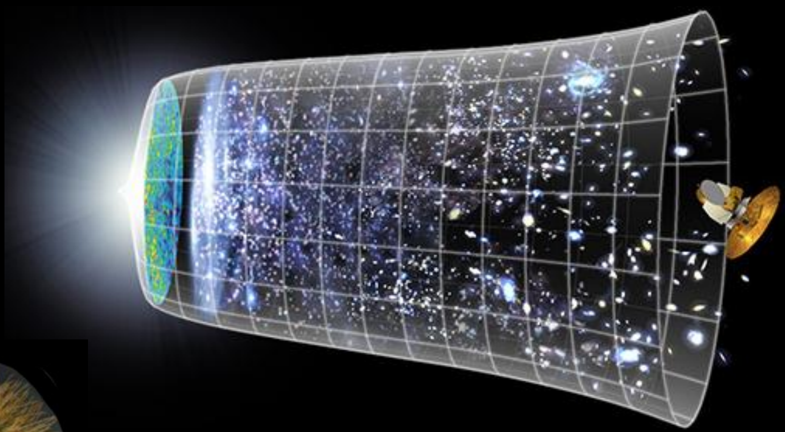
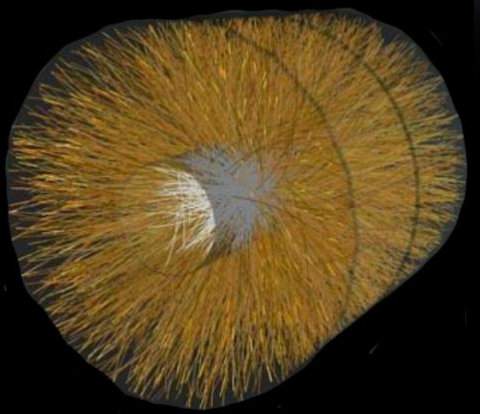
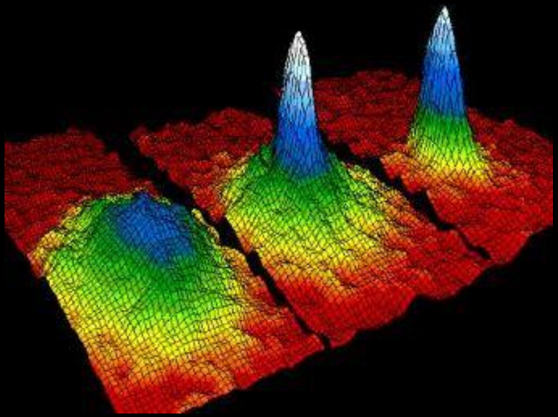
Turbulence and Bose Condensation: From the Early Universe to Cold Atoms



WMAP Science Team



ALICE/CERN



JILA/NIST

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RETUNE 2012, Heidelberg

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I. Nonthermal fixed points

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III. From early universe reheating to ultracold atoms

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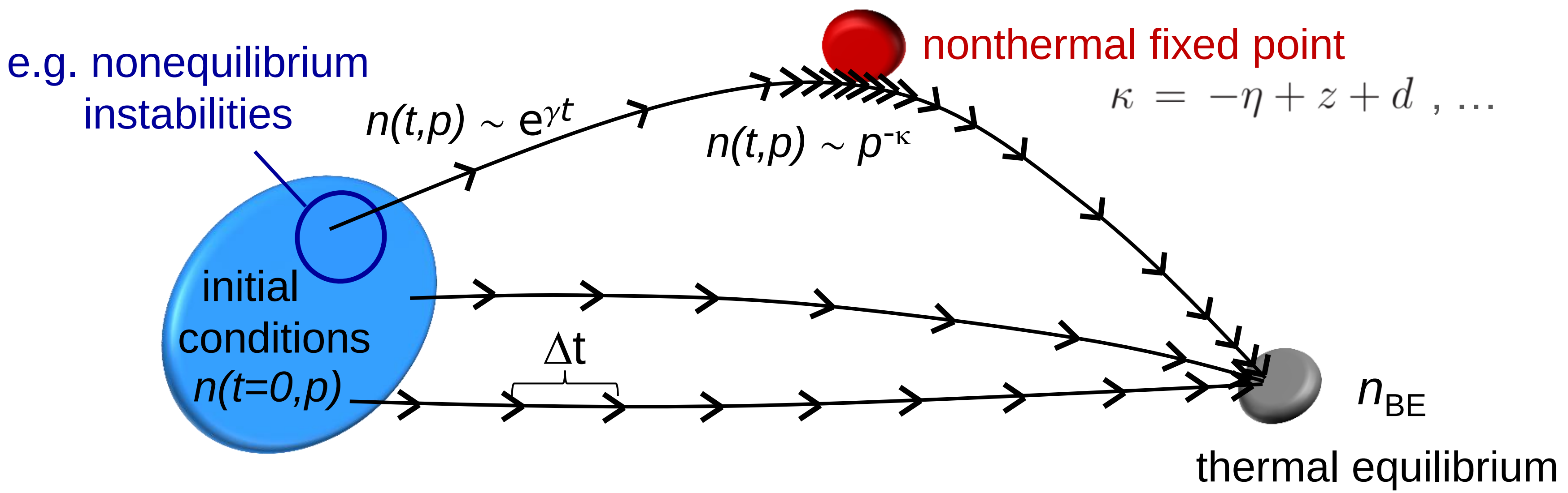
Nonequilibrium initial value problems

Thermalization process in quantum many-body systems?

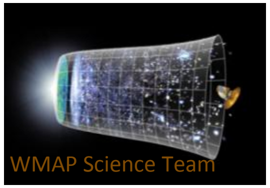
Schematically:



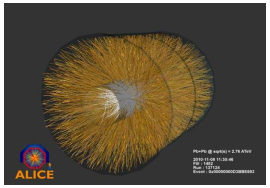
- **Characteristic nonequilibrium time scales? Relaxation? Instabilities?**
- **Diverging time scales far from equilibrium? Nonthermal fixed points?**

Universality far from equilibrium

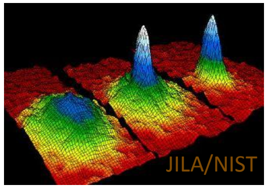
Early-universe preheating
($\sim 10^{16}$ GeV)



Heavy-ion collisions
(~100 MeV)



Cold quantum gas dynamics
($\sim 10^{-13}$ eV)



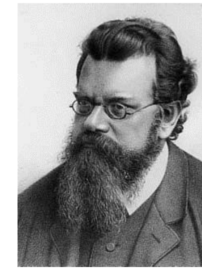
Instabilities, \`overpopulation´, …

Nonthermal fixed points

Very different microscopic dynamics can lead to
same *macroscopic scaling phenomena*

Digression: weak wave turbulence

Boltzmann equation for *relativistic* 2↔2 scattering, $n_1 \equiv n(t,p_1)$:



http://www.aeiou.at/aeiou.encyclop.b/b638771.htm

$$\frac{\mathrm{d}n_1}{\mathrm{d}t} = \int \frac{\mathrm{d}^3 p_2}{(2\pi)^3 2E_2} \int \frac{\mathrm{d}^3 p_3}{(2\pi)^3 2E_3} \int \frac{\mathrm{d}^3 p_4}{(2\pi)^3 2E_4}$$

$$\times \quad \delta^3(p_1 + p_2 - p_3 - p_4)\,\delta(E_1 + E_2 - E_3 - E_4)\,(2\pi)^4 |M|^2$$

momentum conservation — energy conservation — scattering

$$\times \Big(n_3 n_4 (1 + n_1)(1 + n_2) - n_1 n_2 (1 + n_3)(1 + n_4) \Big)$$

“gain“ term — “loss“ term

Different stationary solutions, $\mathrm{d}n_1/\mathrm{d}t = 0$, in the (classical) regime $n(p) \gg 1$:

1. $n(p) = 1/(e^{\beta\omega(p)} - 1)$ — thermal equilibrium

2. $n(p) \sim 1/p^{4/3}$ — turbulent *particle* cascade

3. $n(p) \sim 1/p^{5/3}$ — *energy* cascade

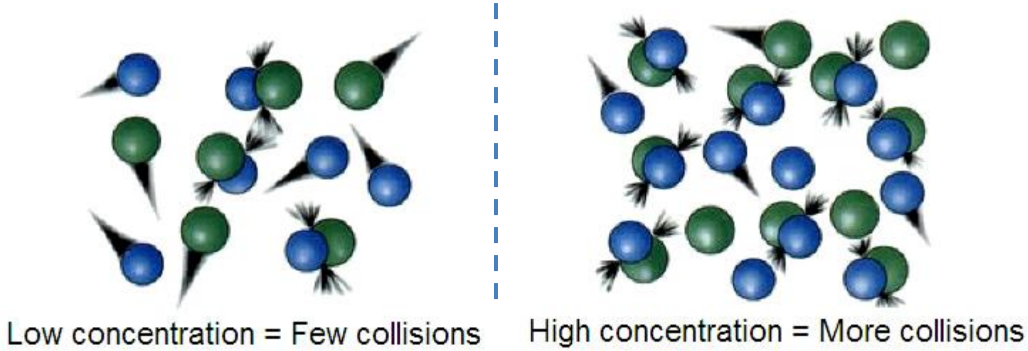
(2 and 3:) *Kolmogorov -Zakharov spectrum*

…associated to stationary transport of conserved quantities

Range of validity of Kolmogorov-Zakharov

E.g. self-interacting scalars with quartic coupling: $|M|^2 \sim \lambda^2 \ll 1$

$n(p) \lesssim 1$ | $1 \ll n(p) \ll 1/\lambda$ | $n(p) \sim 1/\lambda$



`overpopulation´
(non-perturbative)

?

analytically well described by 2PI effective action techniques!

