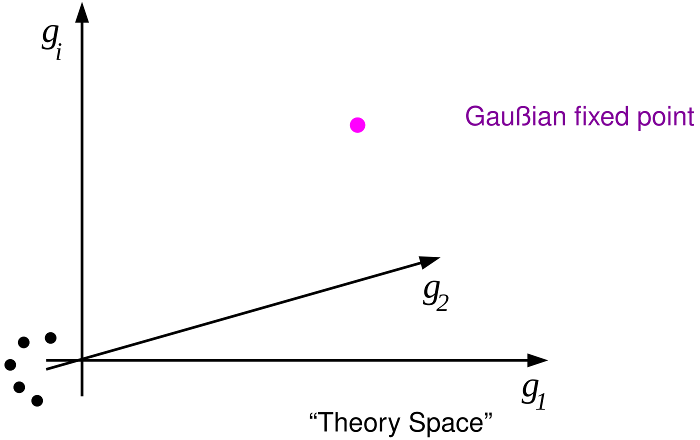
▷ y-axis: m_H observable ✓, x-axis: Λ ?

RG mechanism for “lowering” the lower bound



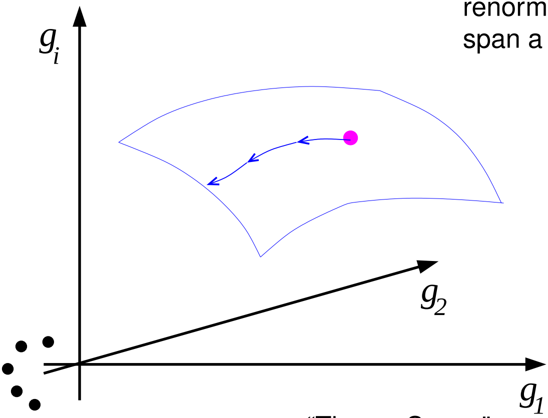
“Theory Space”

RG mechanism for “lowering” the lower bound



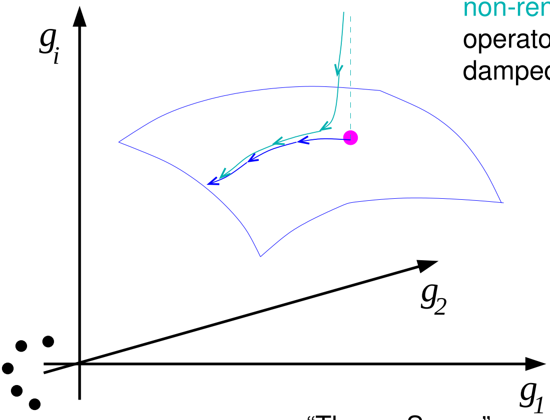
RG mechanism for “lowering” the lower bound

renormalizable operators
span a hyperplane



“Theory Space”

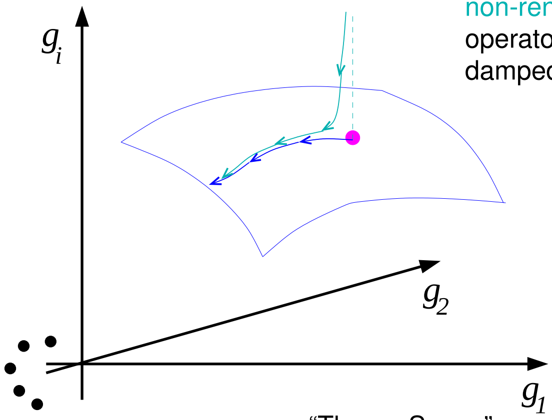
RG mechanism for “lowering” the lower bound



non-renormalizable operators are exponentially damped towards the IR

“Theory Space”

RG mechanism for “lowering” the lower bound



“Theory Space”

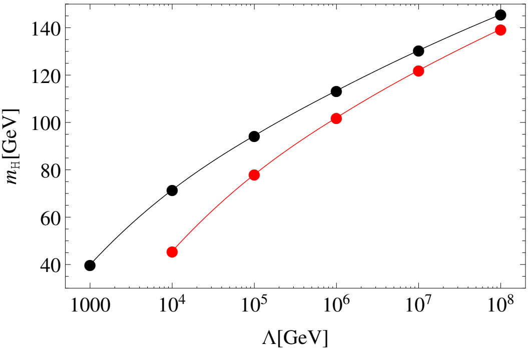
non-renormalizable operators are exponentially damped towards the IR

BUT: RG “time” (scales) is needed for this damping

Lower bounds from generalized bare actions

▷ e.g.,

$$U_\Lambda = \frac{\lambda_{1\Lambda}}{2}\phi^2 + \frac{\lambda_{2\Lambda}}{8}\phi^4 + \frac{\lambda_{3\Lambda}}{24}\phi^6$$



