Instability in an expanding non-Abelian system

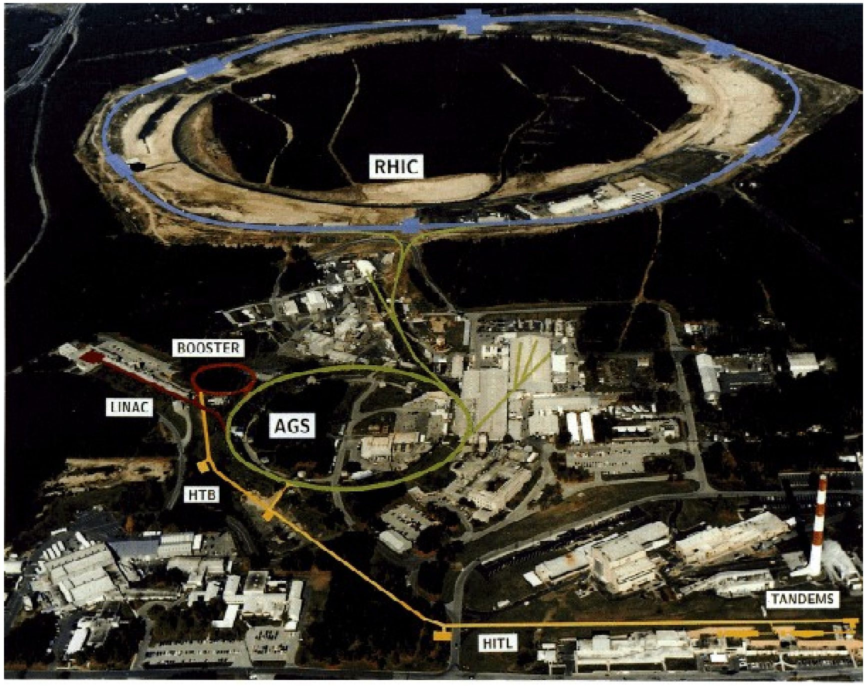
Kenji Fukushima

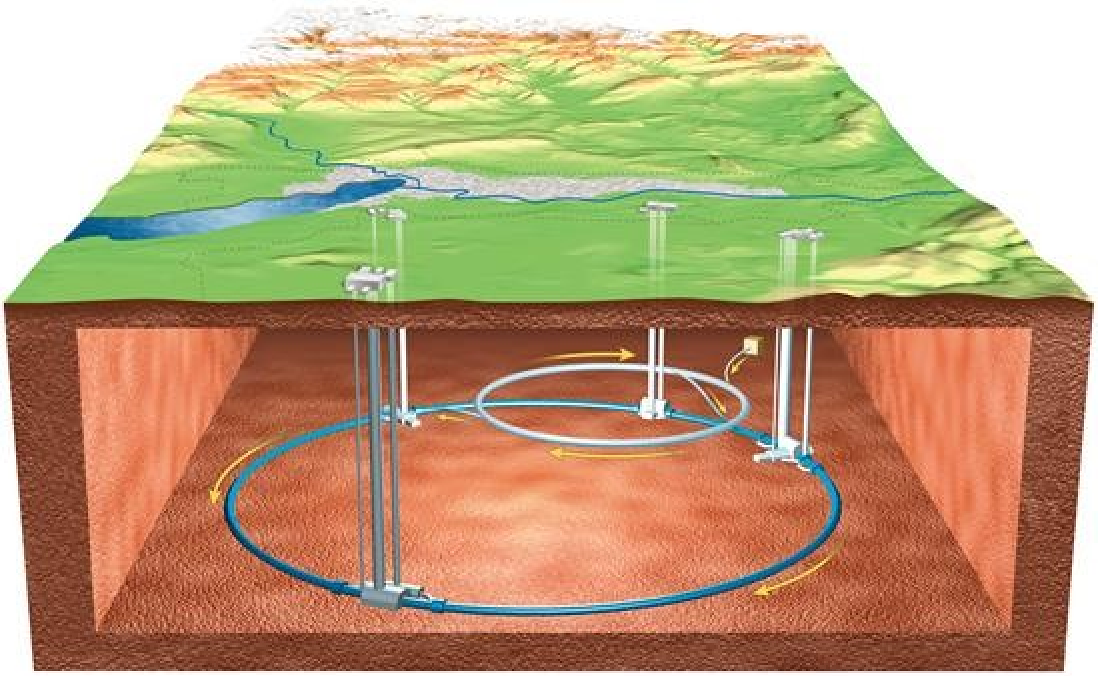
(Department of Physics, Keio University)

Why “expanding” ?

Relativistic Heavy-Ion Collision

RHIC **LHC**



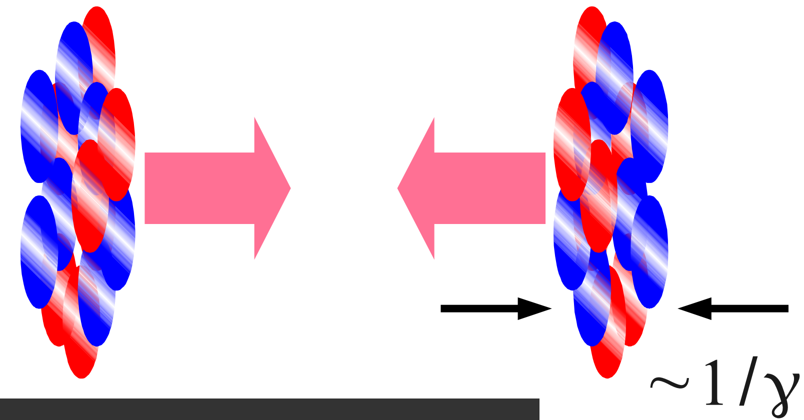


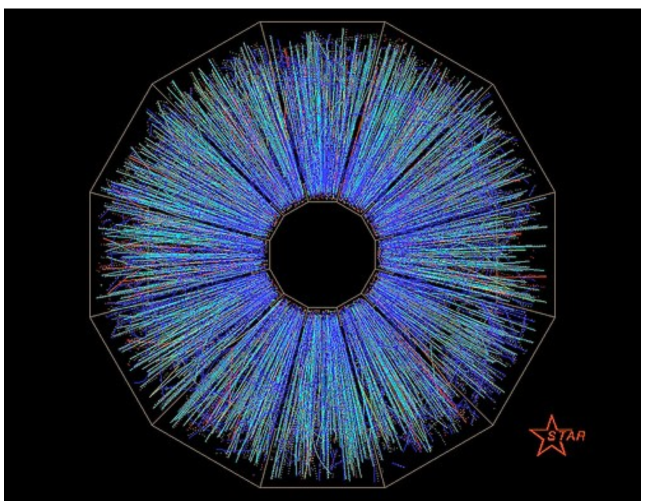
Heavy-ions collide → A new state of matter
(Au, Pb, …) (Quark-gluon plasma)

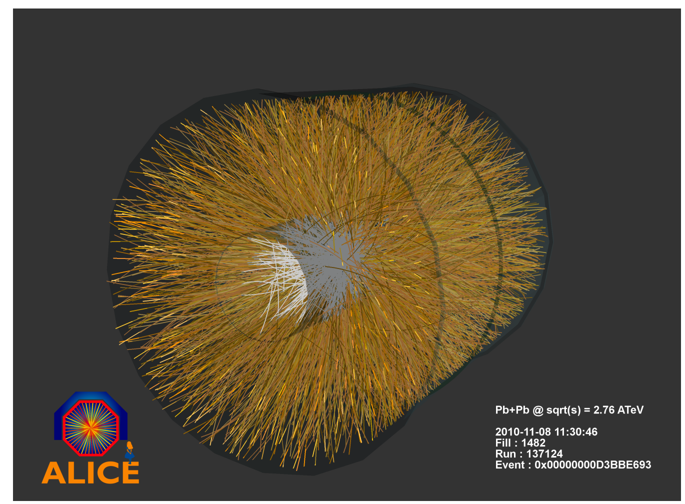
Relativistic Heavy-Ion Collision

LHC: $\sqrt{s_{NN}} = 2.7\,\text{TeV} \rightarrow \gamma \sim 1400$

RHIC: $\sqrt{s_{NN}} = 200\,\text{GeV} \rightarrow \gamma \sim 100$

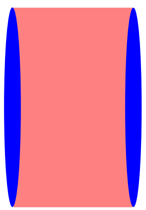






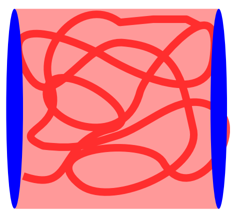
Thermalization achieved (elliptic flow by a hydro-model)
Initial temperature > 200MeV (distribution of thermal photon)

Schematic View of Four Regimes



Soft and coherent gluons
Color Glass Condensate
Initial (quantum) fluctuations

$\tau < Q_s$
~ 0.1 fm/c

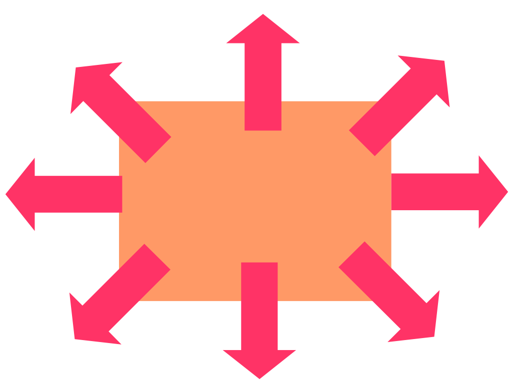


Instabilities → (toward) Isotropization
Glass + Plasma = Glasma
Quantum fluctuations
Particle (entropy) production
→ Thermalization

0.1 fm/c
~ 1 fm/c

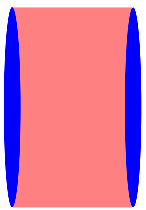
Hydrodynamic evolution + cascade
Relativistic Hydrodynamics

1 fm/c
~ 10 fm/c



Hadronization → Observation
Particle yields, distributions

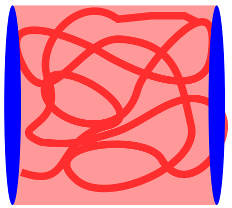
Missing Link



Soft and coherent gluons
Color Glass Condensate (CGC)
Initial (quantum) fluctuations

$\tau < Q_s$
~ 0.1 fm/c

If starting with the CGC what the "theory" predicts?



Instabilities $\rightarrow$ Isotropization
Glass + Plasma = Glasma
Quantum fluctuations
Particle (entropy) production
$\rightarrow$ Thermalization

0.1 fm/c
~ 1 fm/c

Why “non-Abelian” ?

Degrees of Freedom

QED

2 photons

4 electrons (positrons)

QCD

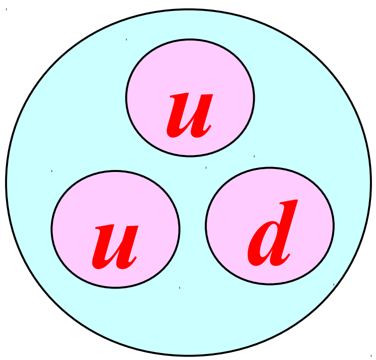
16 gluons

24 ~ 36 quarks

How can we neglect quarks though there are more quarks than gluons in nature?
(c.f. QCD thermodynamics)

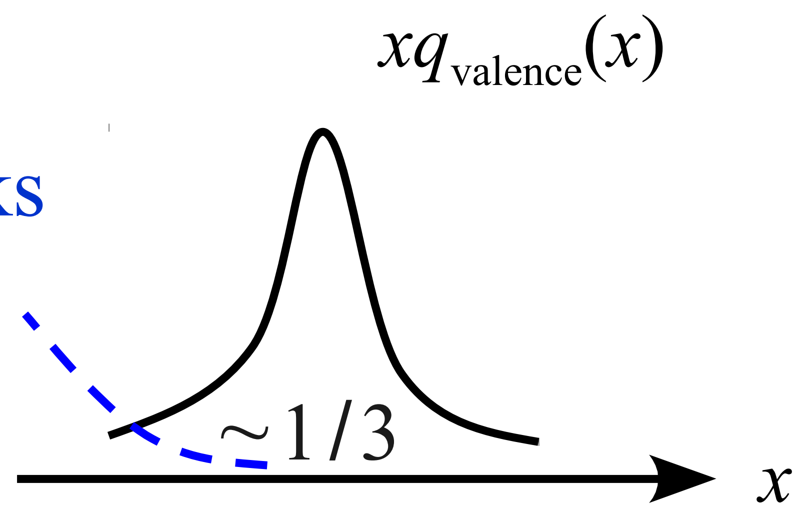
Parton Distribution Function

Valence and Sea Quarks and Gluons



proton

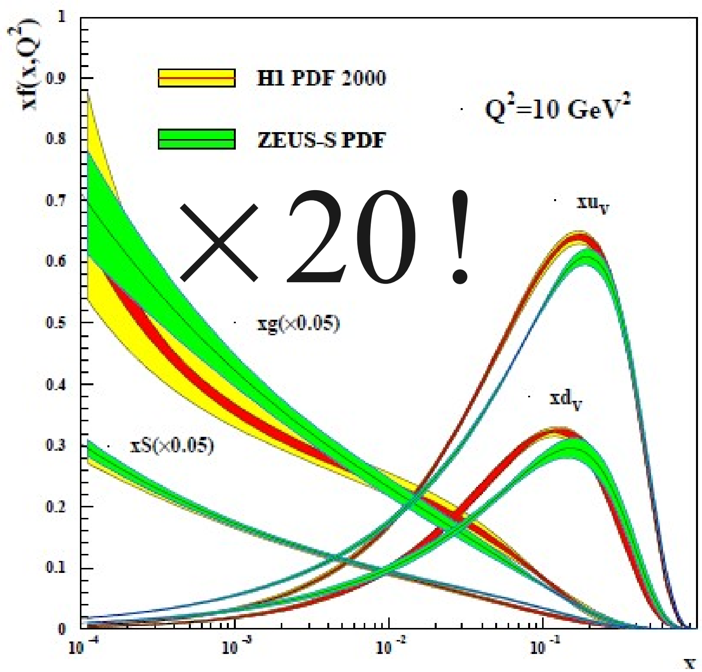
valence quark constituent



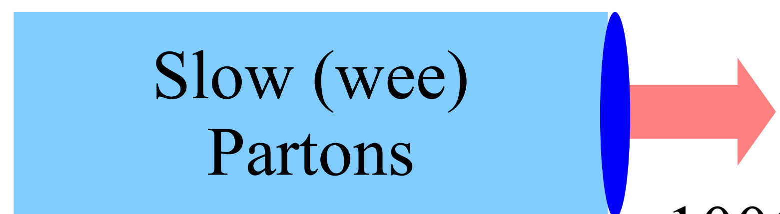
x **: momentum fraction carried by a parton**

Data from HERA

Quantum Evolution of PDFs at fixed Q^2

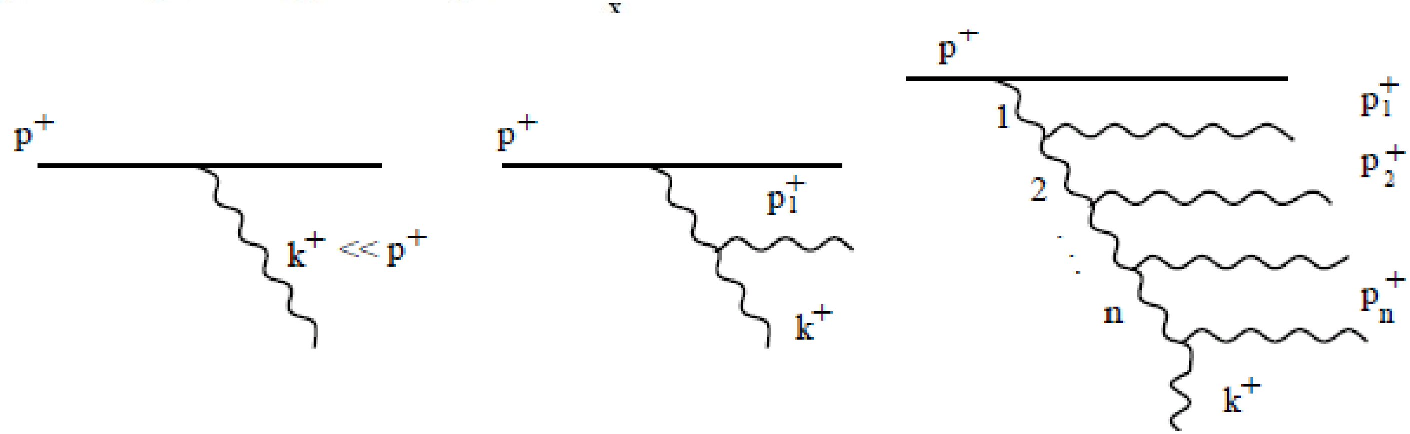


In the soft components of the nucleon wave-function, **gluon** is dominant.



