Dark Energy and Dark matter in

A Superfluid Universe

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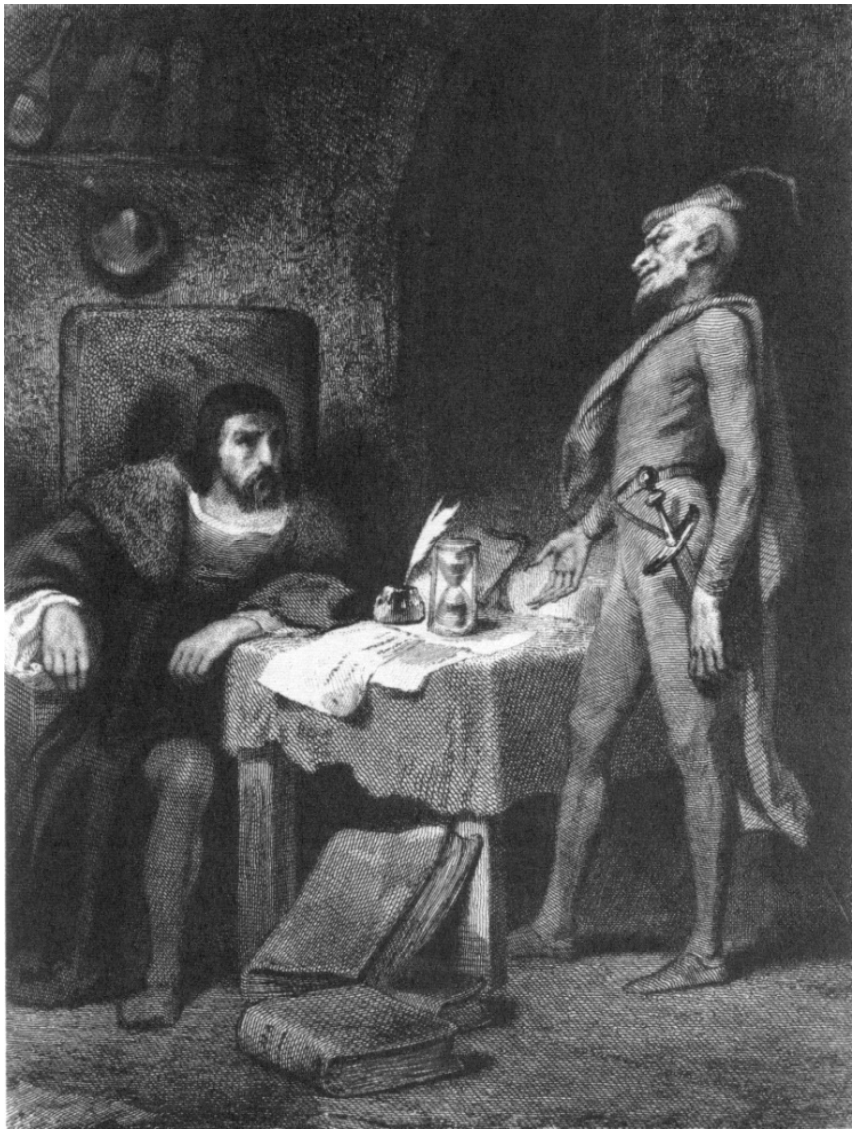


Dr. Johann Faust (Heidelberg 1509)

Goethe

From Goethe's *Faust*,
Quoted by Boltzmann on Maxwell's equations

War es ein Gott der diese Zeichen schrieb,
Die mit geheimnisvoll verborg'nen Trieb
Die Kräfte der Natur um mich enthüllen
Und mir das Herz mit stiller Freud erfüllen?

Was it a god who wrote these signs,
That have calmed yearnings of my soul,
And opened to me a secret of Nature?

Physics in the 20th century

General relativity

- Expanding universe
- Dark energy

Quantum theory

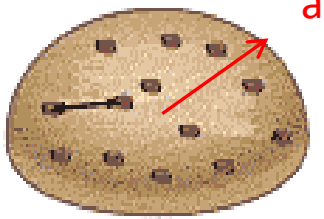
- Superfluidity: Quantum phase coherence
- Dynamical vacuum: Quantum field theory

Expanding universe

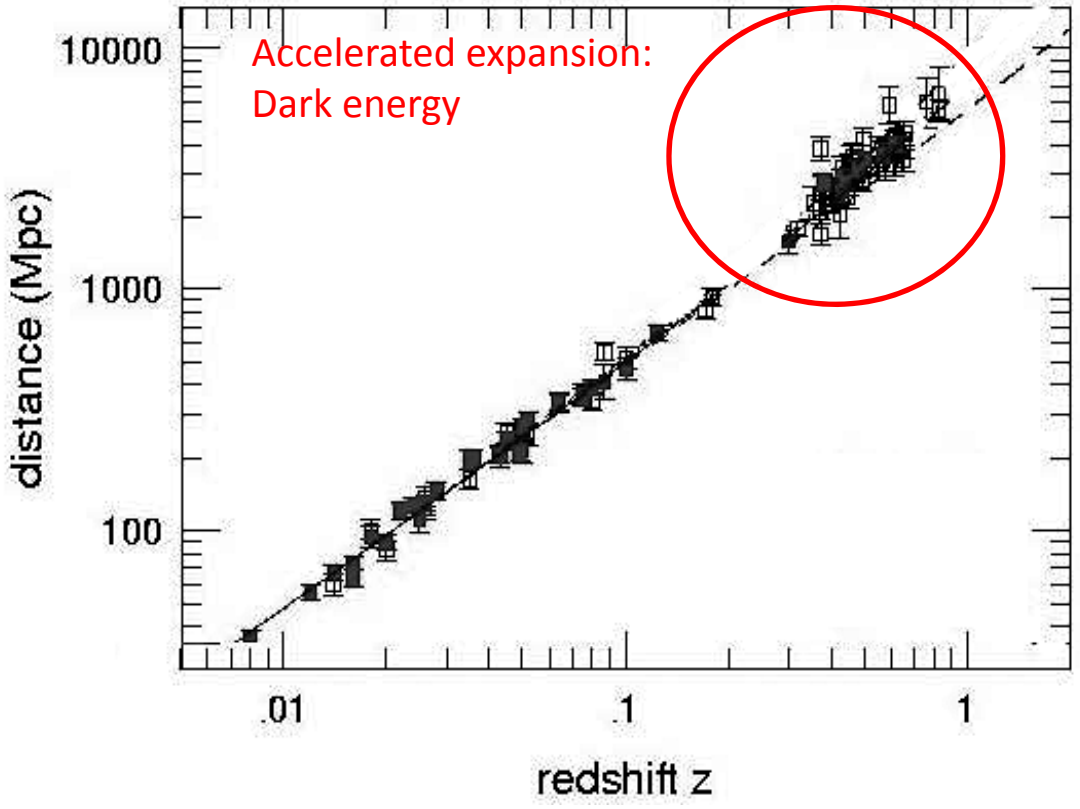
- The more distant the galaxy, the faster it moves away from us.
- Extrapolated backwards to “big bang”

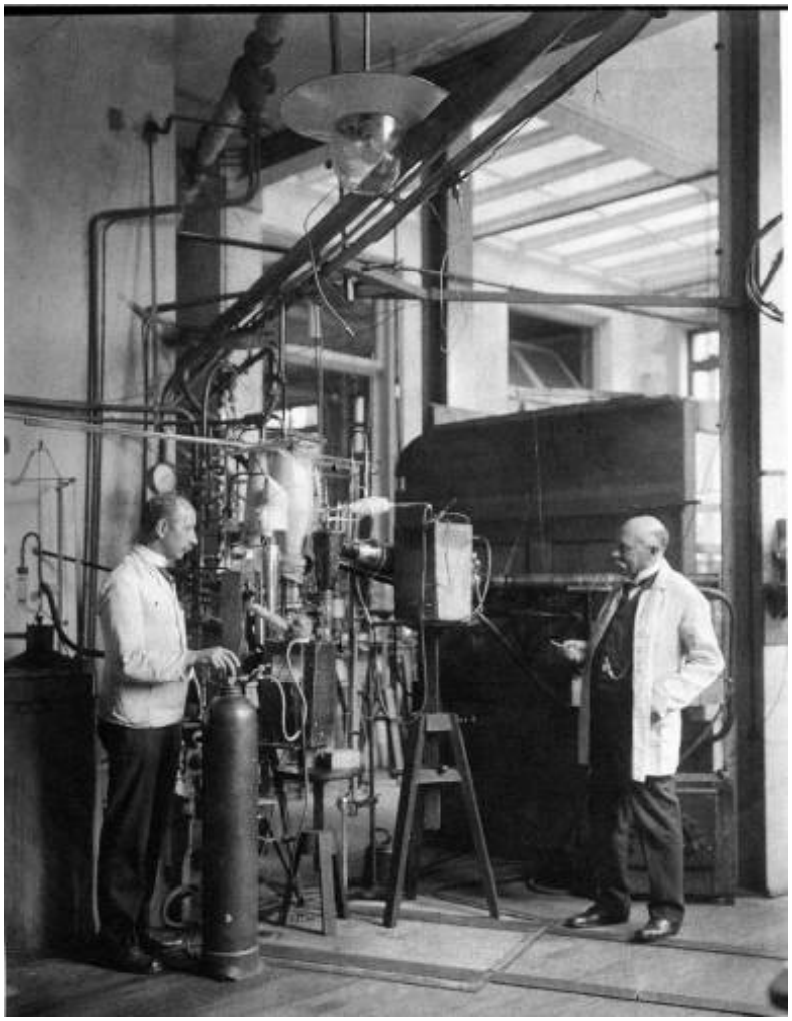
Hubble’s law: Velocity proportional to distance