Very high concentration = ?

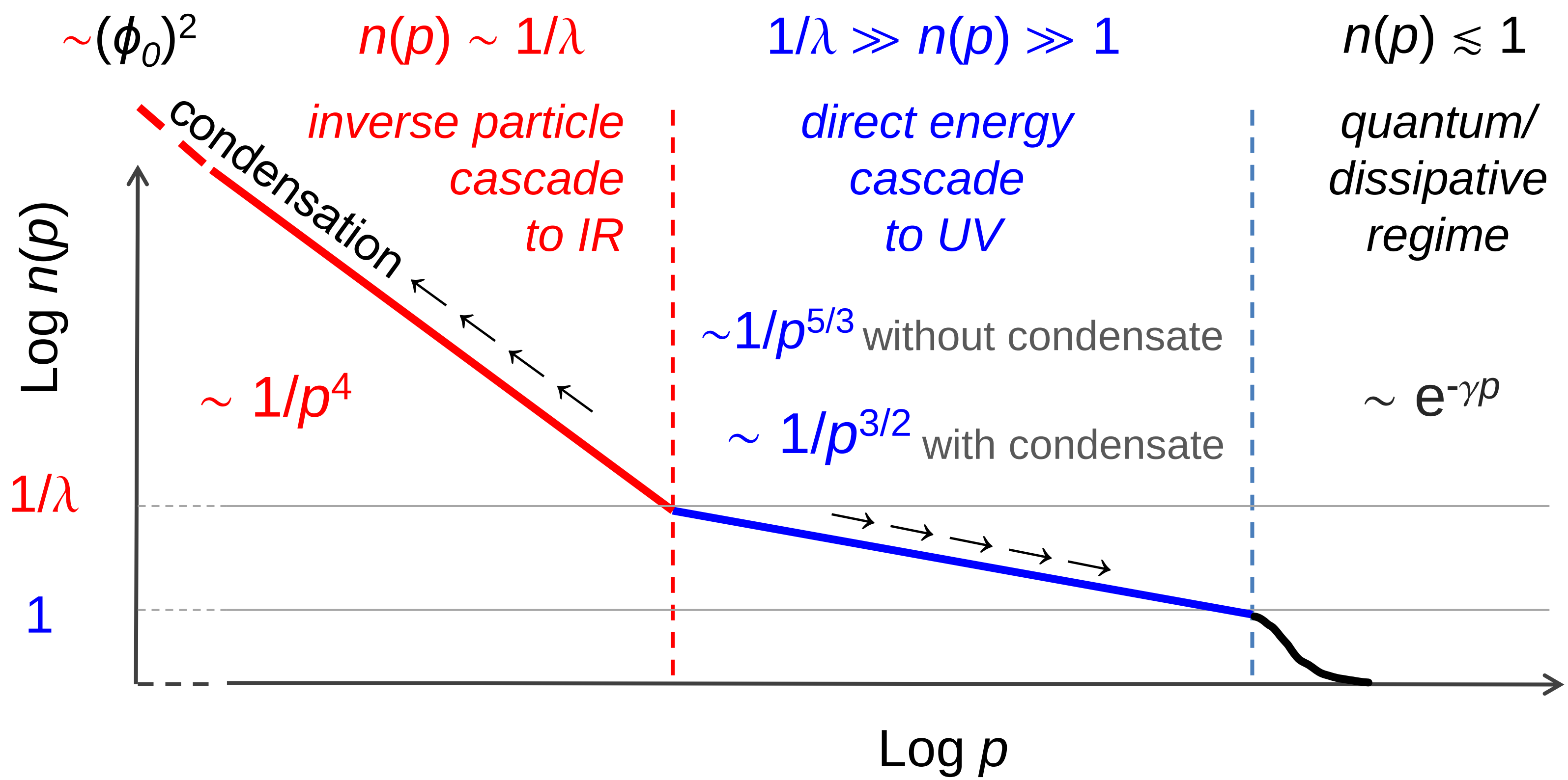
http://upload.wikimedia.org/wikipedia/commons/4/41/Molecular-collisions.jpg

Weak wave turbulence solutions are limited to the “window“

$1 \ll n(p) \ll 1/\lambda$, since for

$n(p) \sim 1/\lambda$ the n↔m scatterings for n,m=1,..,∞ are as important as 2↔2 **!**

Beyond weak wave turbulence: *here relativistic, d=3*



Non-thermal fixed point: $n(p) \sim 1/p^{d+z-\eta}$

Berges, Rothkopf, Schmidt
PRL 101 (2008) 041603

Bose-Einstein condensation from inverse particle cascade:

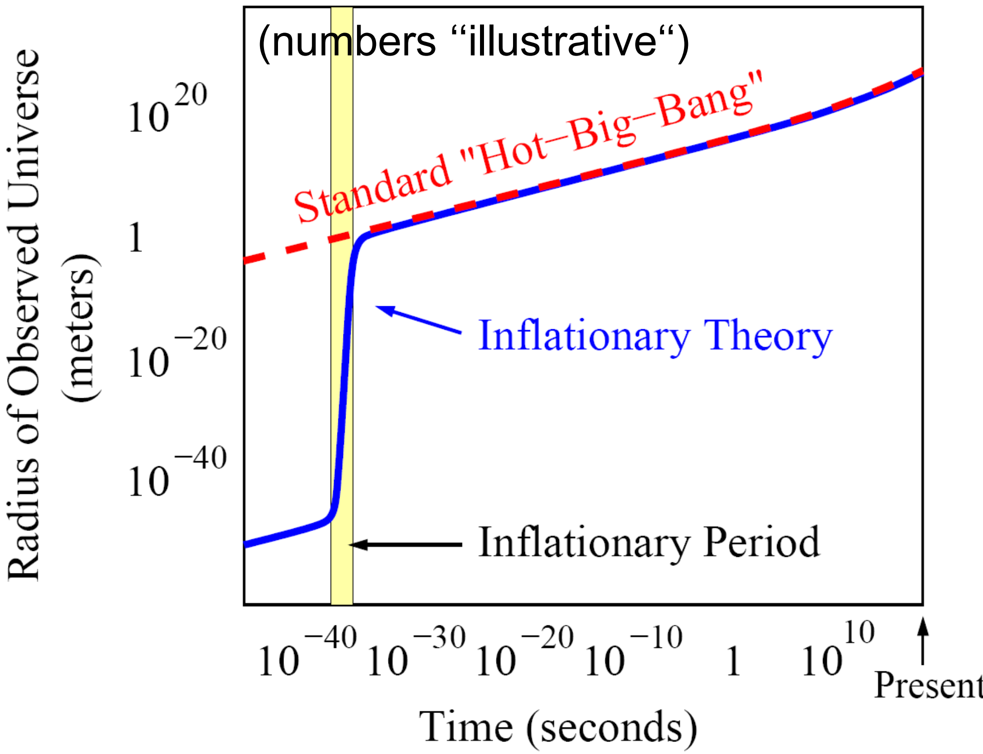
$$\sim \; (2\pi)^d \delta^{(d)}(\vec{p}) \, \phi_0^2(t)$$

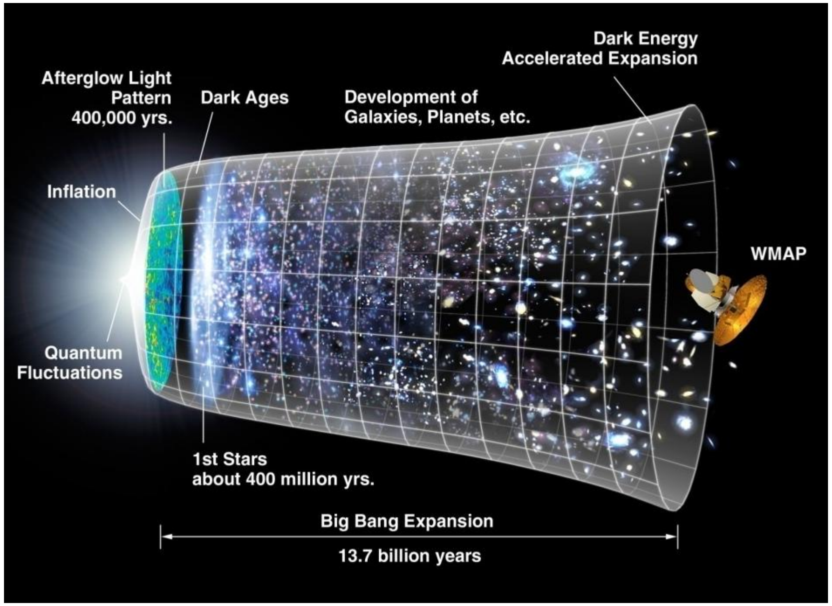
Berges, Sexty, PRL 108 (2012) 161601

Berges, Hoffmeister NPB813 (2009) 383; Nowak et al. PRA85 (2012) 043627; Nowak, Gasenzer arXiv:1206.3181

Heating the Universe after inflation: a quantum example

Schematic evolution:





- Energy density of matter ($\sim a^{-3}$) and radiation ($\sim a^{-4}$) decreases
- Enormous heating after inflation to get 'hot-big-bang' cosmology!

