Phase-Transitions in Ultracold Atomic Gases

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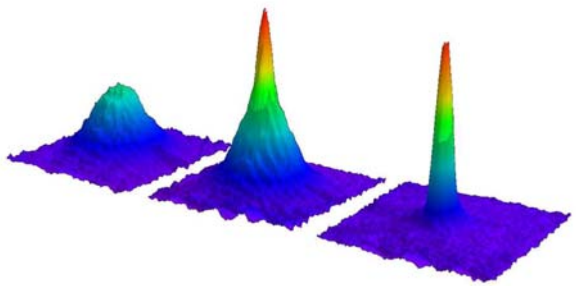
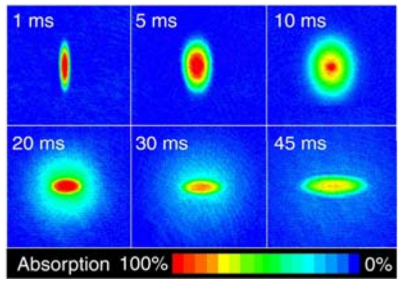
Overview

- Thermal cloud ↔ BEC
- Ultracold atoms in optical lattices:
  - Superfluidity ↔ Mott-Insulator (Bosonic - Fermionic)
  - Supersolid
- BCS-BEC-Crossover
- Critical behavior of strongly interacting ultracold Rydberg Gases
- Self-organization of BEC in a cavity field
- New phases in dipolar quantum gases

Phase-Transition to BEC (1)

- 1995 first Observation of BEC





http://cua.mit.edu/ketterle_group/Nice_pics.htm

BUT: Not much known quantitatively about the transition itself !

- What do we want to know about the transition?

$$\frac{\langle n_{k=0}\rangle}{N_{\text{total}}} = 1 - \left(\frac{T}{T_C}\right)^{\alpha}$$

or

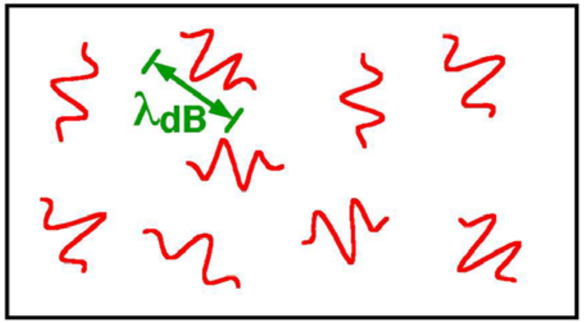
$$\xi \propto \left(\frac{T - T_C}{T_C}\right)^{-\nu}$$

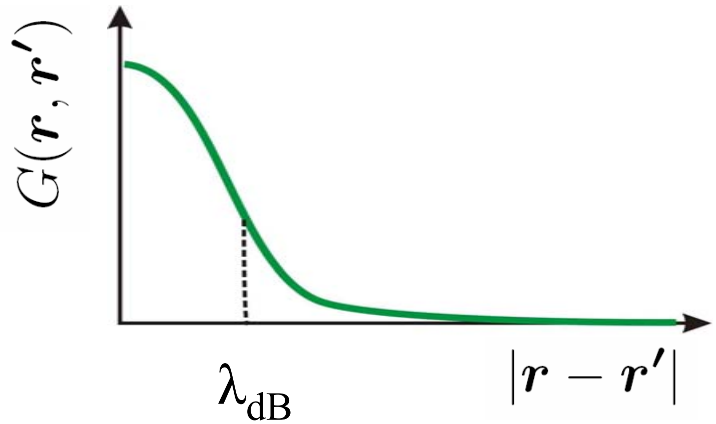
T. Donner *et al*, Science **315**, 1556 (2007)

Phase-Transition to BEC (2)

