Thermalization of the Quark-Gluon Plasma

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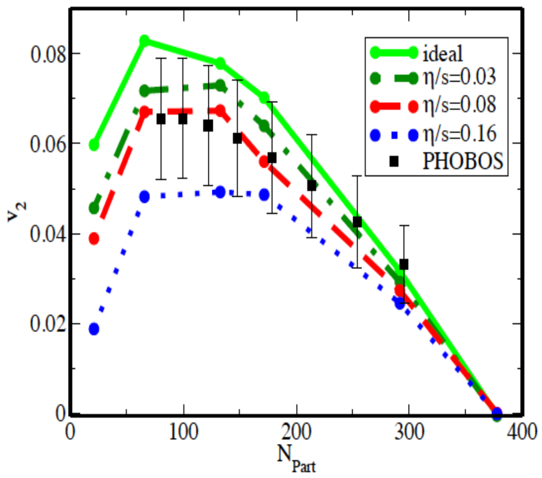
Phenomenological perspective

Empirical evidence from RHIC (and LHC):

- Matter produced in heavy ion collisions exhibits fluid behavior from very early time on (elliptic flow, sensitivity to fluctuations in initial conditions, etc)

- The fluid has very special transport properties, in particular a small value of the shear viscosity to entropy density ratio



Fluid behavior requires (some degree of) local equilibration. How is this achieved?

Elements of a scenario for thermalization in heavy ion collisions

- initial dynamics involves **classical color fields** (CGC), with characteristic **saturation scale** Qs
- instabilities play an important role: **isotropization**, cascade towards the infrared leading to **large occupancy** of (very) **soft modes** [cf. talks by Rebhan and Fukushima]
- because of the large occupation, the system remains **strongly coupled** in spite of the small coupling constant
- a transient **Bose condensate** may form if particle number conserving processes dominate. This may be accompanied by formation of **turbulent cascades** [cf. talk by Berges]

More in the report from the EMMI Rapid Reaction Task Force on 'Thermalization in Non-abelian Plasmas', arXiv: 1203.2042[hep-ph]

What is the fluid made of ?
What are the important degrees of freedom ?

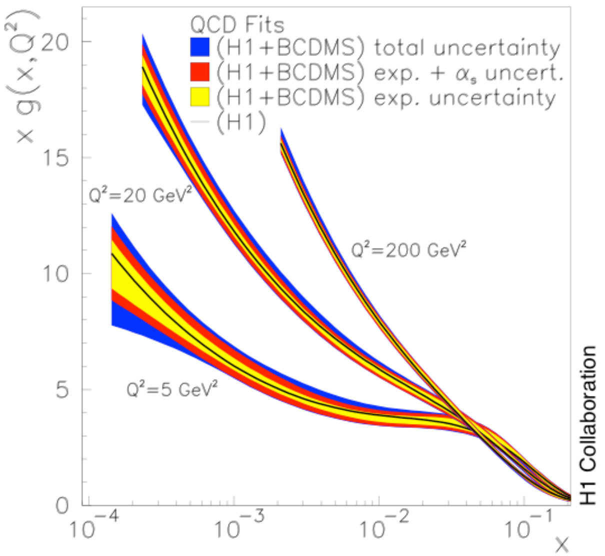
(quasi) particles ? massive quarks and gluons ?

(classical) fields (color field, or AdS)