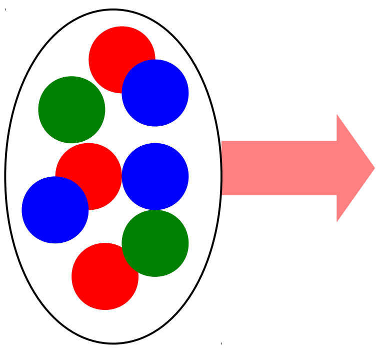
Soft physics < 1GeV

100GeV (RHIC)

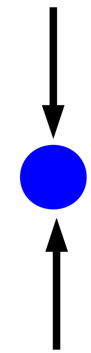


Saturation

Gluons eventually cover the transverse area:



Nucleon moving
at the speed of light



Transverse size of
a gluon ~ $1/Q$

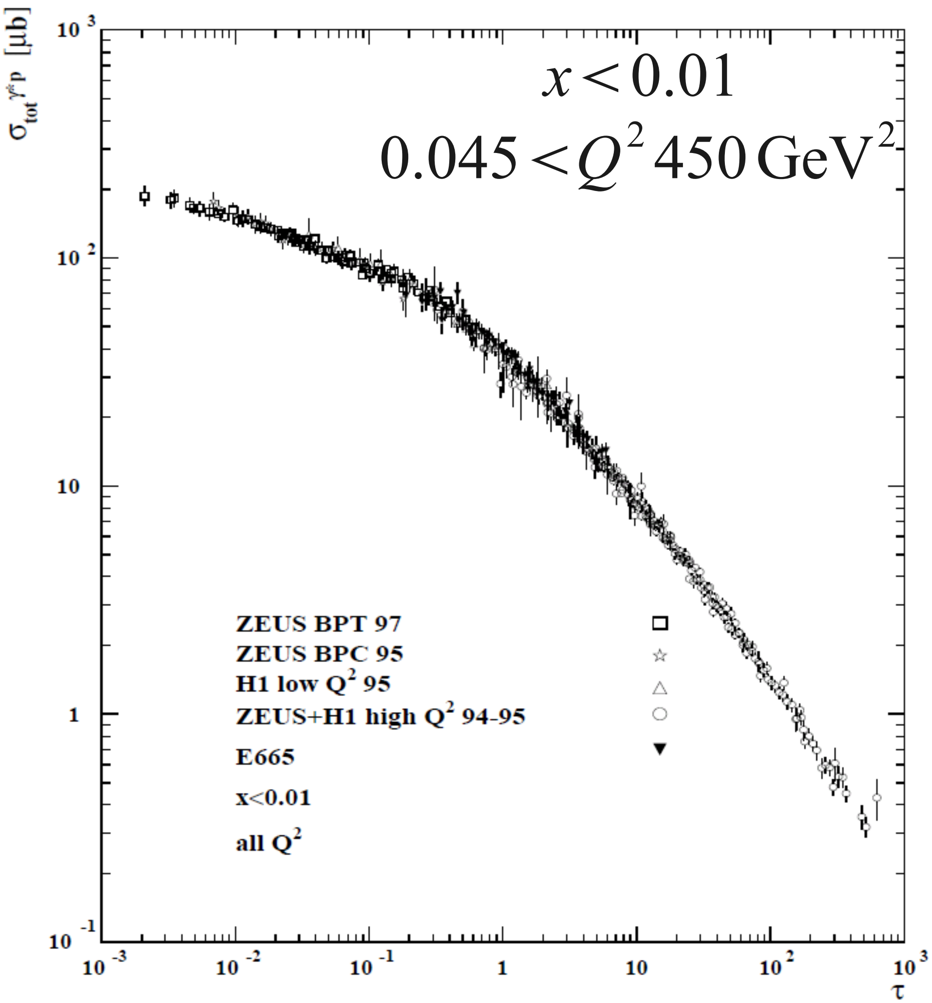
Naive condition for saturation:

$$ xg(x,Q) \Big/ (N_c^2-1)\,Q^2\pi R^2 \sim \frac{1}{\alpha_s N_c} \sim 1 $$

Once it happens, only $Q_s(x)$ fixes the physical scale!

Scaling Behavior

Dipole Cross Section in a Saturation Model



$$\sigma_{\gamma^* p}(x, Q^2) \to \sigma_{\gamma^* p}(Q^2/Q_s^2(x))$$
$$Q_s^2(x) = Q_0^2 (x/x_0)^{-\lambda}$$

Golec-Biernat-Wuesthoff

Stasto-Golec-Biernat-Kwiecinski Plot

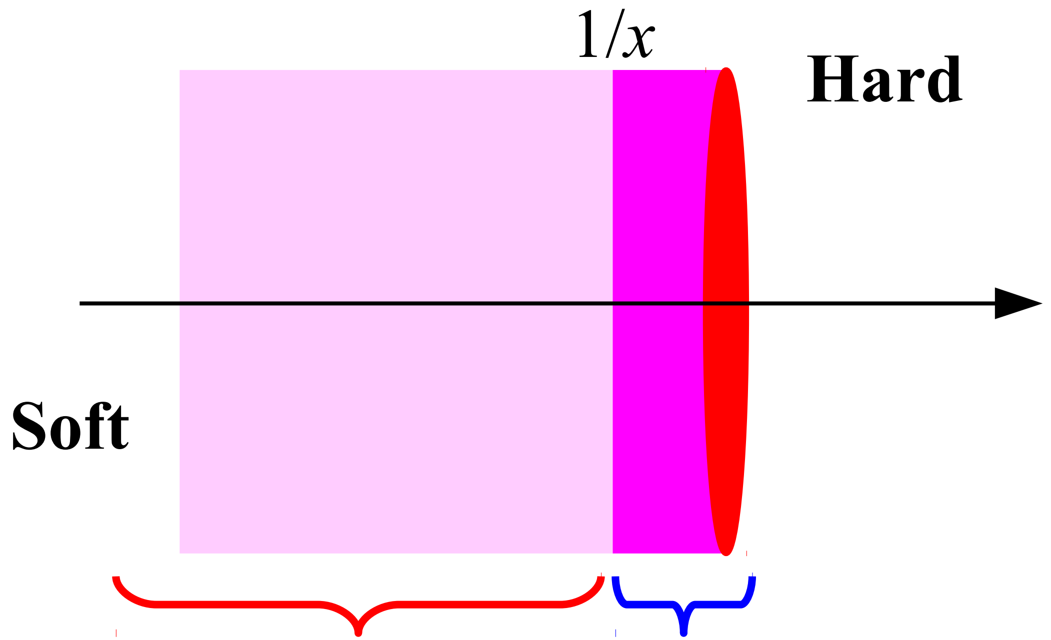
Geometric Scaling

Q_s as a function of x is fixed

$$Q_0 = 1\ \mathrm{GeV}$$
$$x_0 = 3.04 \times 10^{-4}$$
$$\lambda = 0.288$$

Saturation is sufficient for scaling, but not necessary to it.

“Effective Theory” of Saturation



Coherent gluons **Integrated out**

→ Classical fields $\mathcal{A}[\rho]$ **→ Classical source ρ**

Wave-function $W_x[\rho]$ → Classical sol. $\mathcal{A}[\rho]$ → Observable

Kovner, McLerran, Weigert, Iancu, Jalilian-Marian, Leonidov, ...

McLerran-Venugopalan (MV) Model

Gaussian Approximation:

McLerran-Venugopalan (1993)

$$W_x[\rho] = \exp\left[-\int d^3x \frac{|\rho(x)|^2}{2g^2\mu_x^2} \right]$$

μ_x is related to $Q_s(x)$

larger μ_x = larger ρ = dense gluons = larger Q_s

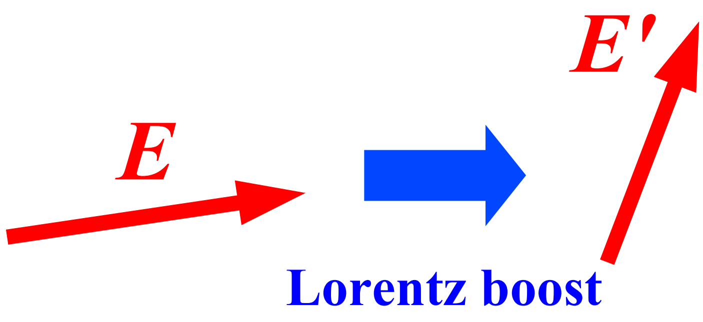
Now we know that fluctuations are important for v_3, v_4, etc, but in a zero-th order approximation, this description should make sense as a good starting point... but!?

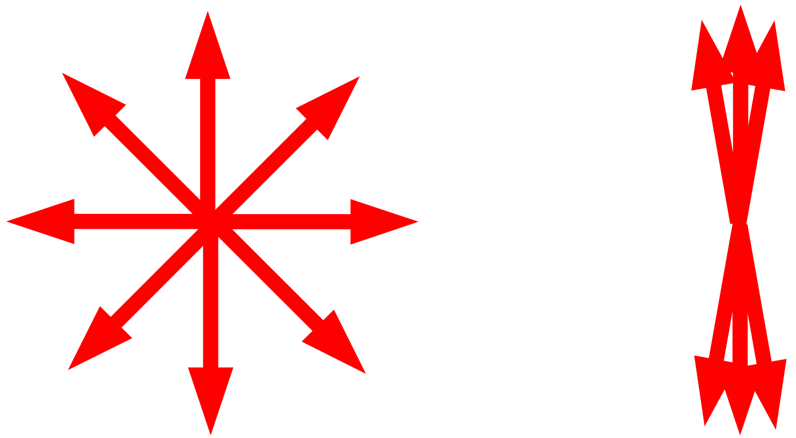
Why “instability” ?

Schematic Picture before Collision

Two nuclei do not talk to each other
Just one color-source problem

No longitudinal fields but only transverse fields attached on the nucleus sheet





Initial Condition

Fields made by colliding two sources

Initial condition is known on the light-cone

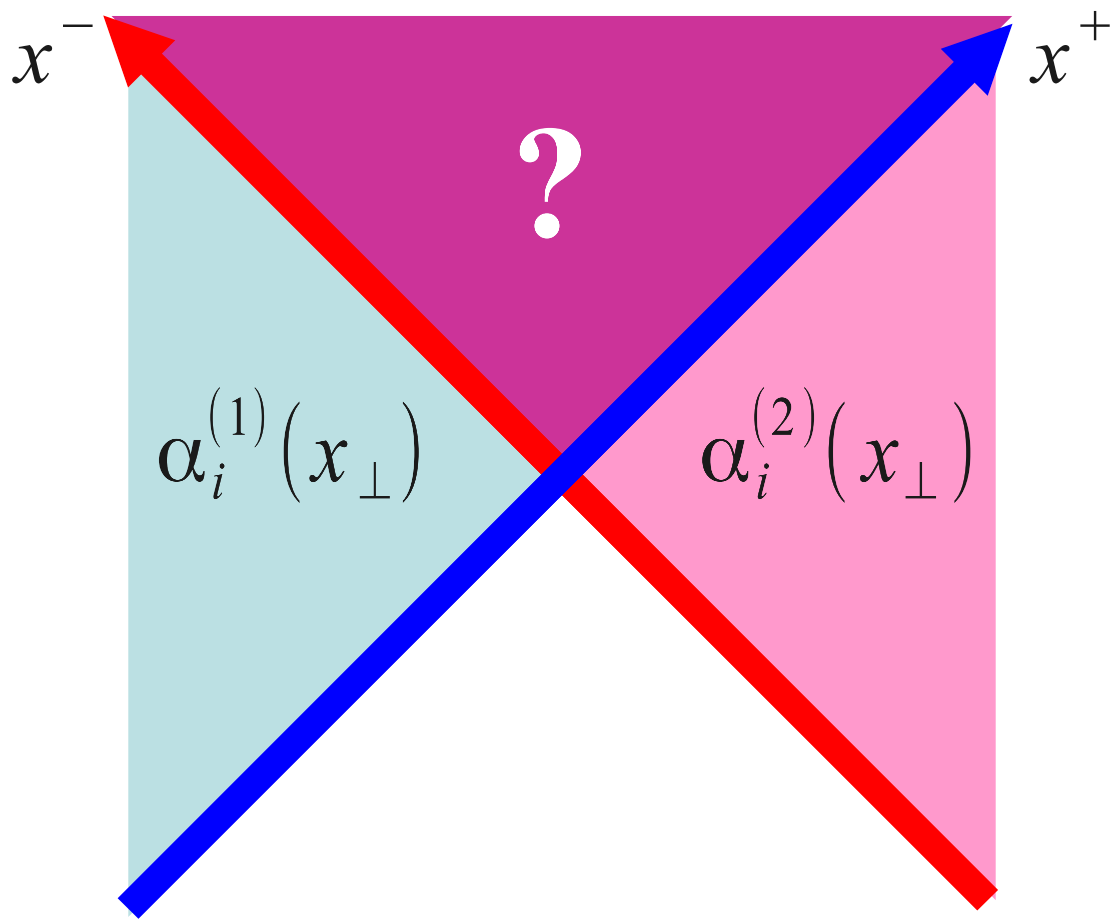
$$\mathcal{A}_i = \alpha_i^{(1)} + \alpha_i^{(2)}$$