Hubble’s parameter: $H = \frac{1}{a}\frac{da}{dt} = \frac{1}{15 \times 10^9 \text{ yrs}}$



Edwin Hubble





H. Kamerlingh Onnes (1908)

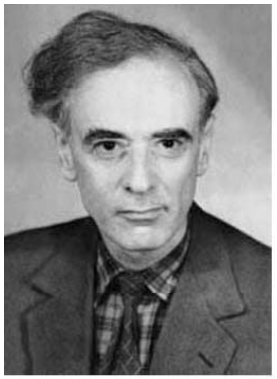
Superconductivity, superfluidity

• Quantum phase coherence over macroscopic distances

• Phenomenological description: order parameter = complex scalar field

$$\phi(x) = F(x)\, e^{i\sigma(x)}$$

$$\mathbf{v}_s = \kappa \nabla \sigma(x)$$



Lev Davidovich Landau
1908-1958

Vitaly L. Ginsburg
(1915-)

Willis E. Lamb
1913-

Polykarp Kusch
1911-1993

Dynamical vacuum --- QFT

- Lamb shift in hydrogen:
  E(2S) – E(2P) = 1060 mhz = 10^{-6} eV
- Electron anomalous moment:
  (g-2)/2 = 10^{-3}
- Vacuum complex scalar field:
  Higgs field in standard model
  Others in grand unified theories

A vacuum complex scalar field makes the universe a superfluid.

We investigate
- Emergence of vacuum scalar field in big bang
- Observable effects

Scalar Field

Lagrangian density :

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

Potential :

$$V(\phi) = \lambda_2\phi^2 + \lambda_4\phi^4 + \lambda_6\phi^6 + \cdots$$

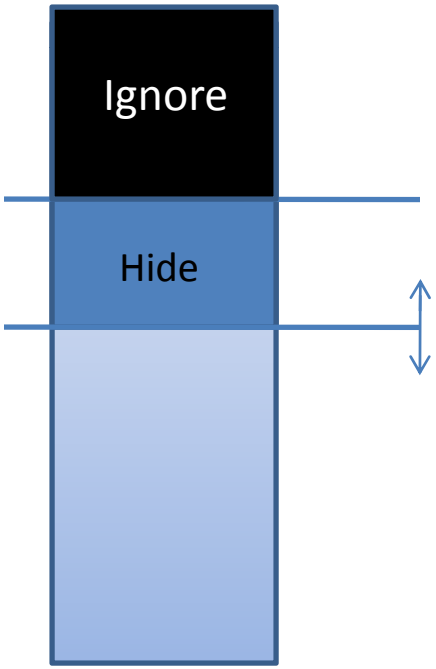
Equation of motion :

$$\partial^2\phi + V\phi = 0$$

- The vacuum field fluctuates about mean value. We can treat it as a classical field by neglecting flucutuations.
- But quantum effect of renormalization cannot be ignored.
- This makes V dependent on the length scale.
- Especially important for big bang, when scale changes rapidly.

Renormalization

- In QFT there exist virtual processes.
- Spectrum must be cut off at high momentum Λ.
- Λ is the only scale in the theory.



Momentum spectrum

Cutoff Λ_0

Effective cutoff Λ (Scale)

Renormalization:
Adjusting couplings so as to preserve theory, when scale changes.

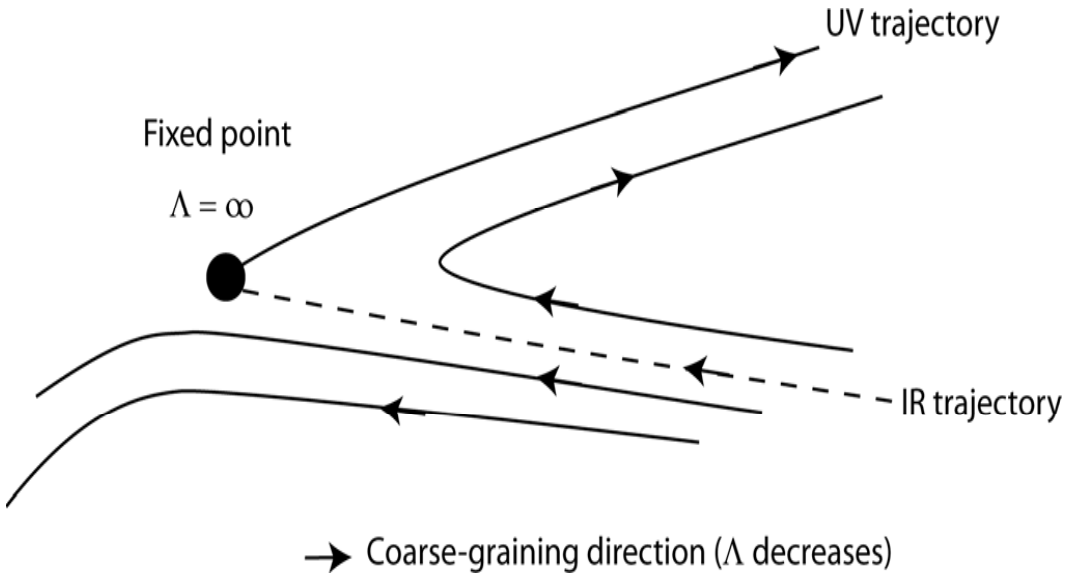
Freeman J. Dyson
1923-

Kenneth G. Wilson
1936-

Renormalization-group (RG) trajectory:

Trajectory of $V(\phi, \Lambda)$ in function space, as scale Λ changes.

Fixed point: system scale invariant, $\Lambda=\infty$.

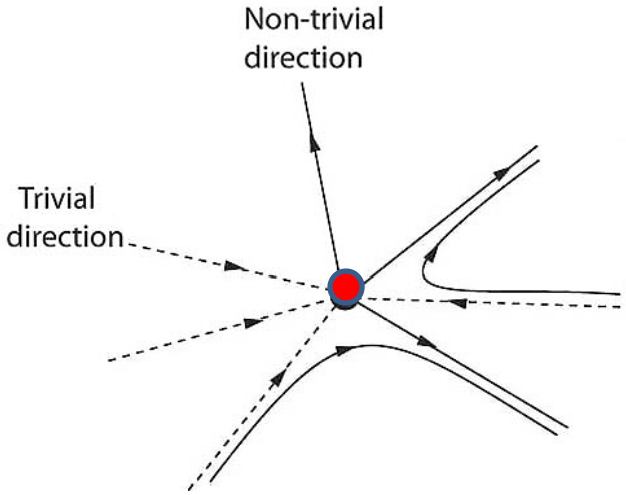


UV trajectory: Asymptotic freedom

IR trajectory: Triviality

The Creation

- At the big bang $\Lambda = \infty$.
- There was no interaction.
- Universe was at the **Gaussian fixed point** ● $(V \equiv 0,$ massless free field).
- It emerges along some direction, on an RG trajectory.
- The direction corresponds to a particular form of the potential V.



In the space of all possible theories

Outgoing trajectory --- Asymptotic freedom

Ingoing trajectory --- Triviality (free field)

The only asymptotically free scalar potential is the **Halpern-Huang** potential:

- Transcendental function (Kummer function)
- Exponential behavior at large fields
- 4D generalization of 2D XY model, or sine-Gordon theory.

Cosmological equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} \quad \text{(Einstein's equation)}$$

$$\partial^2 \phi + V\phi = 0 \quad \text{(Scalar field equation)}$$

Robertson-Walker metric (spatial homogeneity)

Gravity scale = a (radius of universe)

Scalar field scale = Λ (cutoff monentum)

Since there can be only one scale in the universe,

$$\Lambda = \hbar / \mathrm{a}$$

Dynamical feedback:
Gravity provides cutoff to scalar field,
which generates gravitational field.