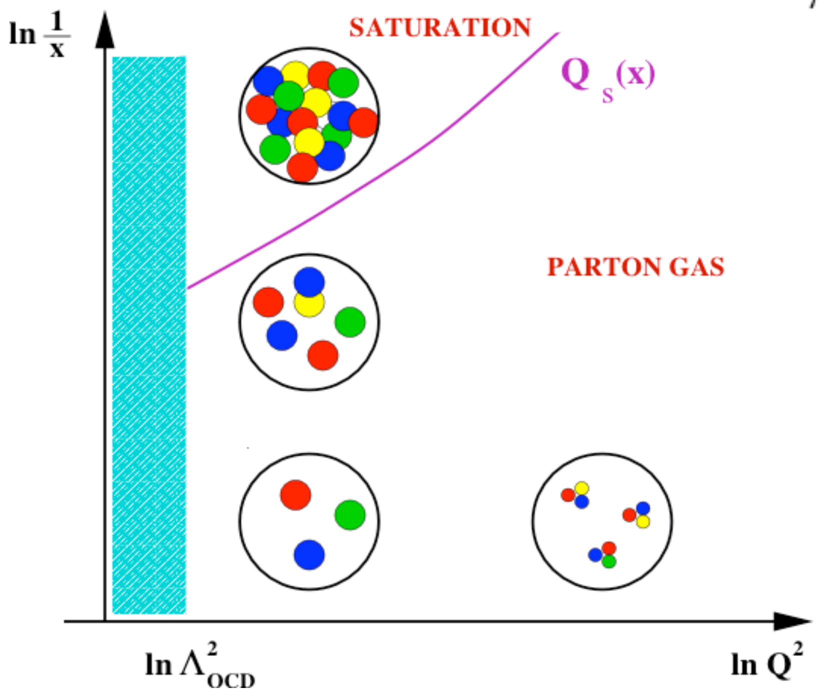
or both? in a plasma, at weak coupling, separation of hard (particles) and soft (collective) modes, that are coupled together (hard loops)

$$j^{\mu}_{ind}(x) = \int \frac{d^3p}{(2\pi)^3} v^{\mu} \delta f(x,p) = \int dy \Pi^{\mu\nu}(x,y) A_{\nu}(y)$$

High density partonic systems



$$Q_s^2 \approx \alpha_s \frac{xG(x,Q^2)}{\pi R^2}$$



Large occupation numbers

$$\frac{xG(x,Q^2)}{\pi R^2 Q_s^2} \sim \frac{1}{\alpha_s}$$

saturation fixes the initial scale

$$\epsilon_0 = \epsilon(\tau = Q_s^{-1}) \sim \frac{Q_s^4}{\alpha_s} \qquad n_0 = n(\tau = Q_s^{-1}) \sim \frac{Q_s^3}{\alpha_s} \qquad \epsilon_0/n_0 \sim Q_s$$

The over-populated quark-gluon plasma

Thermodynamical considerations

Initial conditions $(t_0 \sim 1/Q_s)$

$$\epsilon_0 = \epsilon(\tau = Q_s^{-1}) \sim \frac{Q_s^4}{\alpha_s} \qquad n_0 = n(\tau = Q_s^{-1}) \sim \frac{Q_s^3}{\alpha_s} \qquad \epsilon_0/n_0 \sim Q_s$$

overpopulation parameter $\quad n_0\, \epsilon_0^{-3/4} \sim 1/\alpha_s^{1/4}$

In equilibrated quark-gluon plasma

$$\epsilon_{eq} \sim T^4 \qquad n_{eq} \sim T^3 \qquad n_{eq}\, \epsilon_{eq}^{-3/4} \sim 1$$

mismatch by a large factor (at weak coupling) $\quad \alpha_s^{-1/4}$

Will the system accommodate the particle excess by forming a Bose-Einstein condensate ?

Kinetic evolution dominated by elastic collisions

[work in collaboration with Jinfeng Liao and Larry McLerran]

Boltzmann equation with 2->2 scattering

Gluon distribution function

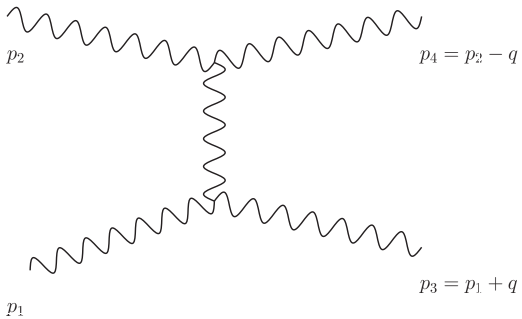
$$f(\boldsymbol{x},\boldsymbol{p}) = \frac{(2\pi)^3}{2(N_c^2-1)}\frac{dN}{d^3\boldsymbol{x}d^3\boldsymbol{p}}$$