$$\mathcal{A}_\eta = 0$$

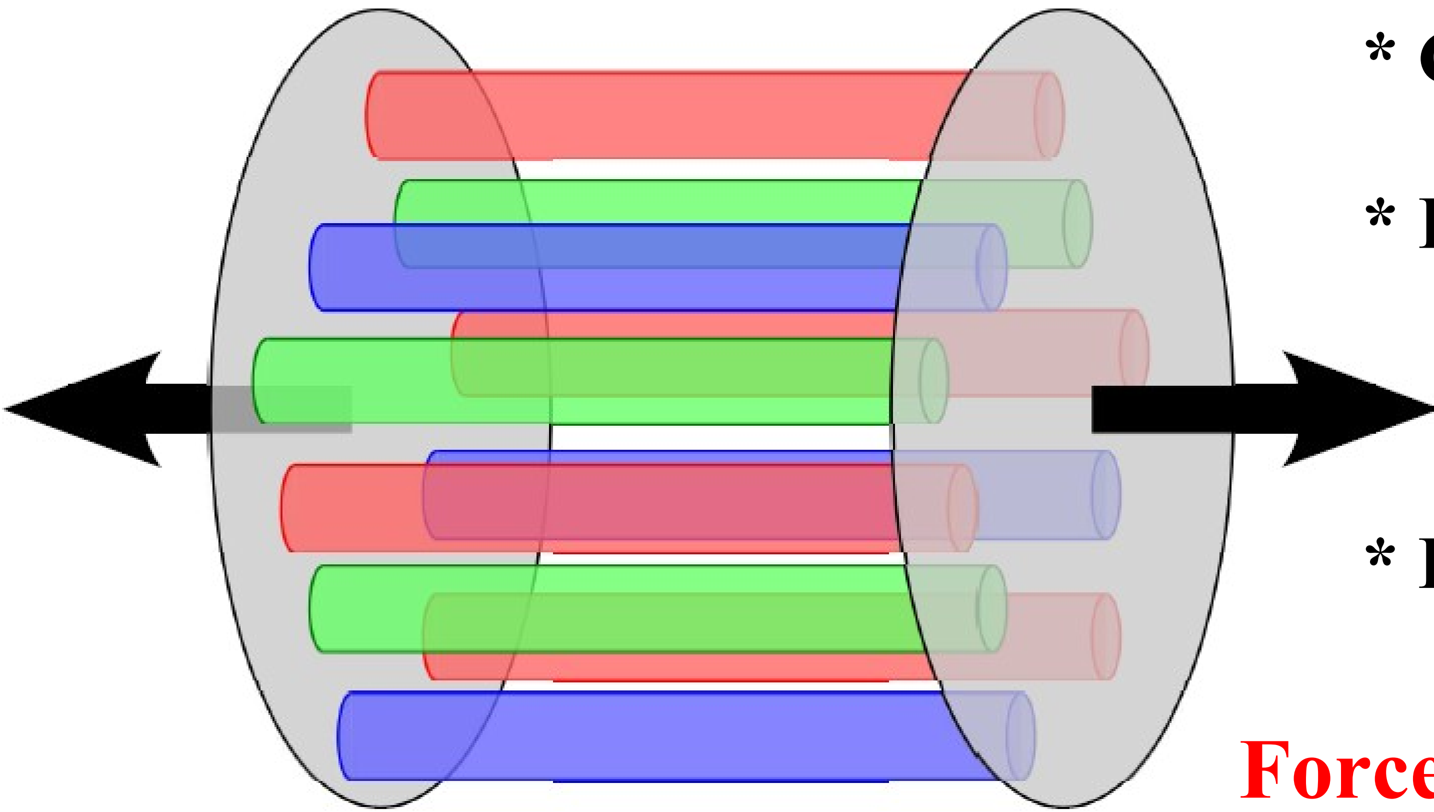
$$\mathcal{E}^i = 0$$

$$\mathcal{E}^\eta = ig\left[\alpha_i^{(1)}, \alpha_i^{(2)}\right]$$

Kovner-McLerran-Weigert (1995)



Intuitive Picture of "Glasma"



* **Boost Invariance**
* **Coherent Fields (amp. ~ $1/g$)**
* **Flux Tube (size ~ $1/Q_s$)**
* **Expanding**

Force from the tube should be overcome

McLerran-Lappi (2006)

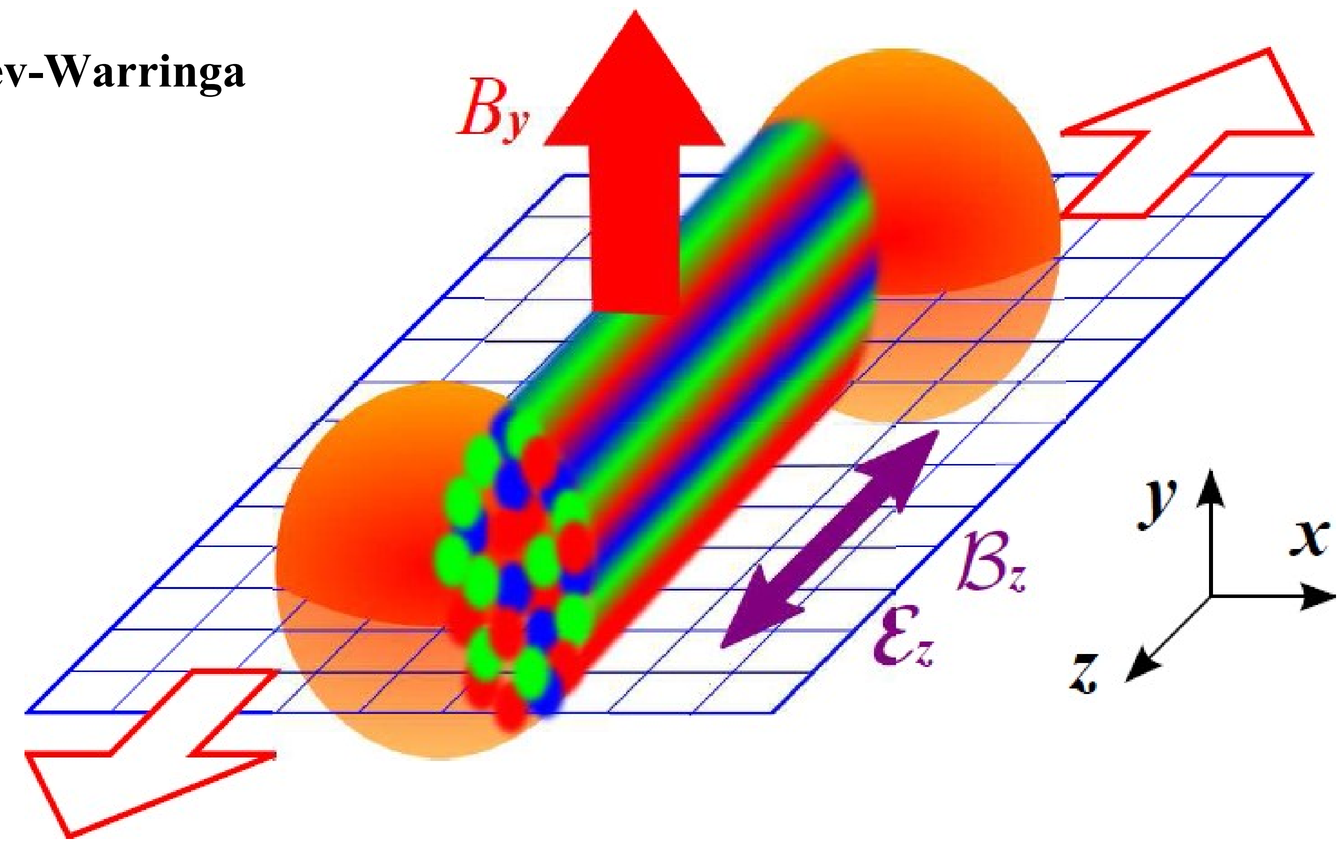
Schematic Picture after Collision

What is the initial condition in the HIC?

Negative longitudinal pressure

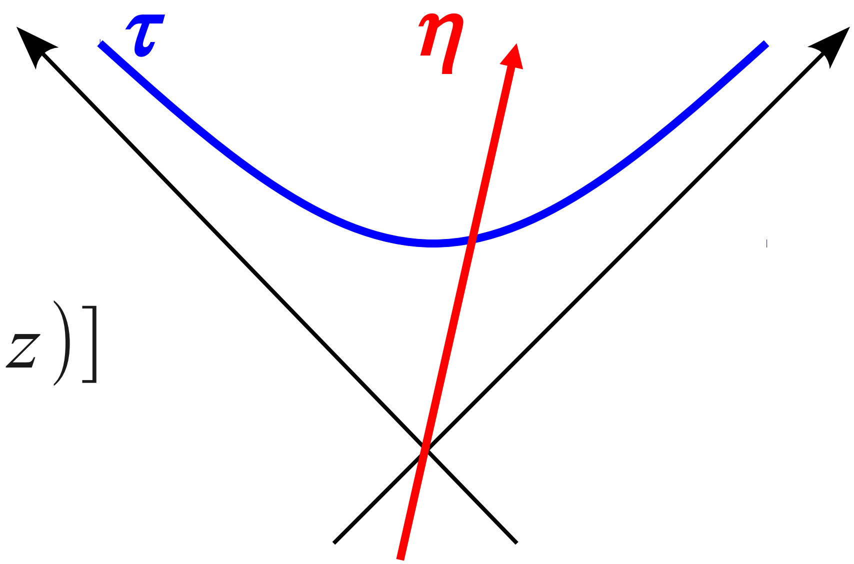
Topological charge density → Chiral magnetic effect

KF-Kharzeev-Warringa



Equations of Motion to be Solved

Coordinates



proper time $\tau = \sqrt{t^2 - z^2}$

rapidity $\eta = \frac{1}{2} \ln\left[(t+z)/(t-z) \right]$

Equations to be solved

$$D_\mu F^{\mu\nu} = j^\nu = 0$$

Formulations

Time Evolution

$$E^i = \tau \partial_\tau A_i\,, \qquad E^\eta = \tau^{-1} \partial_\tau A_\eta$$

$$\partial_\tau E^i = \tau^{-1} D_\eta F_{\eta i} + \tau D_j F_{ji}$$

$$\partial_\tau E^\eta = \tau^{-1} D_j F_{j\eta}$$

Classical Equations of Motion in the Expanding System

(c.f. Gross-Pitaevskii eq.)

Ensemble Average

$$\langle\langle \mathcal{O}[A] \rangle\rangle_{\rho_t,\rho_p} \sim \int D\rho_t\, D\rho_p W_x[\rho_t] W_{x'}[\rho_p]\, \mathcal{O}[\mathcal{A}[\rho_t,\rho_p]]$$

Quantum fluctuations partially included in the initial state

Physical Degrees of Freedom

$$A_x^a(\tau, \eta, x, y) \qquad A_y^a(\tau, \eta, x, y)$$

$$A_\eta^a(\tau, \eta, x, y) \qquad A_\tau^a = 0$$

$$E_x^a(\tau, \eta, x, y) \qquad E_y^a(\tau, \eta, x, y)$$

$$E_\eta^a(\tau, \eta, x, y)$$

η-dep drops from the init. cond.

a = 8 (adjoint with red, green, blue)

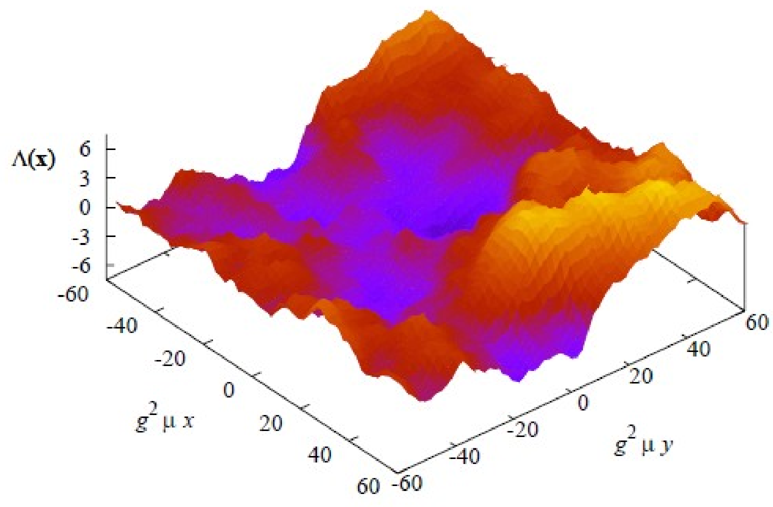
48 fields

→ **Simplify from 48 to 18 by assuming *a* = 3 (2colors)**

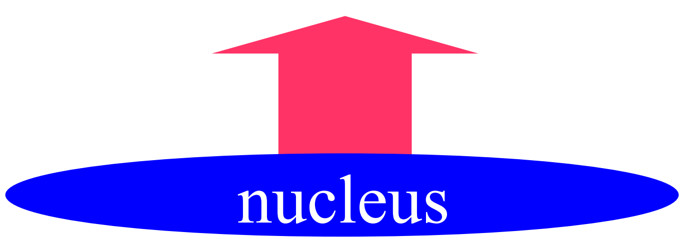
Initial Configurations

Solve the Poisson Eq

$$\partial_\perp^2 \Lambda^{(m)}(\boldsymbol{x}_\perp) = -\rho^{(m)}(\boldsymbol{x}_\perp)$$

