1. Homework sheet for PTP 4 (Quantum mechanics)

Solutions should be handed in during the tutorial of the 15th of 16th of April 2009.

Q 1: *Pauli-matrices* **(4 points)**

The Pauli matrices were introduced in the lectures.

$$\hat{\tau}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{\tau}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \hat{\tau}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

a) Solve the Eigenvalue problem for the Pauli matrix $\hat{\tau}_2$, i.e. find the Eigenvalues and their corresponding Eigenvectors. (2 points)

b) Show that the following commutation relation is satisfied

$$[\hat{\tau}_i, \hat{\tau}_j] := \hat{\tau}_i\hat{\tau}_j - \hat{\tau}_j\hat{\tau}_i = 2i\epsilon_{ijk}\hat{\tau}_k$$

(2 points)

Q 2: *Mixed state spin system* **(8 points)**

A spin system ($s = 1/2$, two-state system, as defined in the lectures) is at a given time in the state

$$|\psi >= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1+i}{2} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad .$$

Consider the operator

$$\hat{\vec{S}} \cdot \vec{e} = \hat{S}_x e_x + \hat{S}_y e_y + \hat{S}_z e_z$$

with the normalised ($|\vec{e}| = 1$) direction vector

$$\vec{e} = \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix}$$

(e_x, e_y and e_z are real valued). $\vec{S} \cdot \vec{e}$ is the direction $\vec{e}$ spin projection operator. For which spatial direction (i.e. for which $\vec{e}$) is the the spin in a pure state, i.e. for which spatial direction is

$$\Delta(\hat{\vec{S}} \cdot \vec{e}) = \sqrt{< \psi|(\hat{\vec{S}} \cdot \vec{e})^2|\psi > - < \psi|(\hat{\vec{S}} \cdot \vec{e})|\psi >^2} = 0 \qquad ?$$

Tip: $\Delta(\hat{\vec{S}} \cdot \vec{e}) = 0$ is satisfied, when $|\Psi >$ is an Eigenvector of the operator $(\hat{\vec{S}} \cdot \vec{e})$.

Please turn over !

Q 3: *Two-state system* **(8 points)**

In a two-state system the orthonormal states $|1>$ and $|2>$ form a basis. In this basis the matrix of the Hamiltonian operator $\hat{H}$ takes the form

$$\hat{H} = \begin{pmatrix} <1|\hat{H}|1> & <1|\hat{H}|2> \\ <2|\hat{H}|1> & <2|\hat{H}|2> \end{pmatrix} = E_0 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad .$$

a) Calculate the Eigenvalues E_i and the corresponding Eigenvectors $|\psi_i>$ of $\hat{H}$ (in the basis $|1>, |2>$). (2 points)

b) At $t = 0$ the system is in state $|1>$. After what time, Δt, is the system in state $|2>$. How large in the energy uncertainty ΔE in state $|2>$? What is given by the product $\Delta E \cdot \Delta t$? (6 points)