Boltzmann equation

$$\mathcal{D}_t f_1 = \frac{1}{2}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}\frac{1}{2E_1} \mid M_{12\to 34}\mid^2$$
$$\times (2\pi)^4\delta(p_1+p_2-p_3-p_4)\{f_3f_4(1+f_1)(1+f_2) - f_1f_2(1+f_3)(1+f_4)\}$$

$$\mathcal{D}_t \equiv \partial_t + \vec{v}_1\cdot\vec{\nabla}$$

$$\mid M\mid^2 = 576 g^4 E_1^2 E_2^2 \left|\frac{1-x^2}{\boldsymbol{q}_0^2-\boldsymbol{q}^2-\Pi_L} - \frac{(1-x^2)\cos\phi_2}{q_0^2-\boldsymbol{q}^2-\Pi_T}\right|^2$$



Small angle approximation

$$\mathcal{D}_t f = C(f) \qquad\qquad C(f) = -\boldsymbol{\nabla}\cdot\boldsymbol{S} = -\frac{\partial S_i}{\partial p_i}$$

Simplified kinetic equation

$$\mathcal{D}_\tau f(\boldsymbol{p}) = \boldsymbol{\nabla}\cdot\left[I_a \boldsymbol{\nabla} f(\boldsymbol{p}) + \frac{\boldsymbol{p}}{p} I_b\, f(\boldsymbol{p})[1+f(\boldsymbol{p})]\right]$$

$$I_a = \int \frac{d^3p}{(2\pi)^3} f(\boldsymbol{p})(1+f(\boldsymbol{p}))$$

$$I_b = \int \frac{d^3p}{(2\pi)^3} \frac{2f(\boldsymbol{p})}{p}$$

$$\tau = 36\pi\alpha^2 \mathcal{L} t \qquad\qquad \mathcal{L} = \int \frac{dq}{q}$$

Some results

Solve

$$\mathcal{D}_\tau f(p) = I_a \frac{\partial^2 f}{\partial p^2} + I_b\,(1+2f)\,\frac{\partial f}{\partial p} + \frac{2}{p}\left[I_a \frac{\partial f}{\partial p} + I_b\, f(1+f)\right]$$