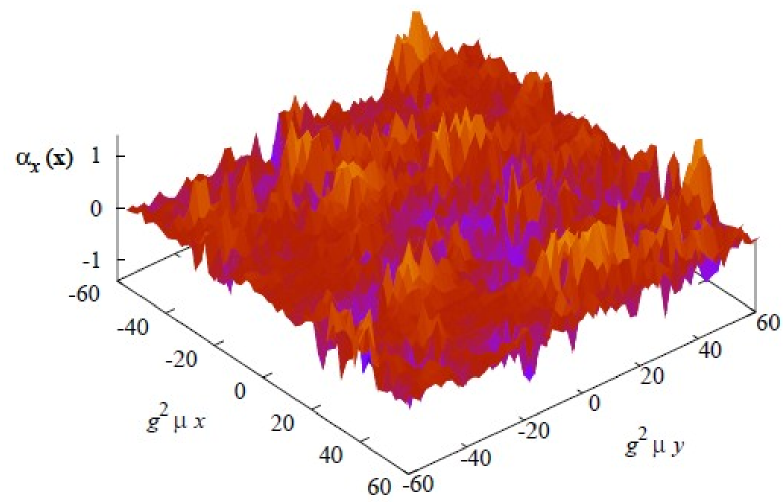


Transverse Distribution



Gauge Configuration

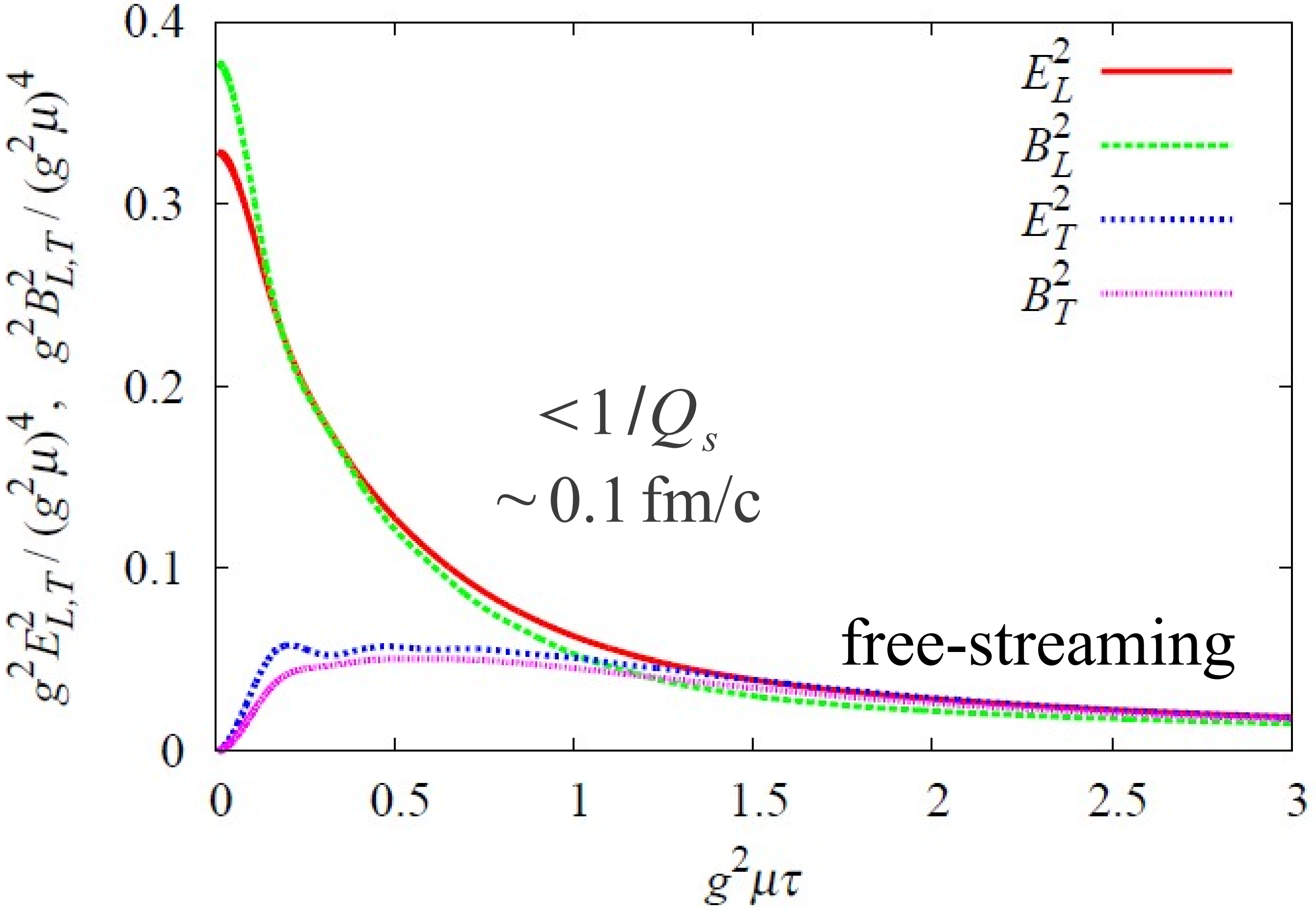
$$e^{-ig\Lambda(\boldsymbol{x}_\perp)} e^{ig\Lambda(\boldsymbol{x}_\perp + \hat{i})} = \exp[-ig\alpha_i(\boldsymbol{x}_\perp)]$$

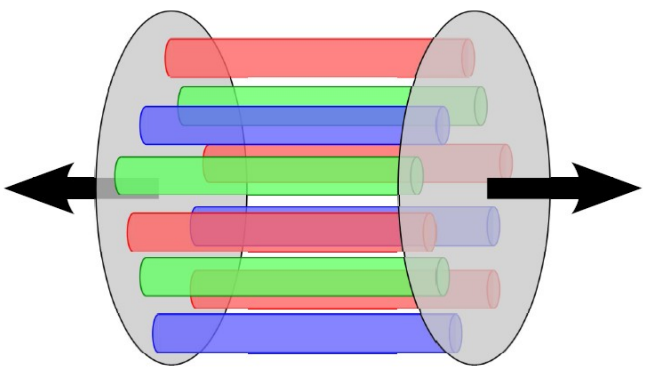


No structure because of
the Gaussian wave-function
(this part improvable)

Chromo-Electric and Magnetic Fields

Longitudinal and Transverse Fields

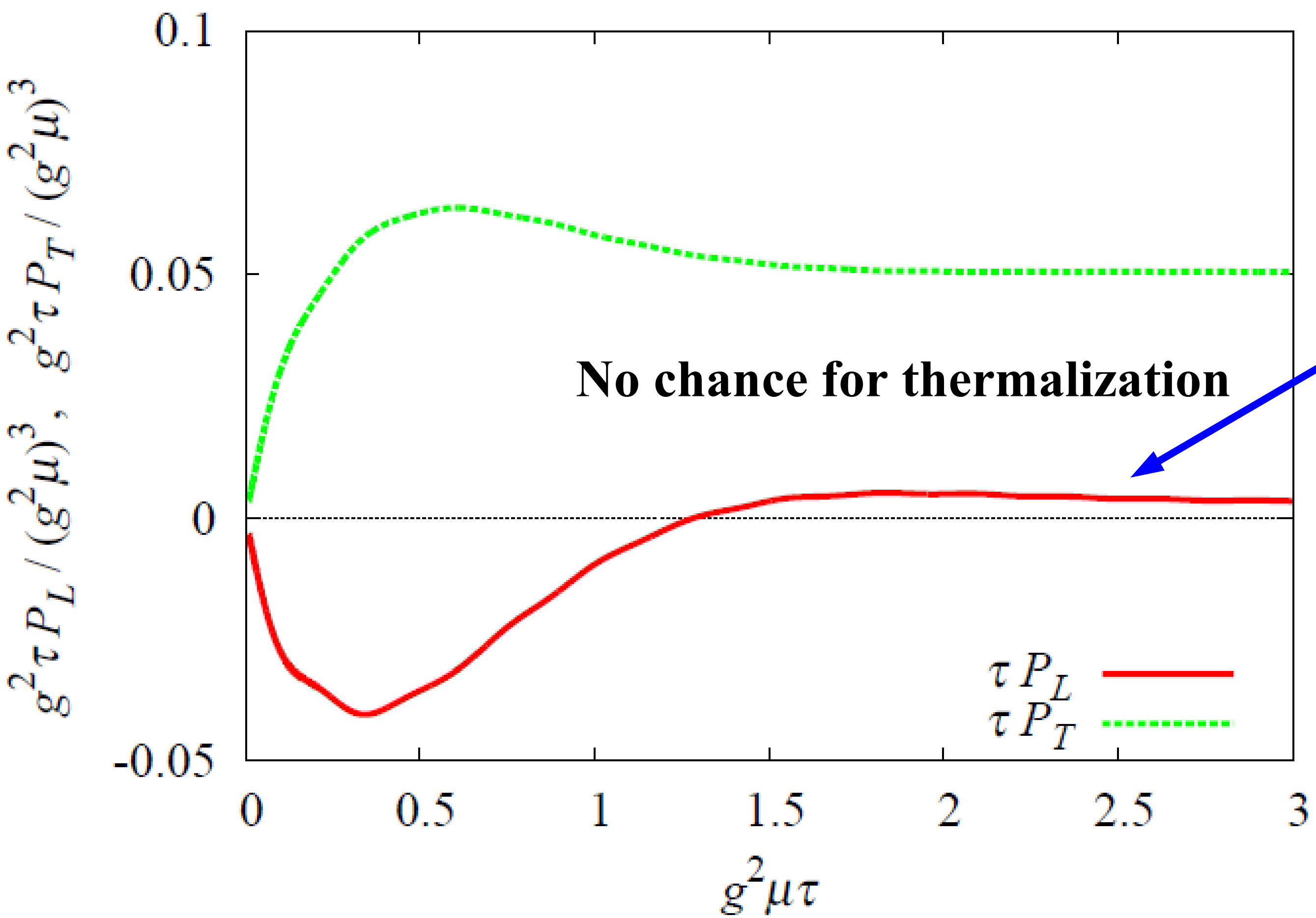




Lappi-McLerran (2006)
Fukushima-Gelis (2011)

Longitudinal and Transverse Pressure

$$P_T = \frac{1}{2}\left\langle T^{xx} + T^{yy} \right\rangle = \left\langle \mathrm{tr}\left[E_L^2 + B_L^2\right] \right\rangle ,$$
$$P_L = \left\langle \tau^2 T^{\eta\eta} \right\rangle = \left\langle \mathrm{tr}\left[E_T^2 + B_T^2 - E_L^2 - B_L^2\right] \right\rangle$$

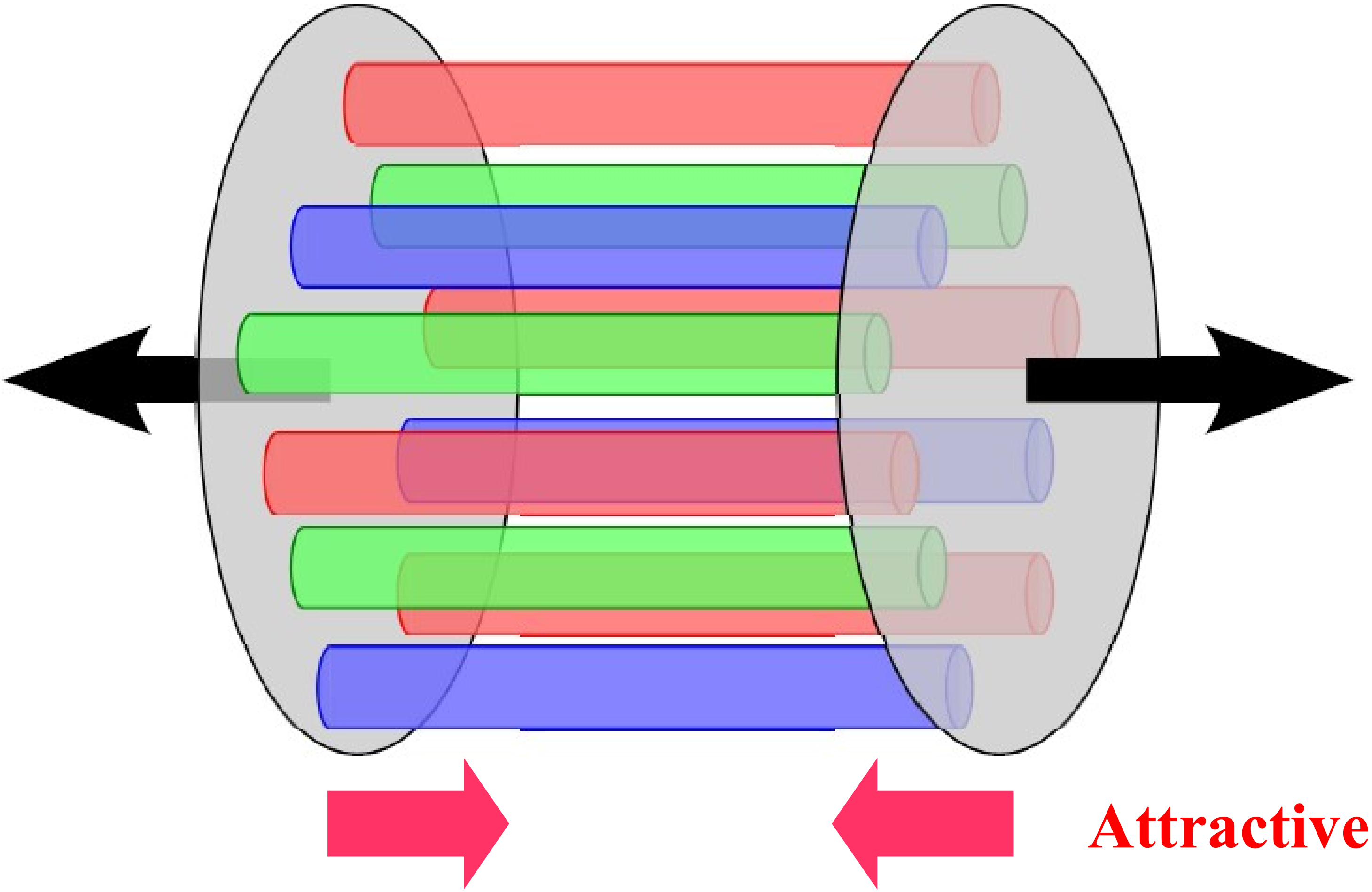


(Almost)
free-streaming

Isotropization
$P_T = P_L$

Fukushima-Gelis (2011)