$\lambda_{3\Lambda} = 0,\ \lambda_{2\Lambda} = 0$

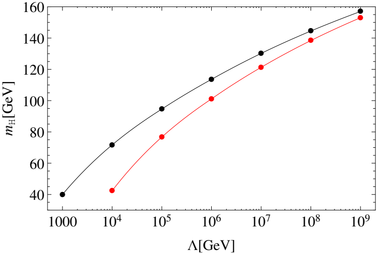
$\lambda_{3\Lambda} = 3,\ \lambda_{2\Lambda} = -0.08$

$\Longrightarrow$ lowered bound $\sim$ shifted Λ axis

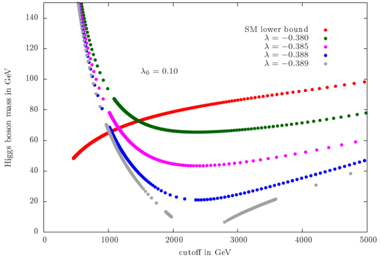
“Lowering” the lower Higgs boson mass bound

▷ chiral model with $\lambda_{6,\Lambda}(\phi^\dagger\phi)^3$ interaction

▷ comparison with lattice data:



(HG,SONDENHEIMER’14IP)



(HEDGE,JANSEN,LIN,NAGY’13)

⟹ RG mechanism confirmed

Summary, Part II

- Bounds on the Higgs boson mass (or any other physical IR observable) arise from a mapping

$$S_{\text{micro}} \quad \to \quad \mathcal{O}_{\text{phys}}$$

. . . provided by the RG

- For "effective quantum field theories" (with a cutoff Λ):

$$\text{bounds on } \mathcal{O}_{\text{phys}} = f[S_\Lambda]$$

. . . full S_Λ not just the "renormalizable" operators

- "lowering" the conventional lower Higgs boson mass bound is possible

. . . without in-/meta-stable vacuum

Implications

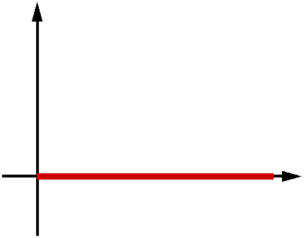
- if $m_\mathrm{H} <$ conventional lower bound:
  - new physics at lower scales
  - first constraints on underlying UV completion

- if m_H exactly on the convenional lower bound:

  . . . "criticality"

  - underlying UV completion has to explain absence of higher dimensional operators

▷ flat interaction potential



Candidates

▷ Asymptotically free YM-Higgs-Yukawa models?

possible, but generically requires O(N) scalars with large N

$\Longrightarrow$ large residual nonabelian subgroup (CALLAWAY'88)

general perturbative UV prediction:

$$\lambda \sim g^2 \to 0$$

...analysis relies on the deep Euclidean region

▷ standard model + asymptotically safe gravity (WEINBERG'76; REUTER'96)

gravity fluctuations induces a UV fixed point $\lambda_* \simeq 0$

(PERCACCI ET AL'03'09)

$\Longrightarrow$ $m_{\rm H}$ put onto conventional lower bound (WETTERICH,SHAPOSHNIKOV'10)

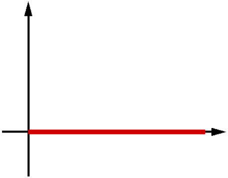
(BEZRUKOV,KALMYKOV,KNIEHL,SHAPOSHNIKOV'12)

Asymptotically free UV gauge scaling solutions?

(Rechenberger,Scherer,HG,Zambelli'13; HG,Zambelli'14ip)

▷ (Almost) flat potentials:

$\Longrightarrow$ large amplitude fluctuations



▷ If flatness is driven by asymptotically free gauge sector:

$$\text{gauge rescaling of fields:} \qquad X \quad = \quad g^{2P} \, \frac{Z_\phi |\phi|^2}{k^2}$$

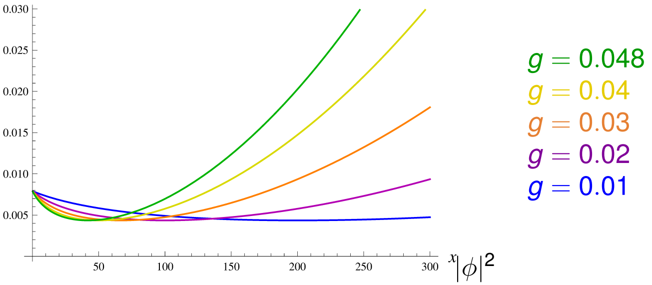
$\Longrightarrow$ solution of fixed-point equation for effective potential ($P = 1$):

$$\text{SU(2) Yang-Mills-Higgs :} \qquad V(X) = \xi X^2 - \left(\frac{3}{16\pi}\right)^2 \left[2X + X^2 \ln\left(\frac{X}{2+X}\right)\right]$$

