Superfluidity in bosonic systems

Rico Pires

PI Uni Heidelberg

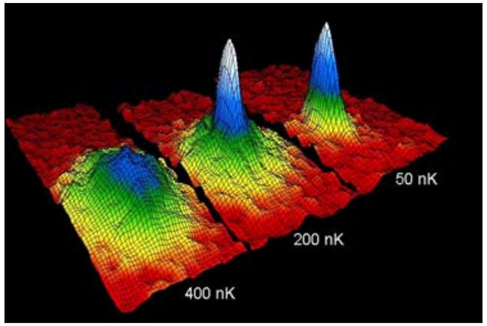
Quantum Dynamics of Atomic and Molecular Systems

Center for Quantum Dynamics

Outline

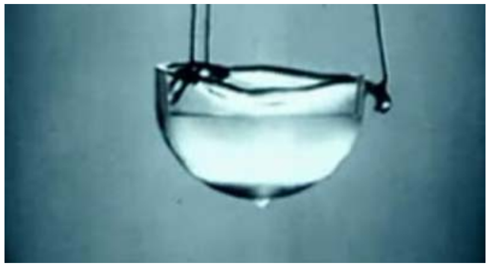
Strongly coupled quantum fluids

2.1 Dilute Bose gases



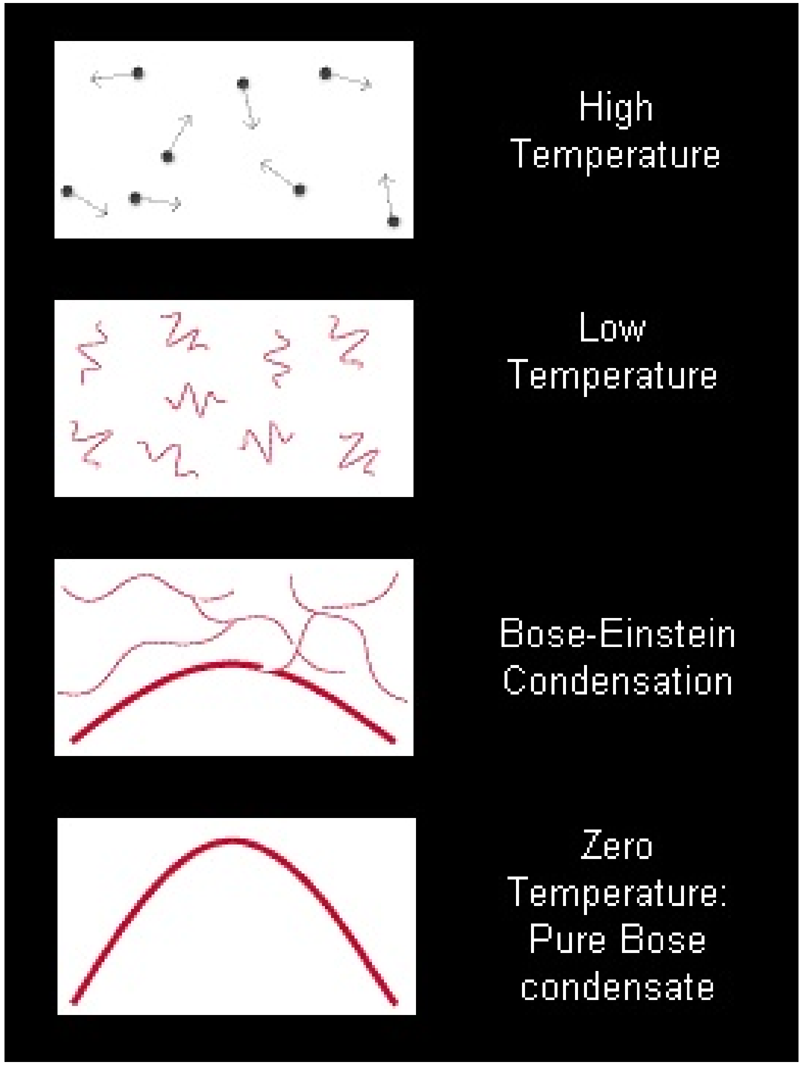
Wieman/Cornell

2.2 Liquid Helium



A. Leitner, from wikimedia

When are quantum effects important??



Pfau webpage

$$\lambda_{DB} = \frac{2\pi\hbar}{\sqrt{2\pi m k_B T}}$$

$$\lambda_{DB} \sim n^{-1/3} \Leftrightarrow n\lambda_{DB}^3 \sim 1$$

Critical Temperature

$$f(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/(kT)} - 1} \qquad N = \sum_{\varepsilon} f(\varepsilon)$$

$$N = N_0 + h^{-3} \int\int f(\varepsilon(\vec{r}, \vec{p})) d^3\vec{r} d^3\vec{p} = N_0 + \int f(\varepsilon)\rho(\varepsilon) d\varepsilon$$

In a 3D box

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_C}\right)^{3/2}$$

In a harmonic potential

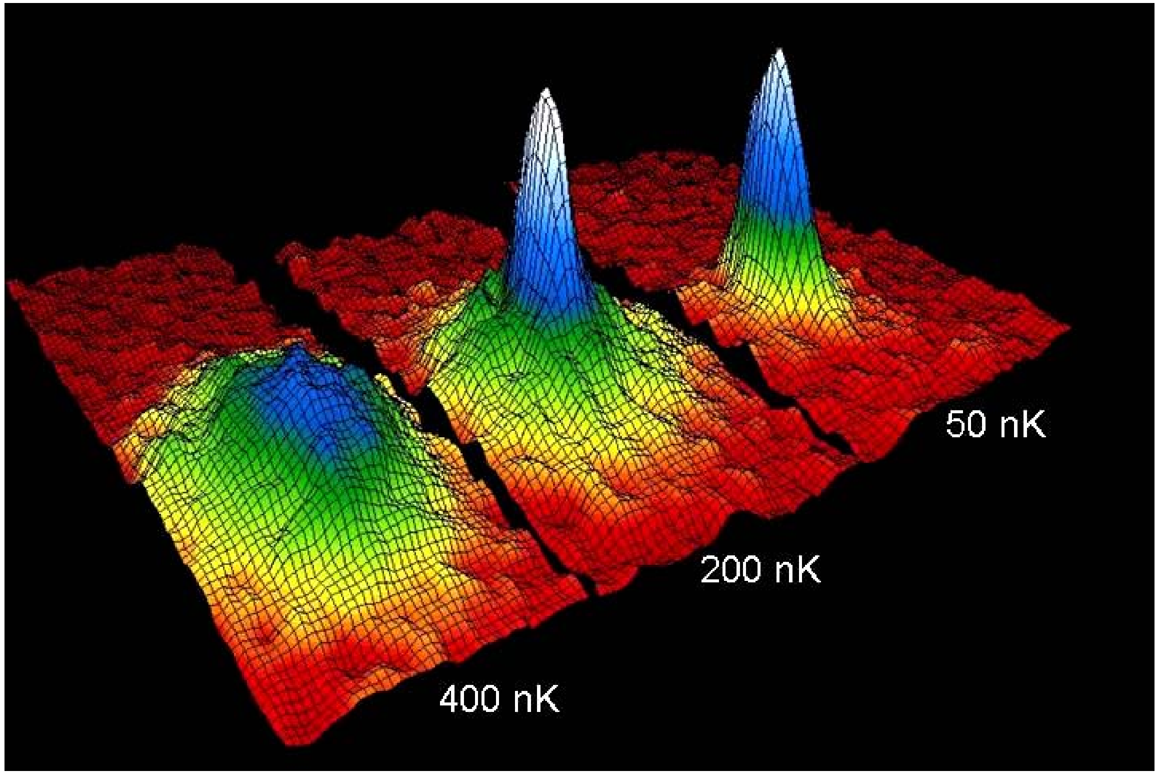
$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_C}\right)^{3}$$

$$T_C = \frac{2\pi\hbar^2}{mk_B}\left(\frac{n}{\xi(3/2)}\right)^{2/3} \qquad \Rightarrow n\lambda_{DB}^3 \approx 2.61$$

Onset of condensation

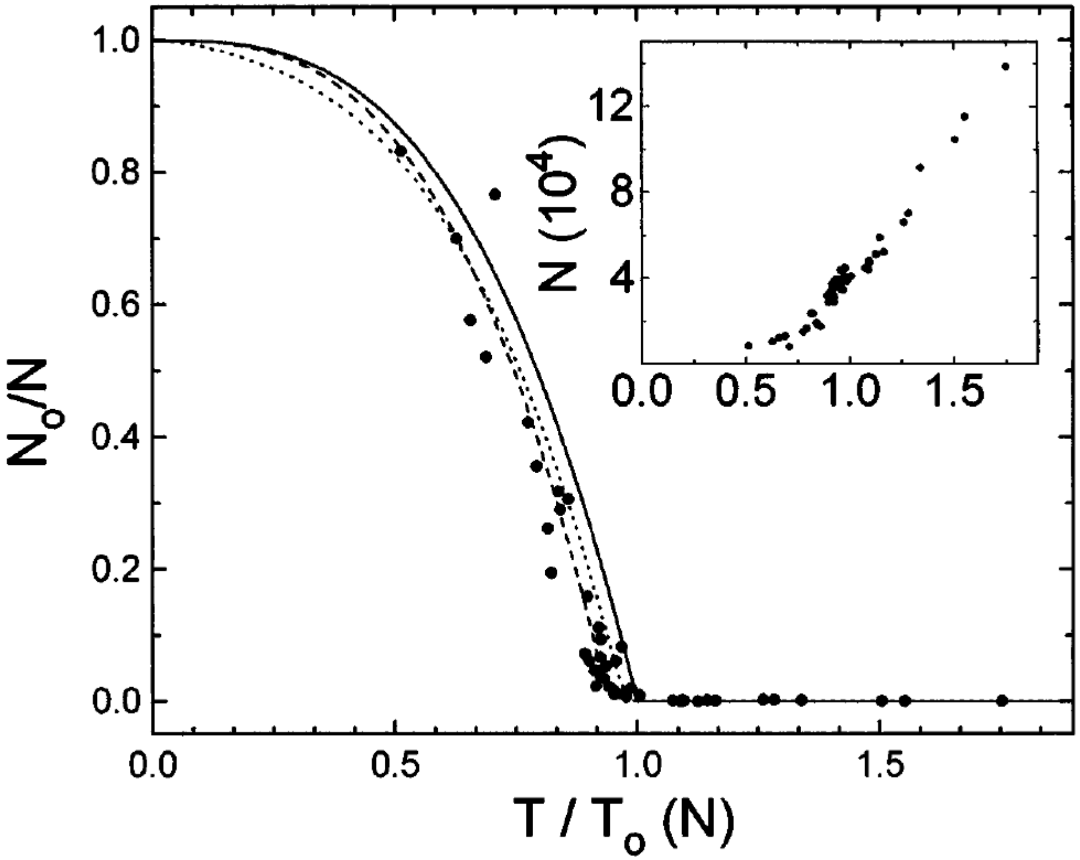
$$n(\vec{r}) = h^{-3} \int f(\varepsilon) d^3 p$$

Sudden jump of n(0)
for $T=T_C$



Wieman/Cornell

^{87}Rb BEC ground state fraction



Ensher et al., PRL 77 (1996)

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_C} \right)^3$$

Gross-Pitaevski Equation

$$\left[\frac{-\hbar^2}{2m}\Delta + U_{trap}(\vec{r}) + g\left|\psi_0(\vec{r},t)\right|^2\right]\psi_0(\vec{r},t) = i\hbar\frac{\partial}{\partial t}\psi_0(\vec{r},t)$$

$$g = 4\pi\hbar^2 a$$

Non-linear Schrödinger equation for the order parameter in zero temperature limit. The solution is the chemical potential!