Preheating by parametric resonance

- Chaotic inflation

Kofman, Linde, Starobinsky, PRL 73 (1994) 3195

$$V(\phi,\chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}\mu^2\chi^2 + \frac{\lambda}{4}\phi^4 + \frac{g}{2}\phi^2\chi^2$$

$$\phi \gg m/\sqrt{\lambda} \quad , \quad \phi_0 \sim M_{\rm P} \quad , \quad \lambda \lesssim 10^{-12} \quad , \quad g^2 \lesssim \lambda$$

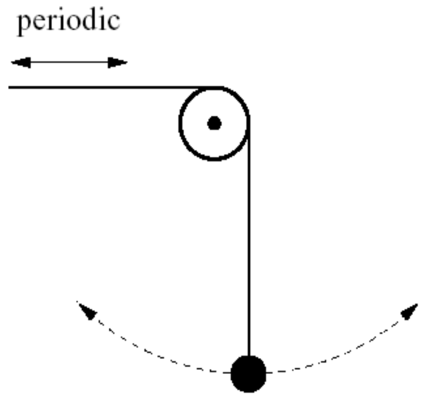
massless preheating: $m = \mu = 0$, conformally equiv. to Minkowski space

$$\frac{d^2\chi_k}{dt^2} + (k^2 + g\phi^2(t))\chi_k = 0$$

Classical oscillator analogue:

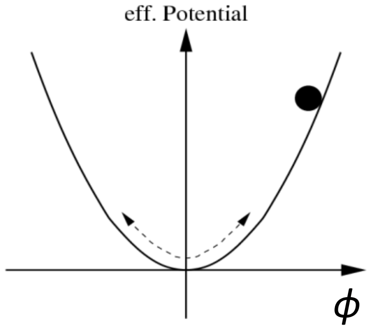
$\omega(t) \leftrightarrow \phi(t)$,
$x(t) \leftrightarrow \chi_{k=0}(t)$

$$\ddot{x} + \omega^2(t)x = 0$$



Dual cascade from chaotic inflation

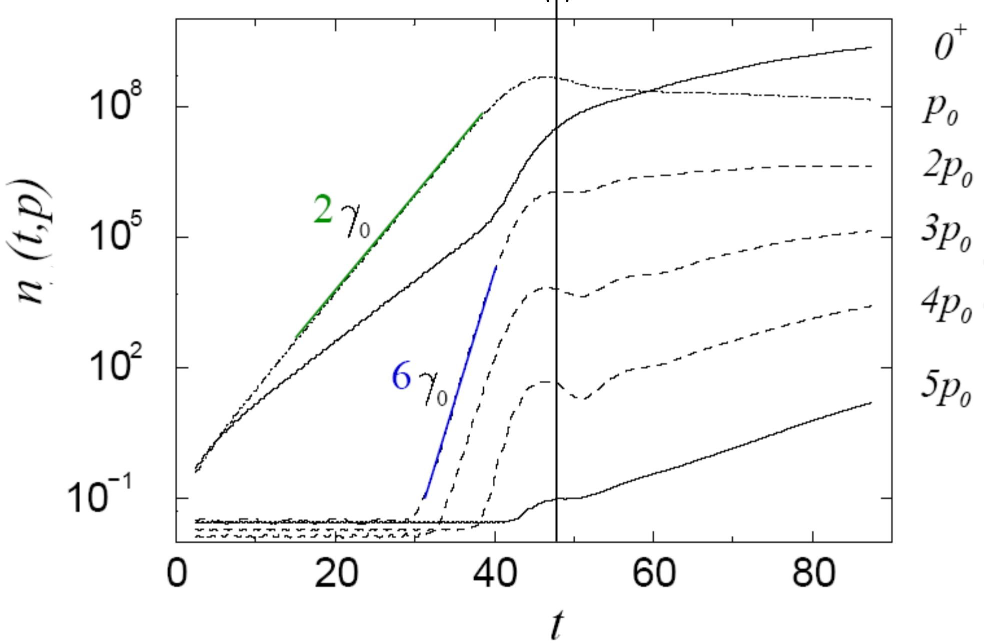
Berges, Rothkopf, Schmidt, PRL 101 (2008) 041603

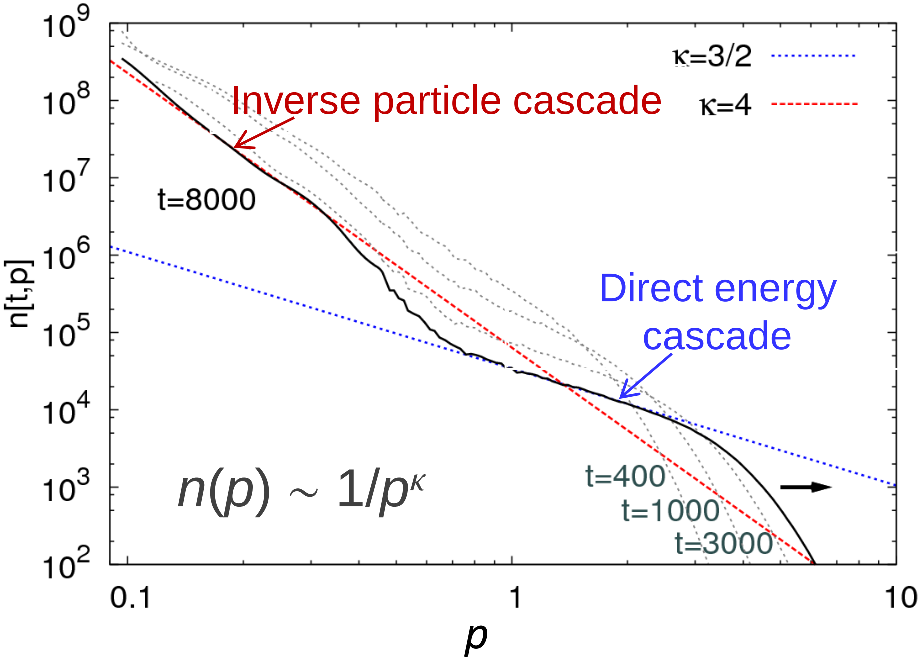


Generalize to N fields (2PI $1/N$ to NLO):

$$\Phi(t,k) = (\ \phi(t,k), \chi_1(t,k), \chi_2(t,k), \ldots, \chi_{N-1}(t,k)\ )$$

instability regime | devel. turbulence





O(N) symmetric with N=4, $\lambda \sim 10^{-4}$, $\phi\ (t) = \sigma(t)\sqrt{6N/\lambda}$ in units of $\sigma(t$=0)

Direct energy cascade: Micha, Tkachev, PRL 90 (2003) 121301 → Talk by I. Tkachev!

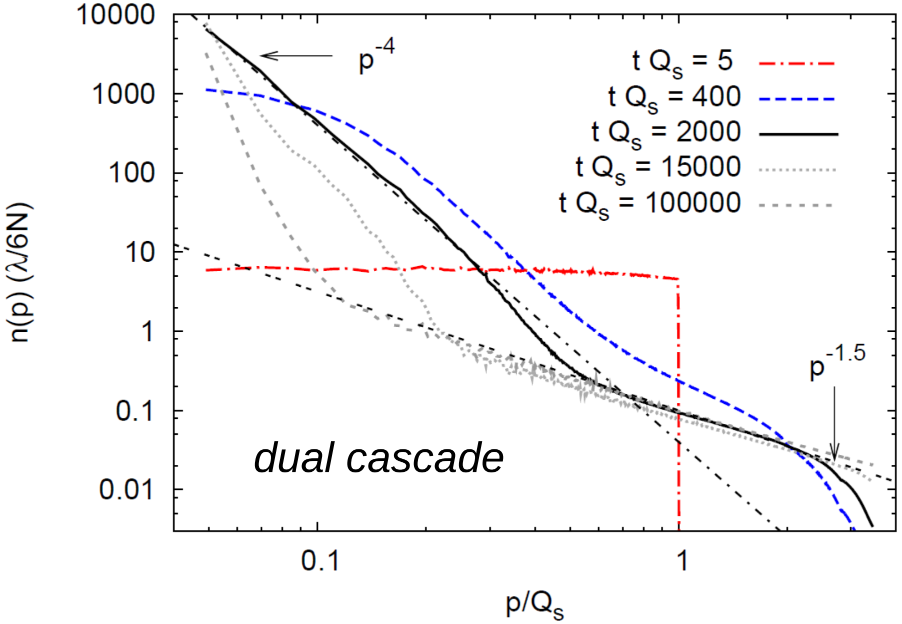
Bose condensation from infrared particle cascade

$$F(t,t';\vec{x}-\vec{y}) = \left\langle \left\{ \hat{\phi}(t,\vec{x}), \hat{\phi}(t',\vec{y}) \right\} \right\rangle$$

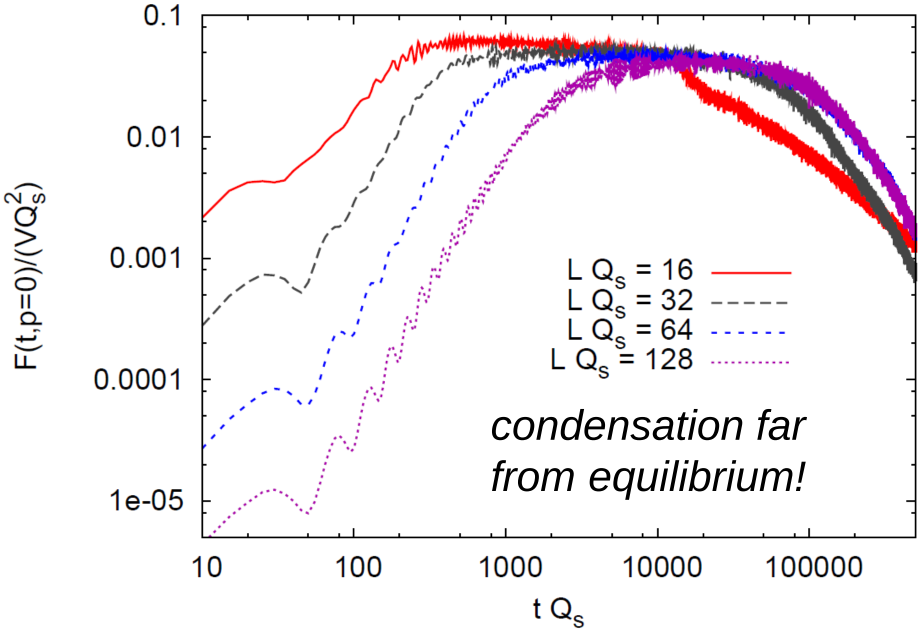
time-dependent condensate

$$F(t,t;p) = \frac{1}{\omega_p(t)} \left(n_p(t) + \frac{1}{2} \right) + (2\pi)^d \delta^{(d)}(\vec{p})\, \phi_0^2(t)$$

starting from initial `overpopulation´:



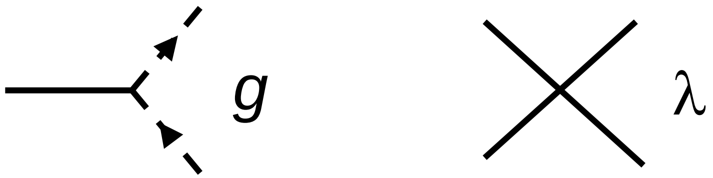
finite volume: $(2\pi)^d \delta^{(d)}(0) \rightarrow V$

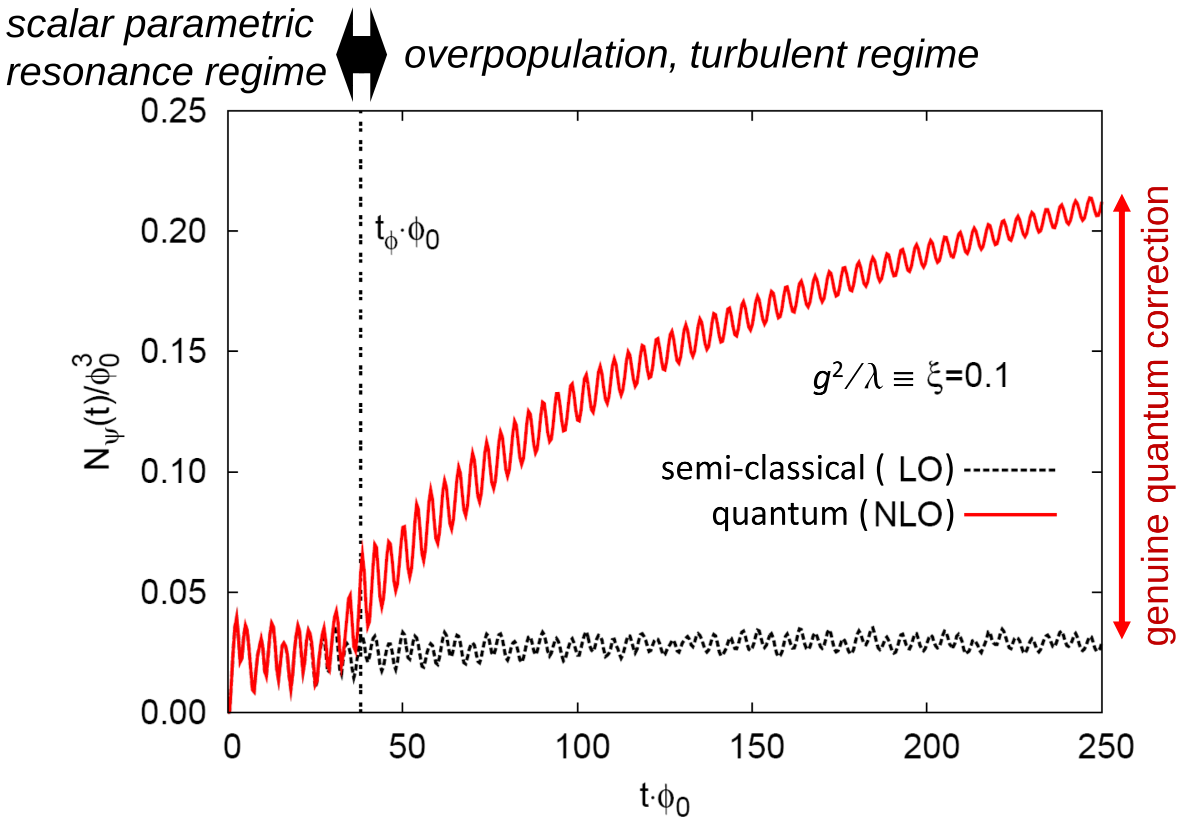


Berges, Sexty, PRL 108 (2012) 161601

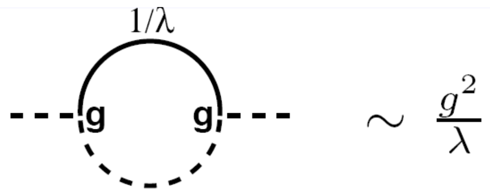
Overpopulation as a quantum amplifier

Inflaton decay into fermions:





2PI-NLO:



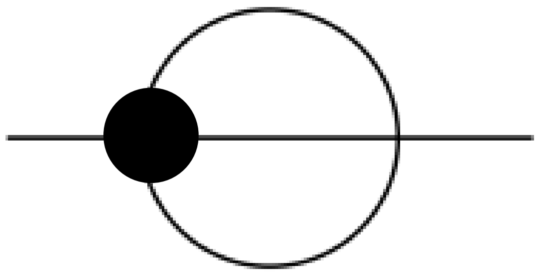
Berges, Gelfand, Pruschke
PRL 107 (2011) 061301

strongly enhanced fermion production rate (NLO): $\sim (g^2/\lambda)\,\phi_0$ **!**

From complexity to simplicity

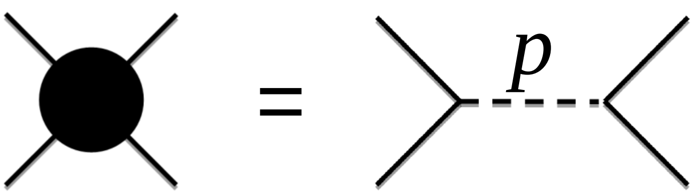
Complexity: many-body *n*↔*m* processes for *n*,*m* = 1,..,∞
as important as 2↔2 scattering (`overpopulation´)!

Simplicity: Resummation of the infinitely many processes leads to *effective kinetic theory* (2PI 1/*N* to NLO) dominated in the IR by



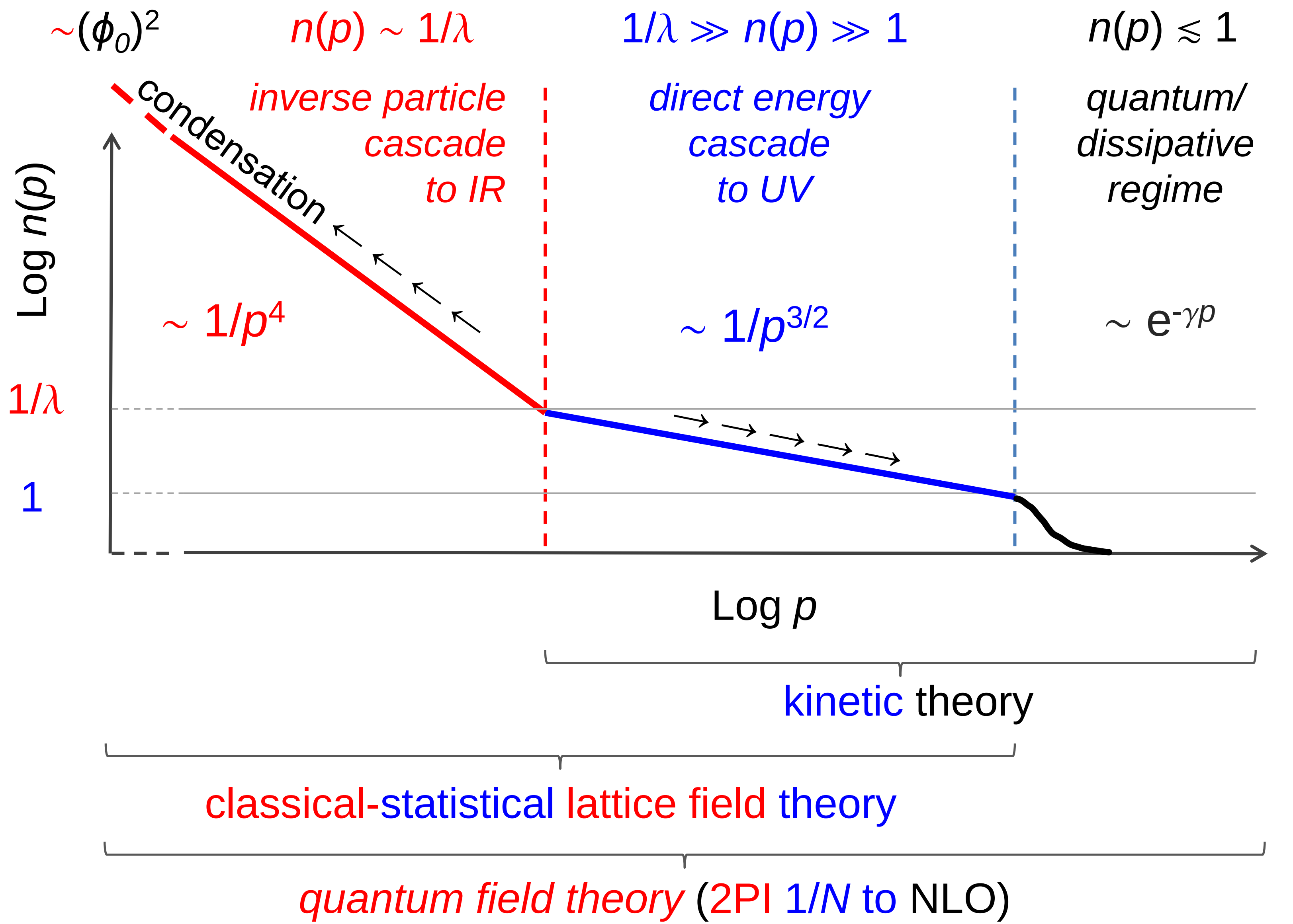
approximately number conserving!
→ particle cascade

