η/s in QCD from kinetic theory

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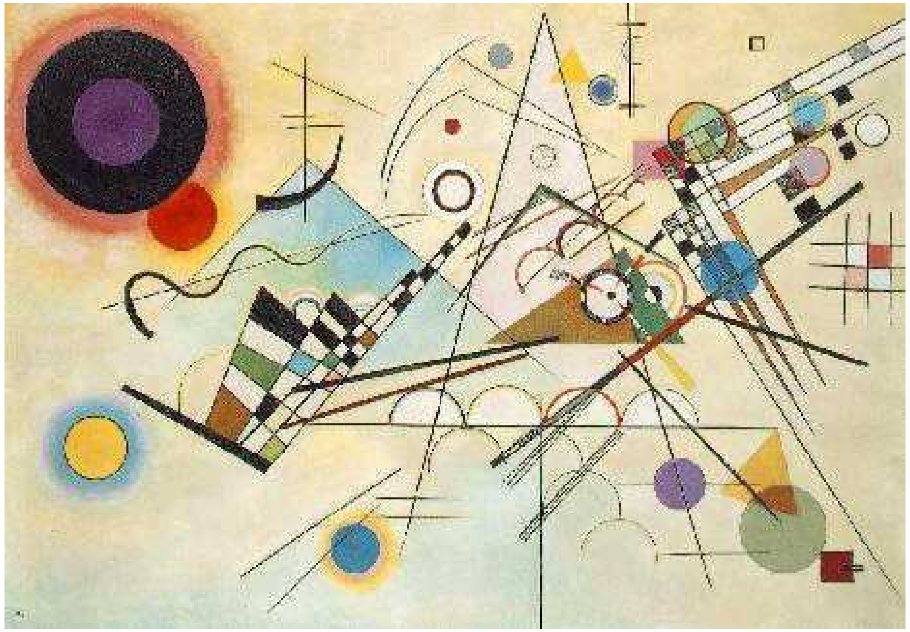


EMMI seminar on Quark-Gluon Plasma and Cold Atoms

Outline

- Basics of kinetic theory
- Spontaneous symmetry breaking and pions in QCD
- η/s from perturbative QCD
- Comparison with $\mathcal{N} = 4$ Super-Yang-Mills

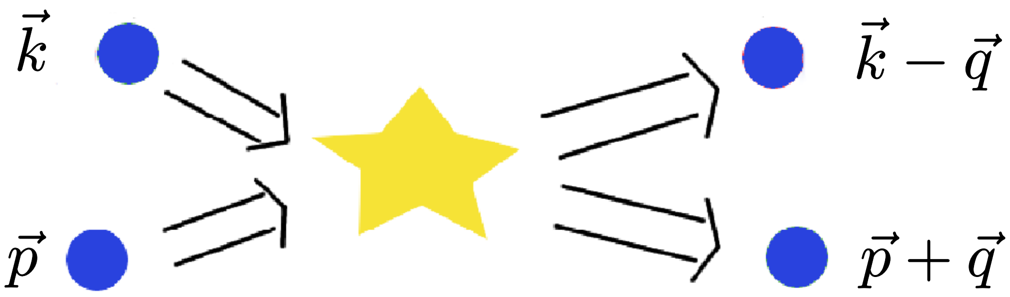
Basics of kinetic theory

- In kinetic theory microscopic quasiparticles evolve freely with localized collisions
- Evolution in phase space is governed by the Boltzmann equation

$$\frac{\partial f_{\vec{p}}}{\partial t} + \vec{v} \cdot \vec{\nabla} f_{\vec{p}} + \vec{F} \cdot \vec{\nabla}_{\vec{p}} f_{\vec{p}} = C[f_{\vec{p}}]$$

- Collision integral

$$C[f_{\vec{p}}] = \int d\vec{k} d\vec{q} \underbrace{P}_{\sim \frac{d\sigma}{dq}} (f_{\vec{p}+\vec{q}} f_{\vec{k}-\vec{q}} - f_{\vec{p}} f_{\vec{k}})$$