The case of finite interaction

$$\delta T_C = \boxed{c} \cdot a n^{1/3} T_c$$

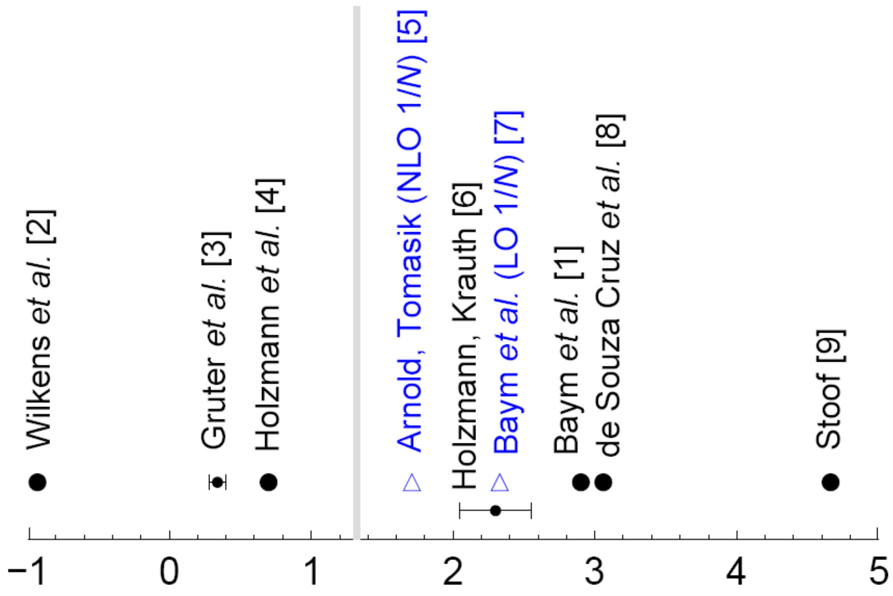


FIG. 1. Estimates from the literature of the constant c in $\Delta T_c/T_0 \to c a n^{1/3}$. The grey bar is the result of this paper.

Arnold and Moore, PRL 87 (2001)

[1] **diagrammatic perturbation theory**, Phys. Rev. Lett. 83, 1703 (1999)

[2] **Perturbation in canonical ensemble**, J. Phys. B 33, L779 (2000)

[3] **Path integral Monte Carlo**, Phys. Rev. Lett. 79, 3549 (1997)

[4] **Ursell operators**, Eur. Phys. J. B 10, 739 (1999)

[5] **O(N) scalar field theory (NLO),** Phys. Rev. A 62, 063604 (2000)

[6] **Path integral Monte Carlo simulation**, Phys. Rev. Lett. 83 2687 (1999)

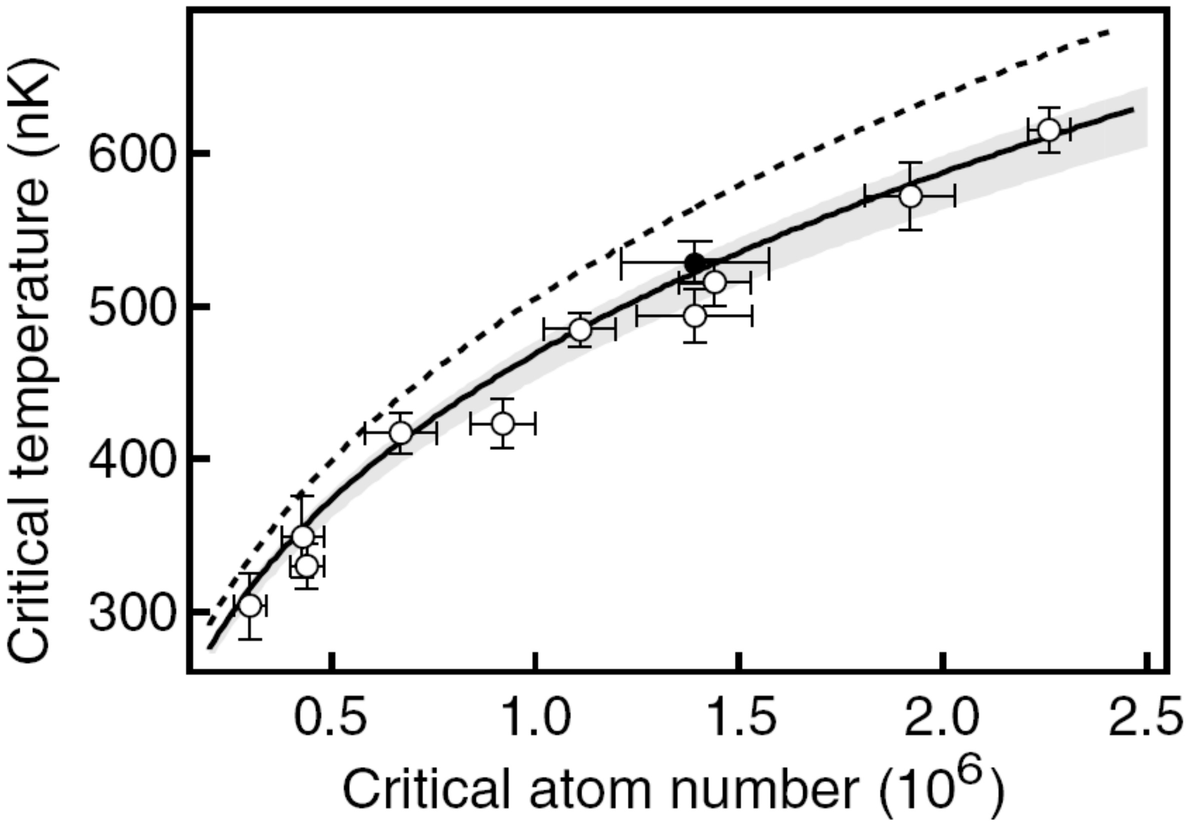
[7] **O(N) scalar field theory (LO),** Europhys. Lett. 49, 150 (2000)

[8] **O(N) scalar field theory linear δ expansion**, cond-mat/0007151 (unpublished).

[9] **Renormalization group**, Phys. Rev. A 45, 8398 (1992)

Lower Critical Temperature (MF approach)

Critical temperature decreased in ^{87}Rb:



Gerbier et al., PRL 92 (2004)

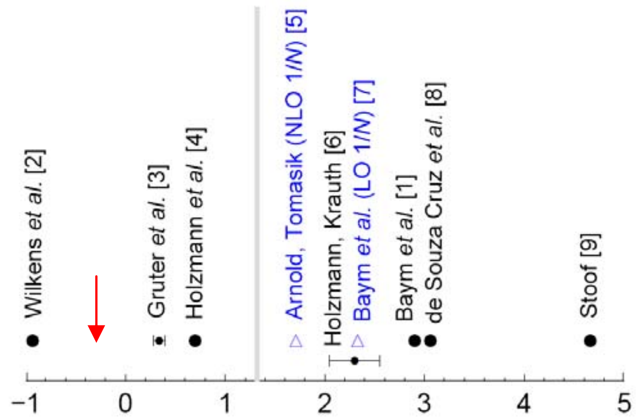
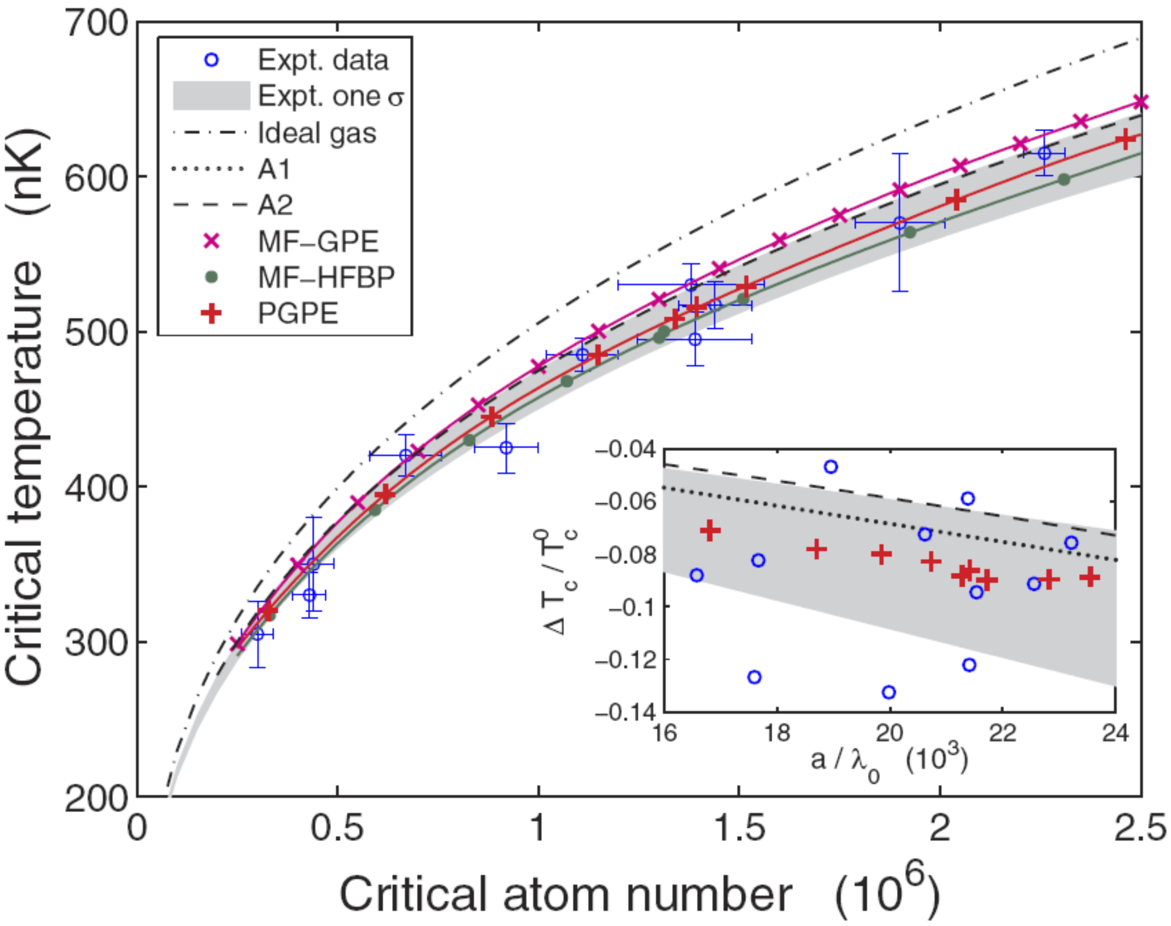


FIG. 1. Estimates from the literature of the constant c in $\Delta T_c/T_0 \to can^{1/3}$. The grey bar is the result of this paper.