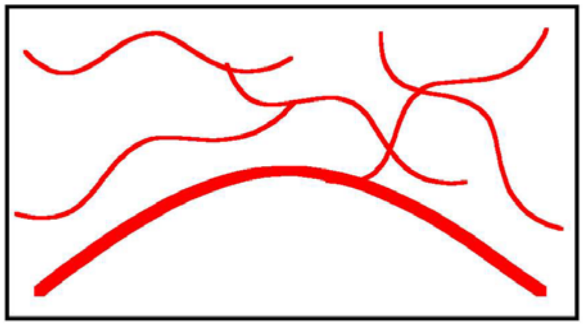
- Correlation function $G(\boldsymbol{r}, \boldsymbol{r}') = \left\langle \hat{\Psi}^\dagger(\boldsymbol{r}) \hat{\Psi}(\boldsymbol{r}') \right\rangle$

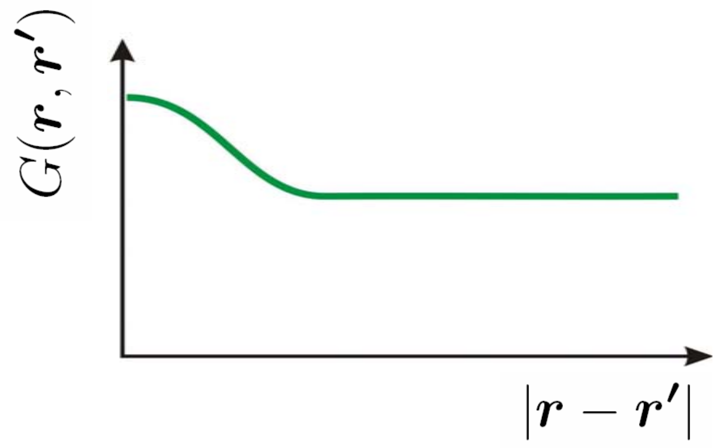
$T>T_C$





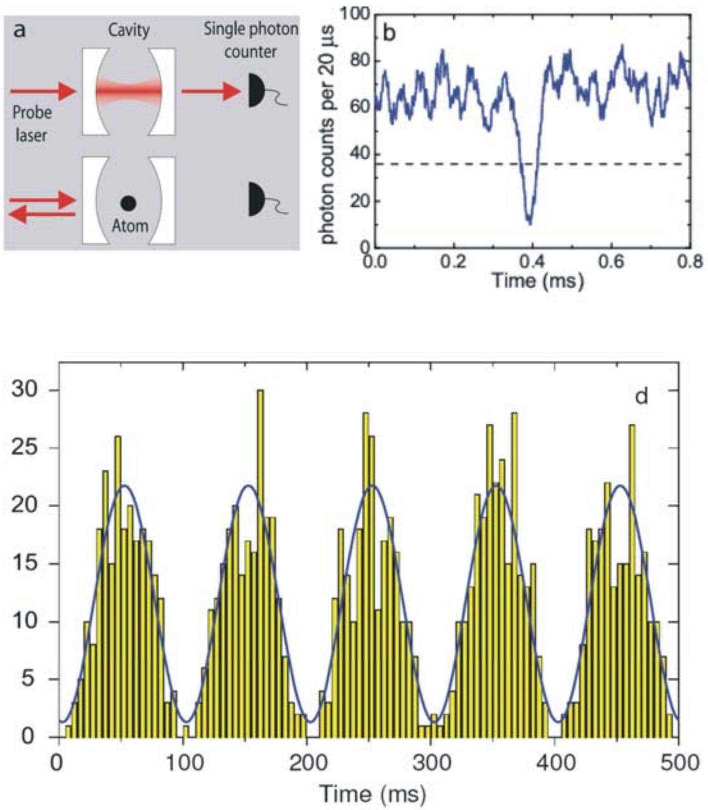
