

3. ÜBUNGSBLATT ZUR VORLESUNG THEORETISCHE PHYSIK IV (STATISTISCHE PHYSIK UND THERMODYNAMIK)

Abgabe: Freitag, 11. Mai 2007 in den Übungen.

Aufgabe 3.1:

Ein Behälter mit Volumen V enthalte N Gasmoleküle. Die Anzahl der Moleküle in einem Teilvolumen v des Behälters sei n . Die Wahrscheinlichkeit, ein bestimmtes Molekül in v zu finden, ist gleich $\frac{v}{V}$ im thermischen Gleichgewichtszustand des Systems.

- (i) Finden Sie die Wahrscheinlichkeitsverteilung $f(n)$ der Zahl n ,
und **(1 Punkt)**
- (ii) berechnen Sie den Mittelwert \bar{n} und $\overline{(n - \bar{n})^2}$. **(1 Punkt)**
- (iii) Zeigen Sie, dass falls N und n beide groß sind, $f(n)$ näherungsweise
Gauß-verteilt ist. **(2 Punkte)**
- (iv) Zeigen Sie, dass im Limes $\frac{v}{V} \rightarrow 0$ und $V \rightarrow \infty$, mit konstantem $\frac{N}{V}$, $f(n)$ die
Poissonverteilung $f(n) = e^{-\bar{n}}(\bar{n})^n/n!$ anstrebt. **(2 Punkte)**

Aufgabe 3.2:

Gegeben sei ein System aus einem Spin-1/2-Teilchen in Ruhe. Die Dichtematrix des Systems sei von der Gestalt:

$$\rho(\kappa) = \frac{1}{2} \begin{pmatrix} 1 & \kappa \\ \kappa & 1 \end{pmatrix} \quad (1)$$

wobei κ ein reeller Parameter ist.

- a) Bestimmen Sie die Eigenwerte der Matrix (1). **(2 Punkte)**
- b) Geben Sie die allgemeinen Bedingungen an, die eine Dichtematrix erfüllen muß.
Für welche Werte von κ erfüllt die Matrix (1) diese Bedingungen? **(2 Punkte)**

Ludwig Boltzmann

Born: 20 Feb 1844 in Vienna, Austria
Died: 5 Oct 1906 in Duino (near Trieste), Austria (now Italy)



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Ludwig Boltzmann's father was a taxation official. Boltzmann was awarded a doctorate from the University of Vienna in 1866 for a thesis on the kinetic theory of gases supervised by Josef Stefan. After obtaining his doctorate, he became an assistant to his teacher Josef Stefan.

Boltzmann taught at Graz, moved to Heidelberg and then to Berlin. In these places he studied under Bunsen, Kirchhoff and Helmholtz.

In 1869 Boltzmann was appointed to a chair of theoretical physics at Graz. He held this post for four years then, in 1873, he accepted the chair of mathematics at Vienna. He did not stay very long in any place and after three years he was back in Graz, this time in the chair of experimental physics.

Boltzmann, at least half jokingly, used to say that the reason he moved around so much was that he was born during the dying hours of a Mardi Gras ball. It was only half joking since he did feel that his nature made him subject to rapid swings between happiness and sadness. His personality certainly had a major impact on the direction that his career took and personal relationships, where he was always very soft-hearted, played a big part. He suffered from an alternation of depressed moods with elevated, expansive or irritable moods. Indeed his physical appearance, being short and stout with curly hair, seemed to fit his personality. His fiancée called him her "sweet fat darling".

After another three years, in 1894, Boltzmann moved back to Vienna, this time to the chair of theoretical physics which became vacant on the death of his teacher Josef Stefan. However, the following year Ernst Mach was appointed to the chair of history and philosophy of science at Vienna. Boltzmann had many scientific opponents but, to Boltzmann, Mach was more than a scientific opponent as the two were on bad personal terms.

In 1900, because of his dislike of working with Mach, Boltzmann moved to Leipzig but here he became a colleague of his strongest scientific opponent Wilhelm Ostwald. Despite their scientific differences Boltzmann and Ostwald were on good personal terms. Despite this, depressed by scientific arguments with Ostwald which are described below Boltzmann unsuccessfully attempted suicide during his time in Leipzig.

