TENTH HOMEWORK SHEET FOR QM

To be handed in on the 16.06

Q 24: *Angular momentum uncertainty* **(5 points)**

Let $\{|lm\rangle, \quad l = 0, \frac{1}{2}, 1, ...; \quad m = -l, -l+1, ..., +l\}$ be a standard basis of angular momentum Eigenstates.

a) Calculate the uncertainties $\Delta\hat{L}_x$ and $\Delta\hat{L}_y$ as well as the product $\Delta\hat{L}_x \cdot \Delta\hat{L}_y$ as functions of the quantum numbers l and m. For which m (and given l) are $\Delta\hat{L}_x$ and $\Delta\hat{L}_y$ are minimal, and give these minimal values?

b) Compare the result of a) with the one of the uncertainty relation for two arbitrary observables (see the lectures) applied on the components $\hat{L}_x$ and $\hat{L}_y$ of the angular momentum operator.

Q 25: **(2 points)**

Prove the identity:

$$\left[\hat{A}, \left[\hat{B}, \hat{C}\right]\right] + \left[\hat{B}, \left[\hat{C}, \hat{A}\right]\right] + \left[\hat{C}, \left[\hat{A}, \hat{B}\right]\right] = 0$$

Q 26: *Time dependent Schrodinger equation* **(6 points)**

Consider a particle of mass m in an even, infinitely deep well potential (see also Q 11 c)

$$V(x) = \begin{cases} 0 & \text{for } |x| \le a \quad \text{with} \quad a \in \mathbb{R}, a > 0 \\ +\infty & \text{for } |x| > a \end{cases} .$$

Now consider the time evolution of the wavefunction $\psi(x,t)$ of a non-stationary state, that at time $t = 0$ is built from the normalised sum of the ground state wavefunction $\psi_0(x)$ and the wavefunction of the first excited state $\psi_1(x)$.

a) Illustrate the time dependence of $|\psi(x,t)|^2$ using sketches for $t/T = 0,\ 1/8,\ 1/4,\ 3/8,\ 1/2,\ 5/8$ and $3/4$, where $T := h/(E_1 - E_0)$.
For Computer generated plots there are **3 extra points**.

b) Compare the time evolution of the expectation value $\langle\psi|\hat{Q}|\psi\rangle$ of the position operator $\hat{Q}$ with the classical particle trajectory (in a sketch).

Q 27: *Heisenberg picture: 1D harmonic oscillator* **(7 points)**

In the lectures you have learnt about the Heisenberg picture. In the Heisenberg picture the complete time dependence is in the operators and the dynamics are described by the Heisenberg equations of motion. Consider now the 1D harmonic oscillator with $\hat{H} = \frac{1}{2m}\hat{P}_H^2 + \frac{1}{2}m\omega^2\hat{Q}_H^2$.

(a) Solve the Heisenberg equations of motion for the operators $\hat{Q}_H(t)$ and $\hat{P}_H(t)$, the position and momentum variables of the harmonic oscillator, and determine $\hat{P}_H(t)$ and $\hat{Q}_H(t)$ as functions of the operators $\hat{P}_S = \hat{P}_H(0)$ and $\hat{Q}_S = \hat{Q}_H(0)$ in the Schrodinger basis.
Tip: Through repeated differentiation of the Heisenberg equations of motion they decouple.

(b) Determine with the result from (a) **two** of the following three commutation relations at different times t_1 and t_2:

$$\left[\hat{P}_H(t_2), \hat{P}_H(t_1)\right], \left[\hat{P}_H(t_2), \hat{Q}_H(t_1)\right], \left[\hat{Q}_H(t_2), \hat{Q}_H(t_1)\right].$$