Higher critical temperature (beyond MF,non-perturbative)



Davis, Blakie PRL 96 (2006)

Only approximation: High modes of Bose quantum field can be approximated by classical field

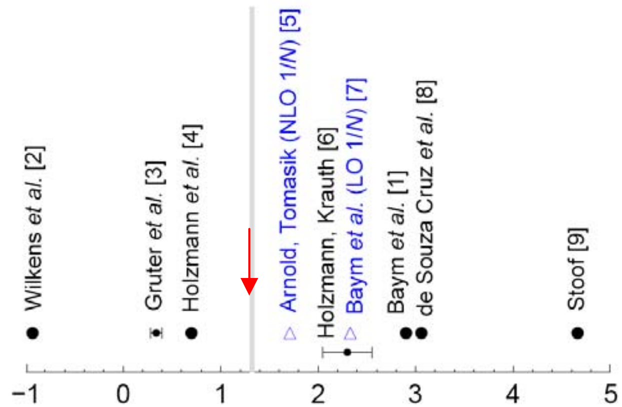


FIG. 1. Estimates from the literature of the constant c in $\Delta T_c/T_0 \rightarrow c a n^{1/3}$. The grey bar is the result of this paper.

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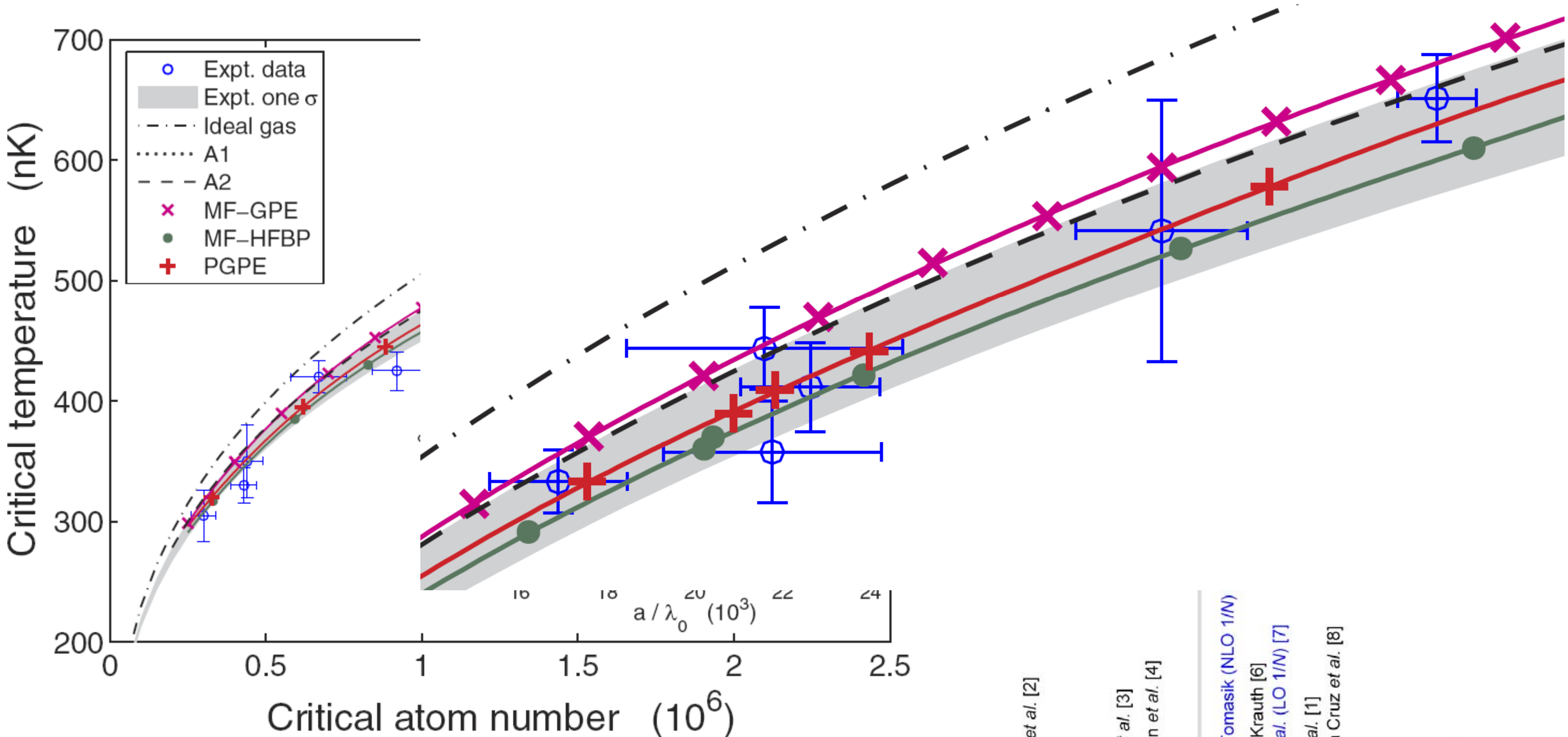
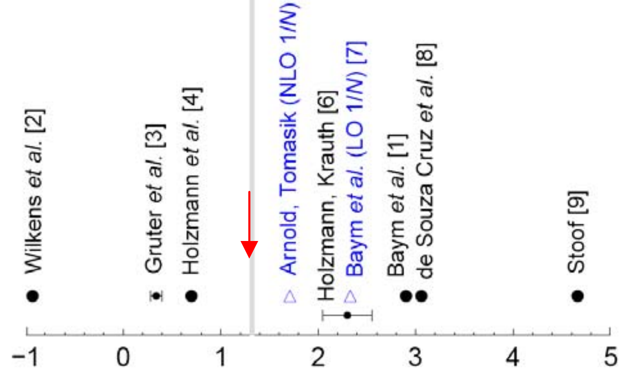


Davis, Blakie PRL 96 (2006)



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Gross-Pitaevski Equation

$$\left[\frac{-\hbar^2}{2m}\Delta + U_{trap}(\vec{r}) + g\left|\psi_0(\vec{r},t)\right|^2\right]\psi_0(\vec{r},t) = i\hbar\frac{\partial}{\partial t}\psi_0(\vec{r},t)$$

$$g = 4\pi\hbar^2 a$$

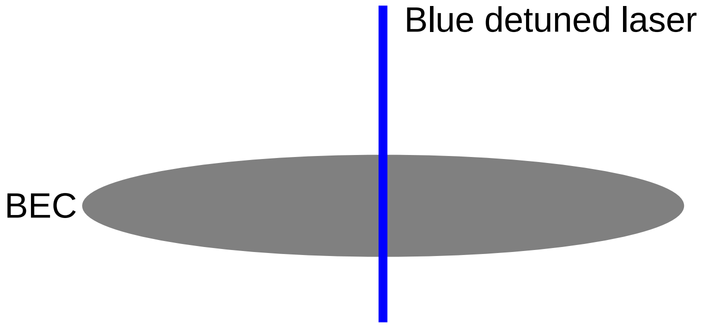
Linearize (Bogoliubov-DeGennes eqs.)

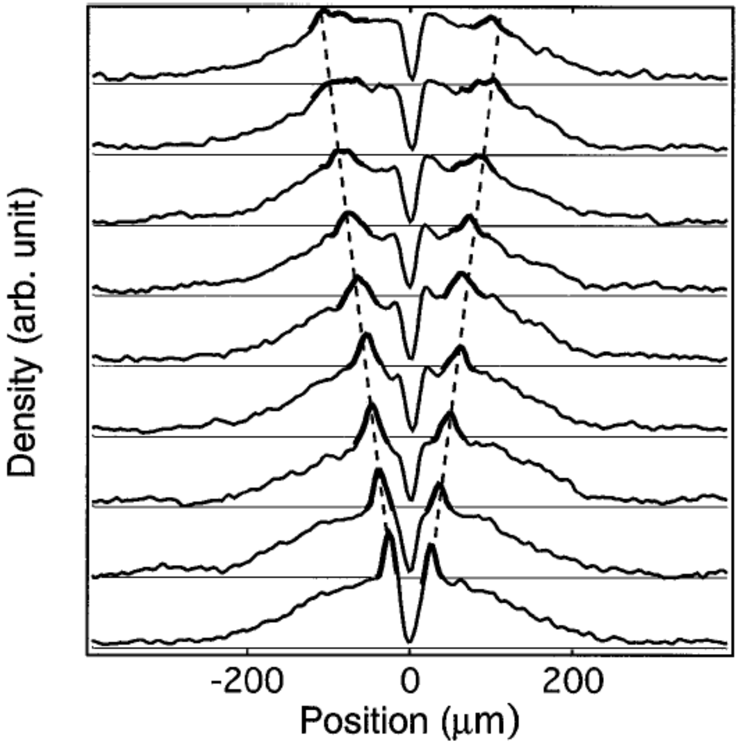
$$\in_p = \frac{1}{2m}\sqrt{(p^2 + 8\pi an)^2 - 8\pi an}$$