with initial condition

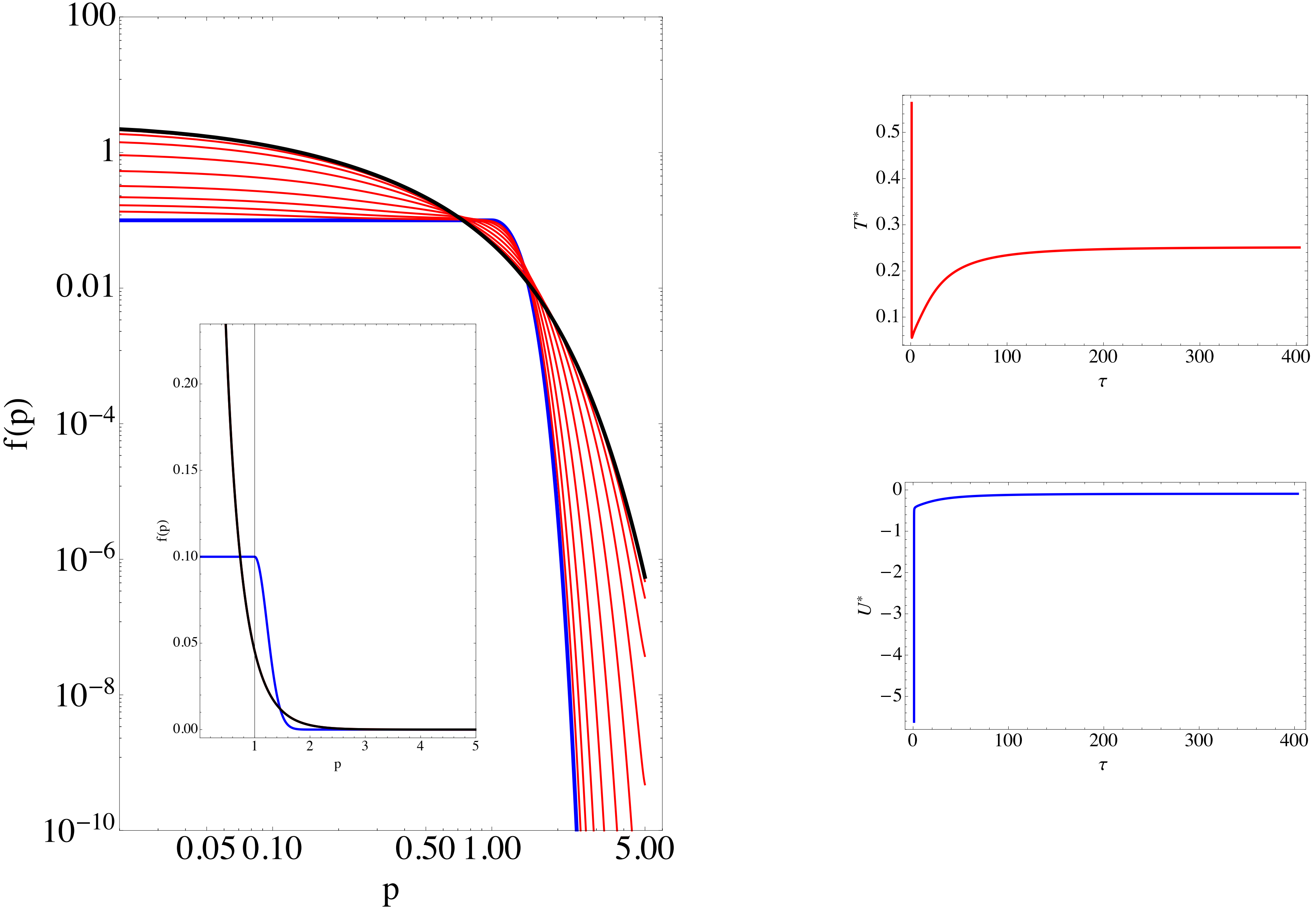
$$f(p) = f_0\,\theta(Q_s - p) \qquad \epsilon_0 = f_0\,\frac{Q_s^4}{8\pi^2} \qquad n_0 = f_0\,\frac{Q_s^3}{6\pi^2}$$

Onset of BEC

$$n_0\,\epsilon_0^{-3/4} = f_0^{1/4}\,\frac{2^{5/4}}{3\,\pi^{1/2}}. \qquad n\,\epsilon^{-3/4}|_{SB} = \frac{30^{3/4}\,\zeta(3)}{\pi^{7/2}} \approx 0.28.$$

$$f_0^c \approx 0.154$$

‘underpopulated’ case - no BEC $(f_0 = 0.1)$



Small momentum behavior

At small momentum $(f \gg 1)$

$$\mathcal{D}_\tau f(p) = I_a \frac{\partial^2 f}{\partial p^2} + \frac{2I_b}{p} f^2 + \left[\frac{2I_a}{p} + 2I_b f\right] \frac{\partial f}{\partial p}$$