Planck units:

$$\text{Planck length} = \sqrt{\frac{\hbar}{c^3} 4\pi G} = 5.73 \times 10^{-35} \text{ m}$$

$$\text{Planck time} = \sqrt{\frac{\hbar}{c^5} 4\pi G} = 1.91 \times 10^{-43} \text{ s}$$

$$\text{Planck energy} = \sqrt{\frac{\hbar c^5}{4\pi G}} = 3.44 \times 10^{18} \text{ GeV} = 5.5 \times 10^{8} \text{ Joule}$$

We shall put

$$4\pi G = c = \hbar = 1$$

Initial-value problem

For illlustration, first use real scalar field.

$$\dot{a} = Ha$$

$$\dot{H} = \frac{k}{a^2} - \dot{\phi}^2 + \frac{a}{3}\frac{\partial V}{\partial a}$$

k = curvature parameter = 0, +1,-1

Trace anomaly

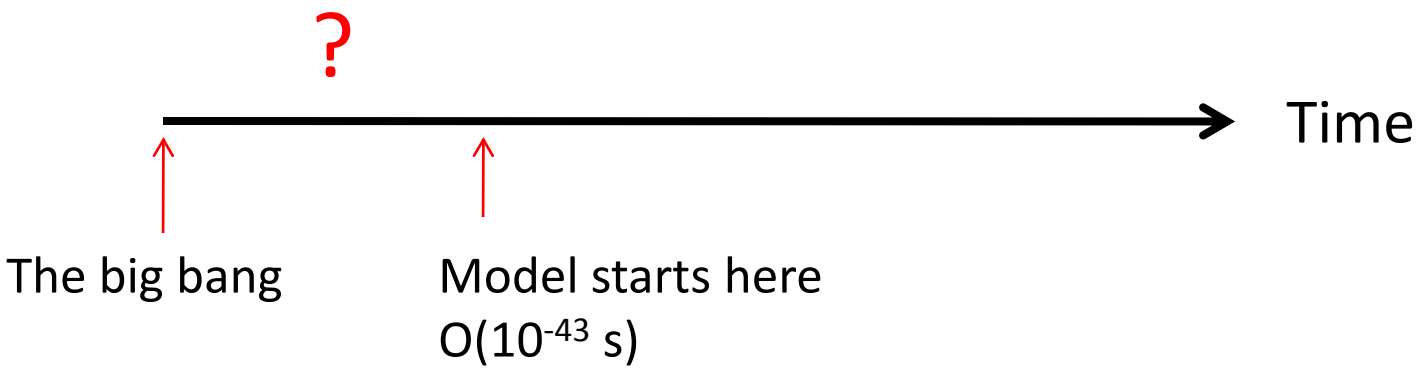
$$\ddot{\phi} = -3H\dot{\phi} - \frac{\partial V}{\partial \phi}$$

$$X \equiv H^2 + \frac{k}{a} - \frac{2}{3}\left(\frac{1}{2}\dot{\phi}^2 + V\right) = 0$$

Constraint equation

$X = 0$ is a constraint on initial values.

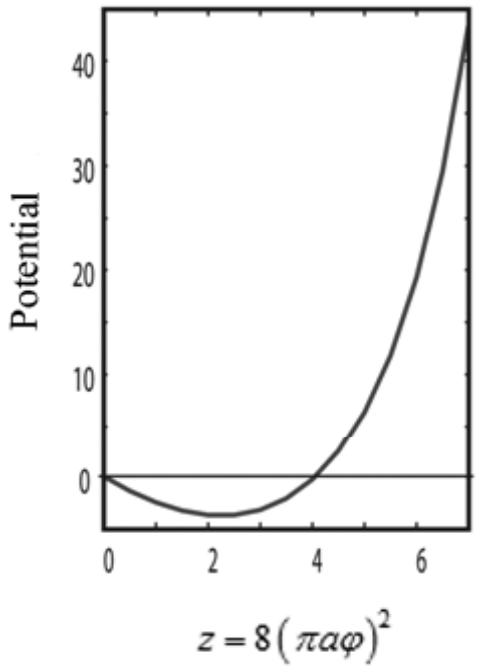
Equations guarantee $\dot{X} = 0$.

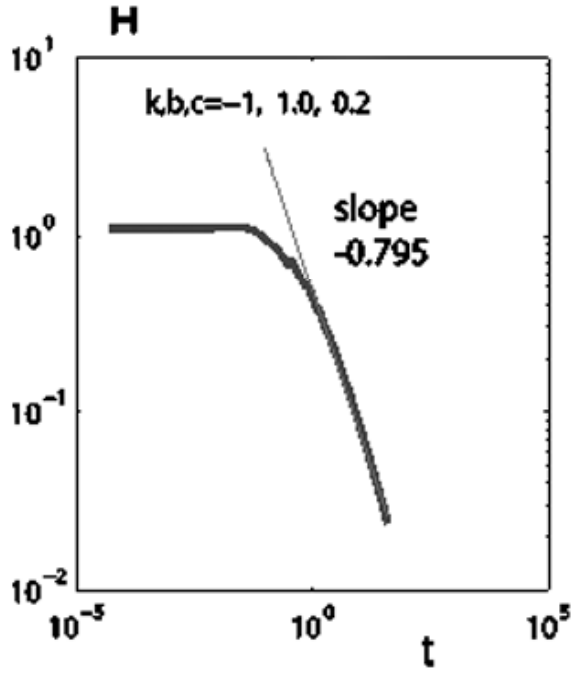


- Initial condition: Vacuum field already present.

- Universe could be created in hot “normal phase”, then make phase transition to “superfluid phase”.

Numerical solutions



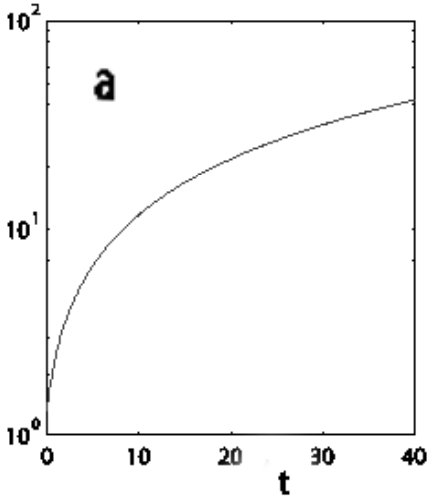


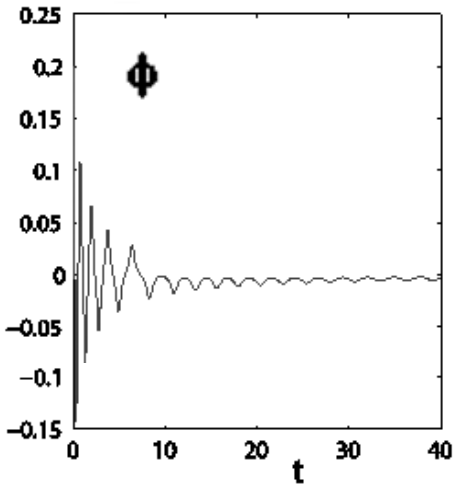
Time-averaged asymptotic behavior :

$$H \approx t^{-p}$$

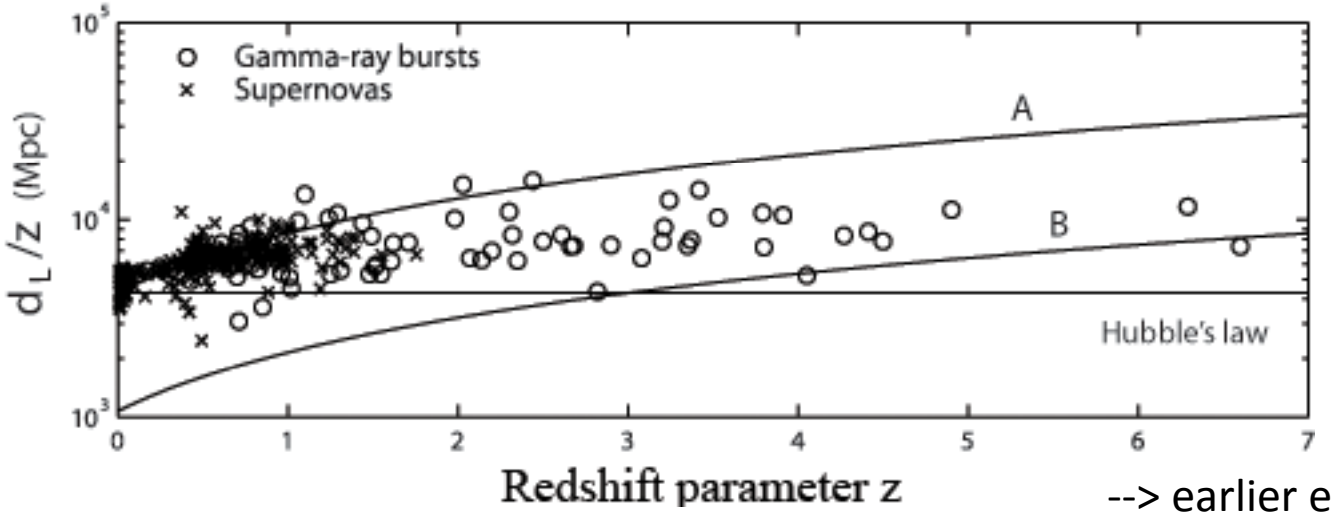
$$a \approx \exp\left(t^{1-p}\right)$$

Gives dark energy without "fine-tuning" problem





Comparison of power-law prediction on galactic redshift with observations



--> earlier epoch

d_L = luminosity distance

Different exponents *p* only affects vertical displacement, such as A and B.

Horizontal line corresponds to Hubble's law.
Deviation indicates accelerated expansion (dark energy).

Indication of a crossover transition between two different phase B -> A.

Cosmic inflation

1. Matter creation

   How to create enough matter for subsequent nucleogenesis before universe gets too large.

2. Decoupling of matter scale and Planck scale

   Matter interactions proceed at nuclear scale of 1 GeV.
   But equations have built-in Planck scale of 10^{18} GeV.
   How do these scales decouple in the equations?