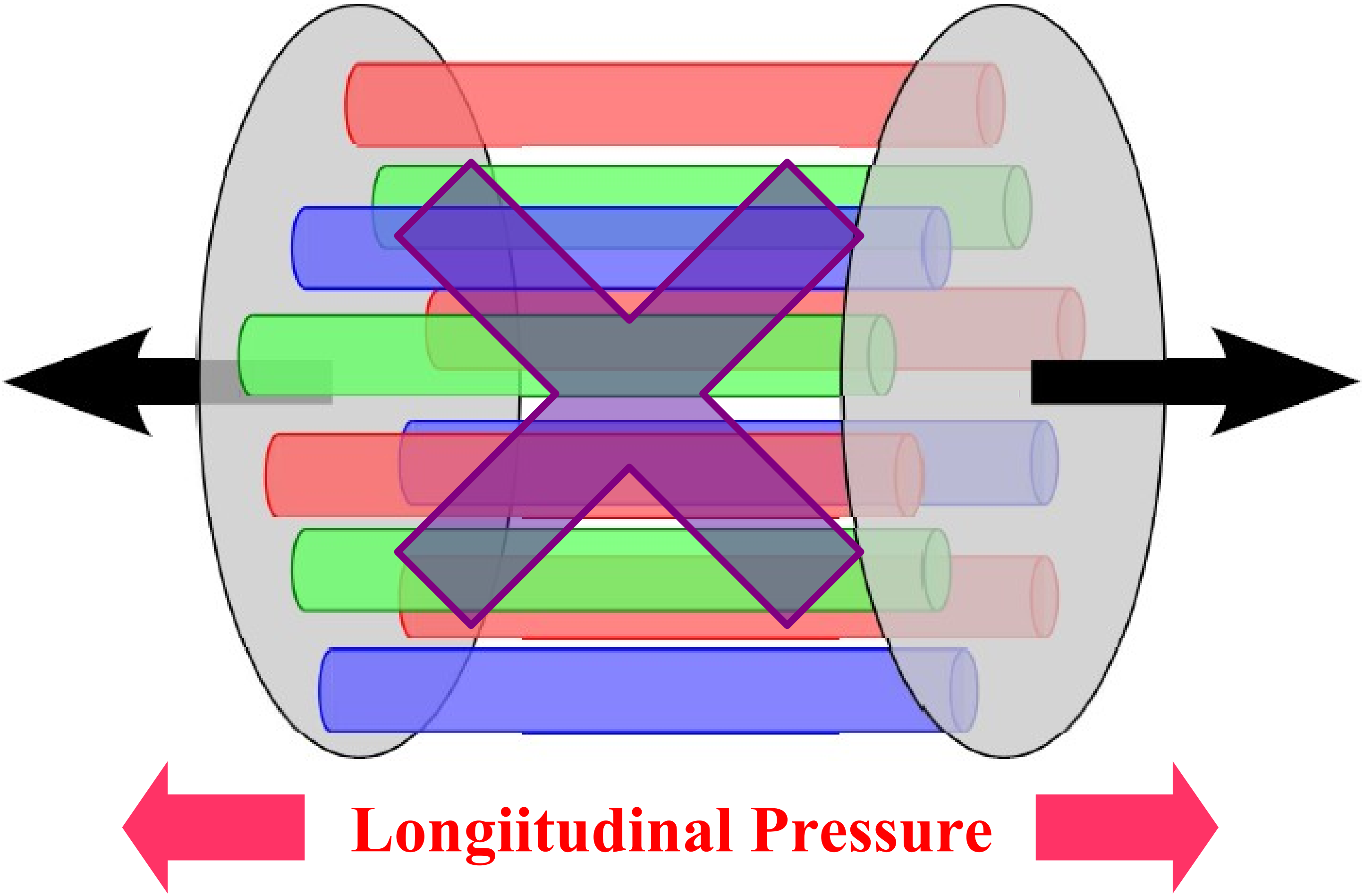
Negative Longitudinal Pressure



Attractive Force

Flux tubes have a positive energy

How It should Look Like Later

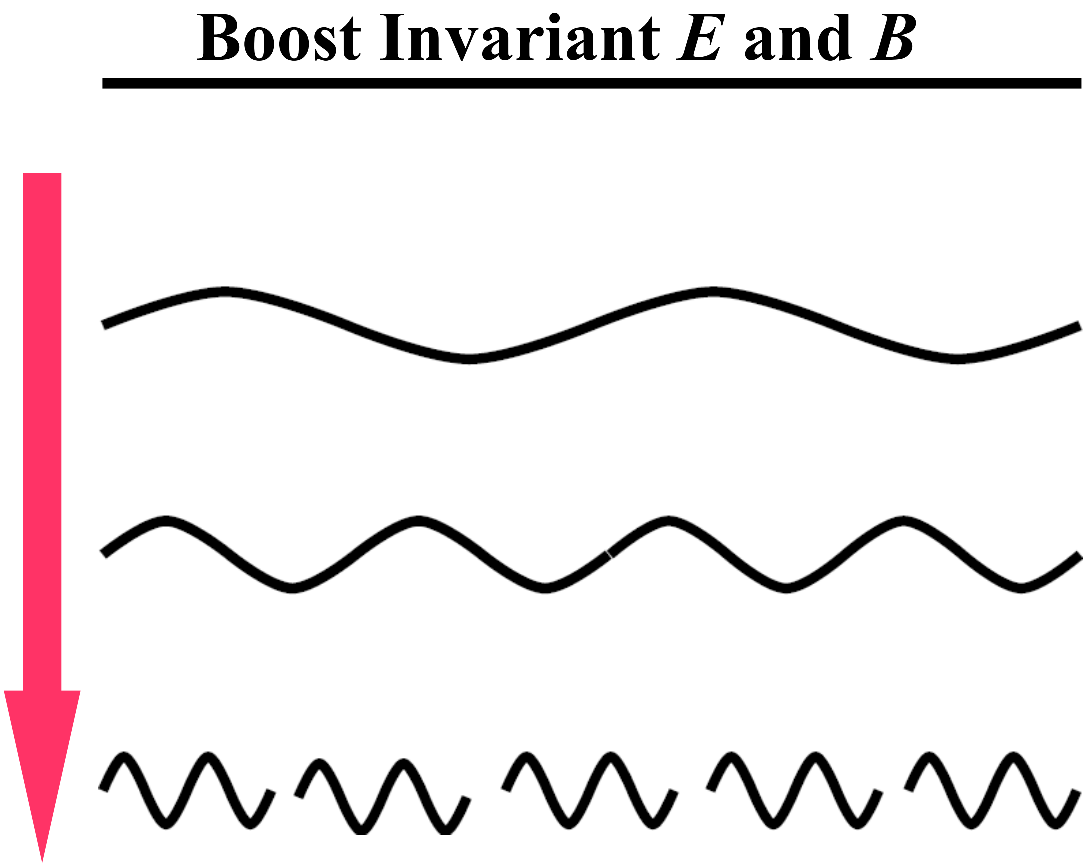


Longiitudinal Pressure

$$\varepsilon \sim \tau^{-1-\delta}$$

What Is Missing

Flux tube **~ QCD string**



Instability

c.f. Plasma instability by Rebhan
Simulations by Berges, Sexty, Kunihiro, Iida

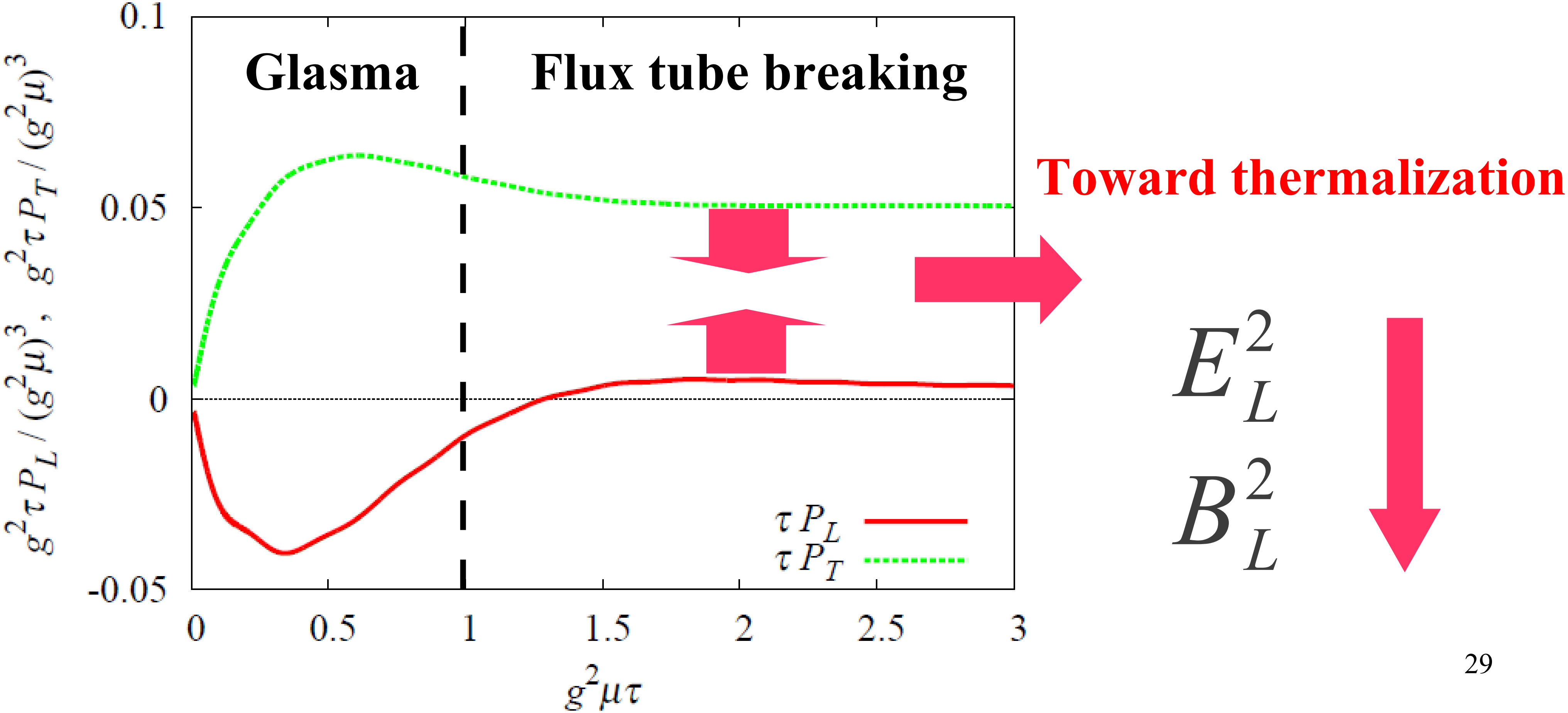
c.f. Deconfinement at high T (entropy wins)

String breaking → Particle production (Schwinger mechanism)

Expectation

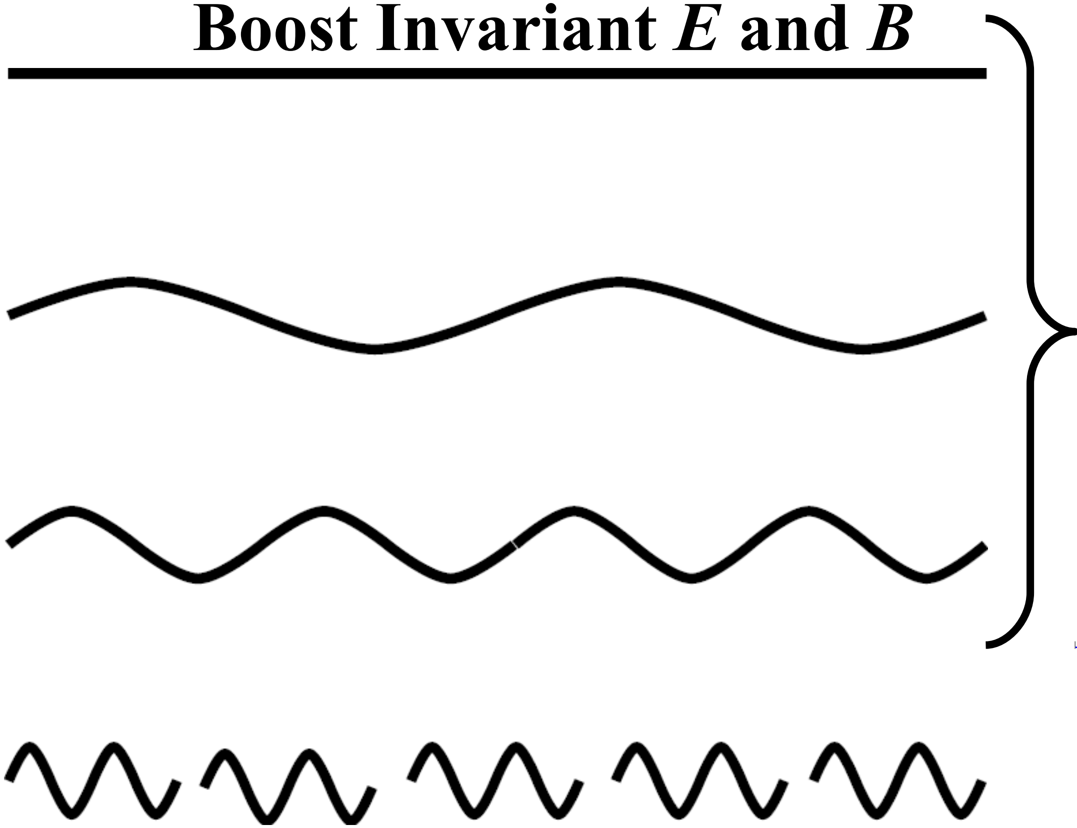
$$P_T = \frac{1}{2}\left\langle T^{xx} + T^{yy}\right\rangle = \left\langle \mathrm{tr}\left[E_L^2 + B_L^2\right]\right\rangle ,$$

$$P_L = \left\langle \tau^2 T^{\eta\eta}\right\rangle = \left\langle \mathrm{tr}\left[E_T^2 + B_T^2 - E_L^2 - B_L^2\right]\right\rangle$$



Classical Statistical Simulation

Boost Invariant *E* and *B*



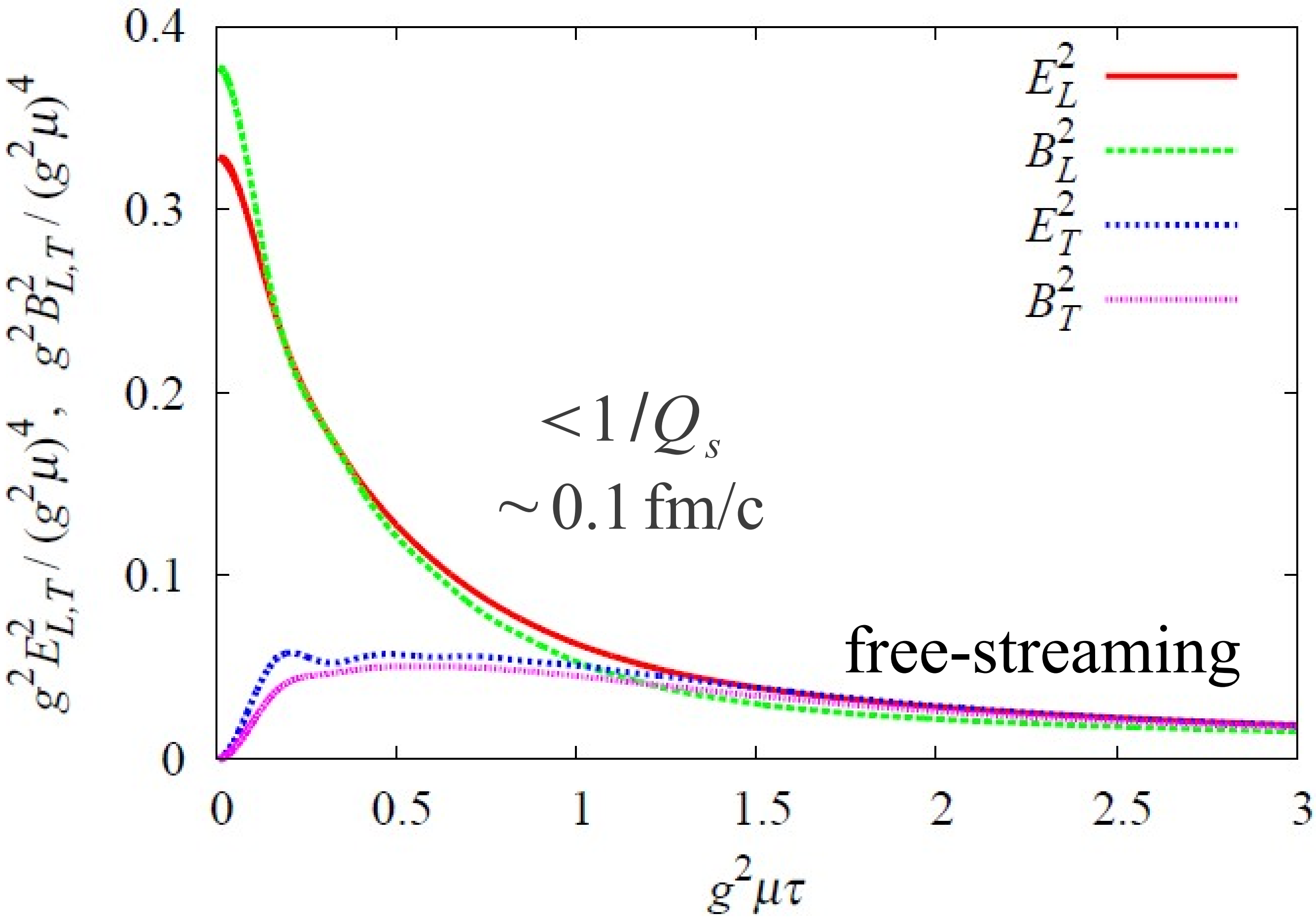
Classical Dynamics
+
Small Fluctuations

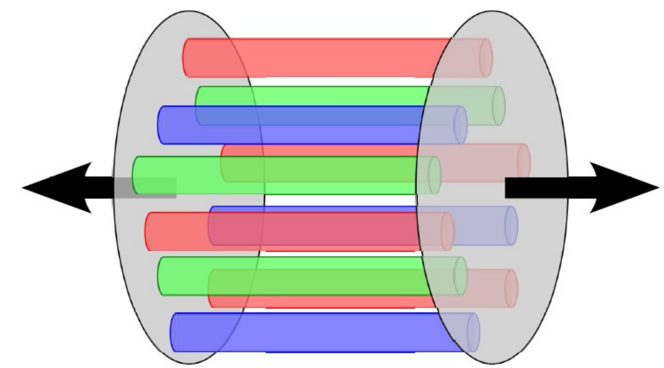
What is the dynamics of the background *E* and *B*?