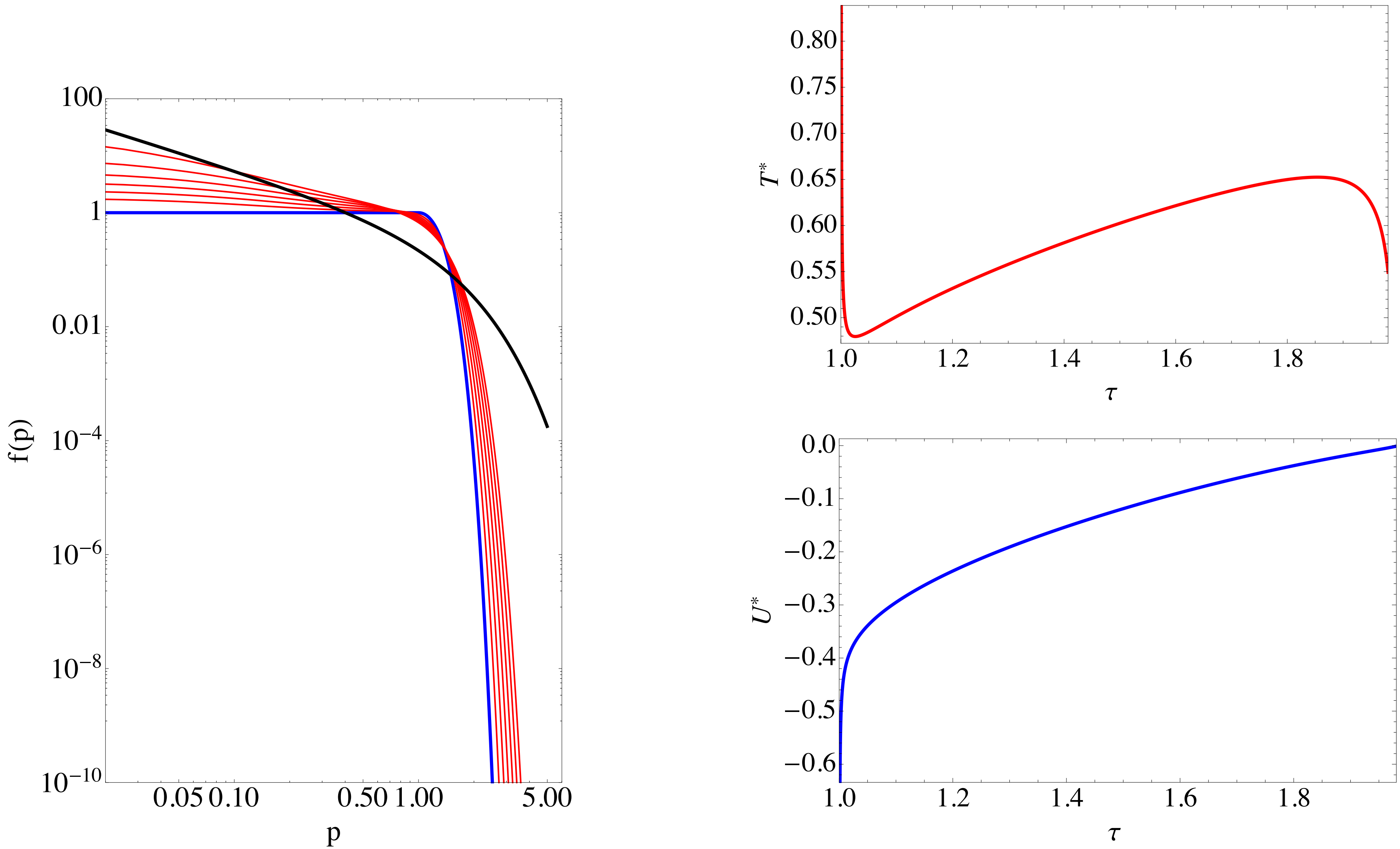
'instantaneous' equilibrium distribution function