$T<T_C$

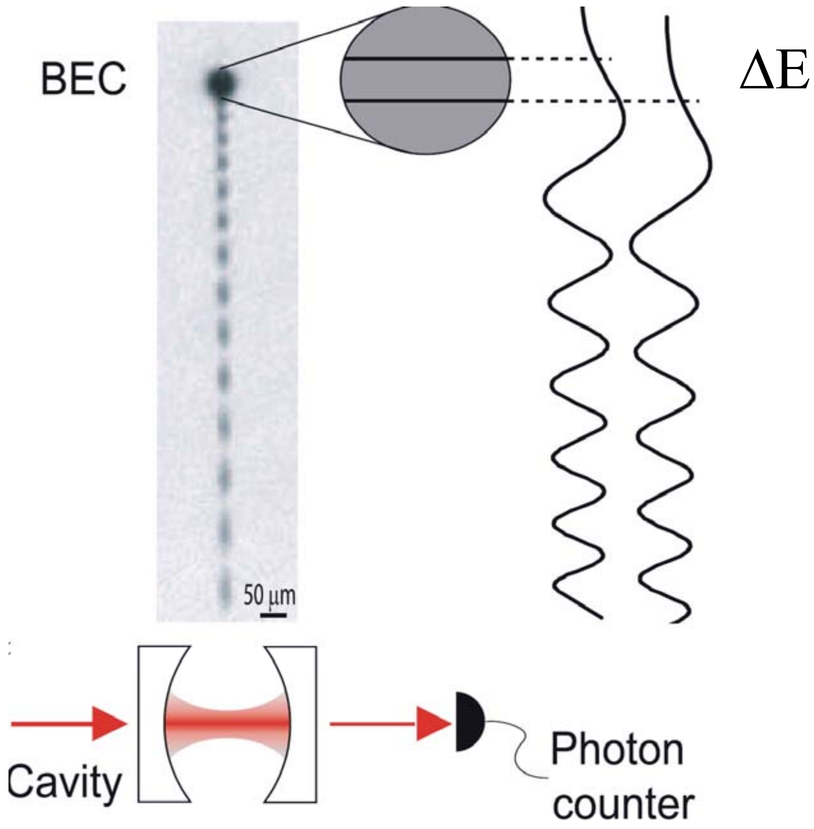




Phase-Transition to BEC (3)

- Double-slit experiment with matter-waves



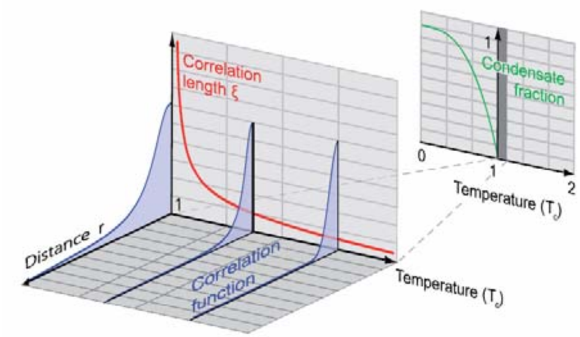


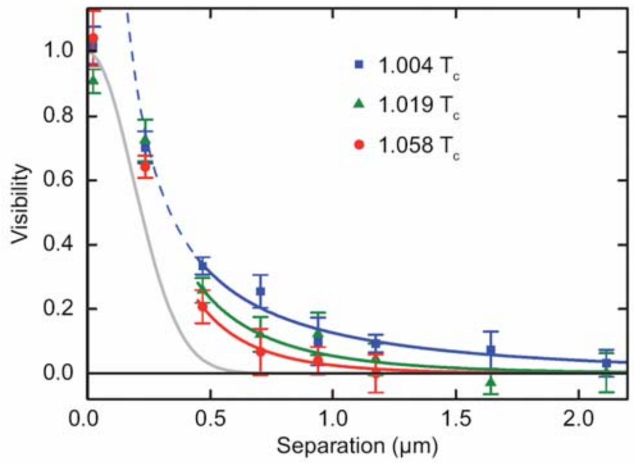
T. Bourdel et al, Phys. Rev. A **73**, 043602 (2006)