Spontaneous symmetry breaking

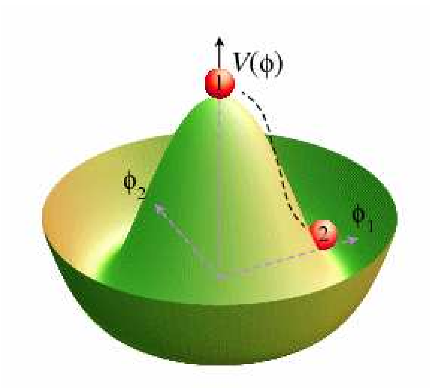
- Simple nonrelativistic example

$$\mathcal{L} = \phi^*(i\partial_t + \frac{\Delta}{2M})\phi + V(\phi^*\phi)$$

- Global U(1) symmetry $\rightarrow$ particle number conservation

$$\phi \rightarrow e^{i\alpha}\phi \qquad \phi^* \rightarrow e^{-i\alpha}\phi^*$$

- Symmetry is spontaneously broken if vacuum is not symmetric
$\langle 0|\phi|0\rangle \neq 0$



- Every broken symmetry $\rightarrow$ gapless Goldstone mode

Effective theory of pions

- QCD has approximate SU(2) chiral symmetry$\leftrightarrow$ u and d are light
- Symmetry is spontaneously broken

$$\langle 0|\bar{\psi}\psi|0\rangle \neq 0$$

- Pions are associated (pseudo)Goldstone bosons
- Low energy effective chiral Lagrangian for pions ϕ^a

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^a)^2 - \frac{1}{2}m_\pi^2(\phi^a)^2 + \frac{1}{6f_\pi^2}\left[(\phi^a\partial_\mu\phi^a)^2 - (\phi^a)^2(\partial_\mu\phi^b)^2\right] + \ldots$$

$$m_\pi \approx 140\text{MeV} \qquad f_\pi \approx 93\text{MeV}$$

- This is similar to effective theory of phonons in cold atoms

η/s from chiral effective theory

- Calculations are simplest in the chiral limit $m_\pi \to 0$
- In the effective theory the $\pi\pi$ cross section

$$\sigma \sim \frac{s}{f_\pi^4} \to \langle\sigma\rangle_T \sim \frac{T^2}{f_\pi^4}$$

- In kinetic theory

$$\eta = \frac{1}{3} n p l_{mfp} \sim \frac{T}{\langle\sigma\rangle_T} \qquad l_{mfp} \sim \frac{1}{n\langle\sigma\rangle_T}$$

- From the Stefan-Boltzmann law

$$s = 3\frac{2\pi^2}{45}T^3$$

- Explicit calculation gives at low energies

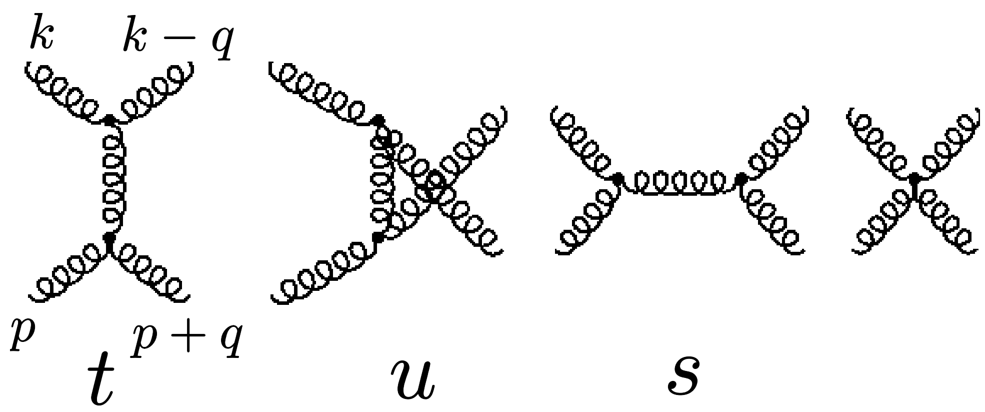
$$\frac{\eta}{s} = \frac{15}{16\pi}\frac{f_\pi^4}{T^4}$$

Pure gauge theory

- First step towards QCD– pure $SU(N_c)$ gauge theory

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2$$

- In kinetic theory η is obtained from binary gluon scatterings



- Scattering amplitude squared

$$|\mathcal{M}|^2 = \frac{9g^4}{2}\left(3 - \frac{ut}{s^2} - \frac{us}{t^2} - \frac{ts}{u^2}\right)$$

