11th QM Homework sheet

To be handed in on the 23.06.

Q 28 *Parity Operator* **(9 points)**

Consider a particle with one degree of freedom $x \in \mathbb{R}$. The operator $\hat{\Pi}$, that in the state space $\mathcal{H}$ of the particle makes the coordinate transformation $x' \equiv \mathcal{P}x := -x$, can formally be defined through its action in the basis system of the Eigenstate $|x\rangle$ of the position operator $\hat{X}$ accordant with $\hat{\Pi}|x\rangle := |\mathcal{P}x\rangle = |-x\rangle$. $\hat{\Pi}$ is referred to as the *parity operator* or point reflection operator. Show that

a) $\hat{\Pi}^{-1} = \hat{\Pi}^{\dagger} = \hat{\Pi}$; **(2 points)**

b) $\hat{\Pi}$ can only take the Eigenvalues $\pi = +1, -1$; **(1 point)**

c) $\langle u|\hat{T}|v\rangle = 0$, if $|u\rangle$ and $|v\rangle$ are Eigenvectors of $\hat{\Pi}$ with the same Eigenvalue π, and $\hat{T}$ is an odd operator defined with respect to $\mathcal{H}$, i.e. an operator for which $\hat{\Pi}\hat{T}\hat{\Pi}^{\dagger} = -\hat{T}$ is true. **(2 points)**

d) Consider a particle in a 1D reflection (mirror) symmetric potential $V(x) = \mathcal{P}V(x) = V(-x)$. Let $\hat{H}$ be the corresponding Hamiltonian operator. What is the given by the commutator $[\hat{H}, \hat{\Pi}]$? **(2 points)**

e) In 3D the Parity operator of the position vector $\vec{x}$ acts by transforming $\vec{x}$ to $-\vec{x}$: $\hat{\Pi}|\vec{x}\rangle = |\mathcal{P}\vec{x}\rangle = |-\vec{x}\rangle$ e.g. by application on a wavefunction $\mathcal{P}\psi(\vec{x}) = \psi(-\vec{x})$. Consider the spherical harmonic $Y_{lm}(\vartheta, \varphi)$, that you know as the position space representation (in spherical coordinates) of the angular momentum functions. Are these functions $Y_{lm}(\vartheta, \varphi)$ Eigenfunctions of $\mathcal{P}$ and if yes, with which Eigenvalue? **(2 points)**

Q 29 *Time independent perturbed system* **(8 points)**

Consider a particle of mass m in a 1D potential

$$V_0(x) = \begin{cases} 0 & \text{for } 0 < x < L \\ +\infty & \text{otherwise.} \end{cases}$$

a) Determine the Eigenfunction $\psi(x)$ (normalised to unity) and the energy Eigenvalues E of the corresponding Hamiltonian operator $\hat{H}_0$. **(2 points)**

b) Which energy Eigenvalues of the corresponding Hamiltonian operator does one have for a particle of mass m in the potential

$$V_b(x) = \begin{cases} -b & \text{for } 0 < x < L \\ +\infty & \text{otherwise,} \end{cases}$$

with the real constant $b > 0$? **(2 points)**

c) Consider now a particle of mass m in the potential $V(x) = V_0(x) + V_1(x)$ with

$$V_1(x) = \begin{cases} -b & \text{for } 0 < x < a \leq L \\ 0 & \text{otherwise,} \end{cases}$$

and the real constants $b > 0$ and $0 < a \leq L$.
Consider $V_1(x)$ as a perturbation of the Hamiltonian operator $\hat{H}_0$ from part a) and calculate the correction to the groundstate energy E_0 at first order in perturbation theory. What is given in the case $a = L$? Compare this to the result of section b).
(4 points)

Q 30: *Time independent perturbed system* **(10 points)**
The Hamiltonian operator of a **2D oscillator** is given by

$$\begin{aligned} \hat{H} &= \hat{H}_0 + \hat{H}_1 \\ \hat{H}_0 &= \frac{1}{2m}\left(\hat{p}_1^2 + \hat{p}_2^2\right) + \frac{m\omega^2}{2}\left(\hat{x}_1^2 + \hat{x}_2^2\right) \\ \hat{H}_1 &= \gamma \frac{m^2\omega^3}{\hbar}\hat{x}_1^2\hat{x}_2^2 \qquad \text{with } \gamma \in R^+ \end{aligned}$$

(a) Determine the solution of the Eigenvalue problem of the unperturbed Hamiltonian $\hat{H}_0$.
(3 points)

(b) Calculate the corrections to the energy at first and second order in perturbation theory for the ground state of $\hat{H}$. **(7 points)**

In den dreißiger Jahren, unter dem demoralisierenden Einfluß der quantenmechanischen Störungstheorie, reduzierte sich die Mathematik, die von einem theoretischen Physiker verlangt wurde, auf eine rudimentäre Kenntnis des lateinischen und griechischen Alphabets.
R. JOST

Aus *R.F. Streater / A.S.Wightman, PCT, Die Prinzipien der Quantenfeldtheorie, BI 1996*