describing 2 ↔ 2 scattering with an *effective coupling:*



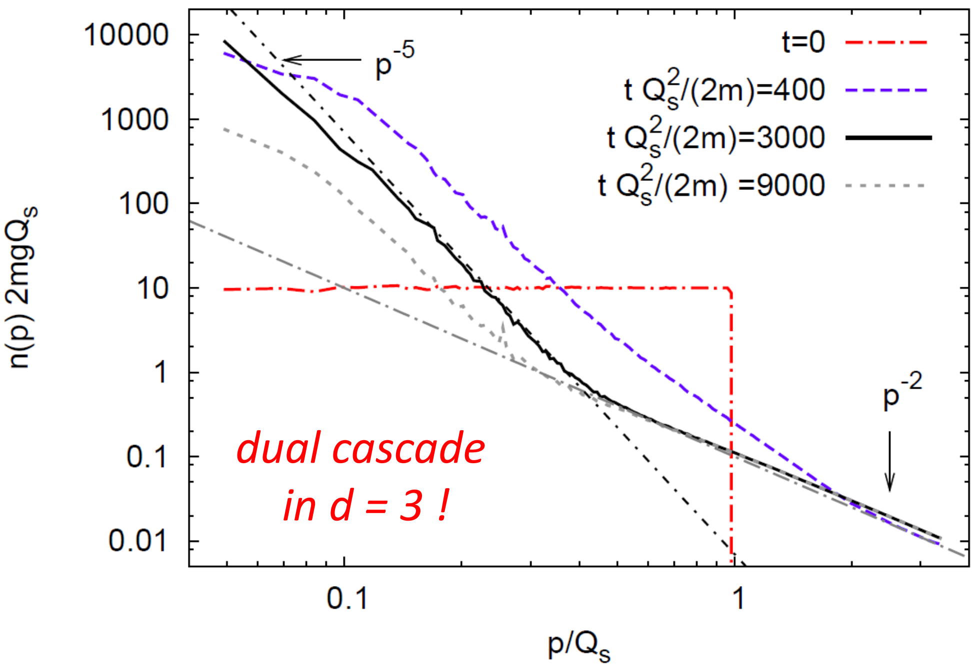
$\sim p^{8-4\eta}$

Methods



Berges, NPA 699 (2002) 847; Aarts, Ahrensmeier, Baier, Berges, Serreau PRD 66 (2002) 045008

Comparison to cold Bose gas (Gross-Pitaevskii)



Berges, Sexty, PRL 108 (2012) 161601

Infrared particle cascade leads to Bose condensation without subsequent decay

(no number changing processes)

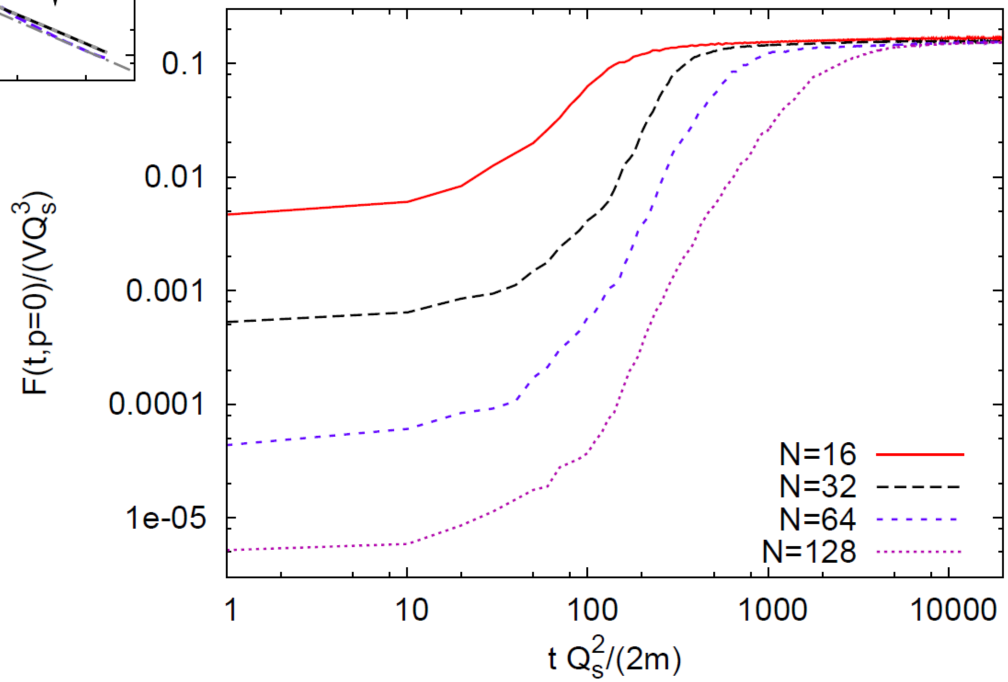
See also talk by B. Nowak!

Expected infrared cascade:

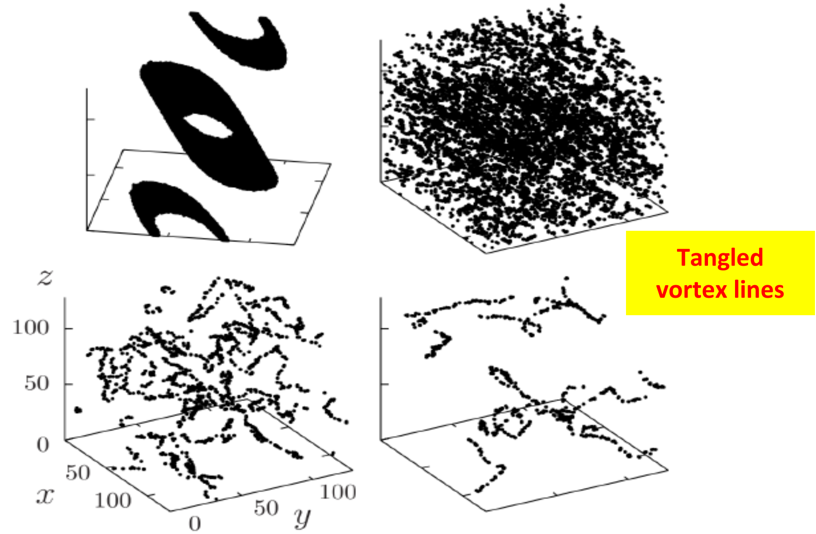
$$n(p) \sim 1/p^{d+2-\eta}$$

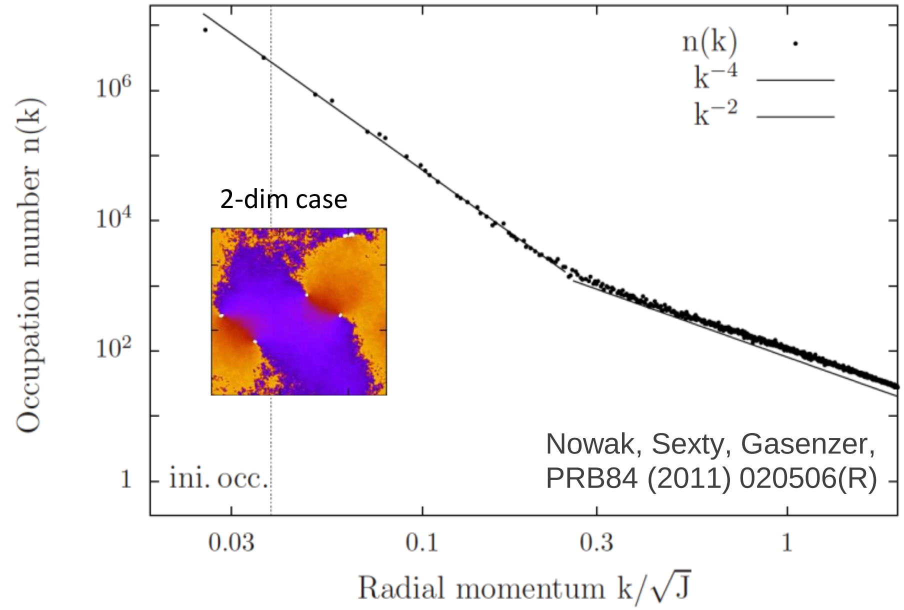
for non-relativistic dynamics

Scheppach, Berges, Gasenzer, PRA 81 (2010) 033611; Nowak, Sexty, Gasenzer, PRB 84 (2011) 020506(R); Nowak, Gasenzer arXiv:1206.3181