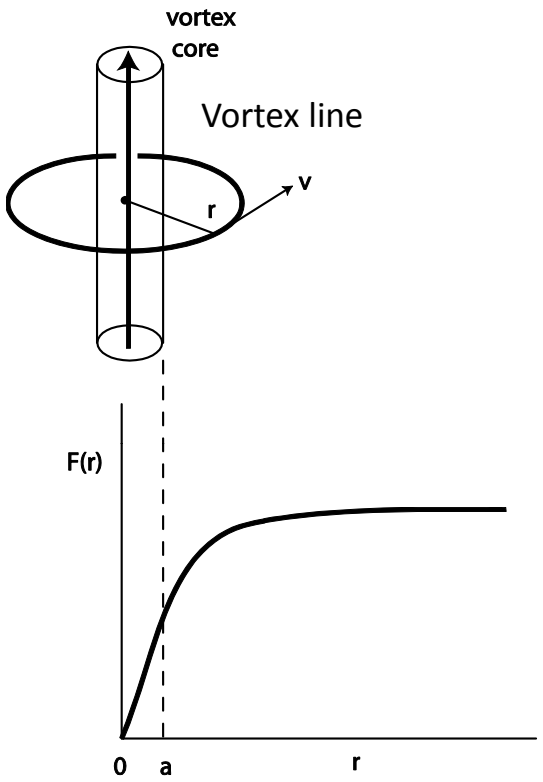
Model with complete spatial homogeneity fail to answer these questions.

Generalization:

Complex scalar field, homogeneous modulus, spatially varying phase.

New physics: Superfluidity, in particular quantum vorticity.

Complex scalar field



$$\phi = Fe^{i\sigma}$$

$v = \nabla\sigma$ (Superfluid velocity)

$F^2v^2 =$ Energy density of superflow

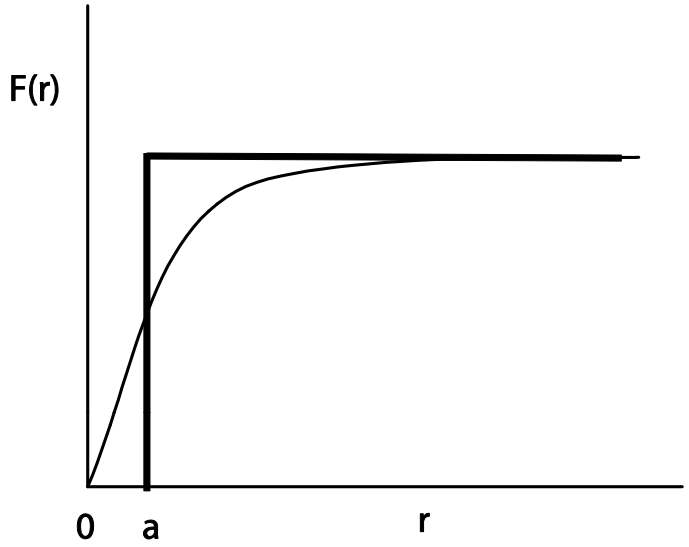
$$\oint_C ds \cdot v = \oint_C ds \cdot \nabla\sigma = 2\pi n$$

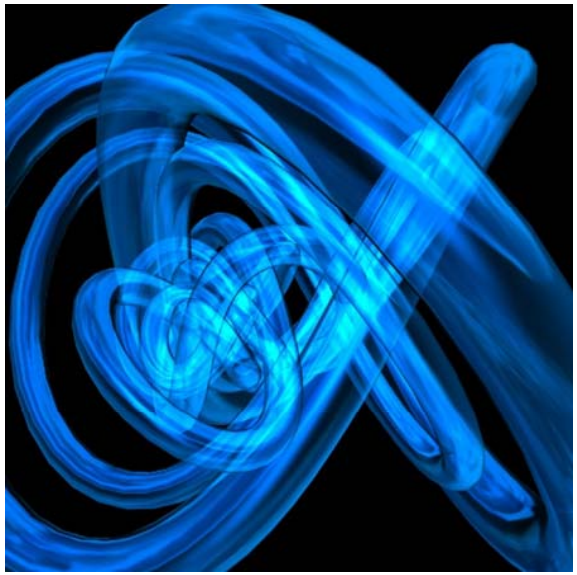
$$2\pi r v = 2\pi n$$

$$v = \frac{n}{r}$$

Like magnetic field from line current

Vortex has cutoff radius of order *a(t).*
Vortex line has energy per unit length.





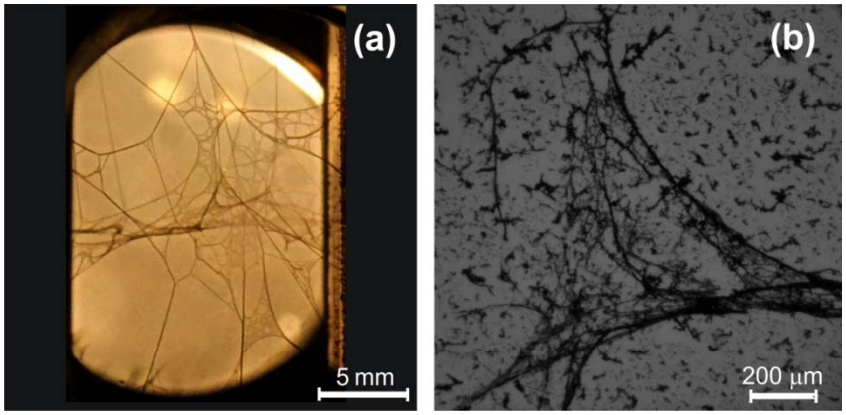
- Replace vortex core by tube.
- Scalar field remains uniform outside.
- Can still use RW metric,
- but space is multiply-connected.

The “worm-hole” cosmos

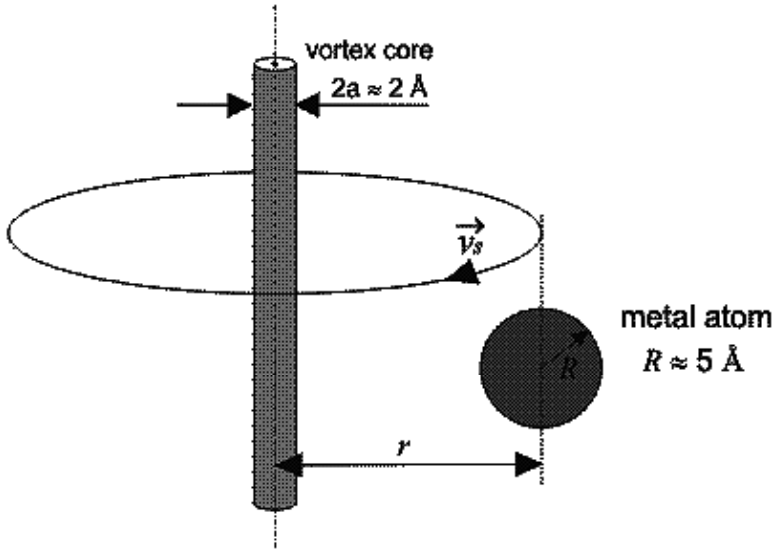
The vortex-tube system represent emergent degrees of freedom.

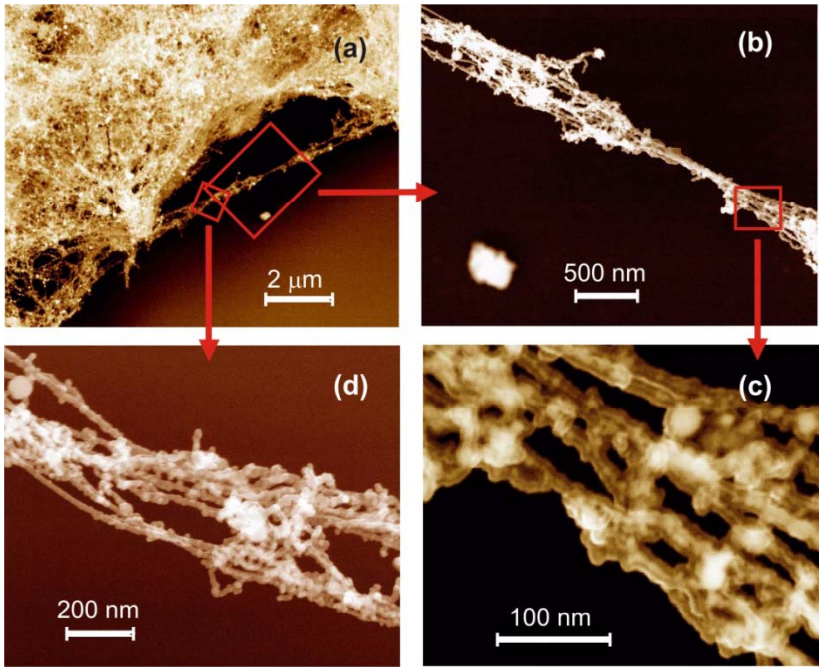
Nanowires

Vortex tubes in superfluid helium made visible by adsorption of metallic powder on surface
(University of Fribourg expt.)