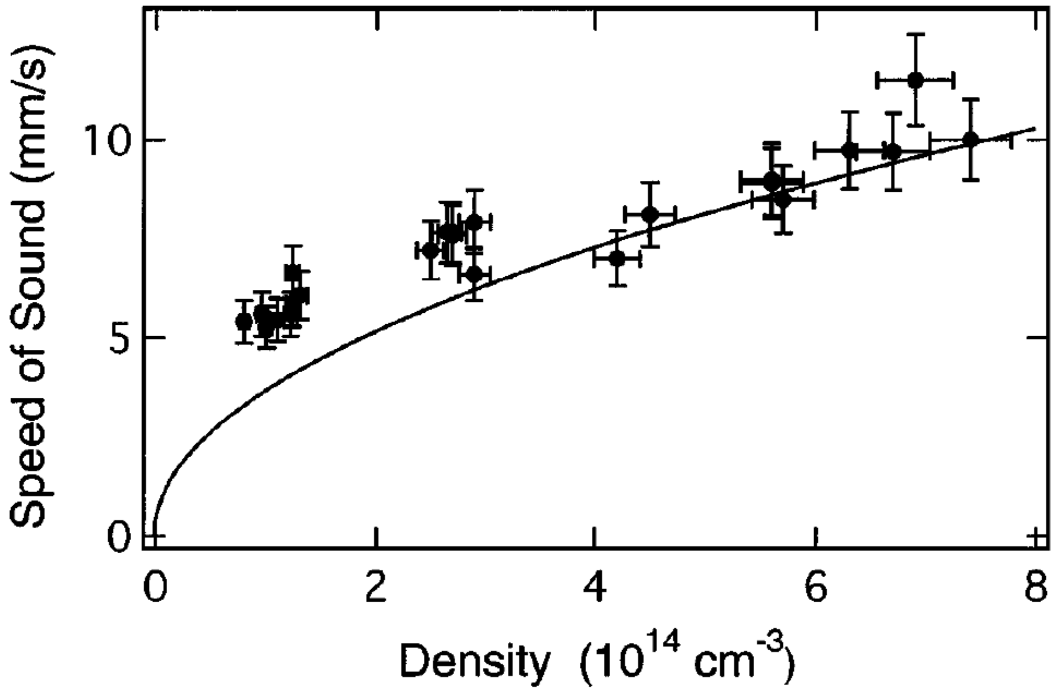
$$\in_p \approx c \cdot p$$

$$\in_p \approx \frac{p^2}{2m} + \mu = \frac{p^2}{2m} + 4\pi\hbar^2 a n_0$$

Sound wave propagation





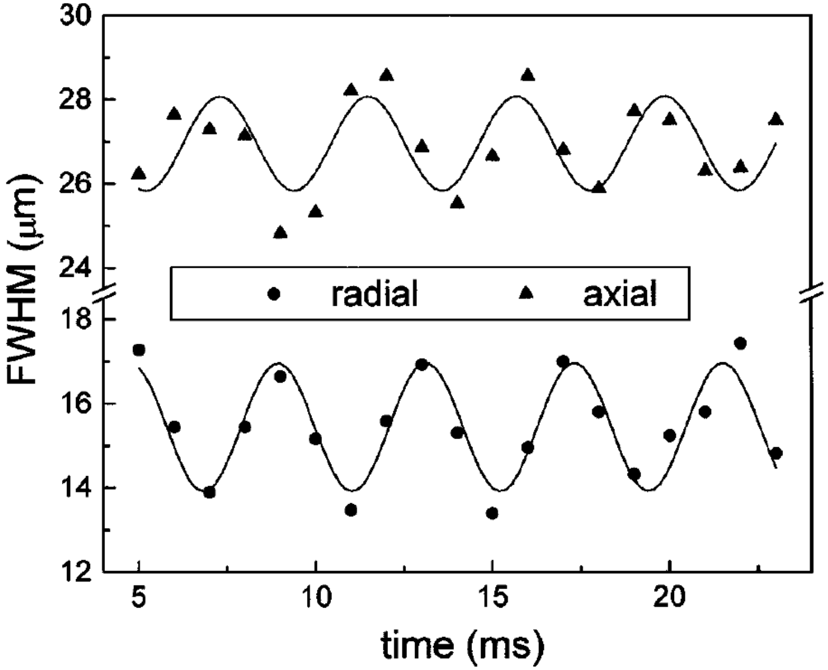


Andrews et al.,PRL 79 (1997)

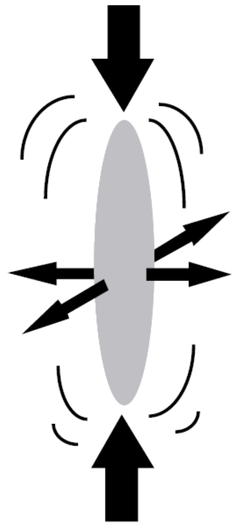
$$c(r) = \frac{\sqrt{n(r) 4\pi\hbar^2 a}}{m}$$

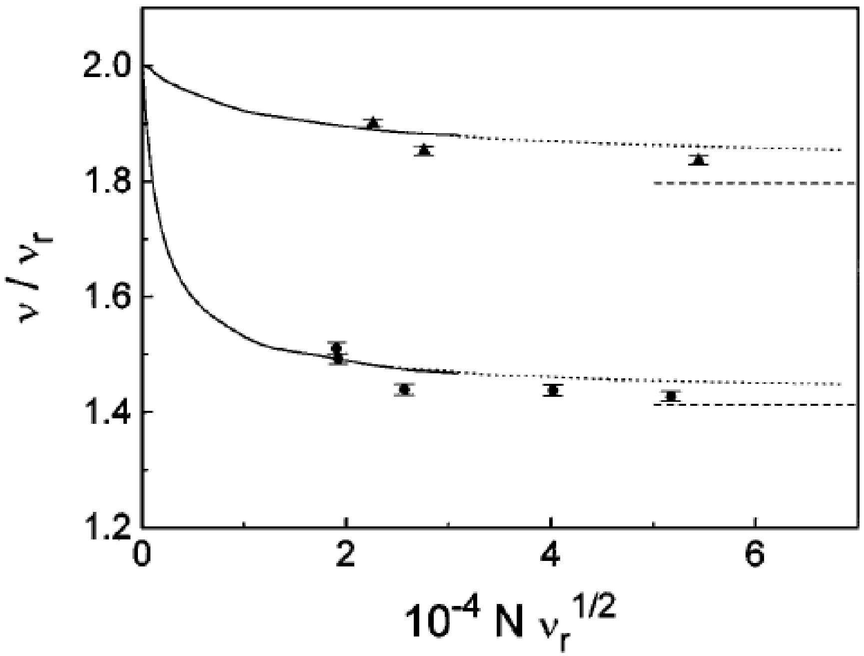
Collective excitations

Perturbing the trapping potential of a ^{87}Rb BEC with a sine wave

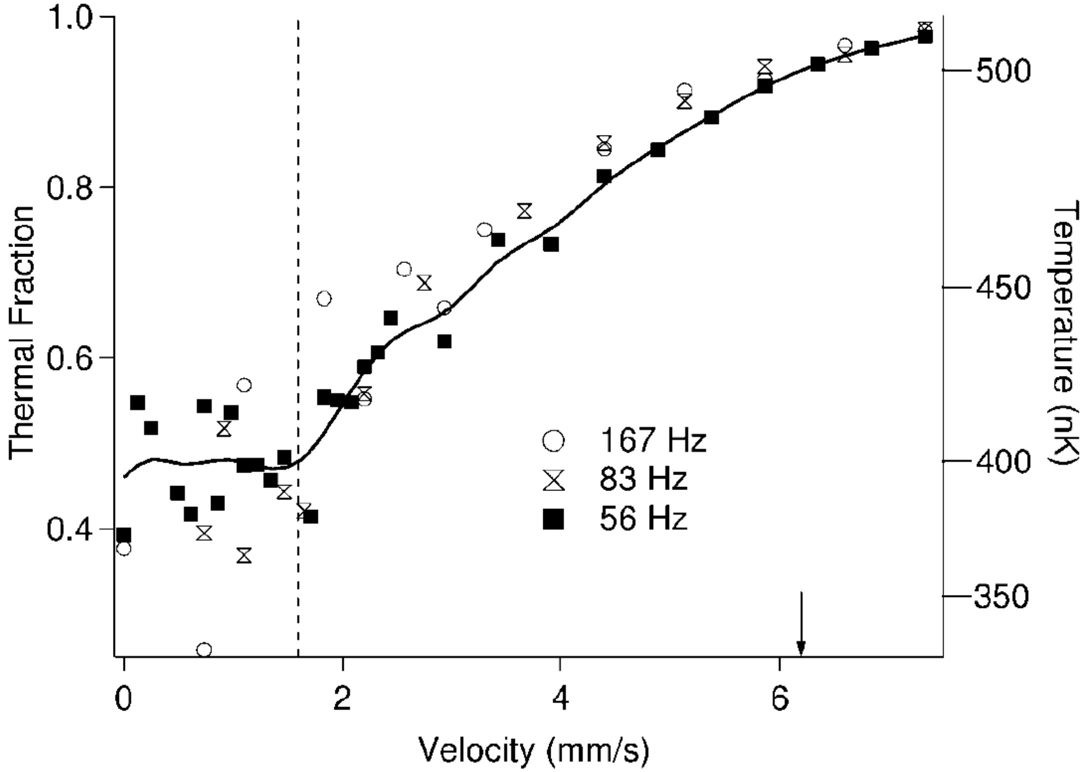


Jin et al., PRL 77 (1996)



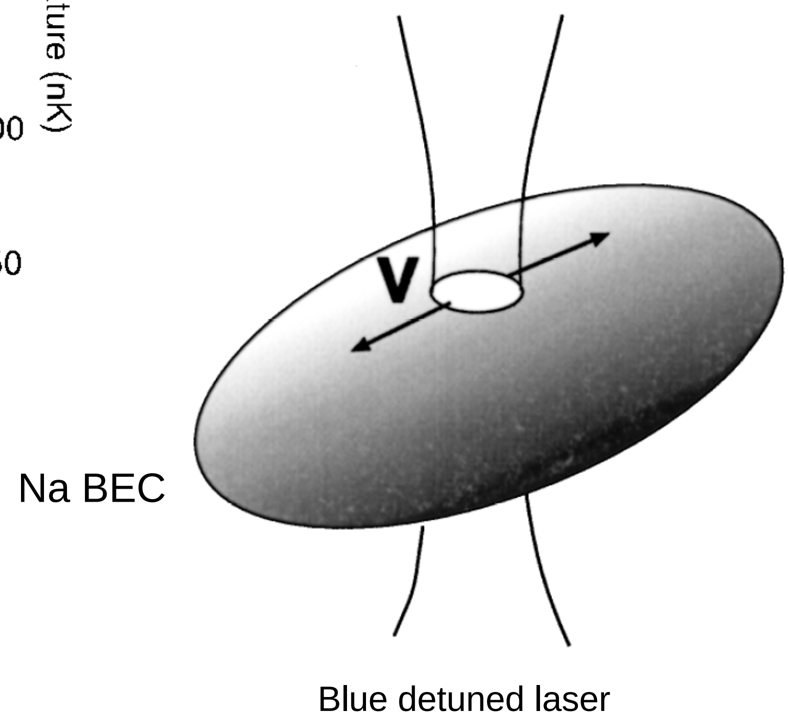


Superfluid behaviour



Raman et al., PRL 83 (1999)