$\Longrightarrow$ Coleman-Weinberg type, one-parameter family ξ

Asymptotically free UV gauge scaling solutions

▷ gauge scaling towards flatness (Rechenberger,Scherer,HG,Zambelli'13; HG,Zambelli'14ip)



▷ approach to UV $k \to \infty$:

$$g^2 \to 0, \quad |\phi_{\min}|^2 \sim \frac{1}{g^2} \to \infty, \quad \underline{\underline{\lambda \sim g^4 \to 0}}, \quad \frac{m_W^2}{k^2} \to \text{const.}$$

$\Longrightarrow$ deep Euclidean region is sidestepped

Summary, Part III

- Numbers matter

  . . . m_{top}, m_{H}

- QFT is more than a collection of recipes

  . . . new insight from new tools

- vacuum stability: no reason for concern

  . . . so far . . .

- UV complete models approaching flat potentials

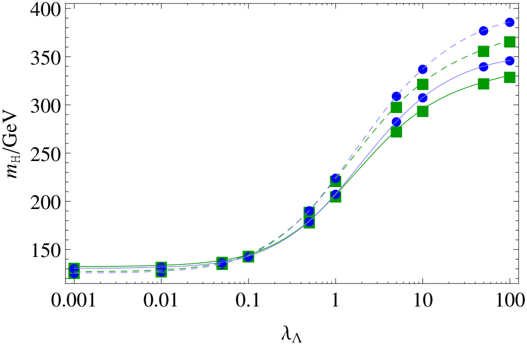
  . . . appealing also in view of current data

The IR window for the Higgs boson mass

▷ $m_H \sim v\lambda_R$ (WETTERICH'87)

mapping: $\lambda_\Lambda \quad \to \lambda_R$ not surjective on $\mathbb{R}_+$

▷ e.g. for ϕ^4 bare potential, fix $\Lambda = 10^7$GeV (HG,GNEITING,SONDENHEIMER'13)



convergence check of

- derivative expansion ΔNLO / LO $\sim$ 10% @ strong coupling
- U_{eff} solver (polynom. exp.)

“Instability” in the Z_2 model

▷ mean-field potential with cutoff Λ (linear regulator):

$$U = \frac{m_\Lambda^2}{2}\,\phi^2 + \frac{\lambda_\Lambda}{24}\,\phi^4 - \frac{1}{16\,\pi^2}\,\left[\Lambda^2\,h_\Lambda^2\,\phi^2 - h_\Lambda^4\,\phi^4\,\ln\left(\frac{\Lambda^2 + h_\Lambda^2\,\phi^2}{h_\Lambda^2\phi^2}\right)\right].$$

$\Longrightarrow$ stable for all ϕ

▷ expand for large Λ:

$$U = \frac{m_\Lambda^2}{2}\,\phi^2 + \frac{\lambda_\Lambda}{24}\,\phi^4 - \frac{1}{16\,\pi^2}\,\left[\Lambda^2\,h_\Lambda^2\,\phi^2 - h_\Lambda^4\,\phi^4\,\ln\left(\frac{\Lambda^2}{h_\Lambda^2\,\phi^2}\right)\right] + \mathcal{O}(\frac{1}{\Lambda^2}).$$

▷ renormalized parameters at scale μ (ignore renormalization of $h_\Lambda \to h$)

$$\begin{aligned} m_\Lambda^2 &= m_\mu^2 + \delta m^2, \\ \lambda_\Lambda &= \lambda_\mu + \delta\lambda. \end{aligned}$$

“Instability” in the Z_2 model

▷ fix finite parts with Coleman-Weinberg renormalization conditions: (at $\mu = v$)

$$U = \frac{m_v^2}{2}\,\phi^2 + \frac{\lambda_v}{24}\,\phi^4 - \frac{h^4\,\phi^4}{16\,\pi^2}\,\left(\ln\frac{\phi^2}{v^2} - \frac{3}{2}\right) - \frac{h^4\,v^2}{8\,\pi^2}\,\phi^2.$$

▷ supposedly accurate for:

$$\lambda_v \ll 1, \quad h^2 \ll 1, \quad \left|\frac{h^4}{16\,\pi^2}\,\ln\frac{\phi^2}{v^2}\right| \ll 1.$$

“Instability” in the Z_2 model

▷ e.g., all conditions satisfiable for

$$1 \gg \lambda_v = \frac{3\,h^4}{4\,\pi^2}, \quad \ln \frac{\overline{\phi}^2}{v^2} = 2$$

$\Longrightarrow$ $U(\overline{\phi}) < U(v)$ Instability !?!