Problem with scattering amplitude

- $|\mathcal{M}|^2$ diverges at small momentum transfer as $\frac{1}{q^4} \sim \frac{1}{\theta^4}$
- Familiar from soft Rutherford scattering
- Apparent problem in calculation of η
- Screening in quark-gluon plasma at $T > 0$

$$\Pi^{abL}(q) = \frac{\delta^{ab}}{\vec{q}^2 + m_D^2} \rightarrow \text{electric}$$

$$\Pi^{abT}(q) = \frac{\delta^{ab}}{\vec{q}^2 + i\frac{\pi}{4} m_D^2 \frac{\omega}{|\vec{q}|}} \rightarrow \text{magnetic}$$

with the Debye mass $m_D^2 = g^2 T^2 \left(1 + \frac{N_f}{6}\right)$

η and s in perturbative QCD

- Perturbation theory predicts for pure gauge theory

$$\eta = k \frac{T^3}{g^4(T) \log(\mu^*/m_D)} \qquad k = 27.13$$

- μ^* is calculated from s- and u-channel diagrams and from gluon bremsstrahlung

$$\mu^* \approx 2.765 T$$

- In QCD theory
  qq and qg scattering in t-channel $\longrightarrow$ affects only k and μ^*
- Entropy– free gas of massless quarks and gluons

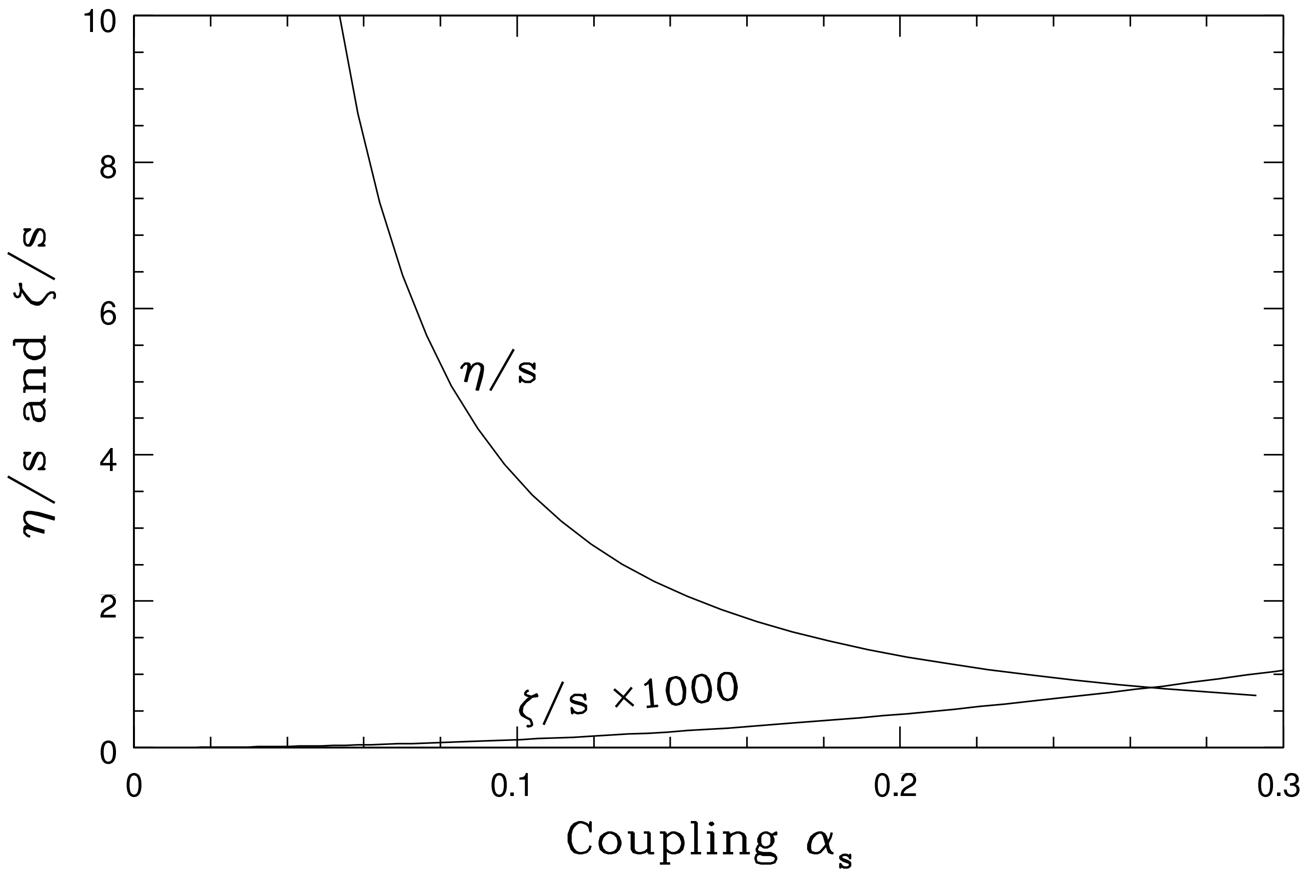
$$s = \frac{2\pi^2}{45} \left(2(N_c^2 - 1) + \frac{7}{8} 4 N_f \right) T^3$$