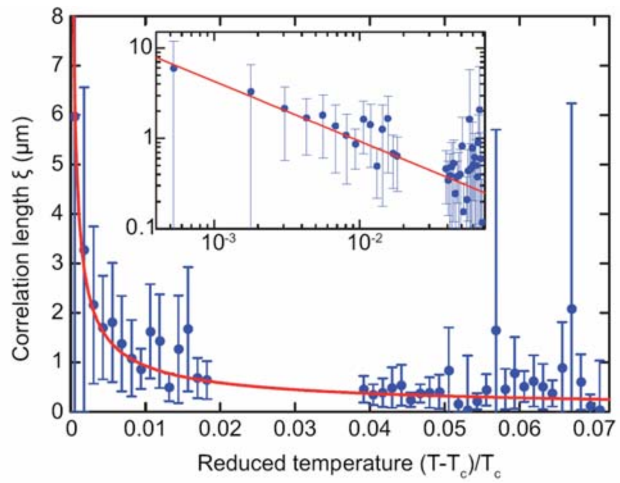
Phase-Transition to BEC (4)

- Correlation function above T_C

$$G(\boldsymbol{r},\boldsymbol{r'}) = \left\langle \hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r'}) \right\rangle \propto \frac{1}{|\boldsymbol{r}-\boldsymbol{r'}|} e^{-\frac{|\boldsymbol{r}-\boldsymbol{r'}|}{\xi}}$$







T. Donner *et al*, Science **315**, 1556 (2007)

$$\boxed{\nu = 0.67 \pm 0.13}$$

Phase-Transition to BEC (5)

- Direct measurement of the critical exponent

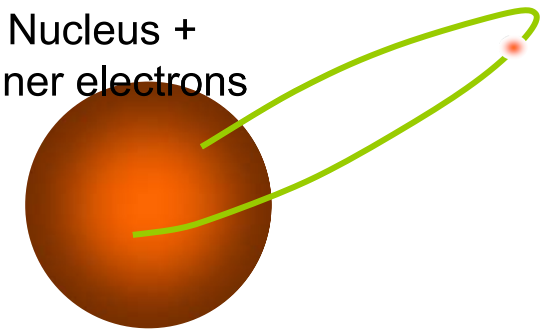
- Confirms universality:
  - weakly interacting Bose-gas
  - liquid ^{4}He
  - 3D-XY-Model Magnet

- Renormalization Theory: $\nu = 0.67$
- ^{4}He (spaceborne experiment): $\nu = 0.67005$
- Weakly interacting Bose-gas : $\nu = 0.67 \pm 0.13$

Critical Behavior in Rydberg Gases (1)

Weimer *et al, Phys. Rev. Lett.* ***101*** *250601 (2008)*

- **Rydberg-atom:**
  - atom with an electron in an highly exited shell
  - large dipole moment $d \sim n^2$
  - large polarizability $p \sim n^7$



→ Strong interactions among Rydberg-atoms:

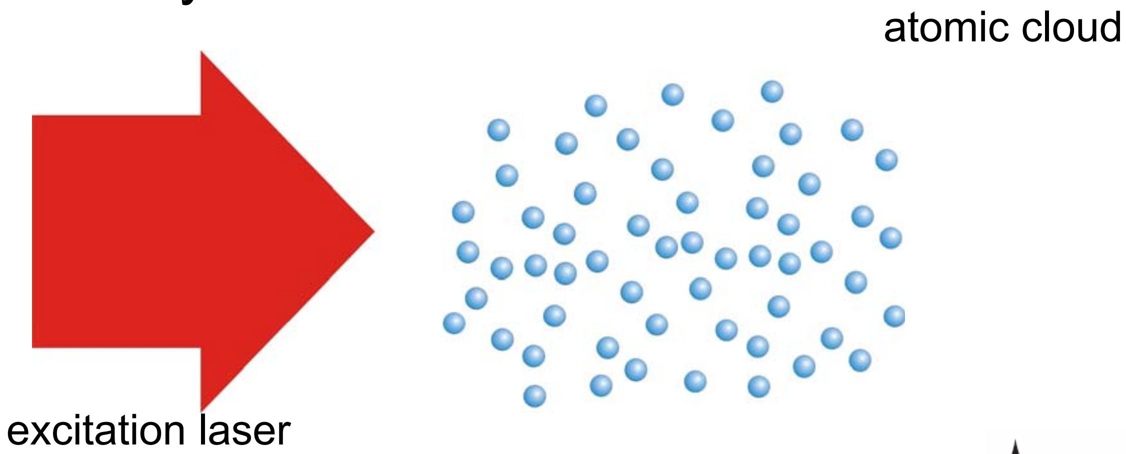
- Van-der-Waals interaction:

