Viscous Hydrodynamics in Heavy Ion Collisions

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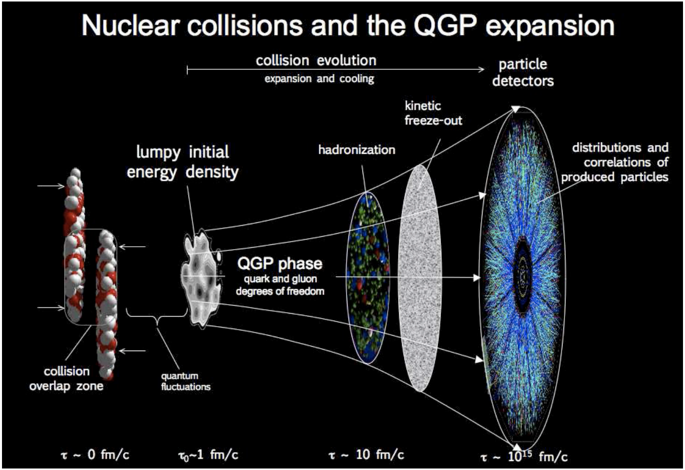
Outline

Stages of a Heavy Ion Collision

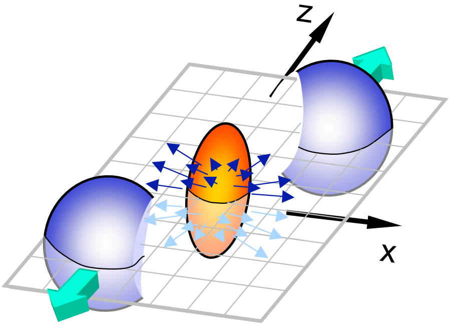
Collision of two Lorentz-contracted nuclei: $t_{\mathrm{coll}} \approx \frac{R}{\gamma}\frac{2}{c}$



Landau already considered hydrodynamics
for high-energy physics (1953)

Hydrodynamic Stage

- explains dynamics:
  pressure gradients
  $\Rightarrow$ expansion
- assumes local equilibration
  $\Rightarrow$ few variables
- amenable to numerics
- access to EoS
- easy to account for
  phase transitions
- fluid-like behaviour (one fluid)
  in contrast to thermal
  equilibrium



m_T spectra

- m_T scaling for indepent particles (thermal source):

$$\frac{\mathrm{d}^3 N}{\mathrm{d}p^3} \propto \exp\left\{-\frac{E}{T}\right\}$$

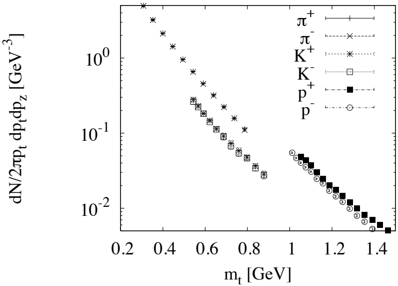
$$\mathrm{d}p^3 = E m_T \,\mathrm{d}m_T \,\mathrm{d}y \,\mathrm{d}\phi$$
$$E = m_T \cdot \cosh y$$

$$\frac{\mathrm{d}^2 N}{m_T \mathrm{d}m_T \,\mathrm{d}y} \propto$$

$$m_T \cdot \cosh y \cdot \exp\left\{-\frac{m_t \cosh y}{T}\right\}$$

- in AuAu: scaling violated → collective motion

pp



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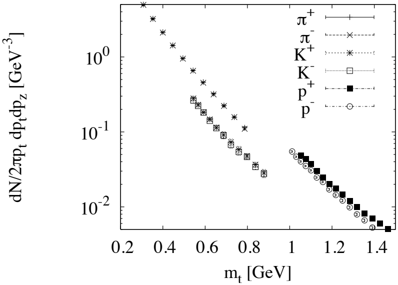
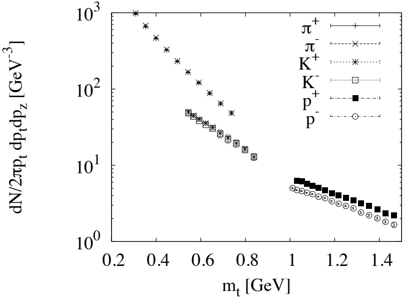