$$f = \frac{T^*}{p - \mu^*} \qquad (\mu^* < 0)$$

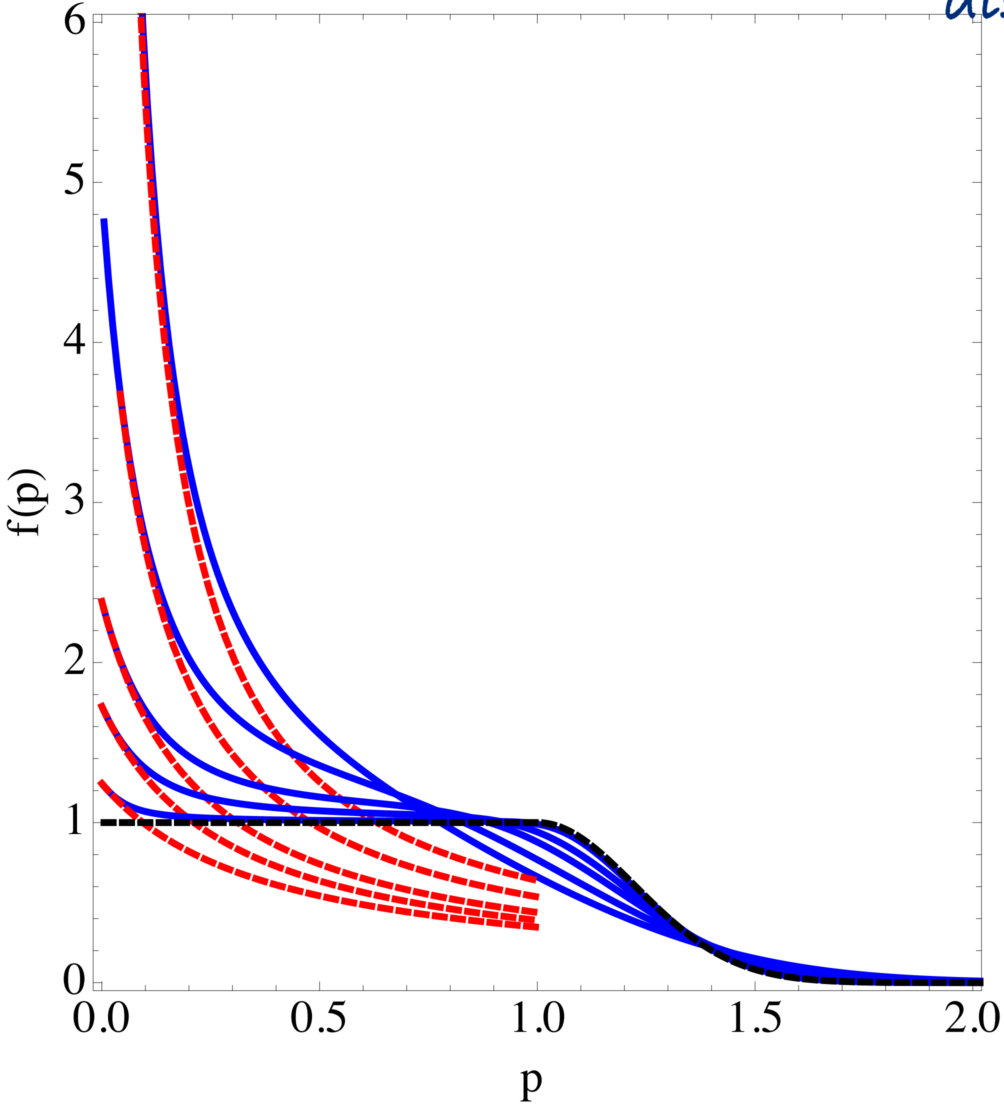
$$\mathcal{D}_\tau f \approx \frac{2T^*}{(p-\mu^*)^3}(I_a - T^* I_b) - \frac{2T^*}{p(p-\mu^*)^2}(I_a - T^* I_b)$$

$$I_a = \int \frac{d^3p}{(2\pi)^3} f(\boldsymbol{p})(1 + f(\boldsymbol{p})) \qquad I_b = \int \frac{d^3p}{(2\pi)^3} \frac{2f(\boldsymbol{p})}{p}$$