$$V(\boldsymbol{r}, \boldsymbol{r}') = \frac{C_6}{|\boldsymbol{r} - \boldsymbol{r}'|^6}$$

with $C_6 \sim n^{11}$

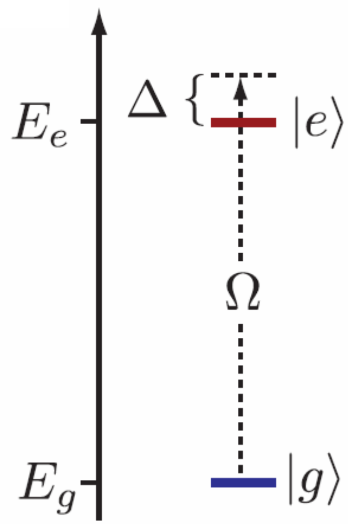
Critical Behavior in Rydberg Gases (2)

- Sketch of the system



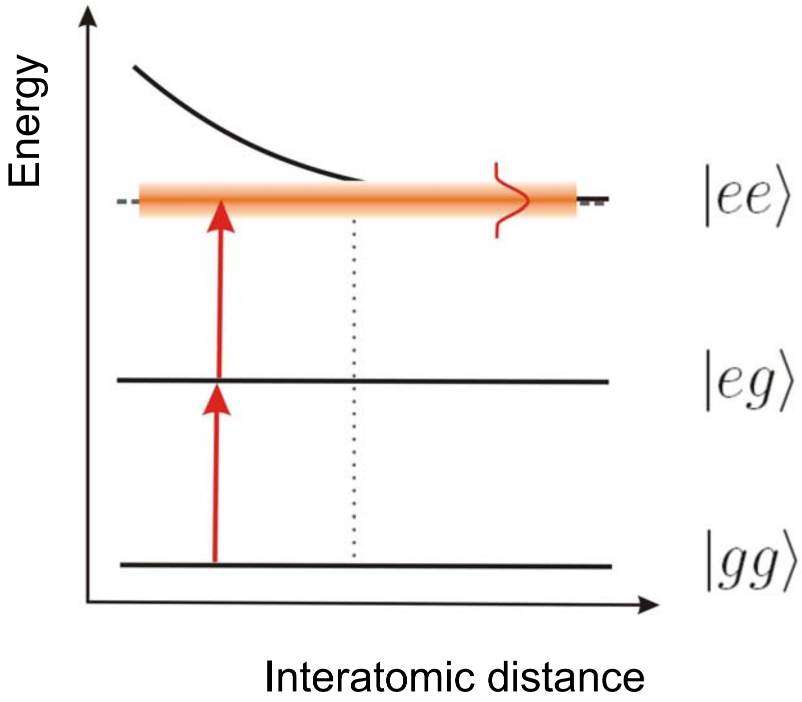
- Driven 2-level system:

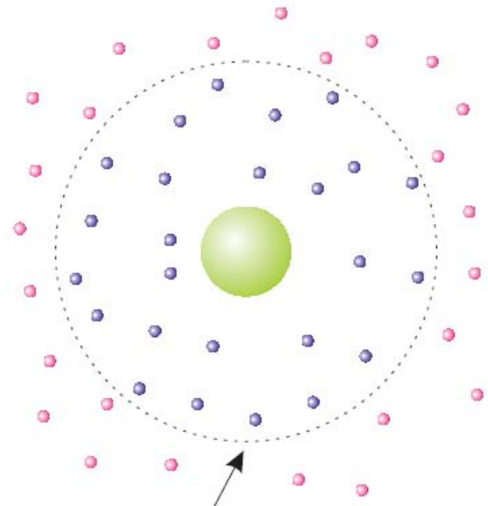
  - Δ detuning
  - Ω Rabi-frequency



Critical Behavior in Rydberg Gases (3)

