Shear Viscosity from Black Holes

Kai Schweda
Physikalisches Institut
University of Heidelberg

HELMHOLTZ GEMEINSCHAFT

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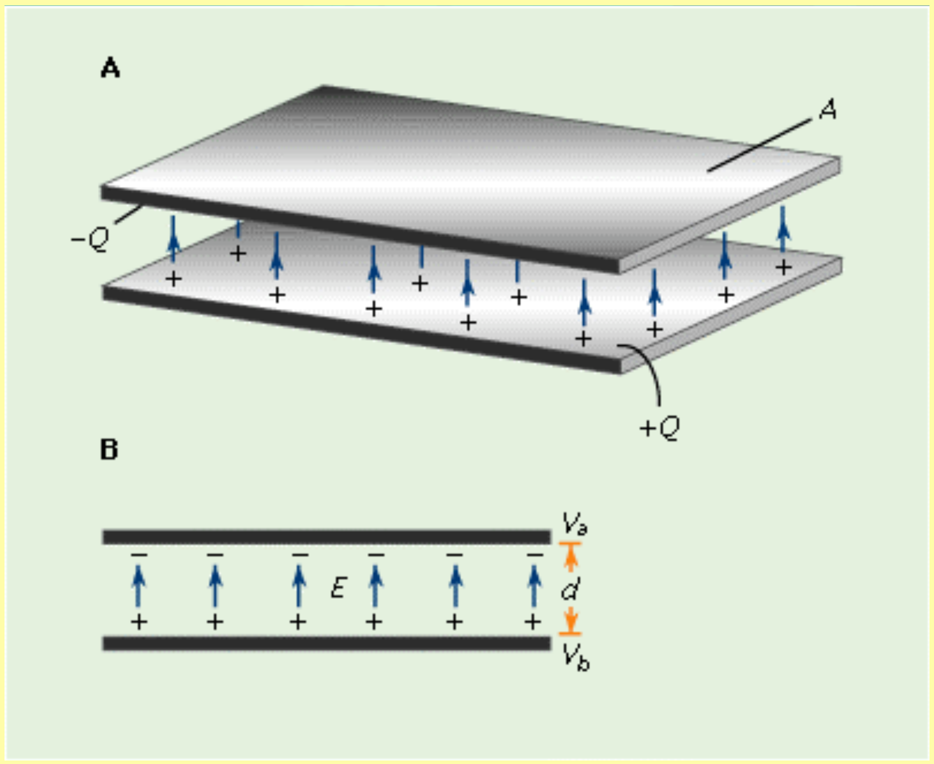
General Considerations

- Strong coupling → quantum effects large
- Use AdS/CFT correspondence
- Holographic duality: relate string theory of higher dimension to 4-d gauge theory on the boundary
- Limit of strong coupling: string theory → classical gravity (GR)

Parallel Plate Capacitor



Source: http://www.britannica.com

- Bulk: 3-d space between plates
- Fluctuations of the field in the bulk induce fluctuations of electric charges on the surface (boundary)
- Correlations of surface charges correlated to bulk field

Black Hole

Source: http://media.photobucket.com

- Black hole, mass M
- Temp. $T = \dfrac{\hbar c^3}{8\pi G M k_B}$
- Entropy
  $S = A/4 \cdot (k_B c^3/G\hbar)$
  A: area of horizon of boundary
- Physics of the interior region projected onto boundary: hologram

Holographic Principle

- Conjectured by ‘t Hooft
- Quantum gravity in (d+1) dimensions $\Leftrightarrow$

  equivalent theory living on d-dimensional boundary

  $\Rightarrow$ holographic dual

AdS/CFT Correspondence

- Fields that propagate in the bulk have well defined values at asymptotic infinity (boundary)
- Asymptotic values behave like field and coupling at the boundary
- Anti-de Sitter spacetime: negative curvature
- Holographic duals are sometimes gauge theories
- E.g. $AdS_5 \Leftrightarrow N=4$ Super Yang-Mills

$AdS_5 \times S_5$ Geometry

- AdS_5: 5 dimensional Anti-de-Sitter space
- Infinitesimal line element

$$ds^2 = \frac{r^2}{L^2}(-dt^2 + dx^2) + \frac{L^2}{r^2}dr^2 + L^2 d\Omega_5^2$$

S_5: 5 dimensional sphere, neglect

- r: radial coordinate
- R = const.: 3+1 dim. flat Minkowski space
- R $\to \infty$: boundary
- L: curvature radius