η/s in perturbative QCD

- From perturbative QCD with three massless quarks

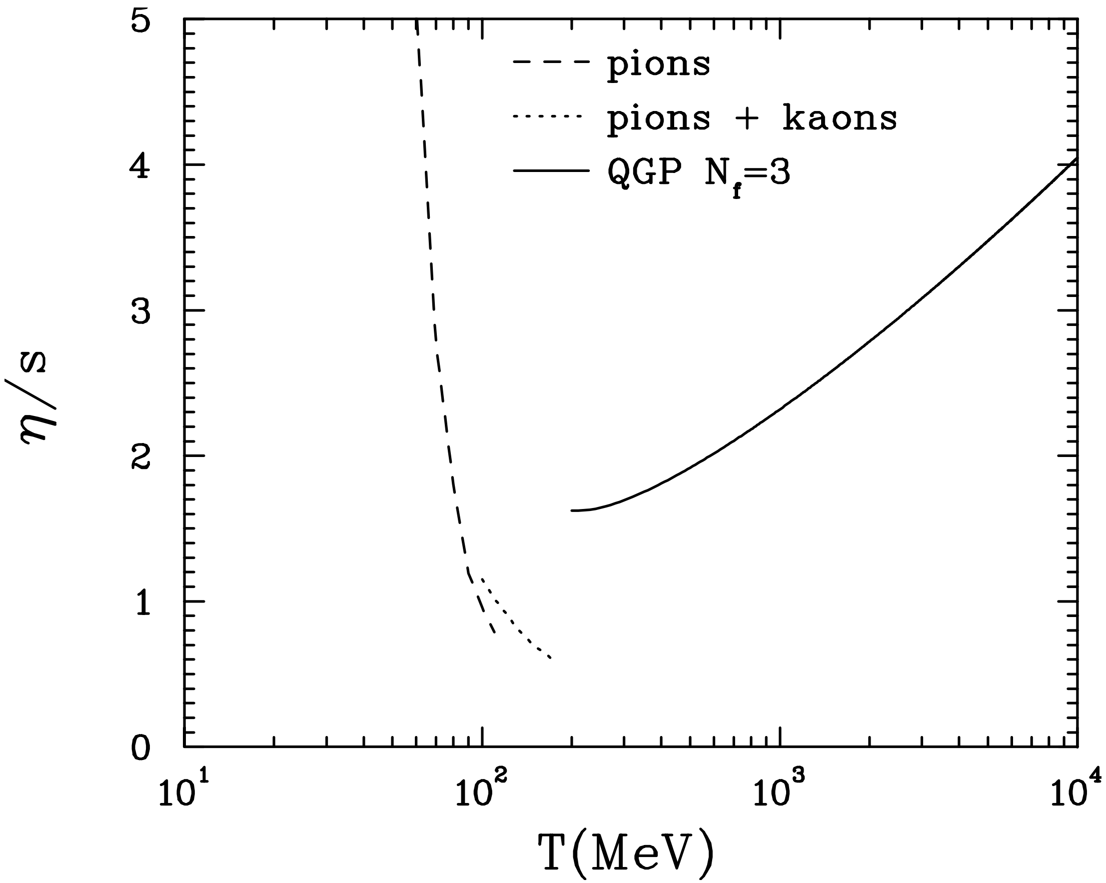
$$\frac{\eta}{s} = \frac{5.12}{g^4(T)\ln(2.42/g(T))}$$