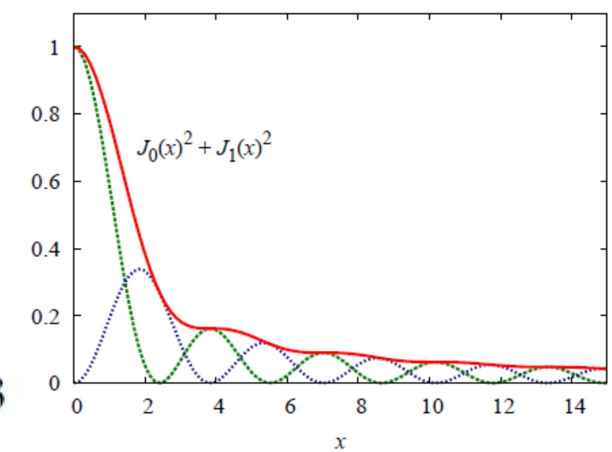
How fluctuations grow?

Boost Invariant Background Again

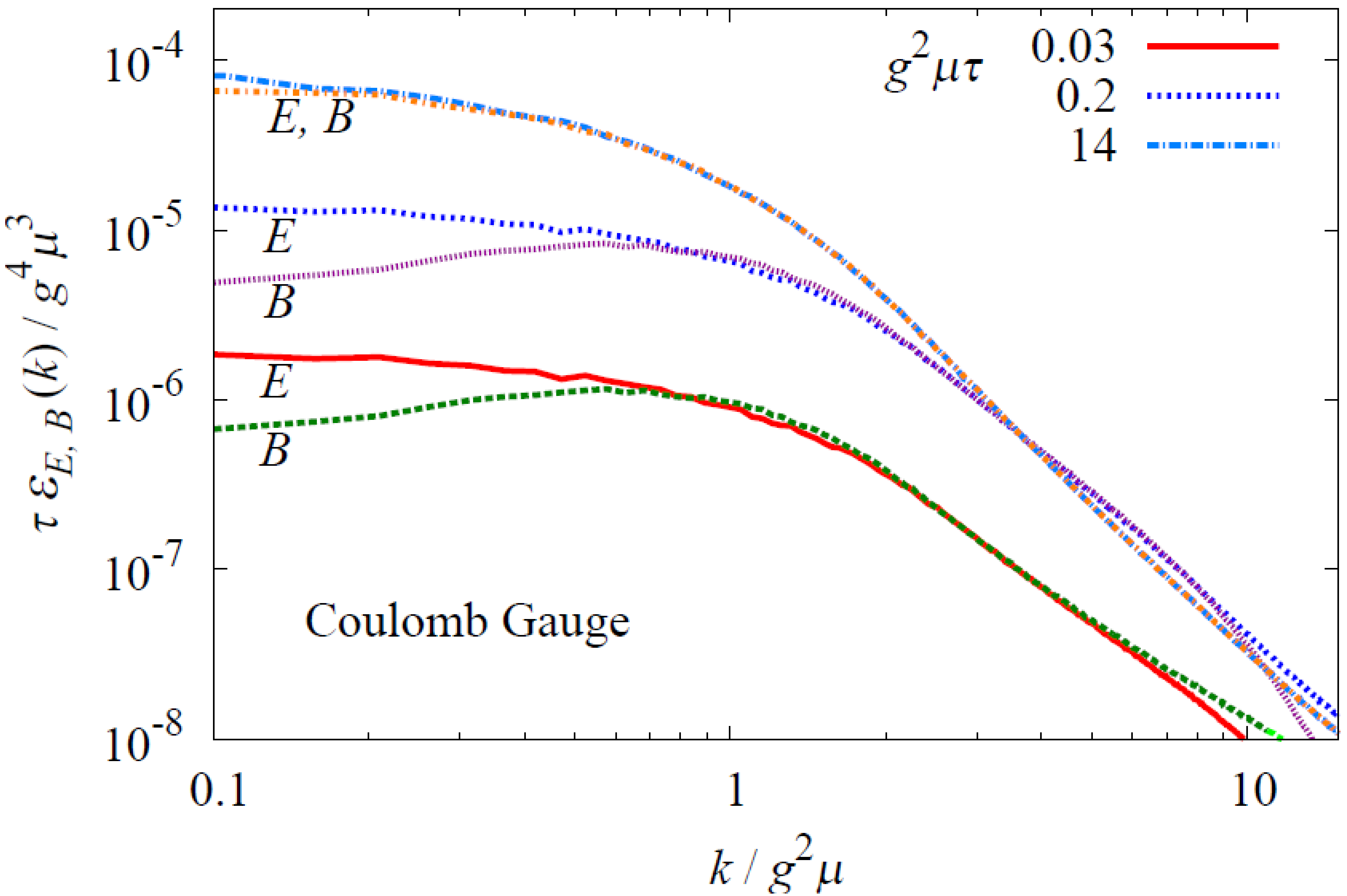
Longitudinal and Transverse Fields



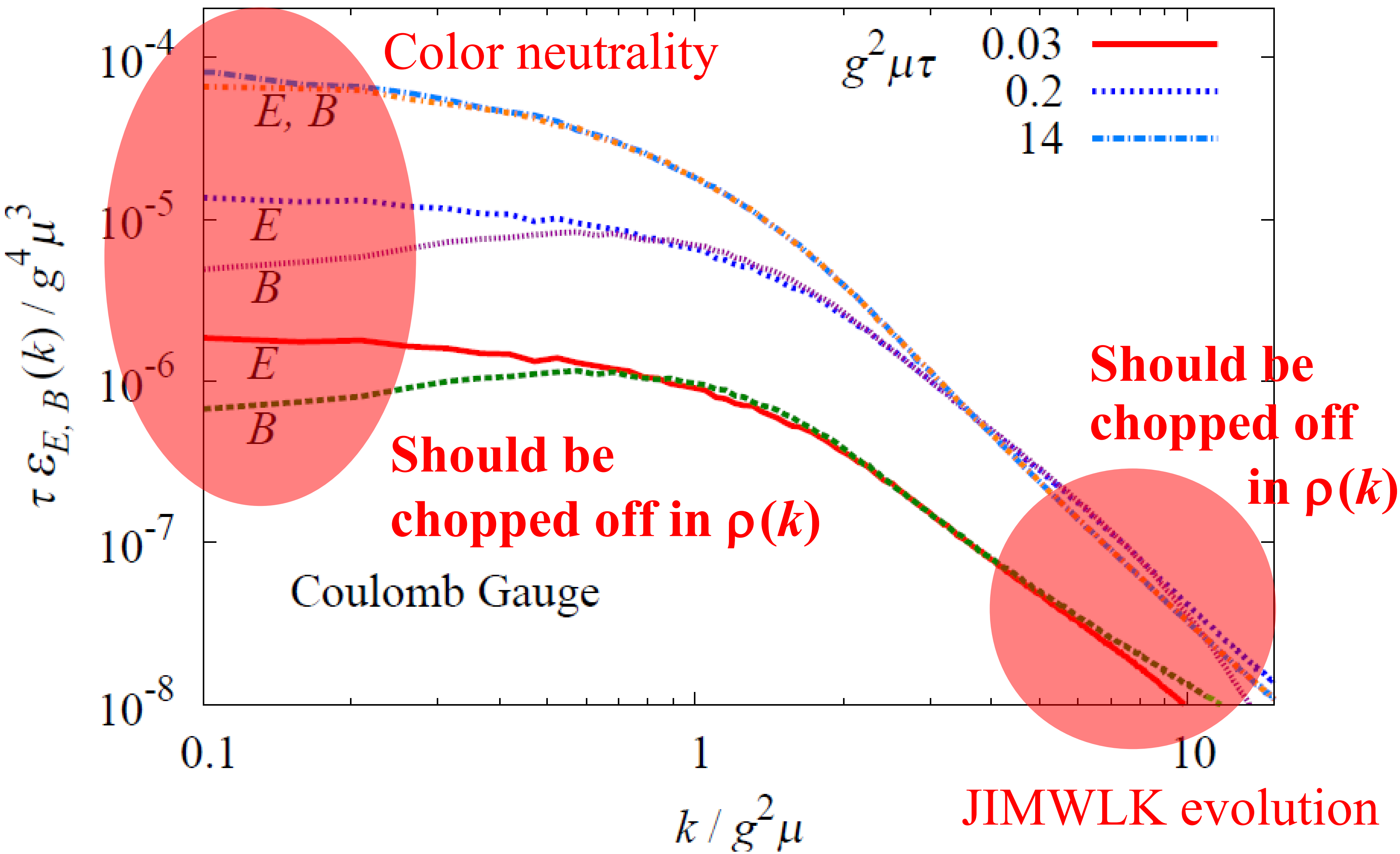




Mode Analysis

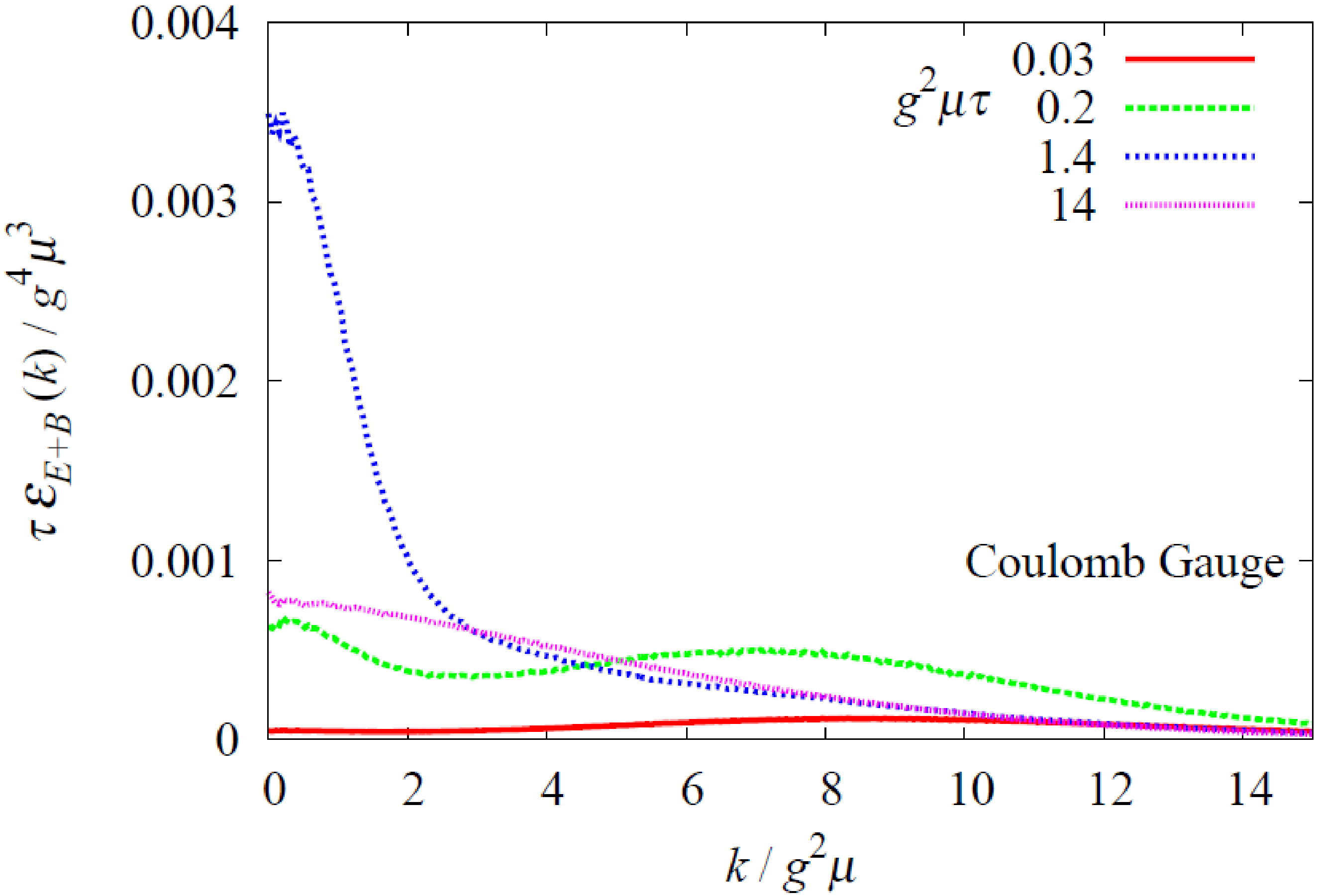


Two Drawbacks



JIMWLK evolution

Wiggle by Hand (just for a test)



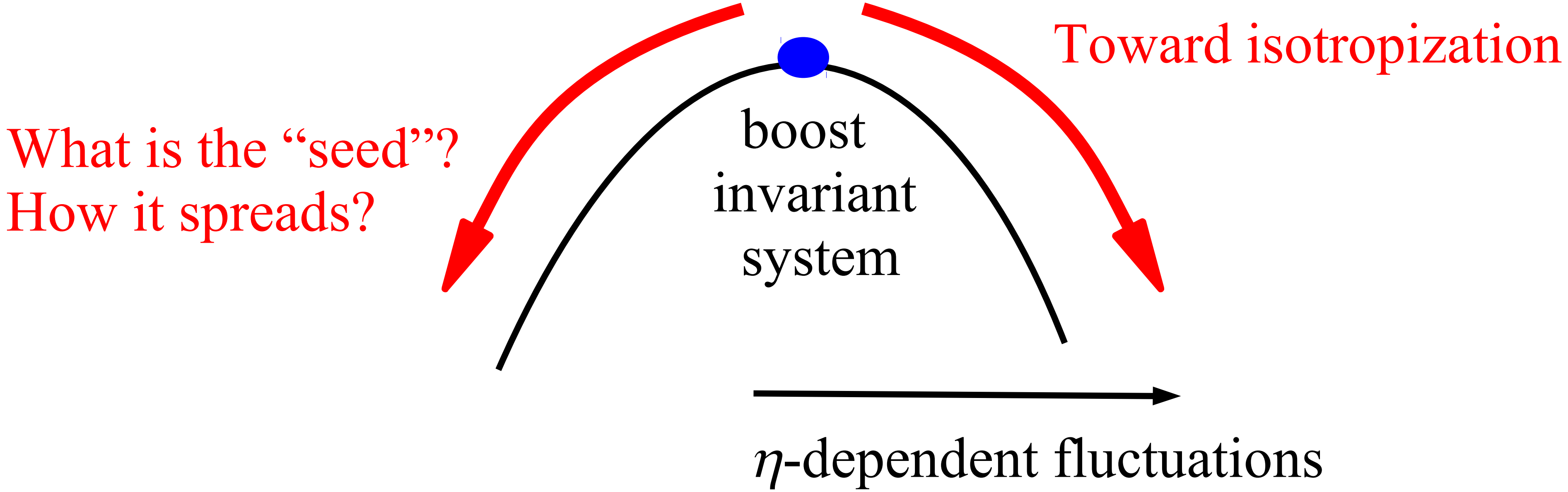
Does this tell us anything?