- Effects due to interaction: **Blockade**





- State of the system: $$|\Psi\rangle = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} |g, g, .., g, e, g, .., g\rangle$$

Critical Behavior in Rydberg Gases (4)

- Hamiltonian of the system

$$\hat{H} = -\frac{\Delta}{2}\sum_i \hat{\sigma}_z^{(i)} + \cancel{\frac{\hbar\Omega}{2}\sum_i \hat{\sigma}_x^{(i)}} + C_6 \sum_{j<i} \frac{\hat{P}_{ee}^{(i)}\hat{P}_{ee}^{(j)}}{|\mathbf{r}_i - \mathbf{r}_j|^6}$$

with $\hat{\sigma}_z^{(i)} = |e\rangle\langle e|_i - |g\rangle\langle g|_i$

- Properties of the Hamiltonian for $\Omega = 0$

$\Delta < 0$: Paramagnet

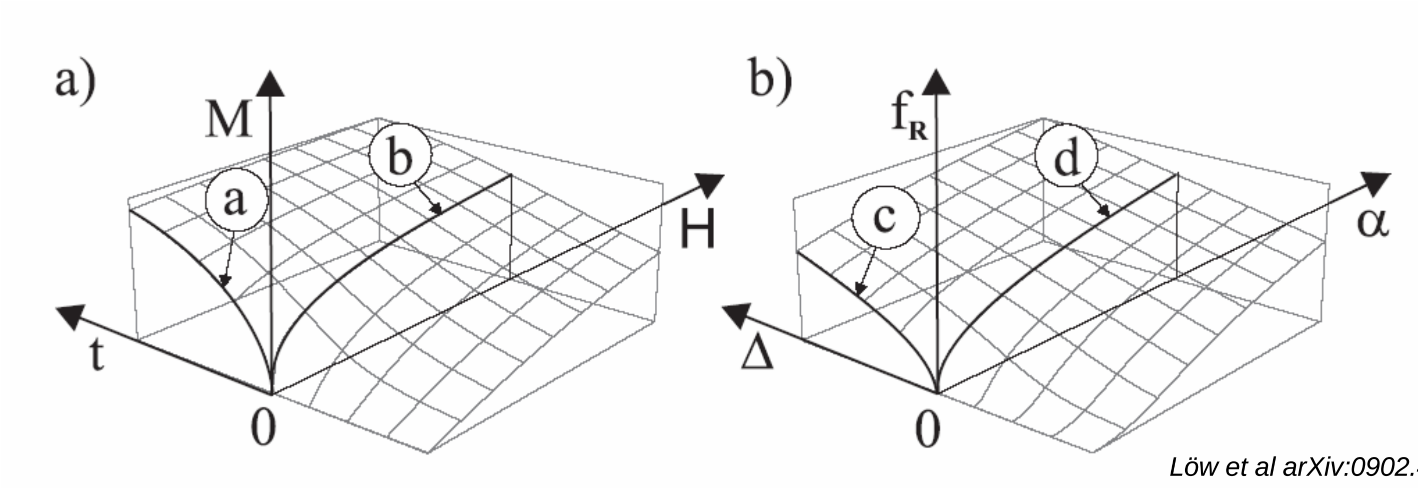
- all particles in the ground state
- $f_R = 0$
- translation symmetry

$\Delta > 0$: Crystalline Phase

- finite number of excitations
- $f_R > 0$
- translation symmetry is broken
- FCC-crystal ???

Critical Behavior in Rydberg Gases (5)

- Comparison: **Ferromagnet** – **Rydberg Gas**



Löw et al arXiv:0902.4523 (2009)

$M \sim t^{\beta}$	$f_R \sim \Delta^{\beta}$
$M \sim H^{1/\delta}$	$f_R \sim \Omega^{1/\delta}$

Critical Behavior in Rydberg Gases (6)

- Determine the critical exponents:

1) $\Omega = 0$: $\beta = ½$ (one can analytically find the ground-state of the system)