- η/s as a function of coupling $\alpha_s = g^2/(4\pi)$



η/s in perturbative QCD

- η/s as a function of temperature T



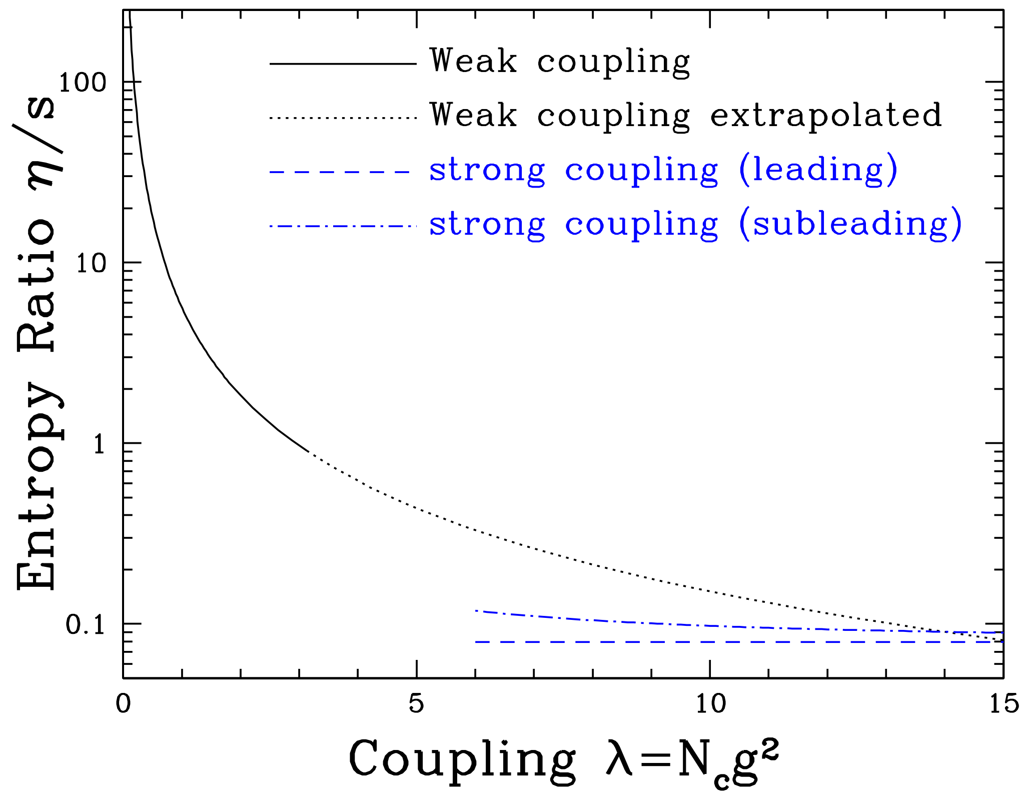
- Minimum for $T \approx T_c$
- Resembles the result for cold fermions at unitarity!

$\mathcal{N} = 4$ Super-Yang-Mills theory

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - i\bar{\lambda}^a_i \sigma^\mu D_\mu \lambda^a_i + D^\mu \phi^{\dagger\, a}_{ij} D_\mu \phi^a_{ij} + \mathcal{L}_{\lambda\lambda\phi} + \mathcal{L}_{\phi^4}$$

- $F^a_{\mu\nu}$– field strength tensor
- λ^a_i– fermionic gluino fields
- ϕ^a_{ij}– colored Higgs fields
- D_μ– covariant derivative
- This theory has a lot of symmetries
  - Lorentz spacetime $\rightarrow$ conformal
  - supersymmetry
  - $SU(N_c)$ gauge
  - $SU(4)$ global R-symmetry

η/s in $\mathcal{N} = 4$ Super-Yang-Mills theory



- Weak coupling calculation from kinetic theory
- Strong coupling calculation from AdS/CFT correspondence $\rightarrow$ KSS bound
- Weak coupling extrapolation breaks down at $\lambda \approx 12$