(a) Copper (b) gold

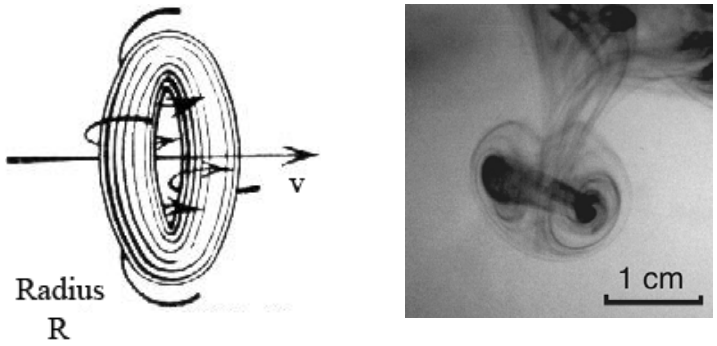




Under electron microscope

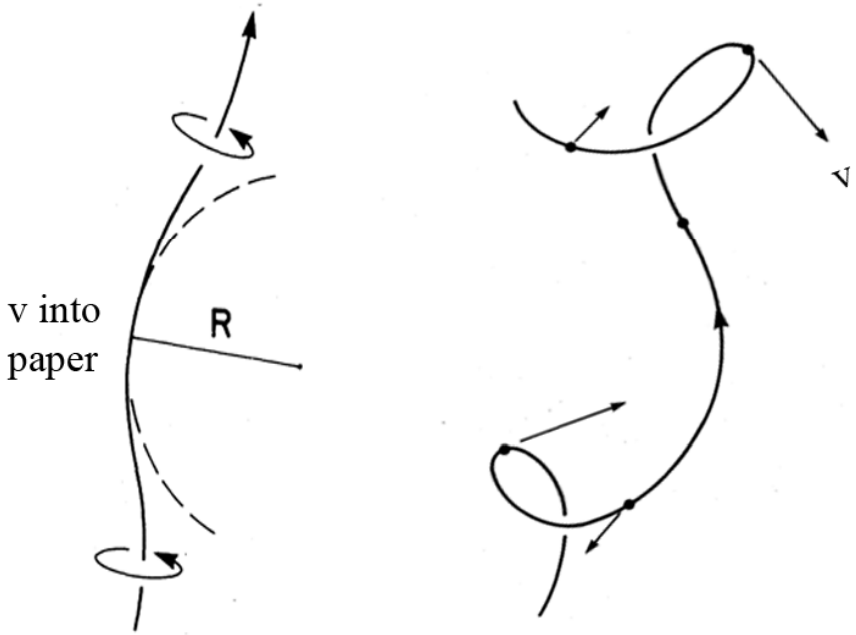
Vortex dynamics

Elementary structure is vortex ring



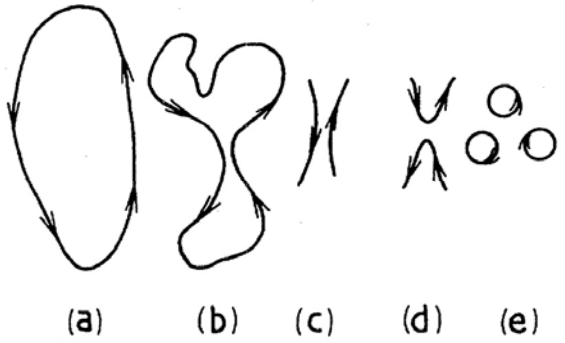
$$v = \frac{1}{4\pi R} \ln \frac{R}{R_0}$$

Self-induced vortex motion

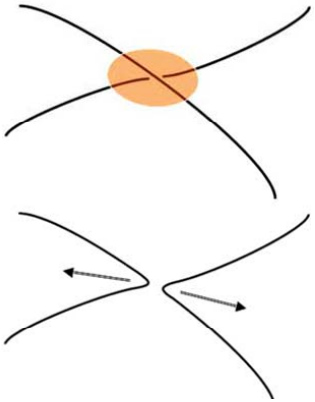


The smaller the radius of curvature R, the faster it moves normal to R.

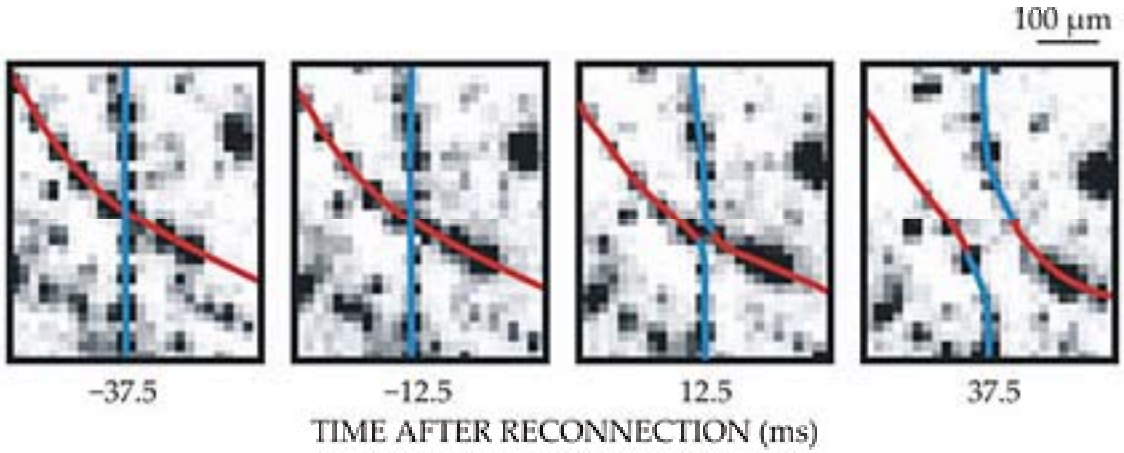
Vortex reconnection



Feynman's conjecture



Signature: two cusps spring away from each other at very high speed (due to small radii), creating two jets of energy.

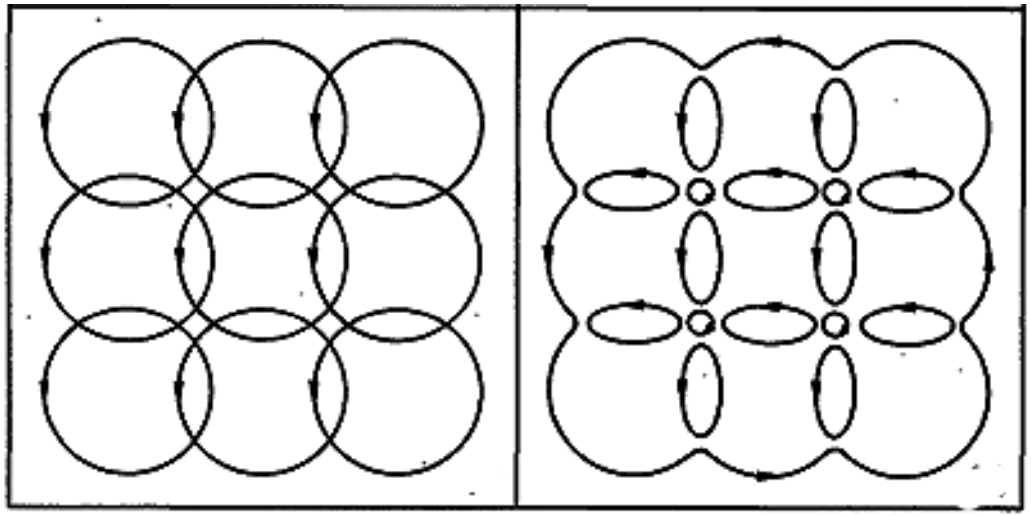


Observed vortex reconnection in liquid helium-- a millisecond event.
D. Lathrop, *Physics Today*, 3 June, 2010.

Magnetic reconnections in sun's corona

Responsible for solar flares.

Change of topology due to reconnections



Microscopic rings eventually decay.

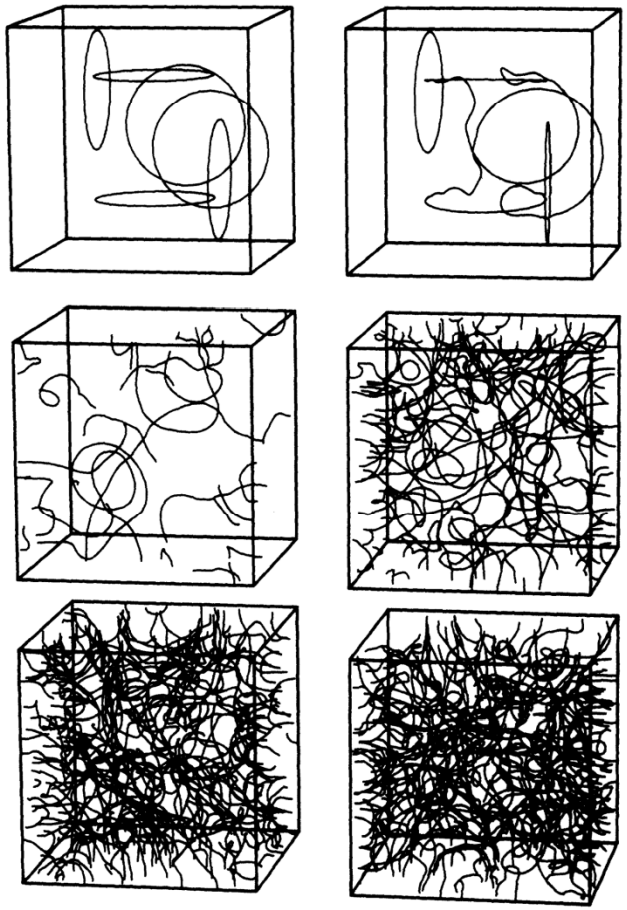
Quantum turbulence: Steady-state “vortex tangle” when there is steady supply of large vortex rings.

K.W. Schwarz, Phys. Rev. Lett. ,**49**,283 (1982).

Simulation of quantum turbulence

Creation of vortex tangle in presence of “counterflow” (friction).

K.W. Schwarz, Phys. Rev. **B 38**, 2398 (1988).