In 1901 Mach retired from Vienna due to ill health, and because of this Boltzmann's reason for moving from Vienna had gone. In 1902 he returned to Vienna to his chair of theoretical physics which had not been filled in the intervening period. In addition to his teaching in mathematical physics, Boltzmann was given Mach's philosophy course to teach. His philosophy lectures quickly became famous with the audience soon being too large for the biggest lecture hall available. In fact because of the fame of these lectures Boltzmann was invited to the Palace of Franz Josef.

Boltzmann's fame is based on his invention of statistical mechanics. This he did independently of Willard Gibbs. Their theories connected the properties and behaviour of atoms and molecules with the large scale properties and behaviour of the substances of which they were the building blocks.

Boltzmann obtained the Maxwell-Boltzmann distribution in 1871, namely the average energy of motion of a molecule is the same for each direction. He was one of the first to recognise the importance of Maxwell's electromagnetic theory.

In 1884 the work of Josef Stefan was developed by Boltzmann who showed how Josef Stefan's empirical T^4 law for black body radiation, formulated in 1879, could be derived from the principles of thermodynamics.

Boltzmann worked on statistical mechanics using probability to describe how the properties of atoms determine the properties of matter. In particular his work relates to the Second Law of Thermodynamics which he derived from the principles of mechanics in the 1890s.

The equations of Newtonian mechanics are reversible in time and Poincaré proved that if a mechanical system is in a given state it will return infinitely often to a state arbitrarily close to the given one. Zermelo deduced that the Second Law of Thermodynamics is impossible in a mechanical system. Boltzmann asserted that entropy increases almost always, rather than always. However he believed that Poincaré's result, although correct in theory, was in practice impossible to observe since the time before a system returns to near its original state was too long.

Boltzmann's ideas were not accepted by many scientists. In 1895, at a scientific meeting in Lübeck, Wilhelm Ostwald presented a paper in which he stated:-

The actual irreversibility of natural phenomena thus proves the existence of processes that cannot be described by mechanical equations, and with this the verdict on scientific materialism is settled.

Sommerfeld, who was at the meeting, described the resulting battle between Ostwald and Boltzmann. Sommerfeld wrote:-

... Boltzmann was seconded by Felix Klein. The battle between Boltzmann and Ostwald

resembled the battle of the bull with the supple fighter. However, this time the bull was victorious ... The arguments of Boltzmann carried the day. We, the young mathematicians of that time, were all on the side of Boltzmann ...

Ostwald led the opposition to Boltzmann's ideas which were opposed by many European scientists, they misunderstood them, not fully grasping the statistical nature of his reasoning. However some, including Mach, thought the arguments were too violent, and this certainly appeared to be the case when Boltzmann attempted suicide while a colleague of Ostwald.

In 1904 Boltzmann visited the World's Fair in St Louis, USA. He lectured on applied mathematics and then went on to visit Berkeley and Stanford. Unfortunately he failed to realise that the new discoveries concerning radiation that he learnt about on this visit were about to prove his theories correct.

Boltzmann continued to defend his belief in atomic structure and in a 1905 publication *Populäre Schriften* he tried to explain how the physical world could be described by differential equations which represented the macroscopic view without representing the underlying atomic structure. :-

May I be excused for saying with banality that the forest hides the trees for those who think that they disengage themselves from atomistics by the consideration of differential equations.

Attacks on his work continued and he began to feel that his life's work was about to collapse despite his defence of his theories. Depressed and in bad health, Boltzmann committed suicide just before experiment verified his work.

On holiday with his wife and daughter at the Bay of Duino near Trieste, he hanged himself while his wife and daughter were swimming. However the cause of his suicide may have been wrongly attributed to the lack of acceptance of his ideas. We will never know the real cause which may have been the result of mental illness causing his depression.

Article by: *J J O'Connor* and *E F Robertson*

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JOC/EFR September 1998

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