‘overpopulated’ case - onset of BEC $(f_0 = 1)$

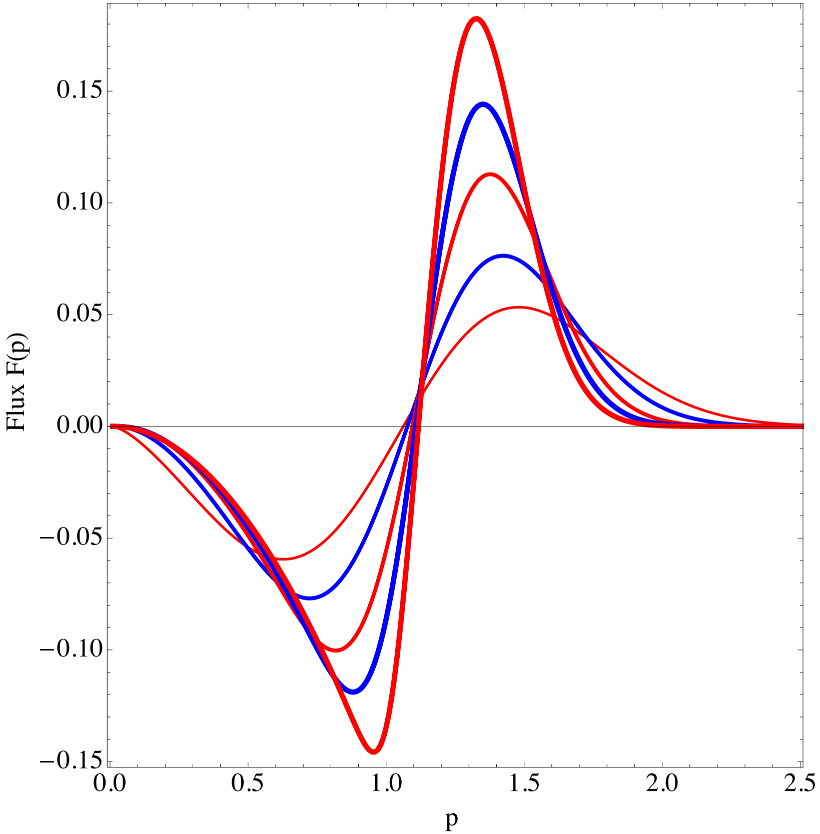


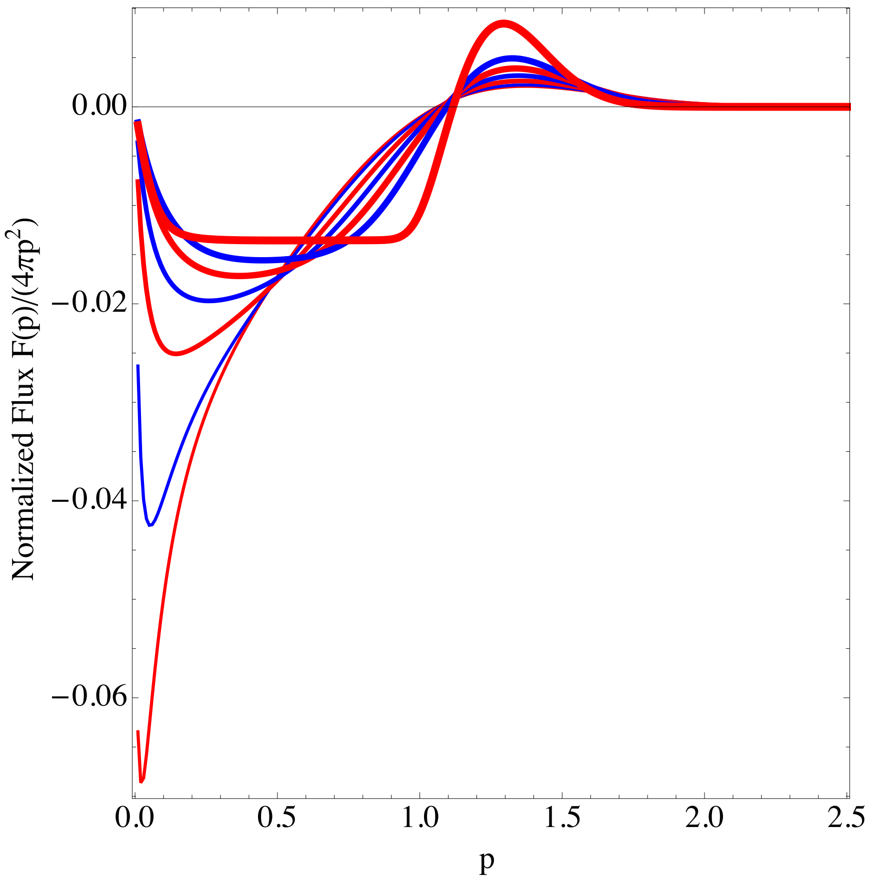
Small momentum behavior well reproduced by 'classical' distribution