Number of reconnections:

0	3
18	844
1128	14781

Fractal dimension = 1.6

D. Kivotides, C.F. Barenghi, and D.C. Samuels. Phys. Rev. Lett. **87**, 155301 (2001).

Cosmology with quantum turbulence

$a(t)$ = Radius of universe

$F(t)$ = Modulus of scalar field

$\ell(t)$ = Vortex tube density

$\rho(t)$ = Matter density

- Scalar field has uniform modulus *F*.
- Phase dynamics manifested via vortex tangle *l*.
- Matter created in vortex tangle (physically, via reconnections).

Equations for the time derivatives:

$\dot{a}(t)$ from Einstein's equation with RW metric.

Source of gravity: $T^{\mu\nu}_{\text{tot}} = T^{\mu\nu}_{F} + T^{\mu\nu}_{\ell} + T^{\mu\nu}_{\rho}$

$\dot{F}(t)$ from field equation.

$\dot{\ell}(t)$ from Vinen's equations in liquid helium.

$\dot{\rho}(t)$ determined by energy-momentum conservation $T^{\mu\nu}_{\text{tot};\mu} = 0$.

Vinen's equation in liquid helium

ℓ = vortex tube density (length per unit spatial volume)

$$\dot{\ell} = \underset{\text{Growth}}{\underset{\uparrow}{A\ell^2}} - \underset{\text{Decay}}{\underset{\uparrow}{B\ell^{3/2}}}$$

In expanding universe this generalizes to

$$\dot{\ell} = -3H\ell + A\ell^2 - B\ell^{3/2}$$

- Proposed phenomenologically by Vinen (1957).
- Derived from vortex dynamics by Schwarz (1988).
- Verified by many experiments.

Put

$E_{\rm v} = a^3 \varepsilon_0 \ell$ (Total vortex energy)

$E_{\rm m} = a^3 \rho$ (Total matter energy)

Cosmological equations: $(4\pi G = c = \hbar = 1)$

Old:

$$\dot{H} = \frac{k}{a^2} - \dot{\phi}^2 + \frac{a}{3}\frac{\partial V}{\partial a}$$

$$\ddot{\phi} = -3H\dot{\phi} - \frac{\partial V}{\partial \phi}$$

Constraint:

$$H^2 + \frac{k}{a} - \frac{2}{3}\left(\frac{1}{2}\dot{\phi}^2 + V\right) = 0$$

Generalized:

$$\frac{dH}{dt} = \frac{k}{a^2} - 2\left(\frac{dF}{dt}\right)^2 + \frac{a}{3}\frac{\partial V}{\partial a} - \frac{1}{a^3}\left(E_{\rm m} + E_{\rm v}\right)$$

$$\frac{d^2F}{dt^2} = -3H\frac{dF}{dt} - \frac{\zeta_0 E_{\rm v}}{a^3}F - \frac{1}{2}\frac{\partial V}{\partial F}$$

Essentially constant

$$\frac{dE_{\rm v}}{d\tau} = -E_{\rm v}^2 + \gamma E_{\rm v}^{3/2}$$

$$\frac{dE_{\rm m}}{d\tau} = \left\langle \frac{\zeta_0}{s_1}\frac{dF^2}{dt} \right\rangle E_{\rm v}$$

- Rapid change
- Av. over t
- of order 10^{18}

Constraint:

$$H^2 + \frac{k}{a^2} - \frac{2}{3}\left(\dot{F}^2 + V + \frac{1+\zeta_0}{a^3}E_{\rm v} + \frac{1}{a^3}E_{\rm m}\right) = 0$$

Decouples into two sets because

$$s_1 = \frac{\tau}{t} = \frac{\text{Planck time scale}}{\text{Nuclear time scale}} = \frac{\text{Nuclear energy scale}}{\text{Planck energy scale}} \sim 10^{-18}$$

Decoupling:

- From the point of view of the cosmic expansion, the vortex-matter system is essentially static.

- From the viewpoint vortex-matter system, cosmic expansion is extremely fast, but its average effect is to give an "abnormally" large rate of matter production.

Inflation scenario:

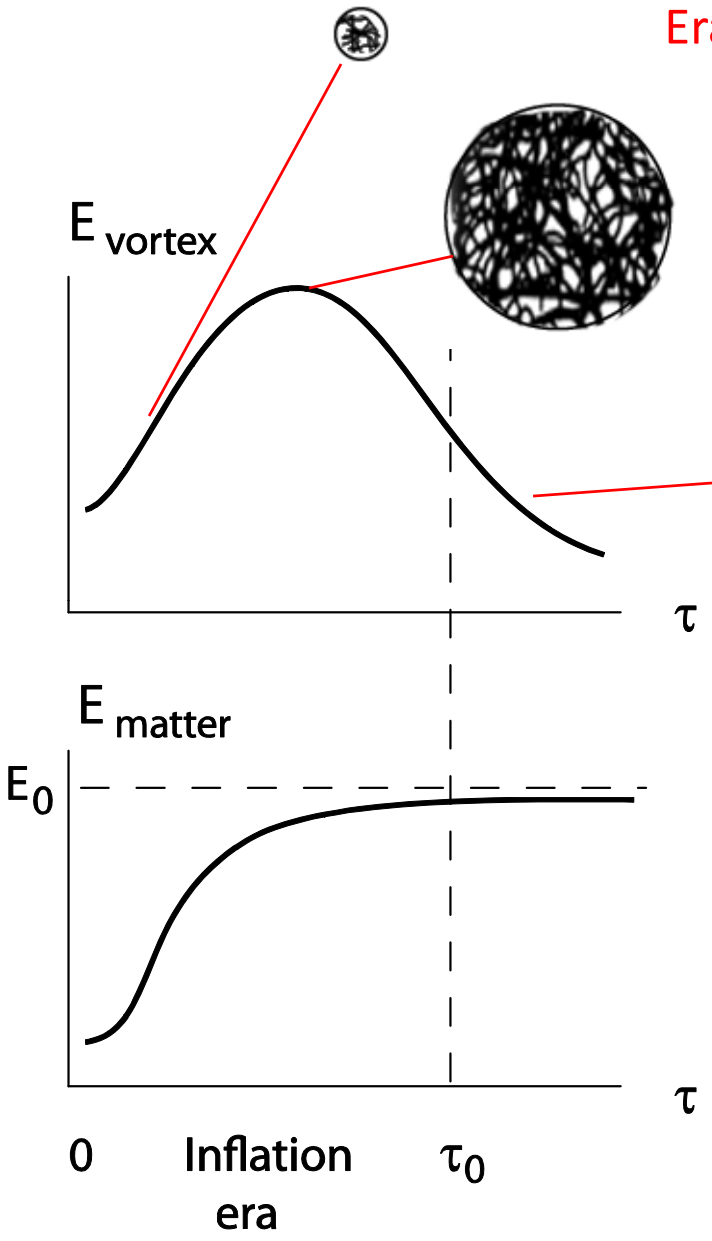
- Vortex tangle (quantum turbulence) grows and eventually decays.

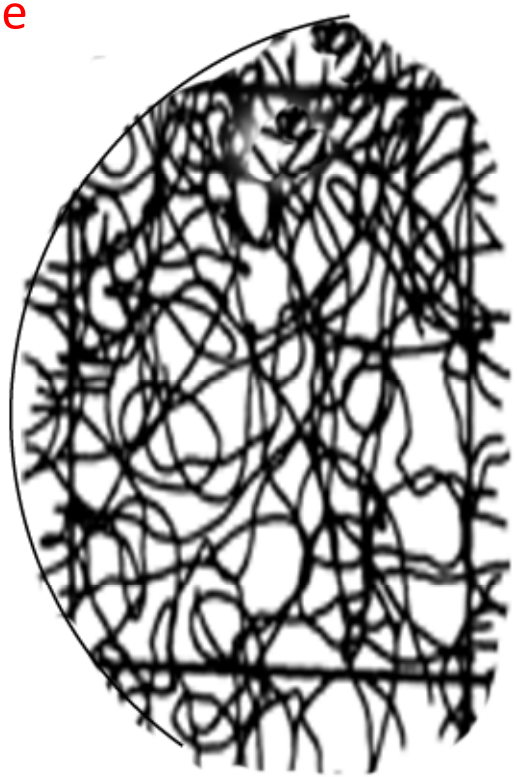
- All the matter needed for galaxy formation was created in the tangle.

- Inflation era = lifetime of quantum turbulence.

After decay of quantum turbulence, the standard hot big bang theory takes over, but the universe remains a superfluid.