Comments

- If a BEC-like content is seen (see a talk by J.-P. Blaizot), it should be in the transverse plane on which the gluon distribution is characterized by Qs.
- This means, even if a BEC-like behavior exists, it has nothing to do with the isotropization.
  It may be on a path to thermalization, but it does not help the problem of negative longitudinal pressure.
- Zero-mode implies homogeneous background fields, which would lead to instabilities (such as one of the Nielsen-Olesen type).

Boost-Invariance Violation

Boost-invariant Glasma sits on the top of the potential maximum (seemingly stable without any perturbation)

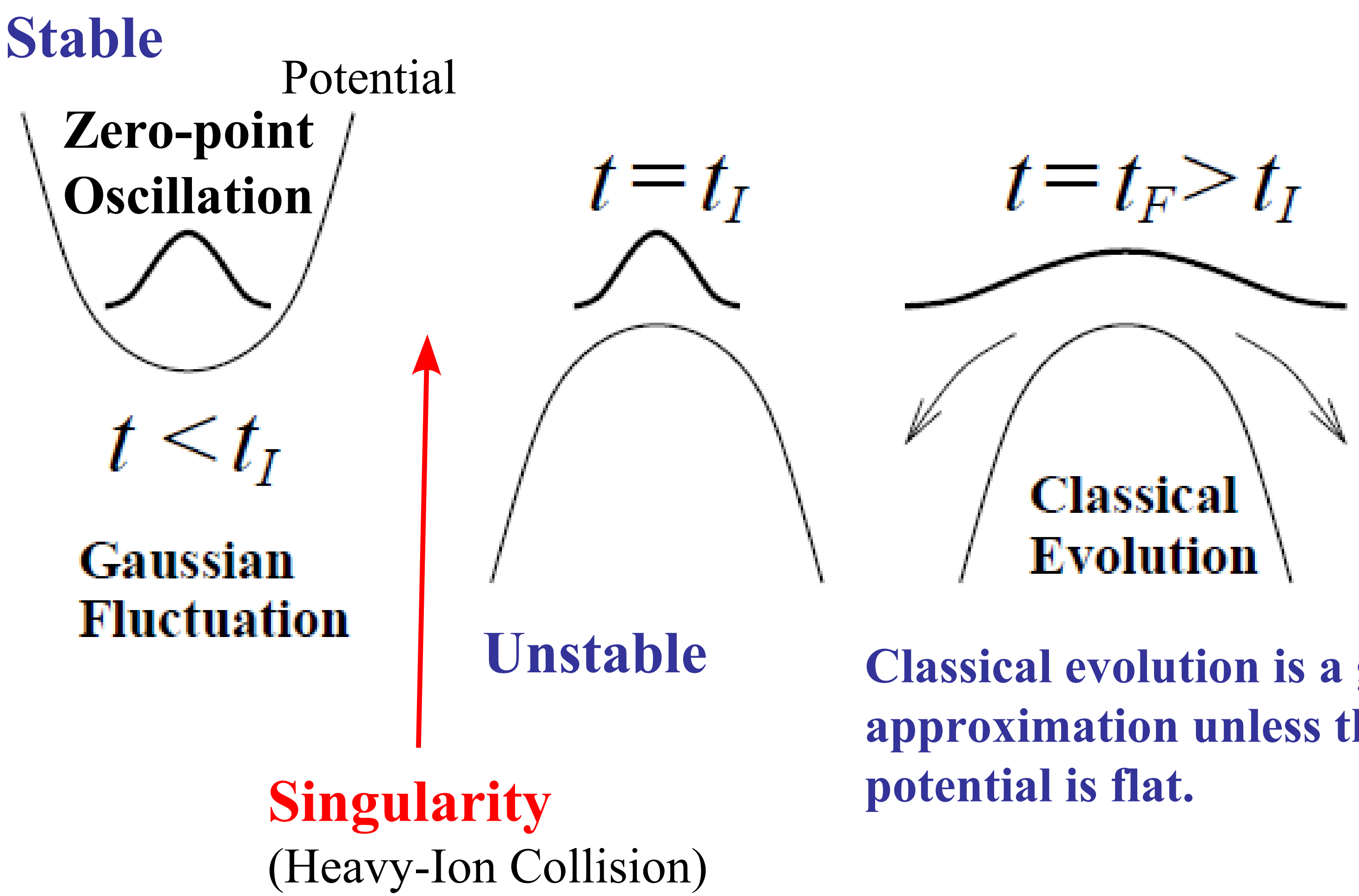


Complete isotropization may not be necessary, nevertheless the free-streaming should not be right.

(How much anisotropy is reasonably accepted?)

Schematic View of Instability

Time Evolution of Fluctuations under Instability



Classical evolution is a good approximation unless the potential is flat.

Physical Degrees of Freedom

$$A_x^a(\tau,\eta,x,y) \qquad A_y^a(\tau,\eta,x,y)$$

$$A_\eta^a(\tau,\eta,x,y) \qquad A_\tau^a = 0$$

$$E_x^a(\tau,\eta,x,y) \qquad E_y^a(\tau,\eta,x,y)$$

$$E_\eta^a(\tau,\eta,x,y)$$

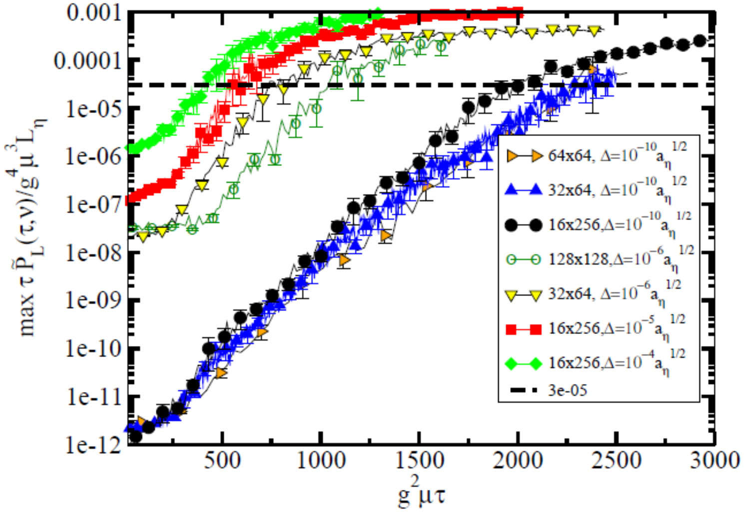
$$\delta E^i(\eta,x,y) \quad \delta E^\eta(\eta,x,y)$$

$$\delta A^i(\eta,x,y) \quad \delta A^\eta(\eta,x,y)$$

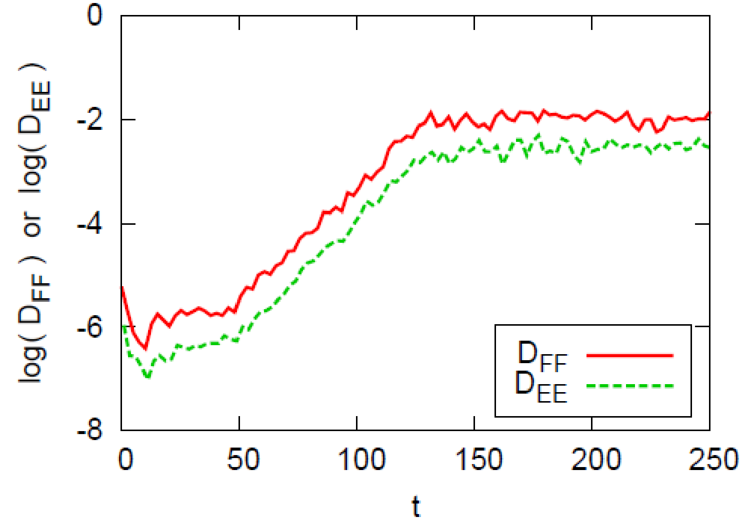
Disturb the system by η-dep fluctuations at $\tau=\tau_0$

Fluctuation patterns: Fukushima-Gelis-McLerran (2006)
Dusling-Gelis-Venugopalan (2011), Dusling-Epelbaum-Gelis-Venugopalan

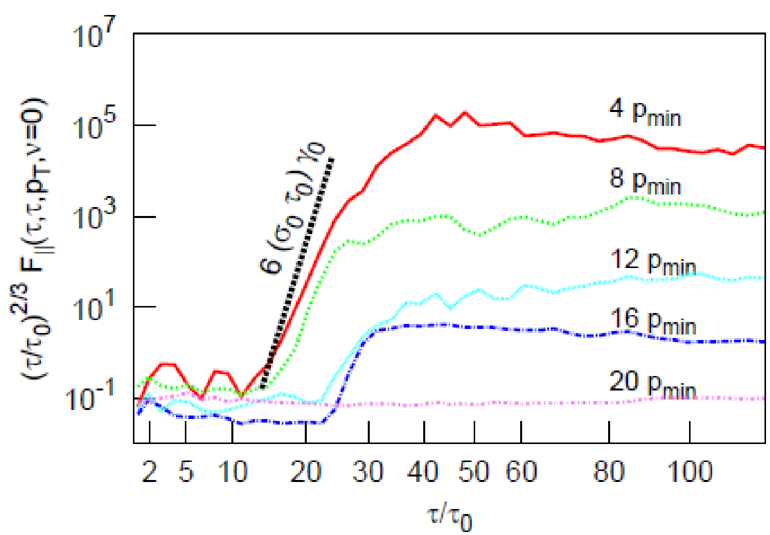
Instabilities in the Classical Pure YM



Romatschke-Venugopalan (2005)



Kunihiro et al. (2010)



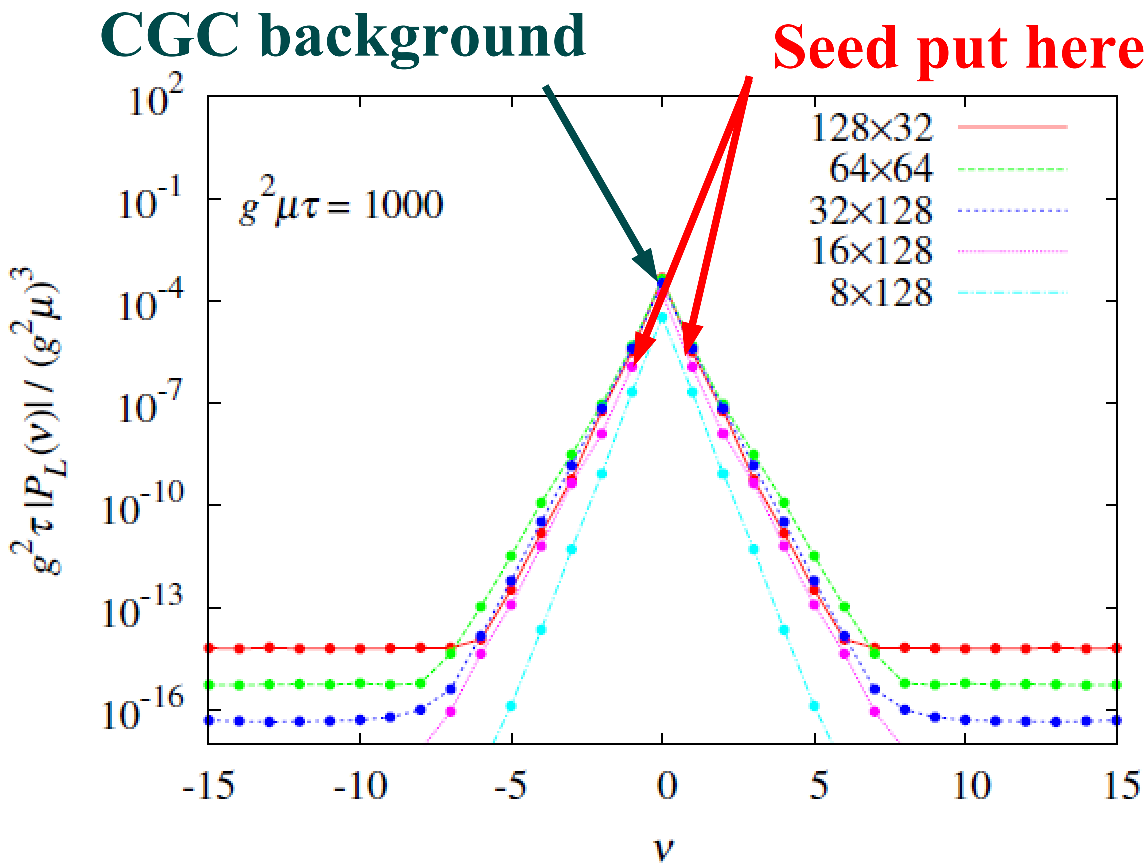
Berges-Boguslavski-Schlichting (2012)

Weibel instability
Nielsen-Olesen instability
Parametric resonance
etc...