2) $\Delta = 0$: Meanfield-model

Rescaled Rabi-frequency $\alpha = \dfrac{\hbar\Omega}{C_6 n^2}$

Blockade: $\left\langle \hat{P}_{ee}^{(j)} \right\rangle \to g_2(\boldsymbol{r}_i, \boldsymbol{r}_j) \left\langle \hat{P}_{ee}^{(j)} \right\rangle$

Power law: $f_R \sim c\,\alpha^{1/\delta}$

Self-consistency: $\boxed{\delta = \dfrac{12 + d}{2d}}$

Critical Behavior in Rydberg Gases (7)

- Solve Schrödinger equation numerically:

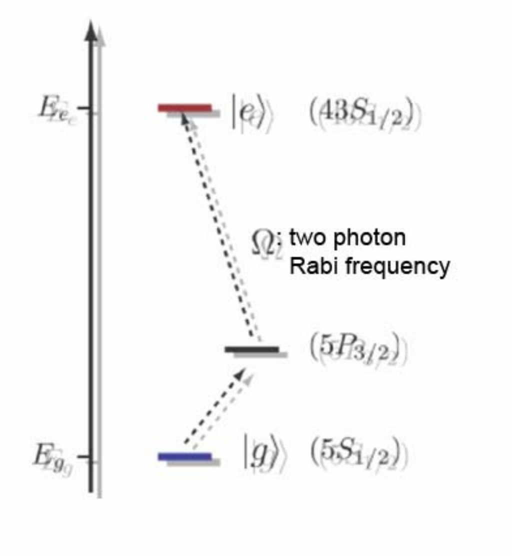
Numerics		Meanfield	
$\delta = (0.404)^{-1}$	3D	$\delta = (0.40)^{-1}$	3D
$\delta = (0.15)^{-1}$	1D	$\delta = (0.15)^{-1}$	1D

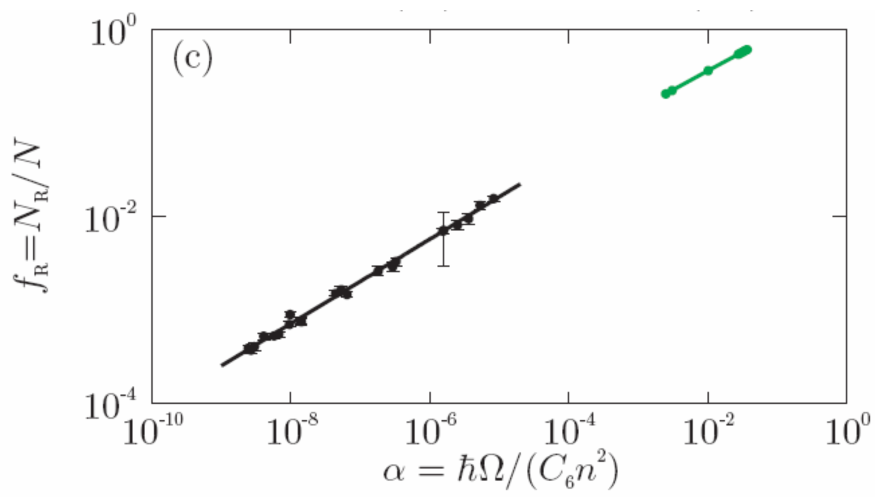
Critical Behavior in Rydberg Gases (8)

- Experiment:

a) ^{87}Rb ($5S_{1/2}$, F = 2, m_F = 2) , N = 1.5*10^7 atoms
b) Landau-Zener-Sweep: Remove particles $\rightarrow$N = 5*10^5 -1.5*10^7 atoms
c) Excite the atoms: $\Omega = 2\,\pi$ (31-154) kHz
d) Field-ionize and detect ions with MCP

Löw et al arXiv:0902.4523 (2009)





$\boxed{\delta^{-1} = 0.45 \pm 0.01 \ \text{(3D)} \qquad \delta^{-1} = 0.16 \pm 0.01 \ \text{(1D)}}$

Critical Behavior in Rydberg Gases (9)

- **Open questions**:

  - Why does meanfield work so well ?

  - Upper critical dimension $d_C = 1$?

  - New universality class ?!