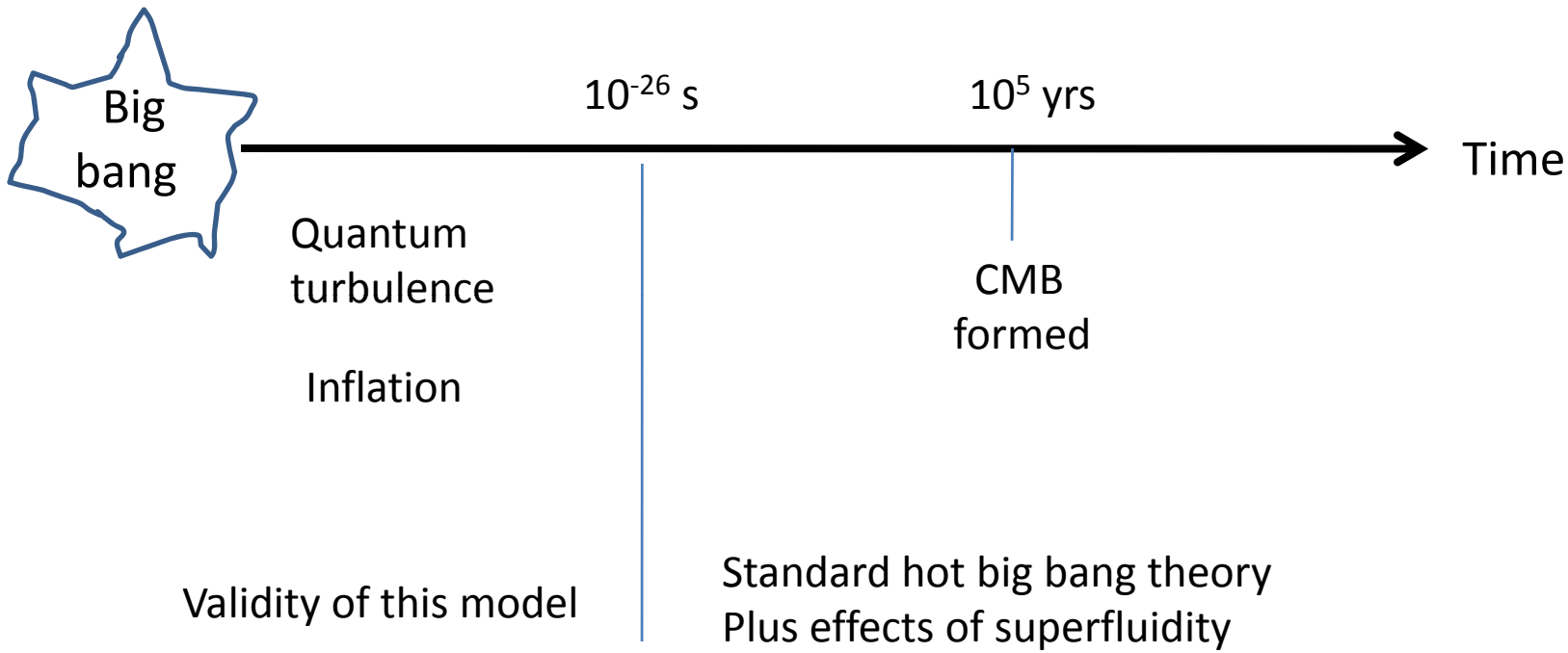
Era of quantum turbulence





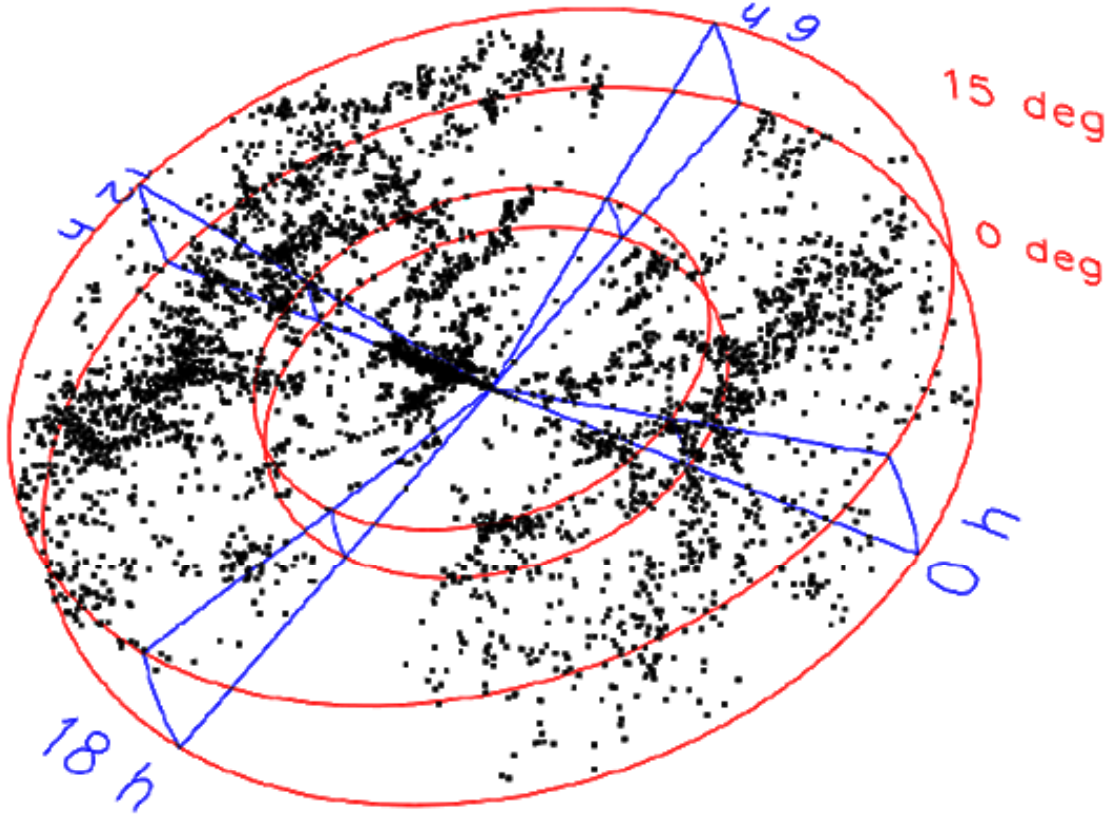
Cosmic inflation:

- Radius increases by factor 10^{27}
- in 10^{-30} seconds.
- Matter created = 10^{22} sun masses
- Eventually form galaxies outside of vortex cores.
- Large-scale uniformity

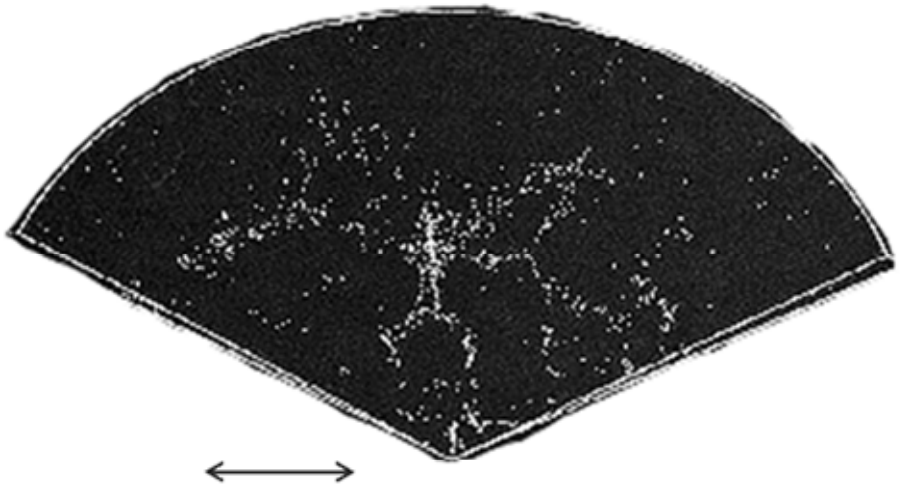

Big bang
10^{-26} s
10^{5} yrs
Time
Quantum turbulence
Inflation
CMB formed
Validity of this model
Standard hot big bang theory
Plus effects of superfluidity

Legacies in the post-inflation universe

Remnant vortex tubes with empty cores grow into cosmic voids in galactic distribution.

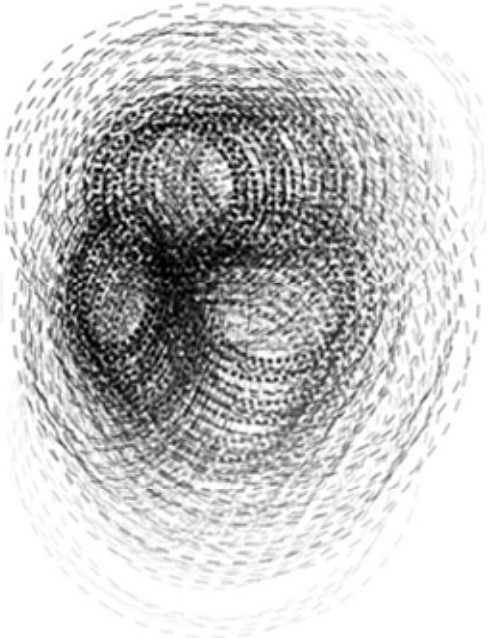


The large-scale structure of the Universe from the CfA2 galaxy survey.