AuAu



Relativistic Hydrodynamics

- energy momentum tensor: $T^{\mu\nu}$
- energy momentum conservation:

$$\partial_\mu T^{\mu\nu} = 0$$

- in ideal hydrodynamics only dependent on ϵ, P.
- in local restframe:

$$T^{\mu\nu} = \begin{pmatrix} \epsilon & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix}$$

- in any frame:

$$T^{\mu\nu} = (\epsilon + P)\, u^\mu u^\nu - P\, g^{\mu\nu}$$

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Analogy to non-relativistic case

$$\partial_\mu T^{\mu\nu} = \partial_\mu \left((\epsilon + P)\, u^\mu u^\nu - P\, g^{\mu\nu} \right) = 0$$

- projection parallel to u^μ:

$$u_\nu \partial_\mu T^{\mu\nu} = \underbrace{D}_{:= u^\mu \partial_\mu} \epsilon + (\epsilon + p)\, \partial_\mu u^\mu = 0$$

- projection perpendicular to u^μ ($\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$):

$$\Delta^\alpha_\nu \partial_\mu T^{\mu\nu} = (\epsilon + p)\, D u^\alpha - \underbrace{\nabla^\alpha}_{:= \Delta^{\alpha\beta}\partial_\beta} p = 0$$

- non-relativistic limit:

$$\begin{aligned} D &\approx \partial_t + \vec{v} \cdot \vec{\nabla} + O(|\vec{v}|^2) \\ \nabla^i &\approx \partial^i + O(|\vec{v}|) \end{aligned}$$

Ideal Relativistic Hydrodynamics

- No dissipative processes:

$$\partial_\mu s^\mu = 0$$

- Additional continuity equations for conserved charges:

$$\partial_\mu N^\mu = 0$$

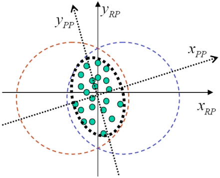
- Equation of State needed, e.g. for ultrarelativistic gas:

$$P = \frac{\epsilon}{3}$$

- Prediction of momentum anisotropy from initial spatial anisotropy

Flow - Introduction

- azimuthal anisotropy of non-central events
- sensitive to early stages (high pressure gradients)
- reaction and participant plane



- in momentum space:

$$E\frac{d^3N}{d^3p} = \frac{1}{2\pi}\frac{d^2N}{p_T\, dp_T\, dy}$$

radial flow

- $v_n(p_T, \eta)$ for given $\sqrt{s_{NN}}$

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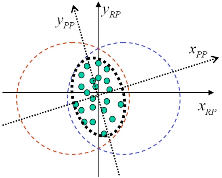

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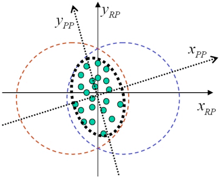

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$$E\frac{\mathrm{d}^3N}{\mathrm{d}^3p} = \frac{1}{2\pi}\frac{\mathrm{d}^2N}{p_\mathrm{T}\,\mathrm{d}p_\mathrm{T}\,\mathrm{d}y}\left(1 + \sum_{n=1}^{\infty} 2v_n\cos(n(\phi - \Psi_\mathrm{RP}))\right)$$

radial flow, directed flow v_1, elliptic flow v_2

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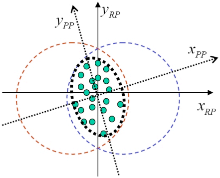

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- $v_n(p_\mathrm{T},\eta)$ for given $\sqrt{s_{NN}}$

Flow - Measurement

- Reconstruction from particle trajectories
- event-plane method:

$$Q_{n,\substack{x\\y}} = \sum_i w_i \substack{\cos\\\sin}(n\phi_i)$$

  reaction-plane:

$$\Psi_n = \frac{1}{n} \arctan \frac{Q_{n,y}}{Q_{n,x}}$$

$$v_n^{\mathrm{obs}} = \langle \cos\left[n(\phi_i - \Psi_n)\right] \rangle$$

- correlation method:

$$\frac{\mathrm{d}N^{\mathrm{pairs}}}{\mathrm{d}\Delta\phi} \propto \left(1 + \sum_{n=1}^{\infty} 2\, v_n^2\, \cos(n\Delta\phi)\right)$$