Heating of the sample due to vortex formation only above v_c



Vortices

From GPE we get: $$\frac{\partial n}{\partial t} + \nabla(n\vec{v}) = 0$$

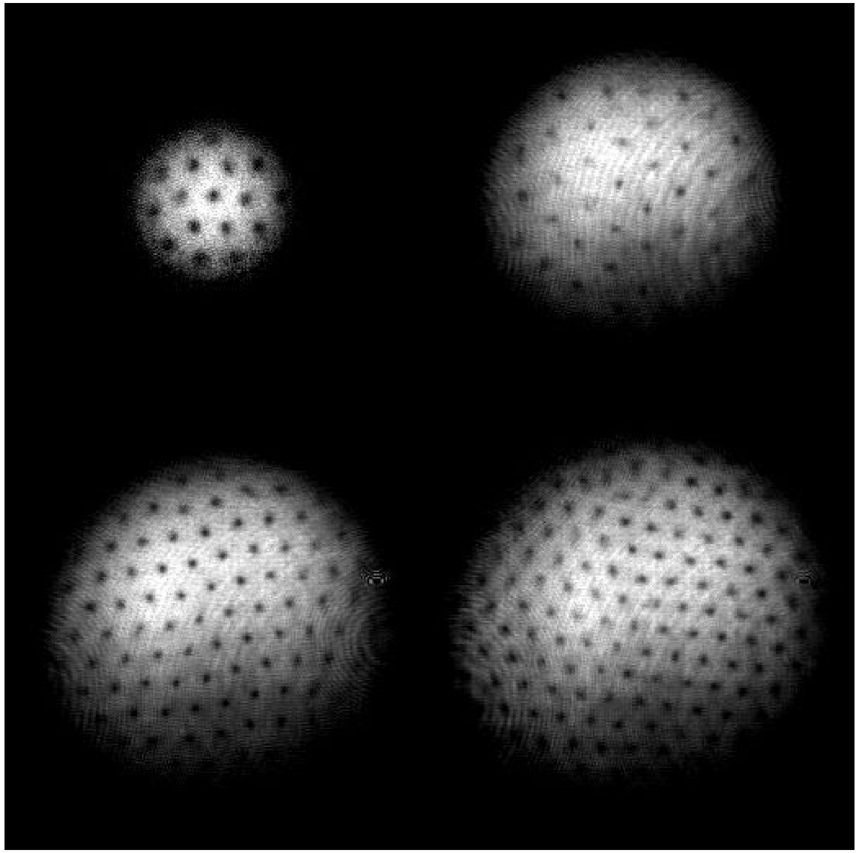
with: $$v = \frac{\hbar}{2mi}\left(\frac{\psi^{*}\nabla\psi - \psi\nabla\psi^{*}}{|\psi|^{2}}\right)$$

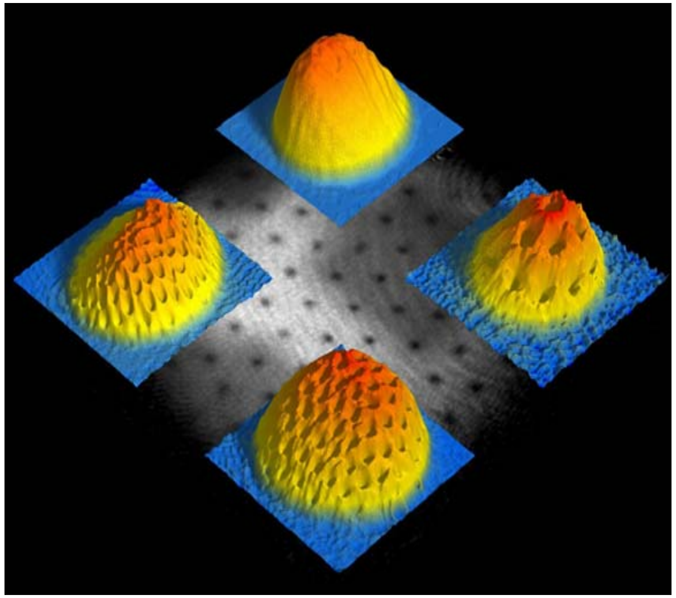
use $\psi(\vec{r}) = \psi(r,z)e^{i\varphi}$: $\vec{v} = \frac{\hbar}{m}\nabla\varphi \qquad \Rightarrow \nabla\times\vec{v} = 0$

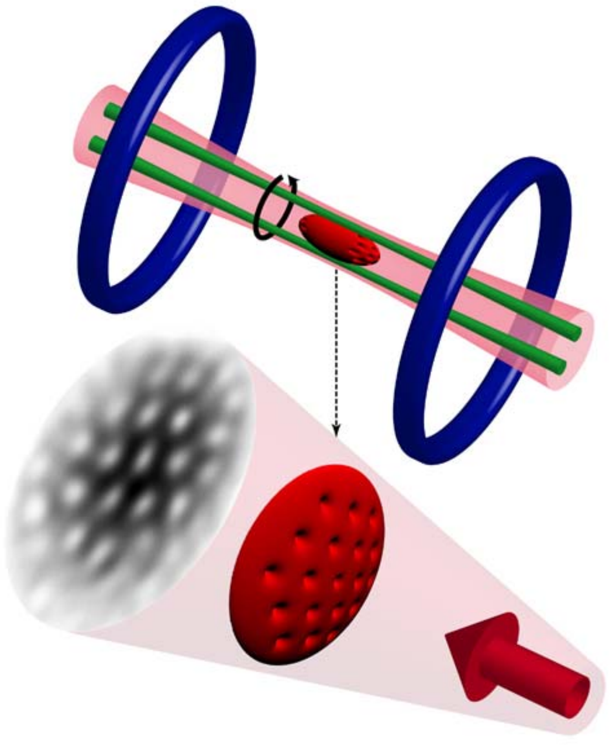
$$\Gamma = \oint v\cdot dr = 2\pi l\frac{\hbar}{m} \qquad \Rightarrow v_{\varphi} = l\frac{h}{2\pi m r}$$

$$\in_{v} = l^{2}\pi n\hbar^{2}\ln\frac{b}{\xi}$$

Vortex lattices

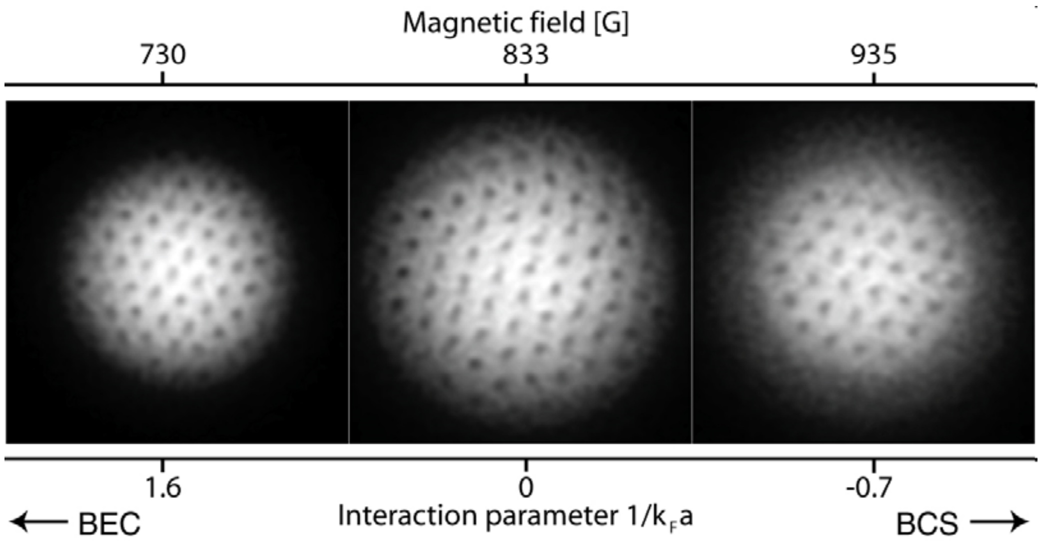




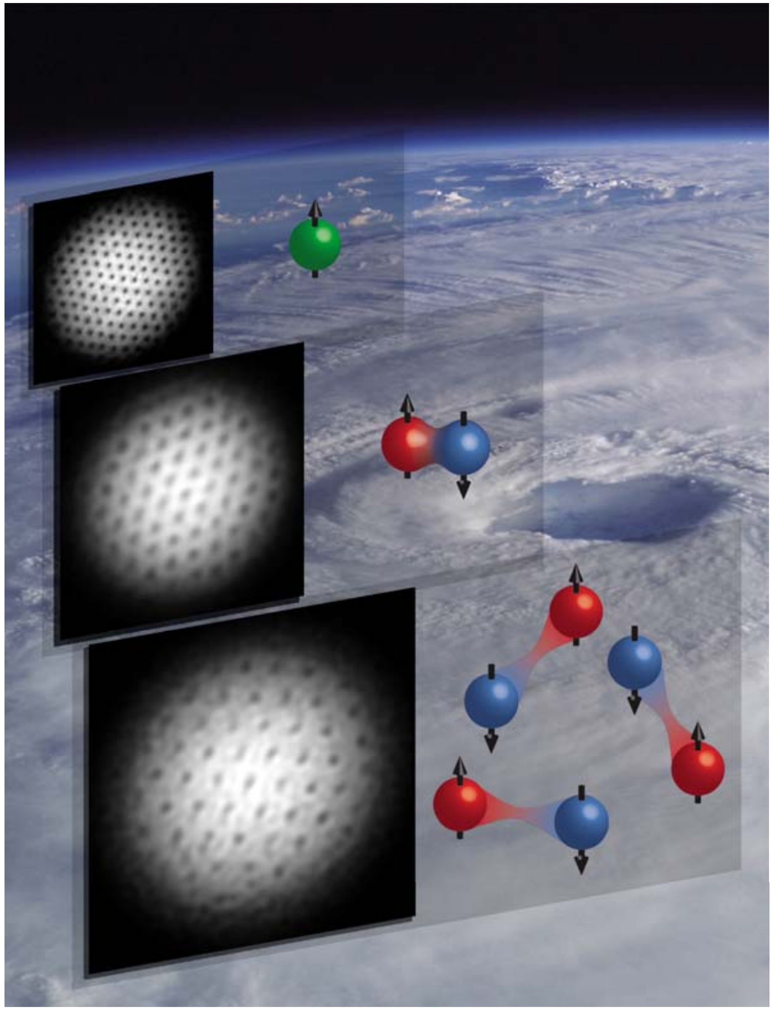


Ketterle et al., Science 292 (2001)