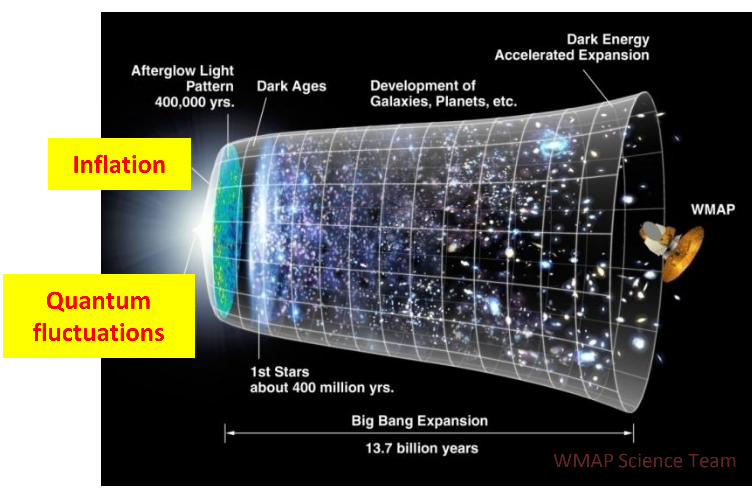


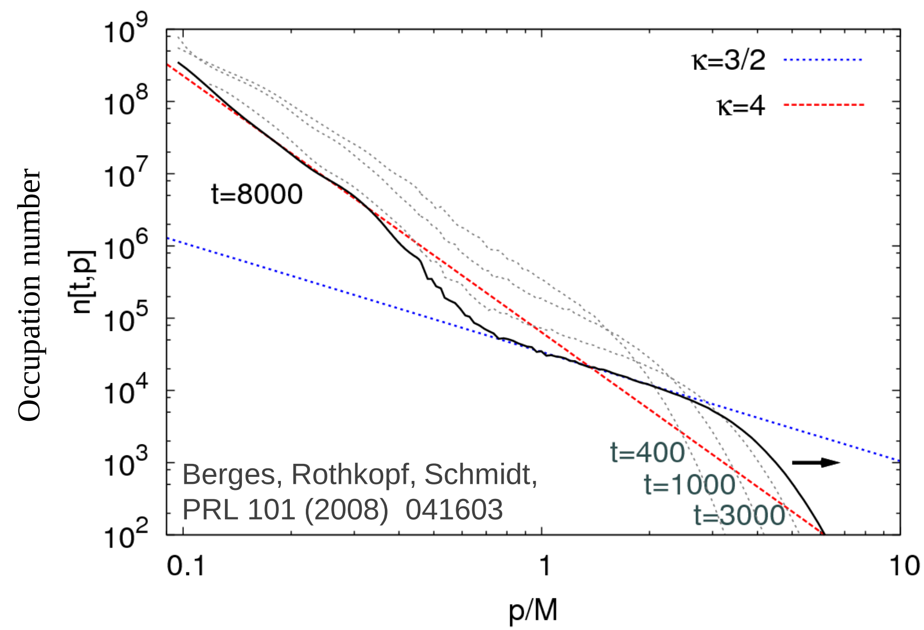
- Quantum turbulence in a cold Bose gas





- Preheating dynamics after chaotic inflation





Turbulence/Bose condensation for gluons?

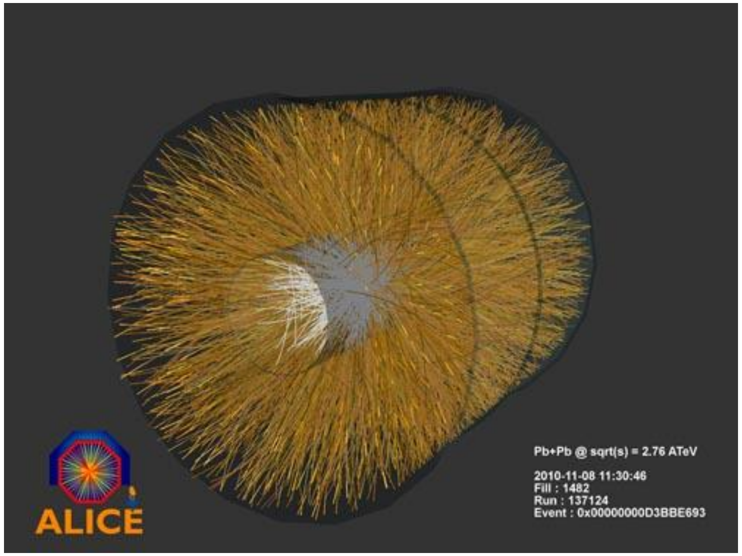
Field strength tensor, here for *SU*(2):

$$F^a_{\mu\nu}[A] = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu$$

Equation of motion:

$$(D_\mu[A] F^{\mu\nu}[A])^a = 0$$

$$D^{ab}_\mu[A] = \partial_\mu \delta^{ab} + g\epsilon^{acb} A^c_\mu$$



Classical-statistical simulations accurate for sufficiently large fields/high gluon occupation numbers:

anti-commutators $\langle\{A, A\}\rangle \gg \langle[A, A]\rangle$ commutators

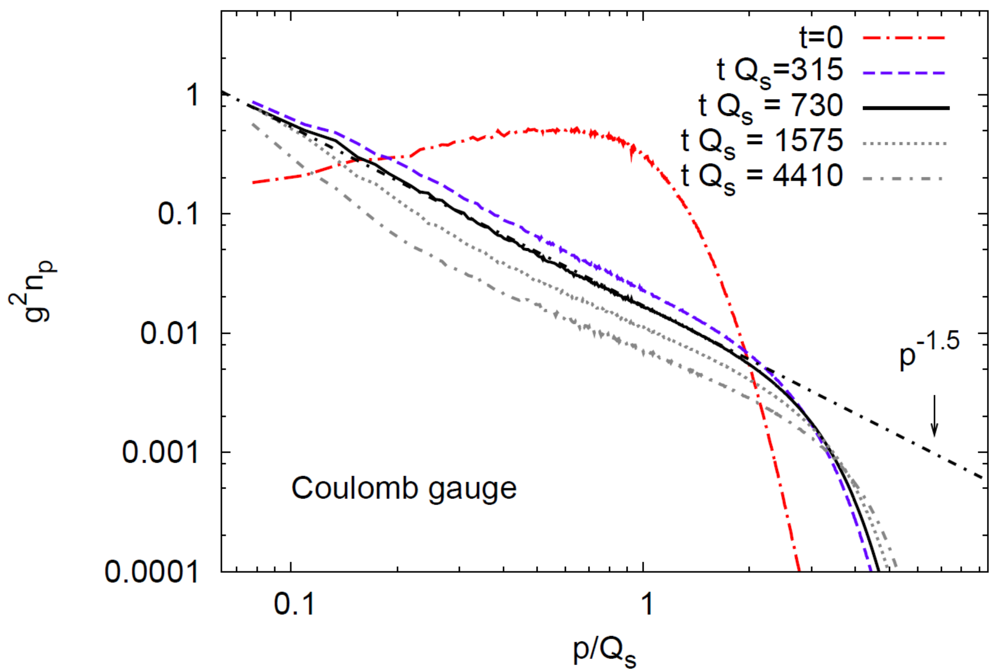
i.e. “n(p)“ $\gg$ 1

See also talk by K. Fukushima!

Classical-statistical lattice gauge theory

Occupancy: $\sim \sqrt{\langle |A^2(p)|\rangle \langle |E^2(p)|\rangle}$

Berges, Schlichting, Sexty, arXiv:1203.4646

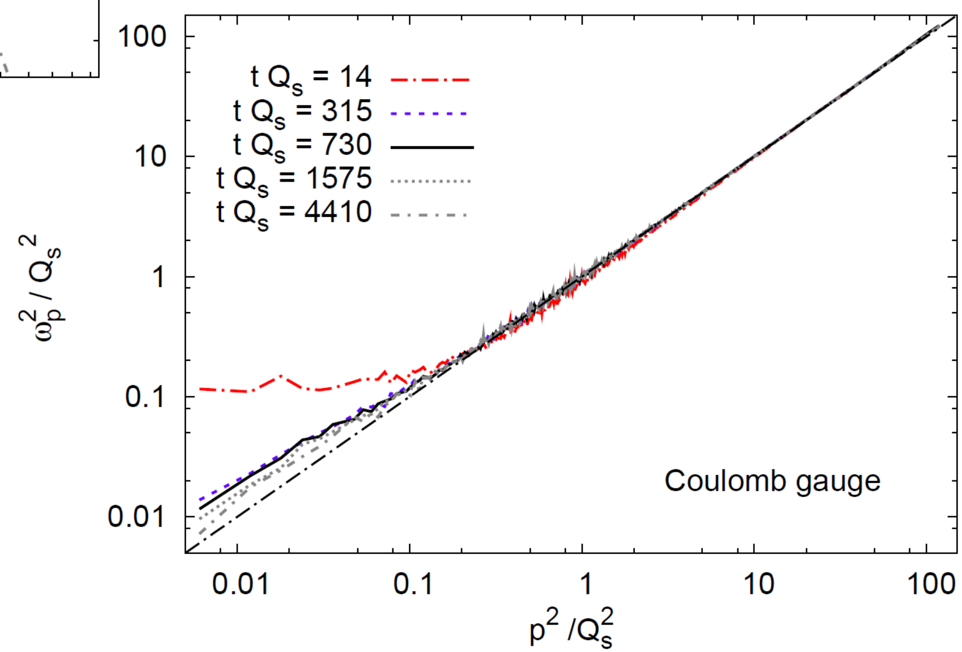


Initial overpopulation:

$$\epsilon \sim \frac{Q_s^4}{g^2} \quad \text{i.e.} \quad n(p \simeq Q_s) \sim \frac{1}{g^2}$$

See also talk by J.-P. Blaizot!

Dispersion: $\sim \sqrt{\langle |E^2(p)|\rangle / \langle |A^2(p)|\rangle}$



- Wave turbulence exponent 3/2
(as for scalars with condensate)!?

- No stable occupation numbers exceeding $g^2 n_p \sim 1$ observed yet

Scaling analysis

Leading (2PI) resummed perturbative contribution ($O(g^2)$):

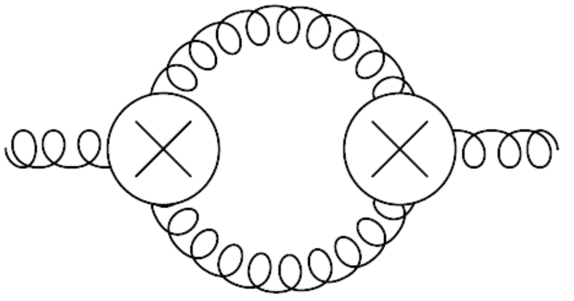


Figure 4: Gluon part of the one-loop contribution to the self-energy with (2PI) resummed propagator lines. The crossed circles indicate an effective three-vertex in the presence of a background gauge field potential.