  fitted to data

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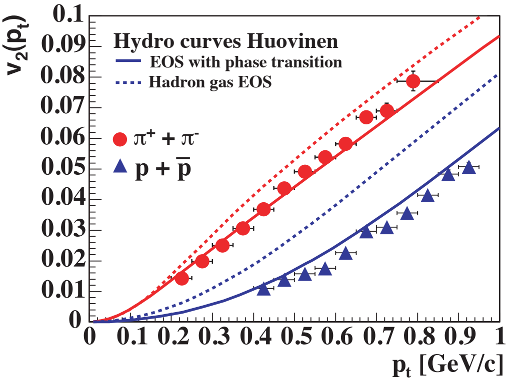
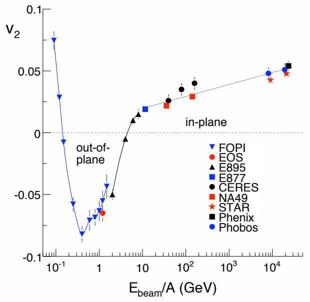
Mass splitting in v_2



- heavy particles shifted to higher p_{T}
- sensitive to equation of state

p_T- and $\sqrt{s_{NN}}$-dependence of v_2

- Change from out-of-plane (squeezing) to in-of-plane



- decreasing slope with increasing p_T
- hint for viscosity: $\frac{\eta}{s} > 0$

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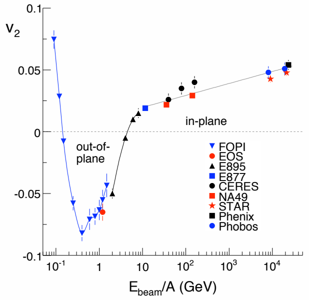

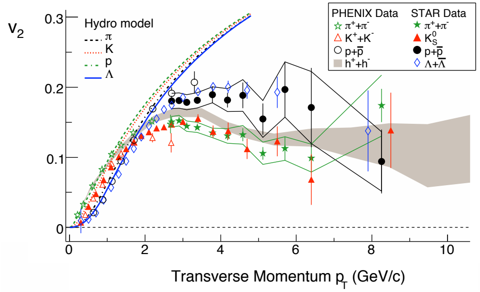

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Relativistic Hydrodynamics - Viscous

$$\partial_\mu T^{\mu\nu} = \partial_\mu \left(T^{\mu\nu}_{\text{ideal}} + \Pi^{\mu\nu} \right) = 0$$

$$\begin{aligned} D\epsilon + (\epsilon + p)\, \partial_\mu u^\mu - u_\nu \partial_\mu \Pi^{\mu\nu} &= 0 \\ (\epsilon + p)\, Du^\alpha - \nabla^\alpha p + \Delta^\alpha_\nu \partial_\mu \Pi^{\mu\nu} &= 0 \end{aligned}$$

- $\Pi^{\mu\nu} = \underbrace{\pi^{\mu\nu}}_{\text{shear}} + \underbrace{\Delta^{\mu\nu}\Pi}_{\text{bulk}}$
- require: $\partial_\mu s^\mu \geq 0$
  given for:
  $$\pi^{\mu\nu} = \eta\, \nabla^{<\mu} u^{\nu>}\,, \quad \Pi = \zeta\, \nabla_\alpha u^\alpha$$
- in non-relativistic limit $\rightarrow$ Navier-Stokes equation:
  $$\Pi^{ki} = \eta \left(\partial^k v^i + \partial^i v^k - \frac{2}{3} \delta^{ki} \partial_l v^l \right) - \zeta\, \delta^{ik} \partial_l v^l$$

Acausality problem

- Consider small perturbation in homogeneous system at rest:

$$\epsilon = \epsilon_0 + \delta\epsilon(t, x) \text{ and } u^\mu = (1, \vec{0}) + \delta u^\mu(t, x)$$

for y-direction leads to:

$$\partial_t \delta u^y - \frac{\eta_0}{\epsilon_0 + p_0} \partial_x^2 \delta u^y = O(\delta^2)$$