BEC-BCS crossover



Ketterle Nature (2005)



Eulers equation for a perfect fluid

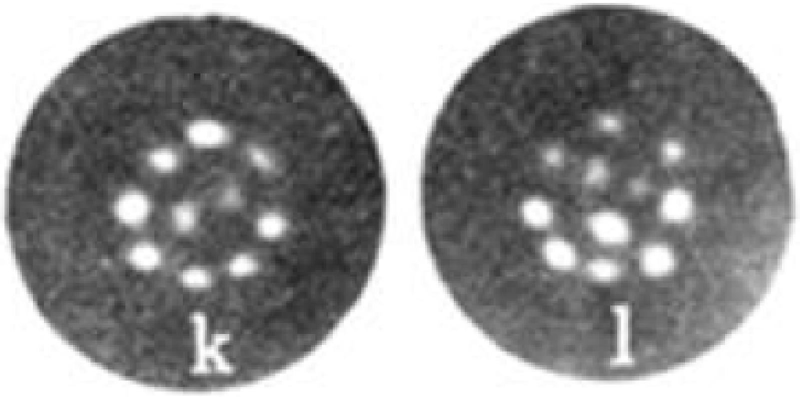
Use trial function in GPE:

$$\frac{\partial \vec{v}}{\partial t} = -\frac{1}{mn}\nabla p - \nabla\left(\frac{v^2}{2}\right) + \underbrace{\frac{1}{m}\nabla\left(\frac{\hbar^2}{2m\sqrt{n}}\nabla^2\sqrt{n}\right)}_{\text{Quantum pressure}} - \frac{1}{m}\nabla V$$

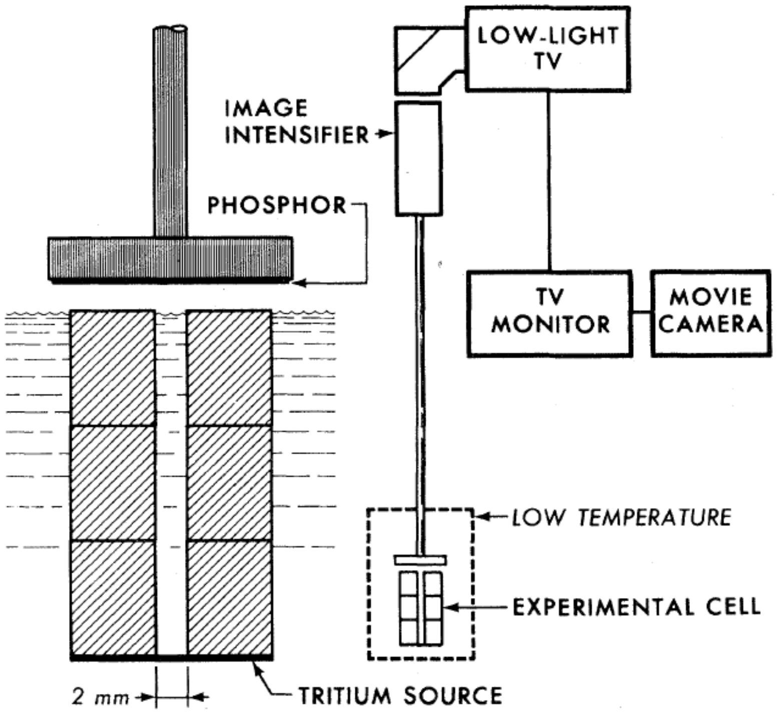
Cf. Eulers eq. For a perfect fluid

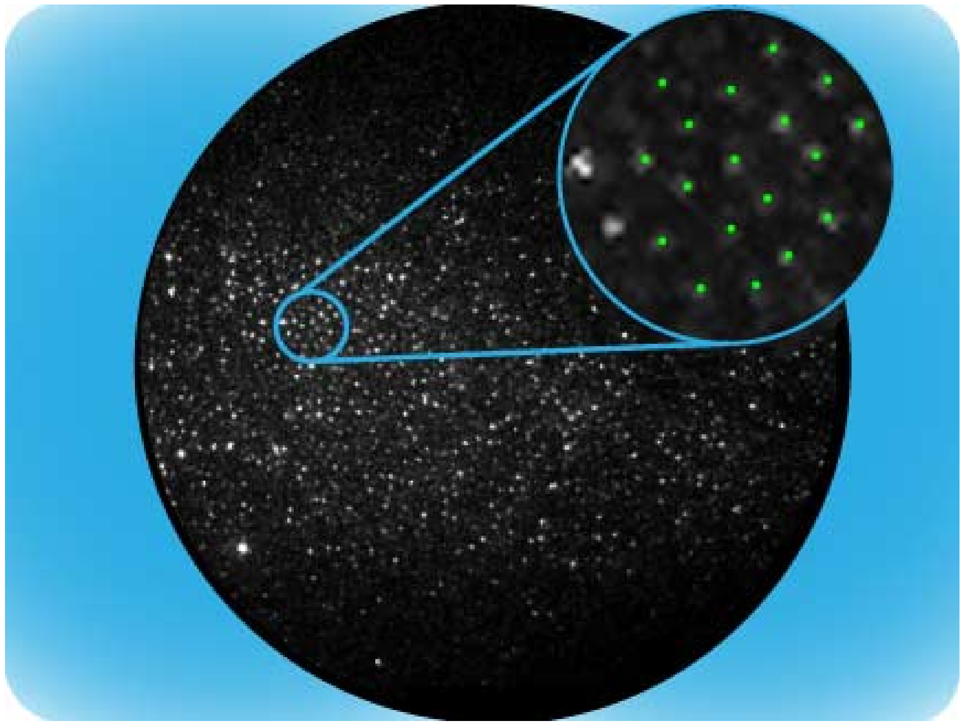
$$\frac{\partial \vec{v}}{\partial t} - \vec{v}\times(\nabla\times\vec{v}) = -\frac{1}{mn}\nabla p - \nabla\left(\frac{v^2}{2}\right) - \frac{1}{m}\nabla V$$

Superfluidity in Helium



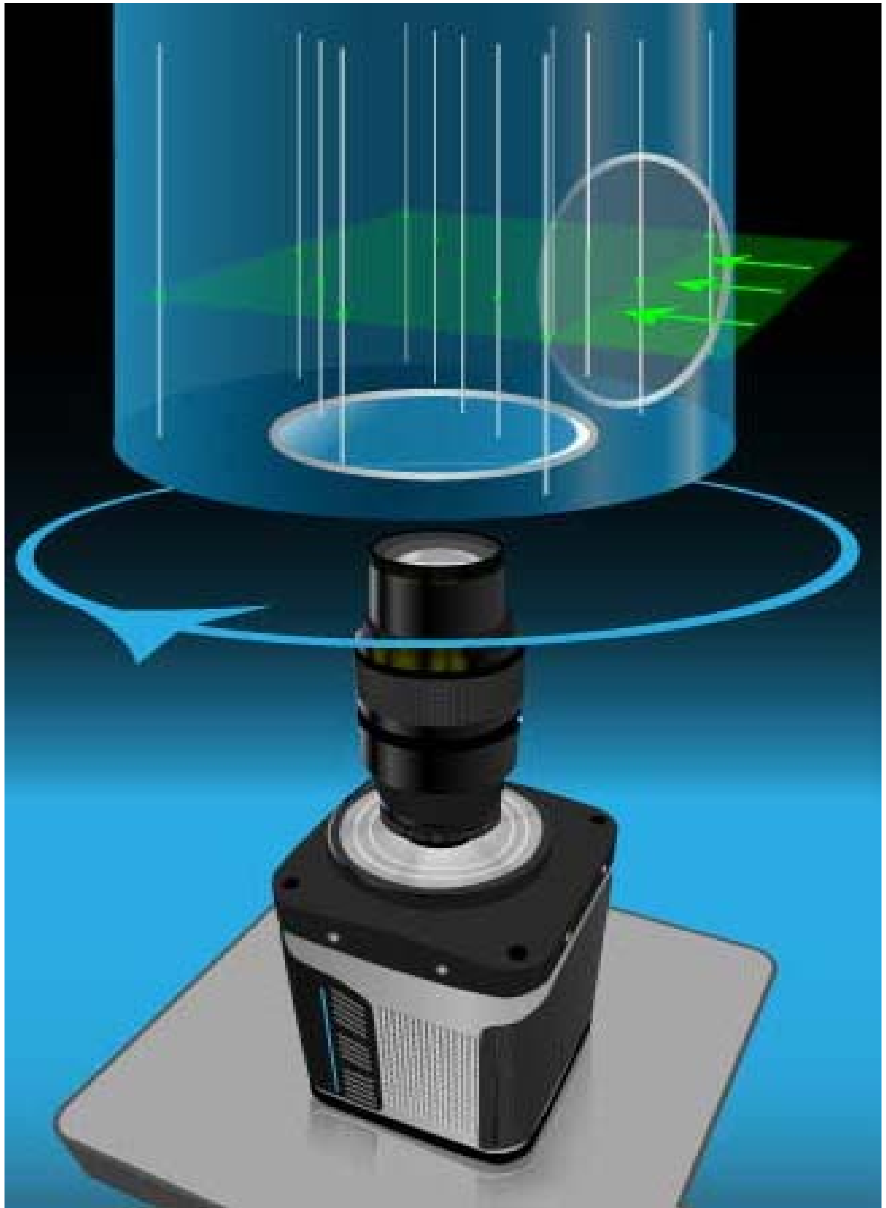
Yarmchuk et al., PRL 43 (1979)