Standard scaling analysis gives ***for slowly varying background field:***

$n(p) \sim 1/p^{\kappa}$

$$\kappa = \frac{3}{2}, \quad \text{or} \quad \kappa = 1$$

energy cascade

particle cascade

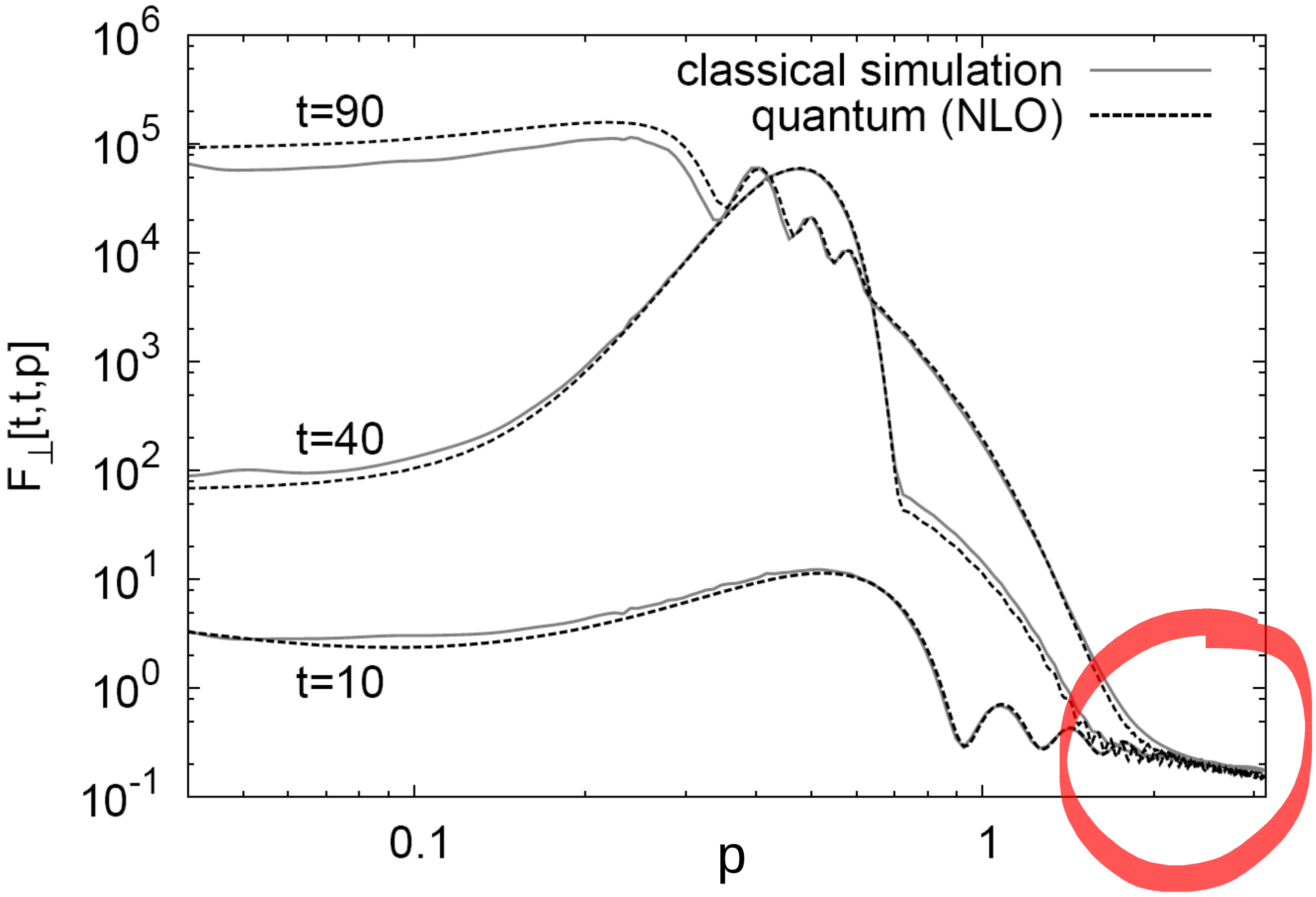
Berges, Schlichting, Sexty, arXiv:1203.4646, Berges, Scheffler, Sexty, PLB 681 (2009) 362

Conclusions

Nonthermal fixed points:

- crucial for thermalization process from instabilities/overpopulation!
- strongly nonlinear regime of stationary transport (*dual* cascade)!
- Bose condensation for scalars from inverse particle cascade!
- large amplification of quantum corrections for fermions!
- gauge theory results indicate the same weak wave turbulence exponents as for scalars!

Comparing classical to quantum

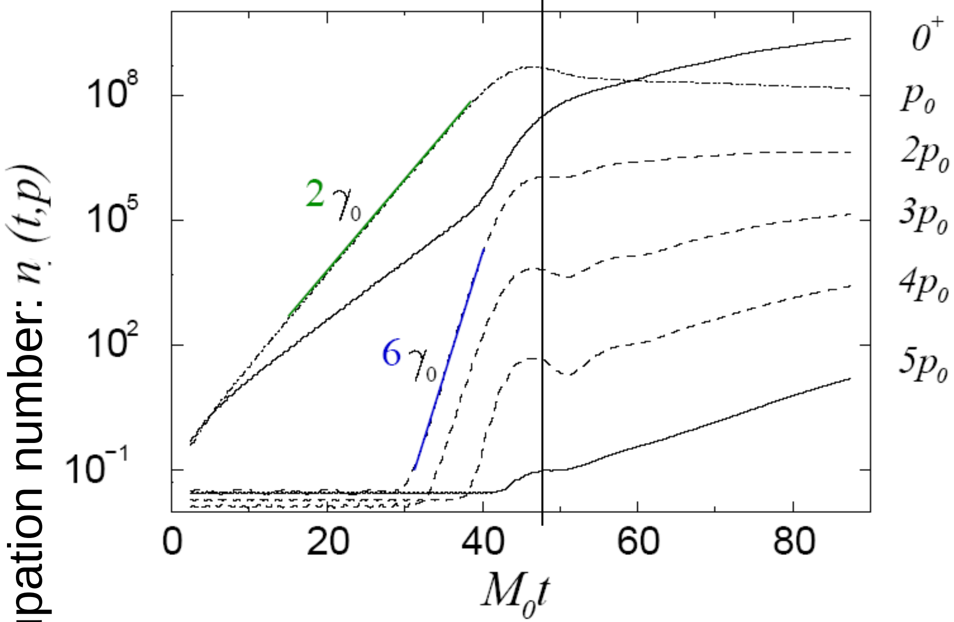


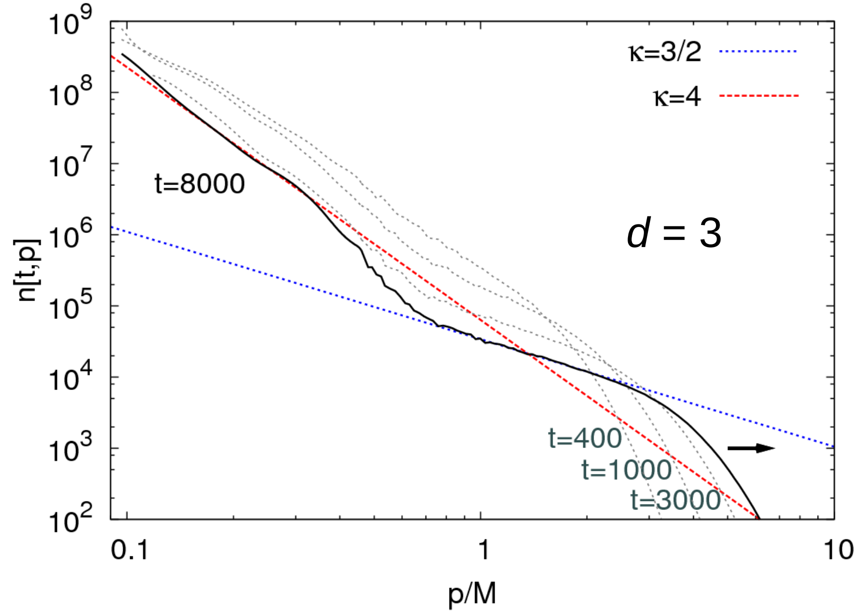
Practically no *bosonic* quantum corrections at the end of preheating

Accurate nonperturbative description by quantum (2PI) 1/*N* to NLO

Dependence on spatial dimension *d*

parametric resonance ⟺ approach to turbulence:

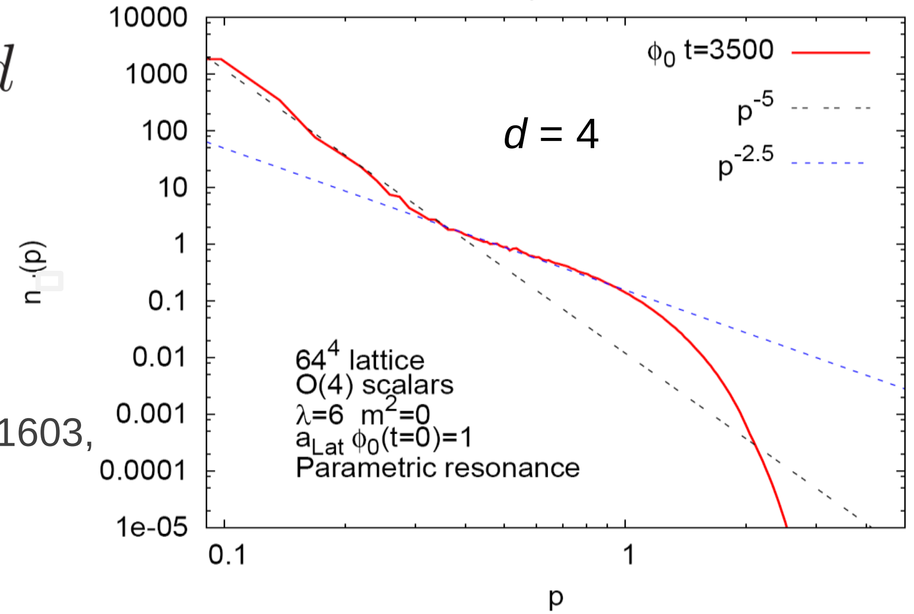




$n(t,p) \sim p^{-\kappa}$ with $\kappa = -\eta + z + d$

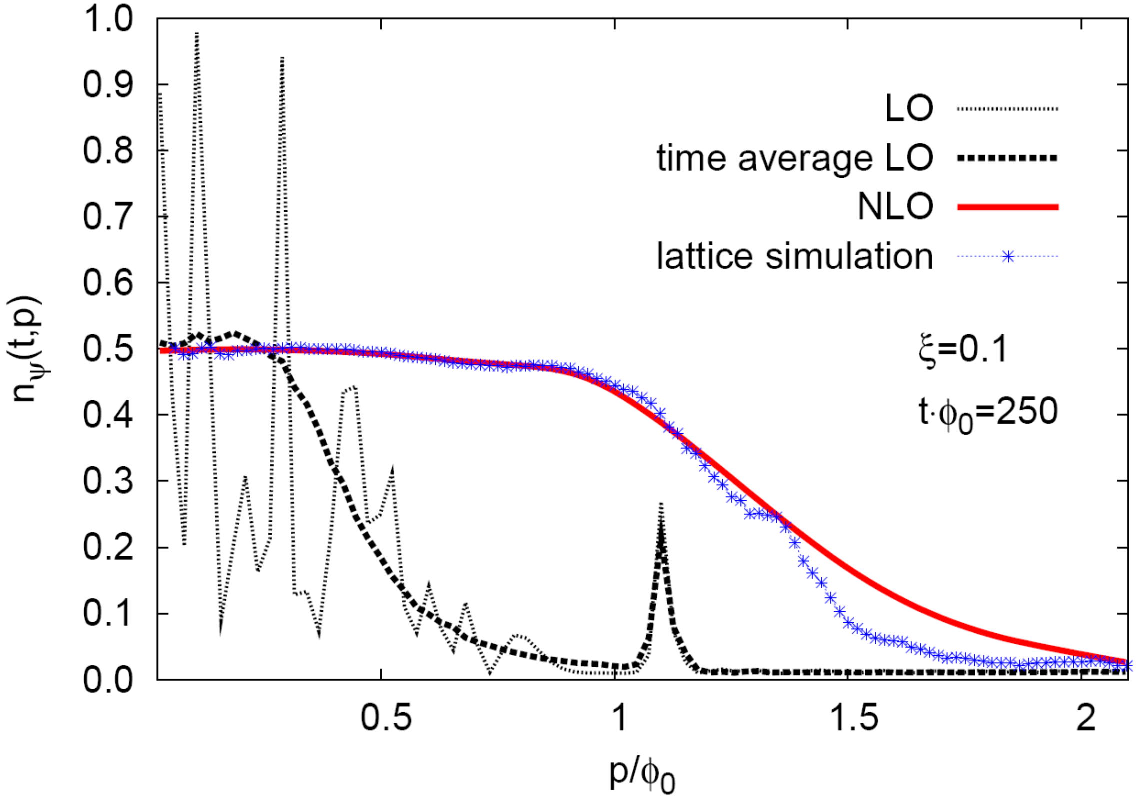
→ κ = 4 for *d* = 3,
κ = 5 for *d* = 4 ✓ IR

for *z* = 1 (relativistic), $\eta = 0$



Berges, Rothkopf, Schmidt, PRL 101 (2008) 041603,
Berges, Hoffmeister, NPB 813 (2009) 383,
Berges, Sexty, PRD 83 (2011) 085004

Real-time dynamical fermions in 3+1 dimensions!



- Wilson fermions on a 64^3 lattice   Berges, Gelfand, Pruschke, PRL 107 (2011) 061301

- Very good agreement with NLO quantum result (2PI) for $\xi \ll 1$
  (differences at larger *p* depend on Wilson term → larger lattices)

- Lattice simulation can be applied to $\xi \sim 1$ relevant for *QCD*

Nonequilibrium fermion spectral function

$$\rho(x,y) = i\,\langle\{\psi(x),\bar{\psi}(y)\}\rangle$$

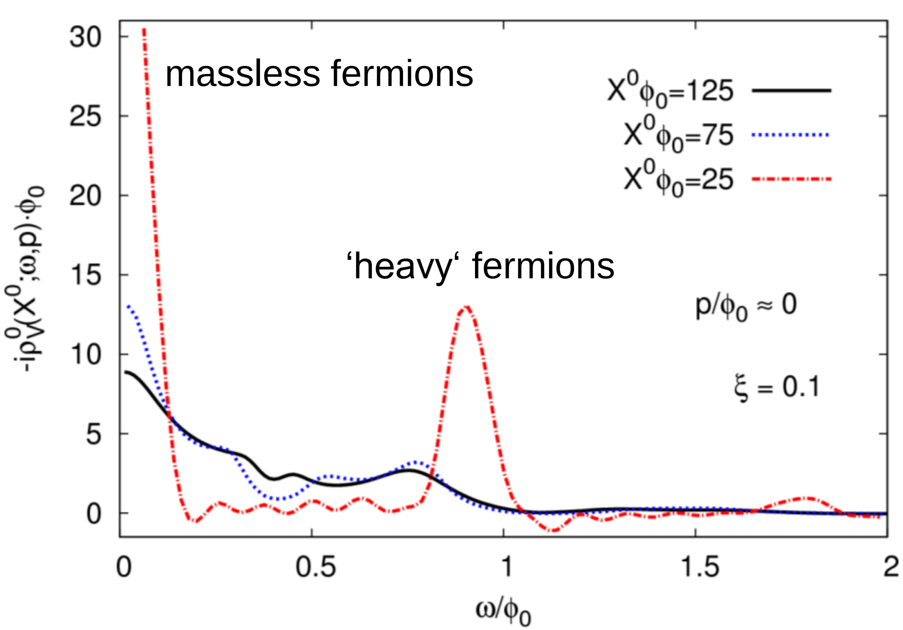
$$\rho_V^\mu = \frac{1}{4}\mathrm{tr}\,(\gamma^\mu\rho)$$ vector components

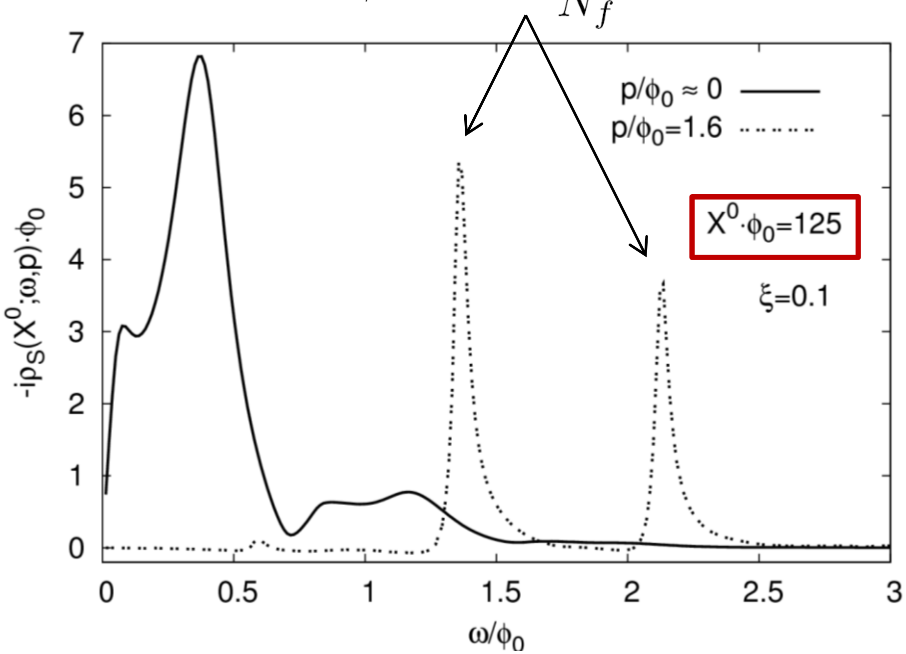
$$\rho_S = \frac{1}{4}\mathrm{tr}\,(\rho)$$ scalar component

quantum field anti-commutation relation: $-i\rho_V^0(t,t;\mathbf{p}) = 1$

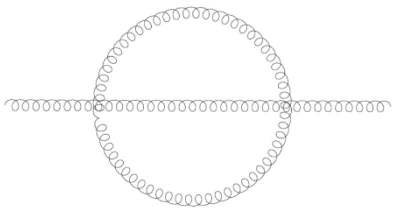
Wigner transform: ($X^0 = (t + t')/2$)

$$M_\psi^{\mathrm{eff}}(t) \simeq \pm\frac{g}{N_f}|\phi(t)|$$





Discussion



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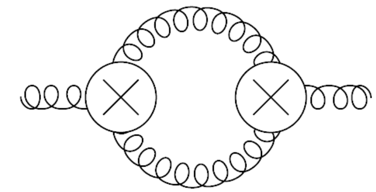

$$\kappa = \frac{5}{3}, \quad \text{or} \quad \kappa = \frac{4}{3}$$

$$\kappa = \frac{3}{2}, \quad \text{or} \quad \kappa = 1$$

‘condensate‘