$$f = \frac{T^*}{p - \mu^*}$$

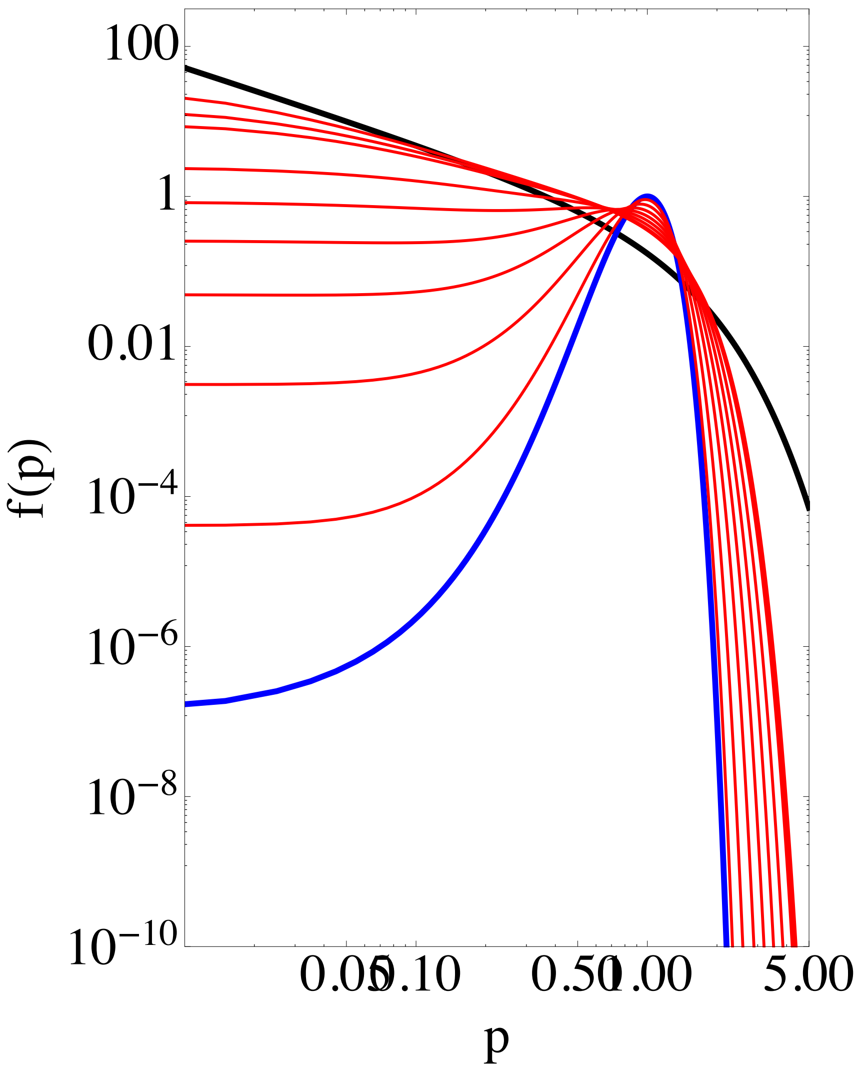
Flux in momentum space





Gaussian initial condition

Same qualitative features



Summary and outlook

- initial wave functions of colliding heavy nuclei at high energy are characterized by **'overpopulated' gluonic states**
- the (dynamical) **growth of (very) soft modes** seems to a be a robust feature.
- because of the large occupation, the system remains **strongly coupled** in spite of the small coupling constant
- a (transient) **Bose condensate** may form if particle number conserving processes dominate. Does the phenomenon survives inelastic collisions, longitudinal expansion? What is the nature of this condensate?