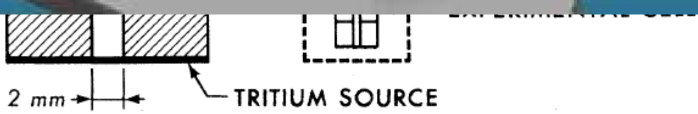


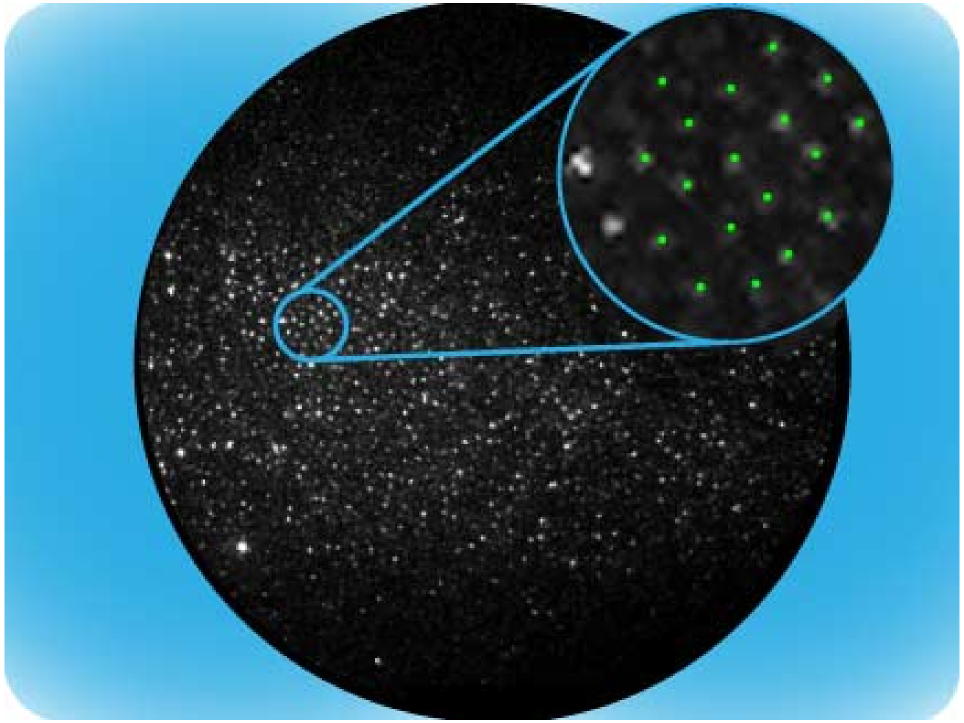


Bewley et al., University of Maryland, from aps.org (2006)

Superfluidity in Helium







Bewley et al., University of Maryland, from aps.org

Why is Helium a quantum fluid?

$$d(He) = 0.265nm$$
$$d(Ne) = 0.296nm$$

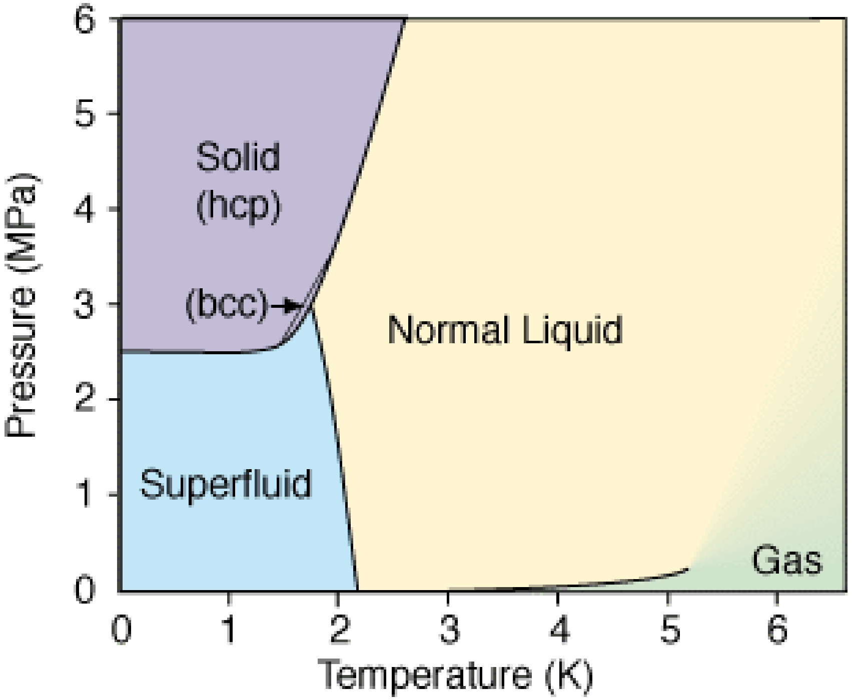
Lennard-Jones type potential (approx.!)

$$V(r) = \varepsilon_0 \left(\frac{d^{12}}{r^{12}} - 2\frac{d^6}{r^6} \right)$$

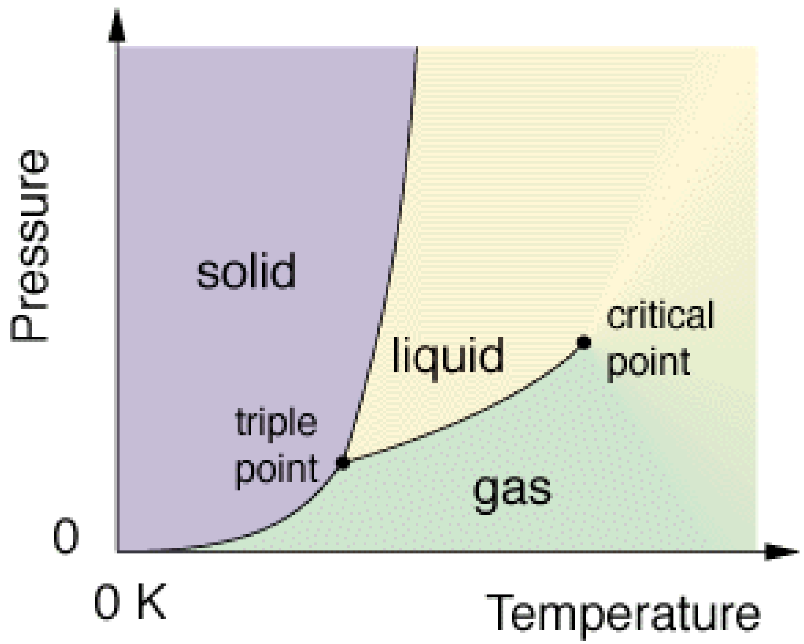
$$\varepsilon_0(He) = 1.03meV$$
$$\varepsilon_0(Ne) = 3.94meV$$

$$\lambda_{DB}(He) \approx 0.4nm > d$$
$$\lambda_{DB}(Ne) \approx 0.07nm < d$$

Helium phase diagram



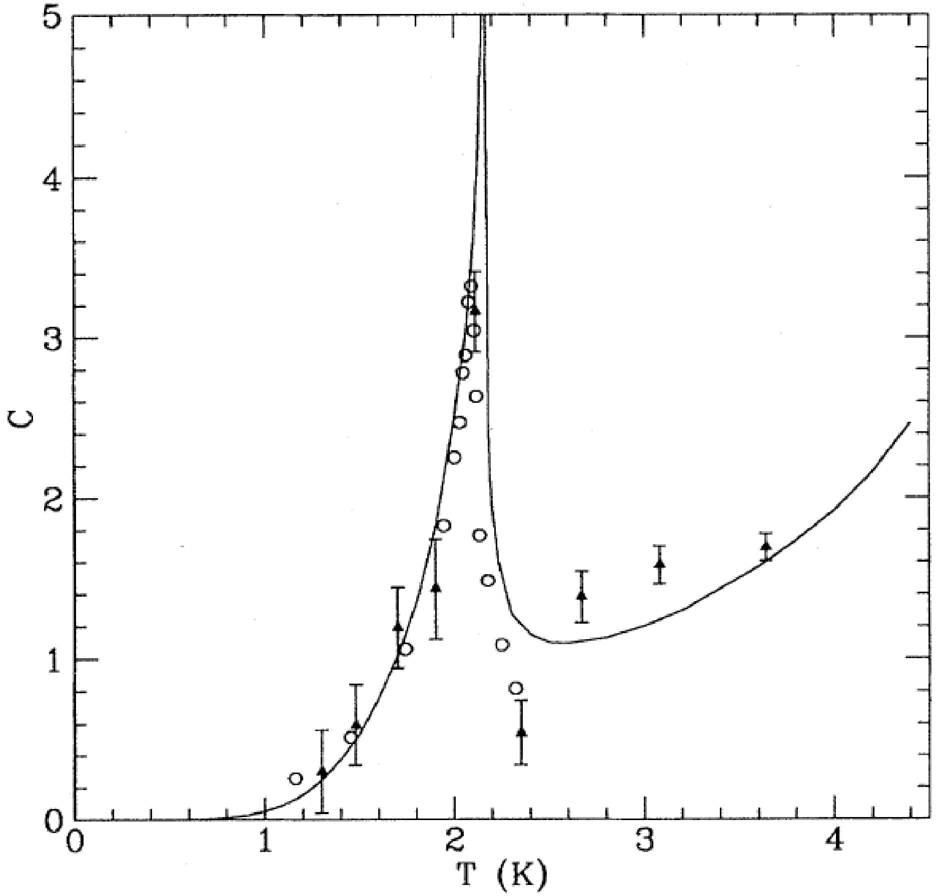
LTT Helsinki



$$E_0 = \frac{3}{2}\hbar\omega_0 \approx 7meV \triangleq 70K$$

Specific heat capacity of ^{4}He

Pulsed heat method compared to PIMC simulation



Ceperley, Rev. Mod. Phys. 67 (1995)

$$C_v = \begin{cases} C(T) + A_+ \left|T - T_C\right|^{-\alpha} & \text{for } (T > T_C) \\ C(T) + A_- \left|T - T_C\right|^{-\alpha} & \text{for } (T < T_C) \end{cases}$$

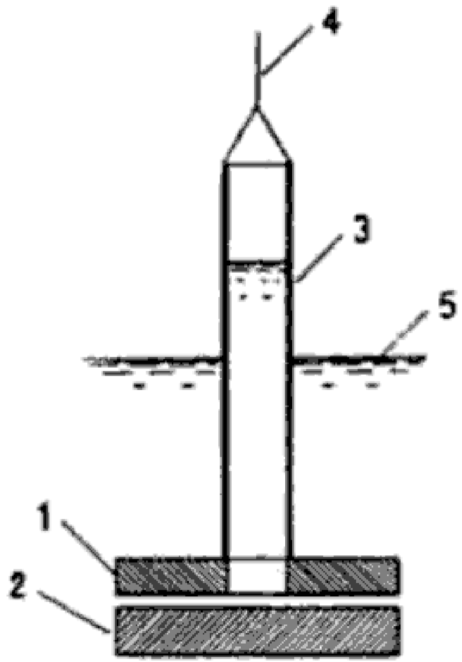
$$\alpha \approx -0.0128$$

Nissen/Israelsson

Heat pulse method in space shuttle

Discovery of superfluidity in Helium

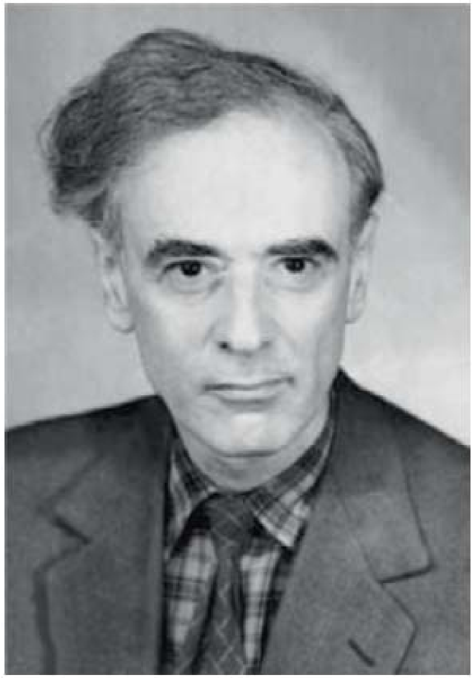
Allen, Misener, Kapitza (NP 1978)