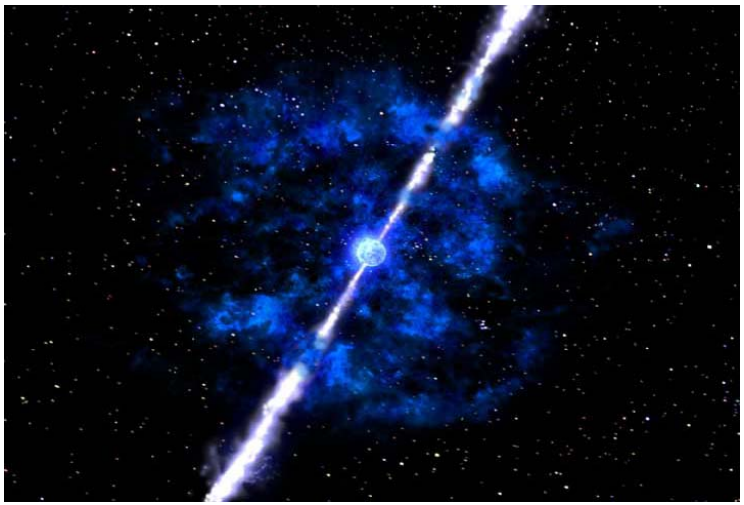
10^7 light years

The observed “stick man”



Simulated with
3 vortex tubes

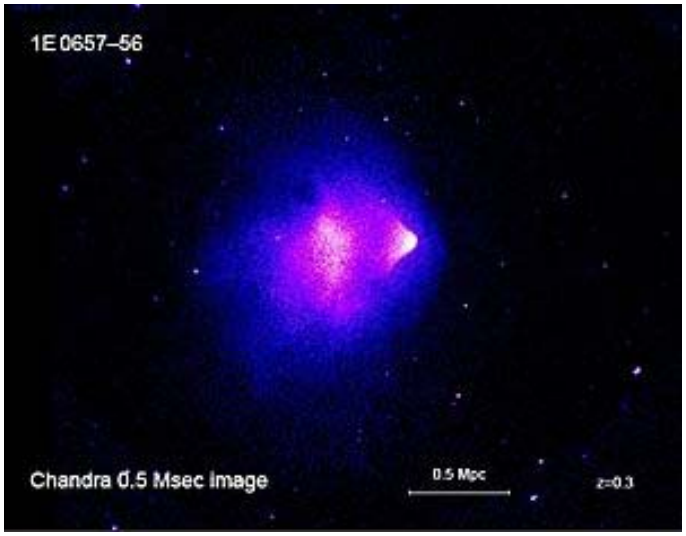
Reconnection of huge vortex tubes
in the later universe will be rare but spectacular.



Gamma ray burst

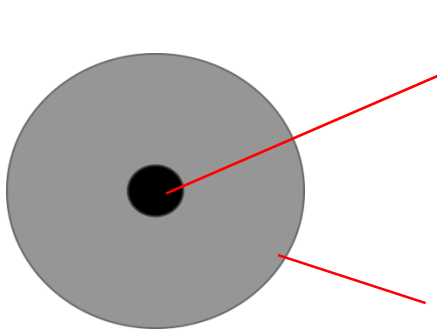
- A few events per galaxy per million yrs
- Lasting ms to minutes
- Energy output in 1 s = Sun's output in entire life (billions of years).

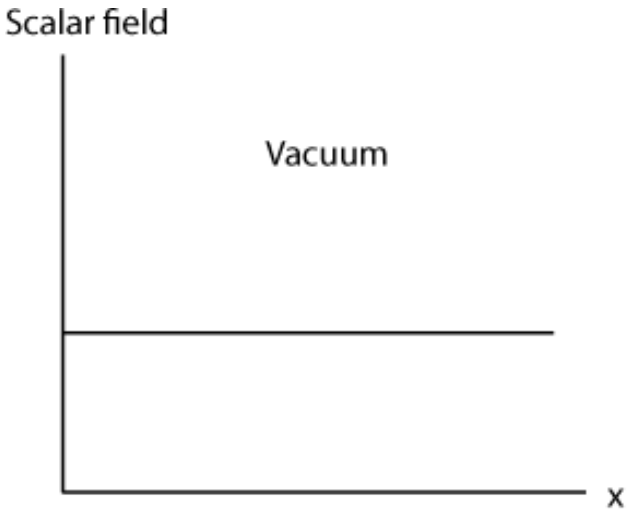
Jet of matter 27 light years long



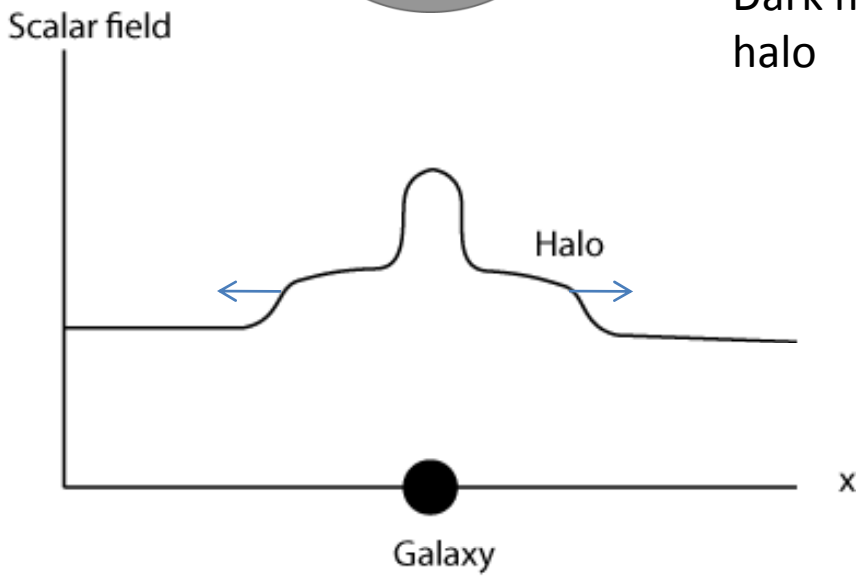
Dark matter

Halo in “bullet cluster” from gravitational lensing (blue)



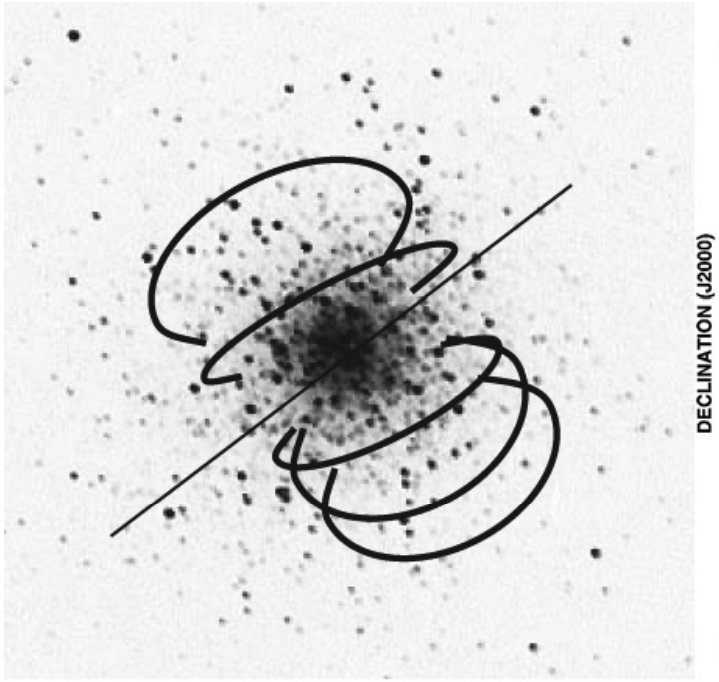


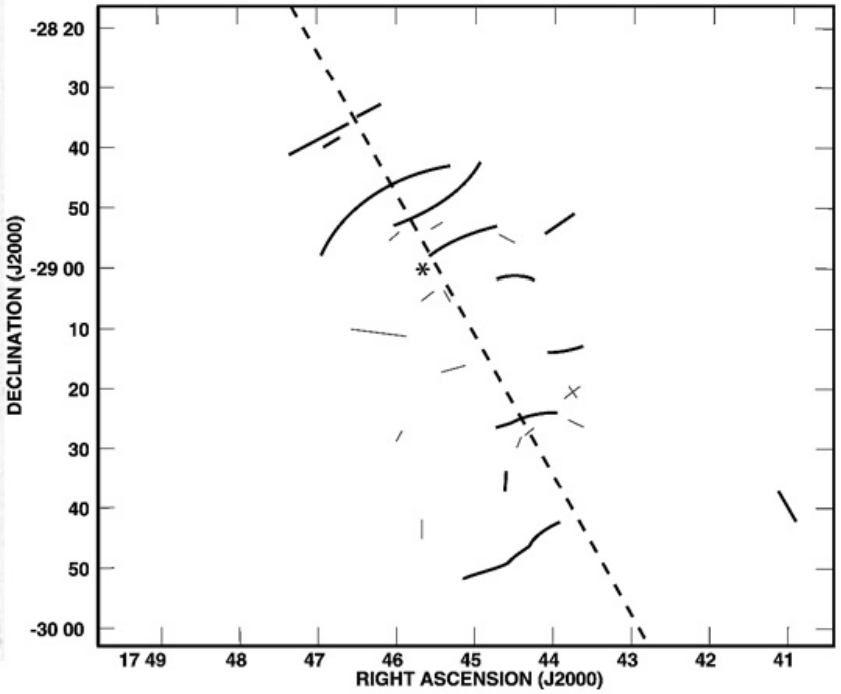
Vacuum field gives dark energy



Deviation from vacuum value, due to presence of galaxy, represents dark matter

“Hair” on black hole





Artist’s conception:
Rotating object in superfluid induce vortex filaments.

Observed:
“Non-thermal filaments" near center of Milky Way.

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KH

References:

K. Huang, H.-B. Low, and R.-S. Tung,

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2. “Scalar field cosmology II: superfluidity and quantum turbulence”, arXiv:1106.5283 (2011).