$AdS_5 \times S_5$ Geometry, cont'ed

$$ds^2 = \frac{r^2}{L^2}(-dt^2 + dx^2) + \frac{L^2}{r^2}dr^2$$

- Require L >> l_s, (classical approx.)
- ‘t Hooft coupling: $\lambda = g^2_{YM} N_c$
- $(L/l_s)^4 = \lambda$
- Classical approx. works at strong coupling

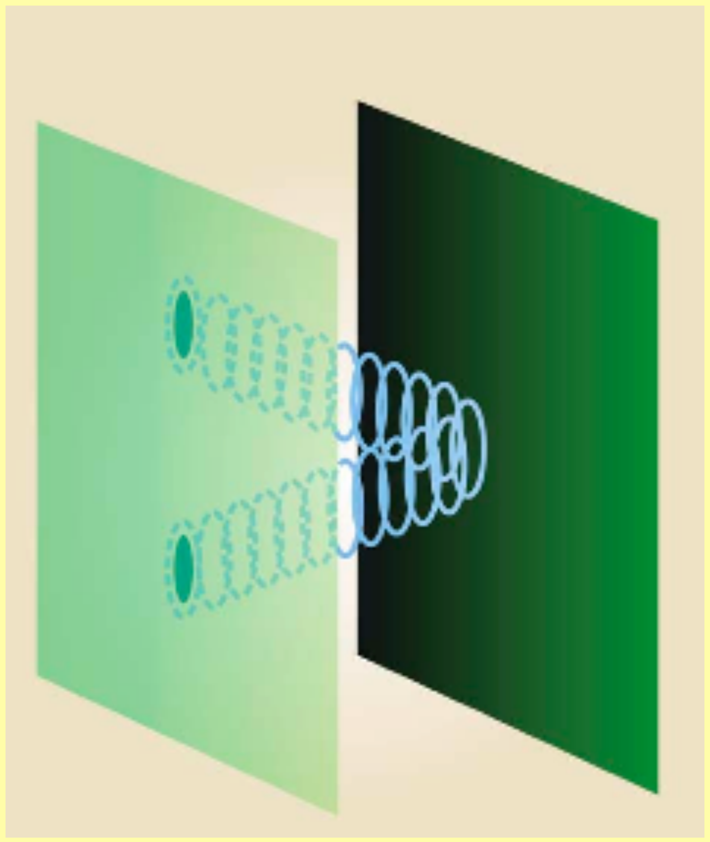
$AdS_5 \times S_5$ Geometry, cont'ed

- Rewrite for AdS_5 black hole

$$ds^2 = \frac{(\pi TL)^2}{u}(-(1-u^2)dt^2 + dx^2) + \frac{L^2}{4u^2(1-u^2)}du^2$$

- $u = (r_0/r)$, r_0: Schwarzschild (horizon) radius
- Horizon at $u = 1$
- Boundary limit: $u = \varepsilon$, then $\varepsilon \to 0$

Ask the AdS/CFT Dictionary…



Source: Physics Today, p29, May 2010

- η from $T^{\mu\nu}$ (Kubo's formula)
- $T^{\mu\nu}$ corresponds to graviton $h^{\mu\nu}$
- Graviton is disturbance in $g_{\mu\nu}$
- Graviton at boundary propagates in the bulk and is scattered back
- Cross section $\propto$ surface A
- Entropy s $\propto$ surface A
- η/s does not depend on A

KSS bound on η/s

$$\frac{\eta}{s} \geqq \frac{1}{4\pi} \cdot \frac{\hbar}{k_B} \left\{ 1 + \frac{15\zeta(3)}{\lambda^{3/2}} + \dots \right\}$$

Classical approximation

Conjectured lower bound
from string theory

$$\frac{\eta}{s} = \frac{1}{4\pi} \cdot \frac{\hbar}{k_B} \left(1 - \frac{1}{2N_C} \right)$$

Potentially lower bound from SU(2)

Some remarks

- Relativistic fluid, but bound does not depend on speed of light
- N=4 Super Yang-Mills is **not** QCD
- N_c = 3, not large
- No confinement
- Quarks are massless
- However, details might not matter too much, system driven by temperature and degrees of freedom

References

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