Complementary to the plasma inst. (Rebhan)

Small (Minimal) Disturbance

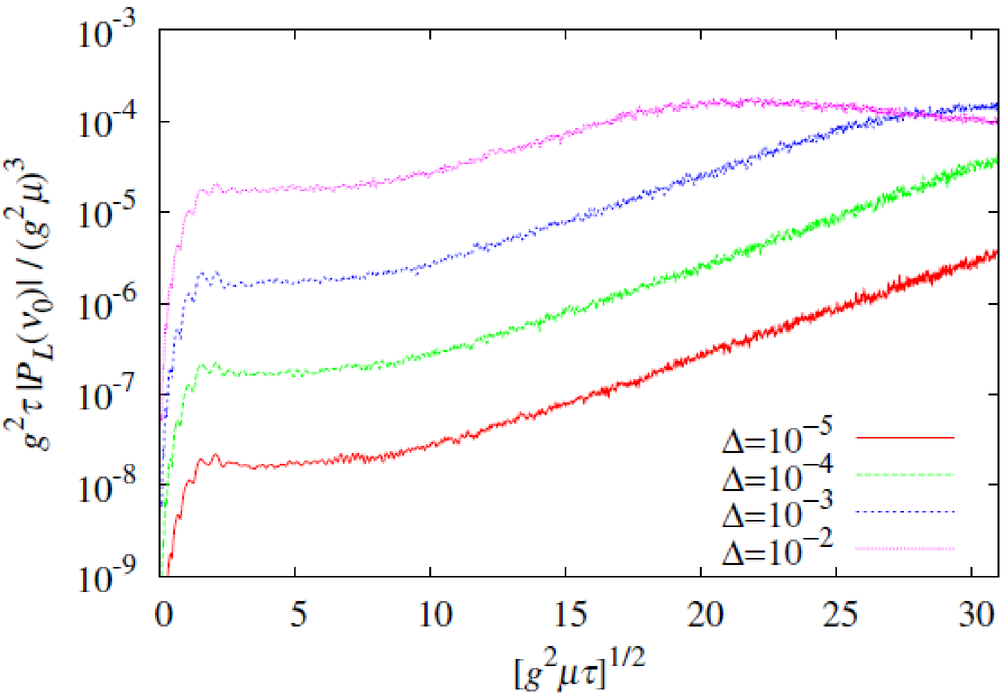
Amplitudes spread from lower to higher wave-number modes

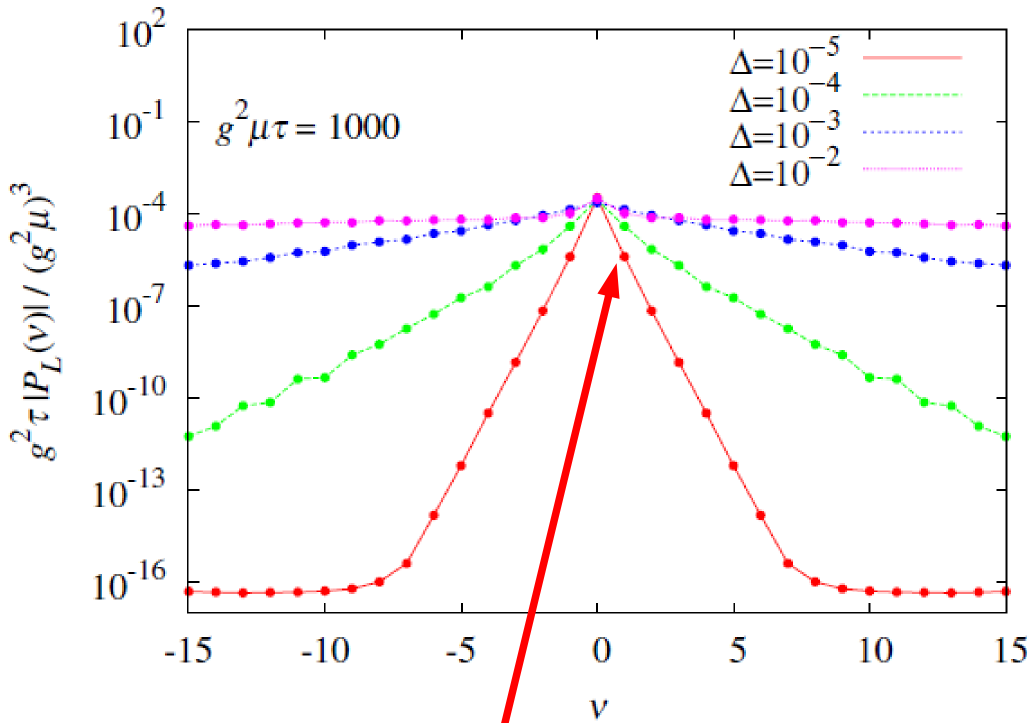


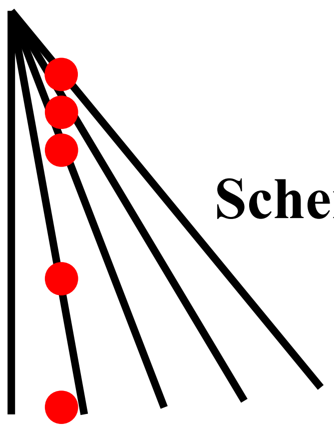
Because the zero-mode background is so huge, it keeps supplying the energy (or particle) injection.

Fourier-mode of the longitudinal pressure

Amplitude Decay from Zero-Mode



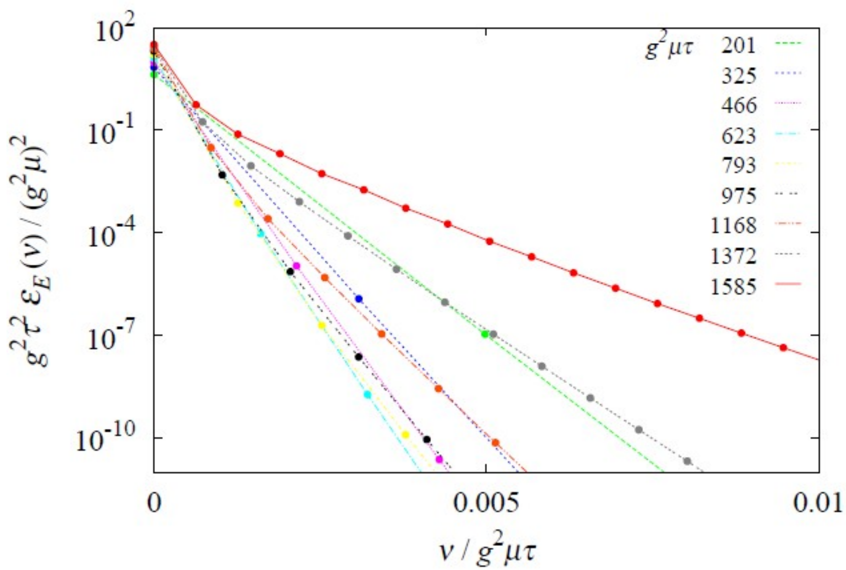


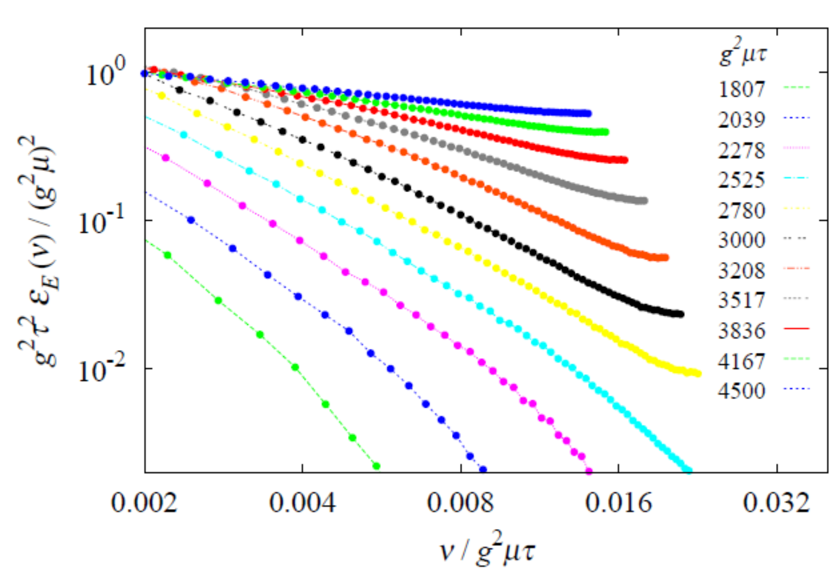


Schematic Behavior

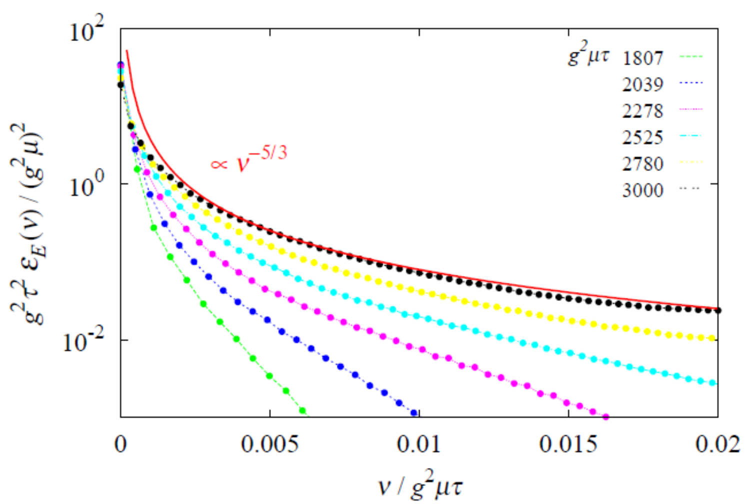
How this mode grows

Evolution of Longitudinal Spectrum





Some scaling seen at unphysical late time



Weak non-linearity remains even in a dilute system

Comments

Expanding systems are *simpler* at large time scale.

$$E^i = \tau \partial_\tau A_i , \qquad E^\eta = \tau^{-1} \partial_\tau A_\eta$$

$$\partial_\tau E^i = \tau^{-1} D_\eta F_{\eta i} + \tau D_j F_{ji}$$

$$\partial_\tau E^\eta = \tau^{-1} D_j F_{j\eta}$$

Asymptotic Behavior

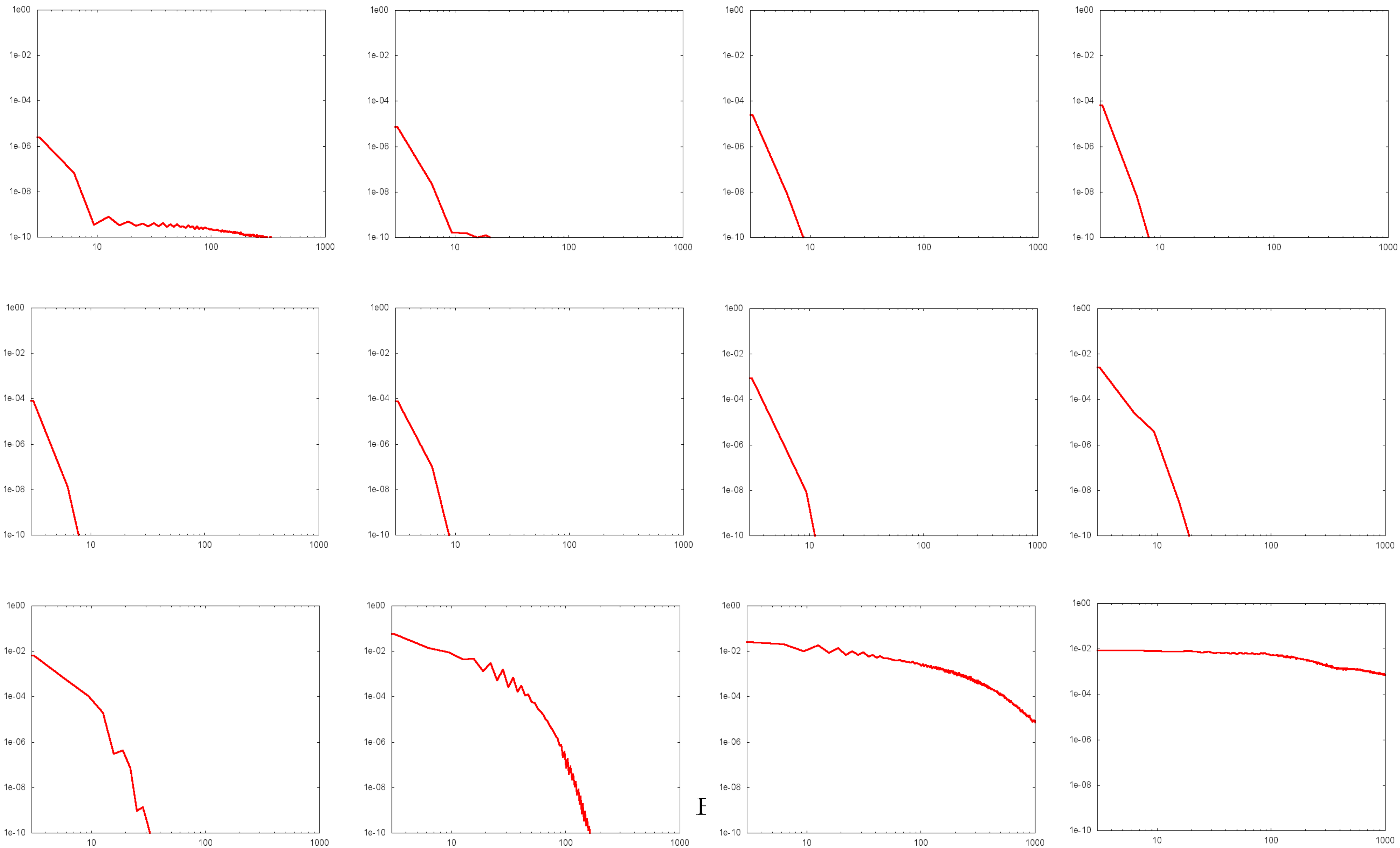
$$E^i \sim \tau^{1/2} \qquad E^\eta \sim 1/\tau^{1/2}$$

$$A_i \sim 1/\tau^{1/2} \qquad A_\eta \sim \tau^{1/2}$$

Leading-order is "free" equations
→ Bessel functions

Soft-modes dominant in non-linearity
→ Zero-mode

Mode Decaying from IR to UV in 1D



Summary I

- Boost-invariant background fields should be a right description for the relativistic heavy-ion collision in the first approximation at infinitely high energy.
- Only one characteristic scale Q_s in this limit.
- Background fields have a peculiar pattern – strong longitudinal E and B fields – which should be disturbed by fluctuations and particle productions.
- Transverse and longitudinal dynamics so different. Entangled to speed up isotropization.

Summary II

- There are (almost always) choices that lead to desired results. Need careful considerations.
  - Choice of the universal parameter
  - Choice of the fluctuation strength
  - Choice of the background fields

- Nevertheless, the (classical) pure YM system is a complicated non-linear system and it is still interesting to investigate long-time behavior.
  - Many types of instabilities
  - Strong-coupling limit (from a holographic dual)
  - Chaos, Topological defects, Turbulence