Experiments on flow through capillary

$$\frac{\Delta P}{L} \sim \eta \frac{v}{R^2}$$

Theoretical understanding of superfluids



London, Tisza and Landau

$$\vec{j} = \vec{j}_s + \vec{j}_n$$

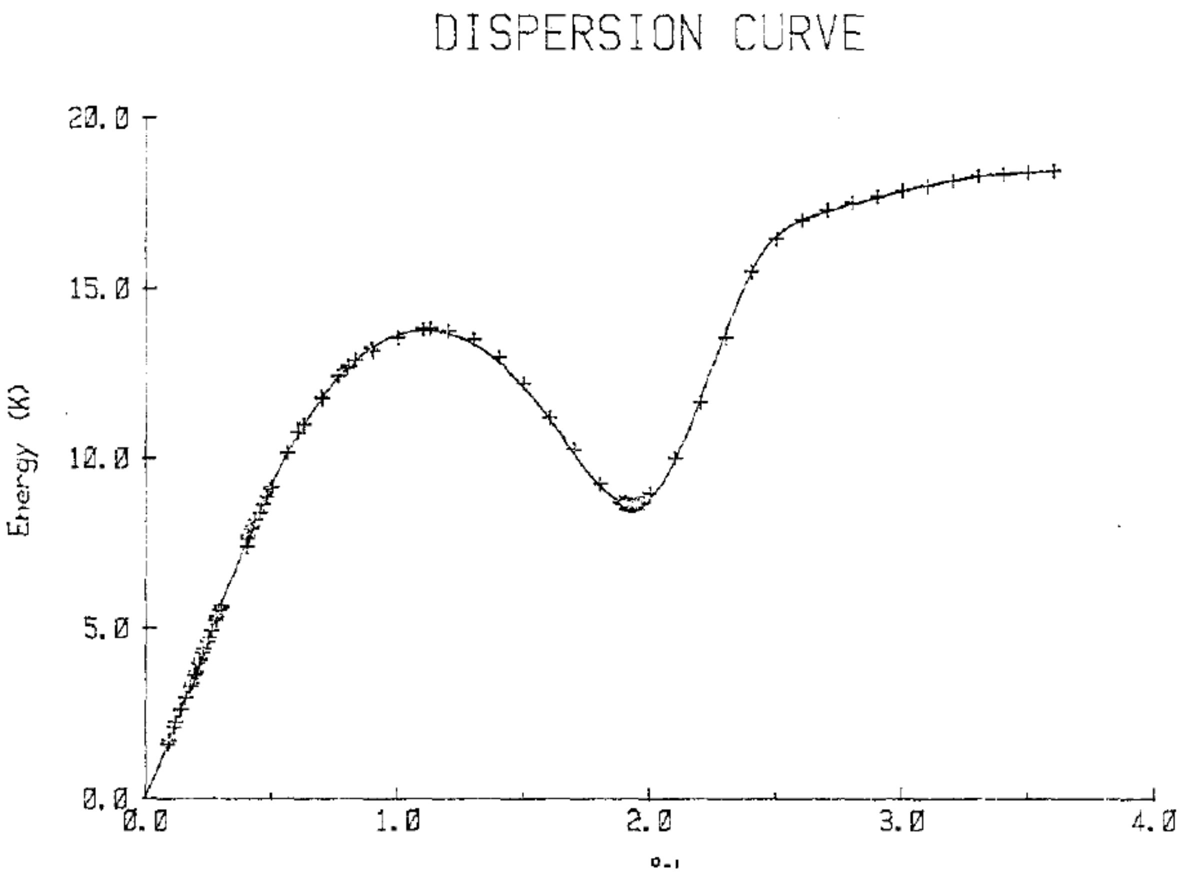
$$n_S \sim \begin{cases} B(T_C - T)^{\nu} & T < T_C \\ 0 & T > T_C \end{cases}$$

Landaus dispersion curve

Neutron scattering experiments

$$\varepsilon_f = \varepsilon_i - \vec{v}(\vec{p}_i - \vec{p}_f)$$

$$\varepsilon(p) = \begin{cases} |c|\, p & p \text{ small} \\ \dfrac{p^2}{2m} & p \text{ large} \end{cases}$$



Donnelly et al., J. Low Temp. Phys. 44 (1981)

Grand canonical ensemble

$$Z = \sum_{n,N} e^{-\beta(E_n^{(N)} - \mu N)}$$

$$\langle N \rangle = k_B T \frac{\partial \ln Z}{\partial \mu} \qquad U = \langle H \rangle = \mu \langle N \rangle - \frac{\partial \ln Z}{\partial \beta}$$

Exact solutions for no interactions

Solve with perturbation theory for weak interactions

QMC simulations for strong interactions

$$\Rightarrow n_0 \approx 0.1 n$$

Ceperley and Pollock, PRL 56 (1986)
Path integral

Neutron Scattering

R. A. Cowley and A. D. B. Woods, Can. J. Phys. 49, 177 (1971)

H. R. Glyde, J. Low Temp. Phys. 59, 561 (1985)

R. N. Silver, Phys. Rev. B 37, 3794 (1988);ibid. 38, 2283 (1988)

A. S. Rinat, Phys. Rev. B 42, 9944 (1990)

Correlation function and structure factor

$$\rho_1(\vec{r}_1 - \vec{r}_1') = N \int \psi_0^*(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N)\psi_0(\vec{r}_1', \vec{r}_2, \ldots, \vec{r}_N) d^3\vec{r}_2 \ldots d^3\vec{r}_N$$

$$n_0 = \lim_{|\vec{r}_1 - \vec{r}_1'| \to \infty} \rho_1(\vec{r}_1 - \vec{r}_1')$$

$$P(\vec{p})d^3p = \frac{Vd^3p}{(2\pi\hbar)^3} \langle \Psi | \hat{n}_K | \Psi \rangle$$

$$\langle \Psi | \hat{n}_K | \Psi \rangle = \int \rho_1(\vec{r}) e^{i\vec{k}\vec{r}} d^3r \propto S(\vec{k})$$

Static structure factor

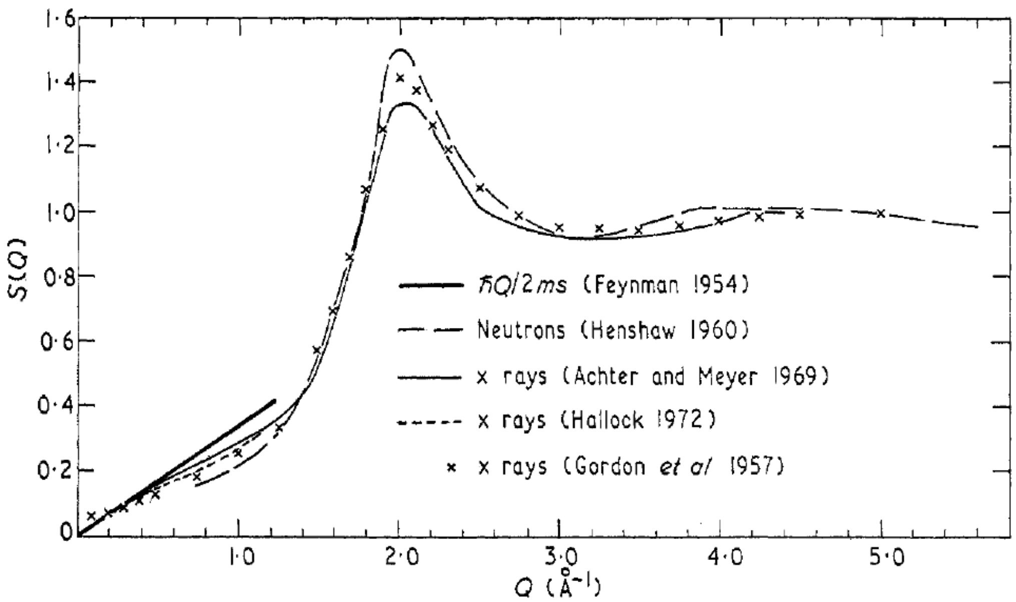


Figure 5. The static structure factor, $S(Q)$, determined by various measurements

Woods et al.,Rep. Prog. Phys. 36, 1135 (1973)

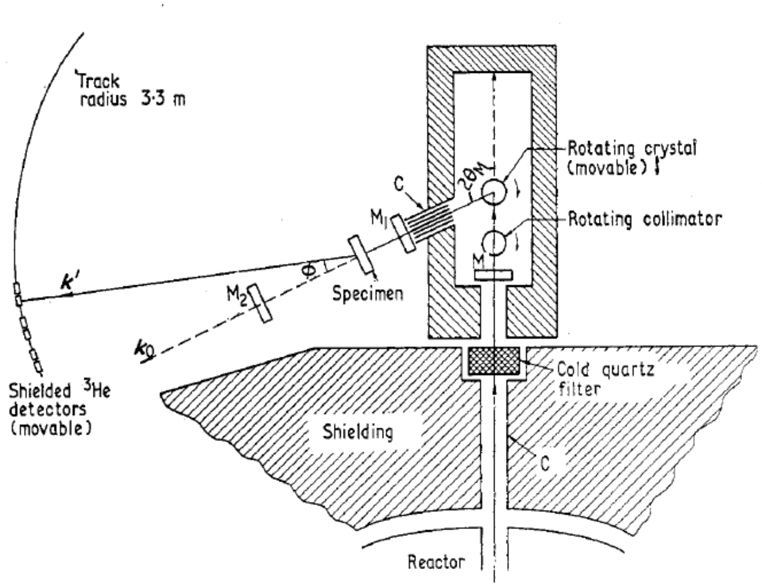


Figure 2. Schematic diagram of rotating crystal spectrometer (RCS). The collimators are denoted by C and monitor counters by M, M_1 and M_2. From Cowley and Woods (1971).

The End

Thank you for your attention!