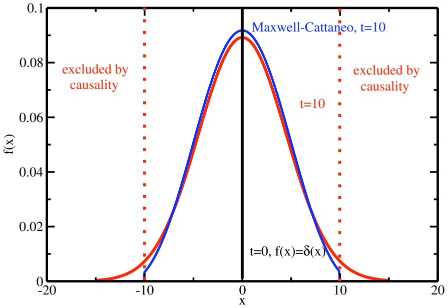
ansatz: $\delta u^y = e^{-\omega t + ikx} f_{\omega,k}$

$$\Rightarrow \omega = \frac{\eta_0}{\epsilon_0 + p_0} k^2$$

$$v_T(k) = \frac{\partial \omega}{\partial k} = 2 \frac{\eta_0}{\epsilon_0 + p_0} k \stackrel{k \to \infty}{\to} \infty$$

- perturbations travel faster than c

Acausality (cont.)



- Maxwell-Cattaneo:

$$\tau_\pi \partial_t^2 \delta u^y + \partial_t \delta u^y - \frac{\eta_0}{\epsilon_0 + p_0} \partial_x^2 \delta u^y = 0$$

  "artificial" solution
- higher orders in viscosity

Higher orders in viscosity

- classification as gradient expansion
- ideal hydro, zeroth order (complete)

$$\pi^{\mu\nu} = 0$$

- Navier-Stokes equation, first order (complete)

$$\pi^{\mu\nu} = \eta \, \nabla^{<\mu} u^{\nu>}$$

- second order:
  complete $\pi^{\mu\nu}$ constructable from symmetry considerations

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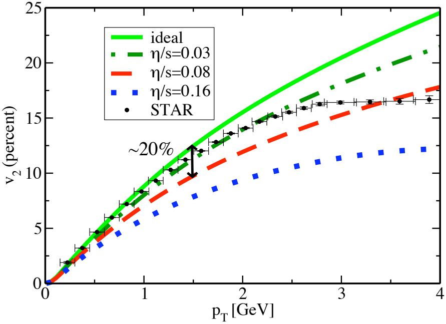
Numerical implementation

- calculations on discretized space-time
- ideal hydro: turbulences
  discretization of space-time (and derivations) adds numerical viscosity
- viscous hydrodynamics (in first order) acausal $\Rightarrow$ numerically problematic
- higher order viscous hydrodynamics well-behaved

Viscous v_2

- Significant reduction of v_2



- Indication for small viscosity close to theoretical boundary of $\frac{\eta}{s} = \frac{1}{4\pi}$

Caveats

- Determination of initial conditions:
  Monte-Carlo Glauber calculations
- Freeze-out
  at some temperature trying to match $T^{\mu\nu}$
- How to disentangle different stages?

Conclusions

- Hydrodynamic description of Heavy Ion Collisions promising
- Viscosity seems to be needed:
  - How large is η?
  - bulk viscosity?
- Constraints from theory (next talks)
- Caveats for quantitative interpretations

References

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Elliptic Flow: A study of space-momentum correlations in relativistic nuclear collisions
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New Developments in Relativistic Viscous Hydrodynamics
http://arxiv.org/abs/0902.3663

Landau vs. Eckart restframe

- with conserved charges restframe not uniquely defined any more
- Landau:
  Consider energy flos

$$u_\mu T^{\mu\nu} = \epsilon u^\nu$$

- Eckart:
  Consider conserved charge:

$$u_\mu J^\mu = j^0$$

$\pi^{\mu\nu}$ in second order

not to be discussed here:

$$\pi^{\mu\nu} = \eta\nabla^{<\mu}u^{\nu>} - \tau_\pi\left[\Delta^\mu_\alpha\Delta^\nu_\beta u^\lambda\partial_\lambda\pi^{\alpha\beta} + \frac{4}{3}\pi^{\mu\nu}(\nabla_\alpha u^\alpha)\right] + \frac{\kappa}{2}\left[R^{<\mu\nu>} + 2u_\alpha R^{\alpha<\mu\nu>\beta}u_\beta\right] - \frac{\lambda_1}{2\eta^2}\pi^{<\mu}_\lambda\pi^{\nu>\lambda} - \frac{\lambda_2}{2\eta}\pi^{<\mu}_\lambda\Omega^{\nu>\lambda} - \frac{\lambda_3}{2}\Omega^{<\mu}_\lambda\Omega^{\nu>\lambda}$$

with:

$$\Omega_{\mu\nu} = \nabla_{[\mu}u_{\nu]}$$

by Baier et al., 2007