▷ BUT: reexpressing this inequality in terms of bare quantities:

$$\lambda_\Lambda + \frac{3\,h^4}{2\,\pi^2} \ln \frac{\Lambda^2}{\overline{\phi}^2} < 0.$$

▷ for $\lambda_\Lambda \geq 0$: $\overline{\phi} > \Lambda$ required

$\Longrightarrow$ in contradiction with large Λ expansion!

Towards the standard model

▷ chiral Yukawa model: (HG,SONDENHEIMER'14IP)

$$S = \int \Big[\partial_\mu \phi^\dagger \partial^\mu \phi + U(\phi^\dagger \phi) + \bar{t} i \partial\!\!\!/ t + \bar{b} i \partial\!\!\!/ b$$
$$+ i h_b (\bar{\psi}_L \phi b_R + \bar{b}_R \phi^\dagger \psi_L) + i h_t (\bar{\psi}_L \phi_{\mathcal{C}} t_R + \bar{t}_R \phi_{\mathcal{C}}^\dagger \psi_L) \Big]$$

$$\phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_4 + i\phi_3 \end{pmatrix} \quad \psi_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

▷ enforce decoupling of Goldstone bosons ($m_G = 0$)

$$\frac{k^2}{k^2 + m_G^2} \quad \rightarrow \quad \frac{k^2}{k^2 + m_G^2 + g v_k^2}$$

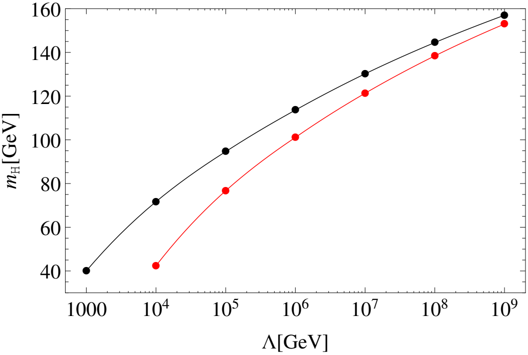
▷ choose "gauge boson" masses $g v_k^2 = (80.4\,\mathrm{GeV})^2$

cf. lattice model (GERHOLD,JANSEN'07'09'10)

“Lowering” the lower Higgs boson mass bound

▷ generalized bare potential with $\lambda_{6,\Lambda}(\phi^\dagger\phi)^3$ interaction:

(HG,SONDENHEIMER'14IP)

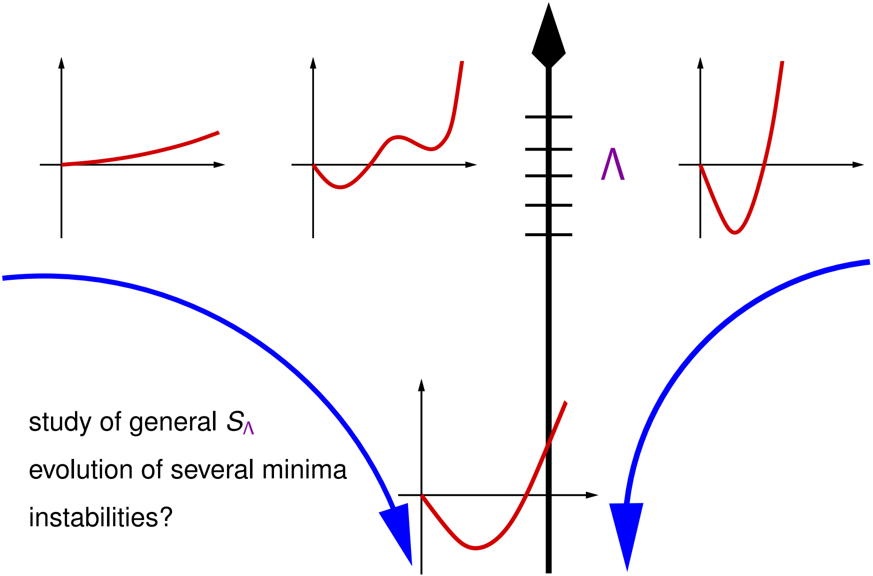


$\lambda_{3\Lambda} = 0,\ \lambda_{2\Lambda} = 0$

$\lambda_{3\Lambda} = 3,\ \lambda_{2\Lambda} = -0.1$

$\Longrightarrow$ same RG mechanism at work

TODO list



study of general S_Λ

evolution of several minima

instabilities?