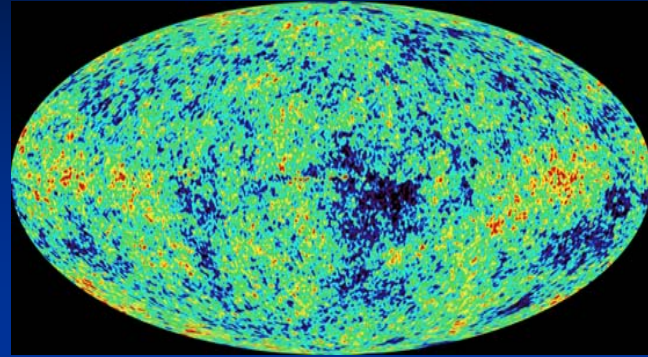


**Quintessence -
a fifth force from variation of
the fundamental scale**

$$\Omega_m + X = 1$$

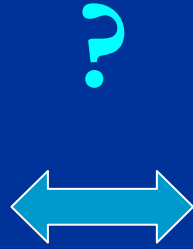
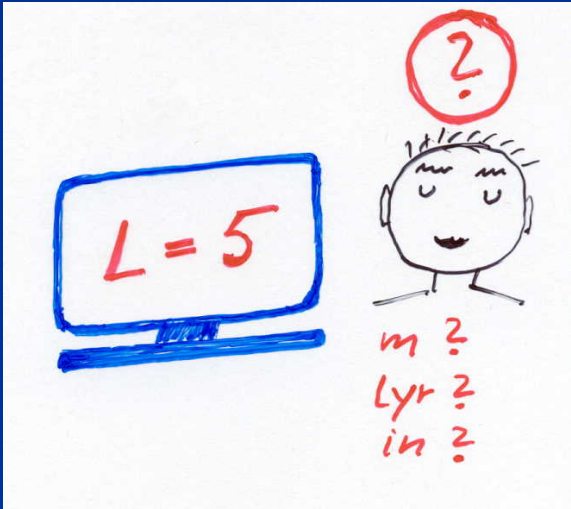


$$\Omega_m : 25\%$$



$$\Omega_h : 75\%$$

Dark Energy



Quintessence

C. Wetterich

A. Hebecker, M. Doran, M. Lilley, J. Schwindt,
C. Müller, G. Schäfer, E. Thommes,
R. Caldwell, M. Bartelmann, K. Karwan

Dark Energy dominates the Universe

Energy - density in the Universe

=

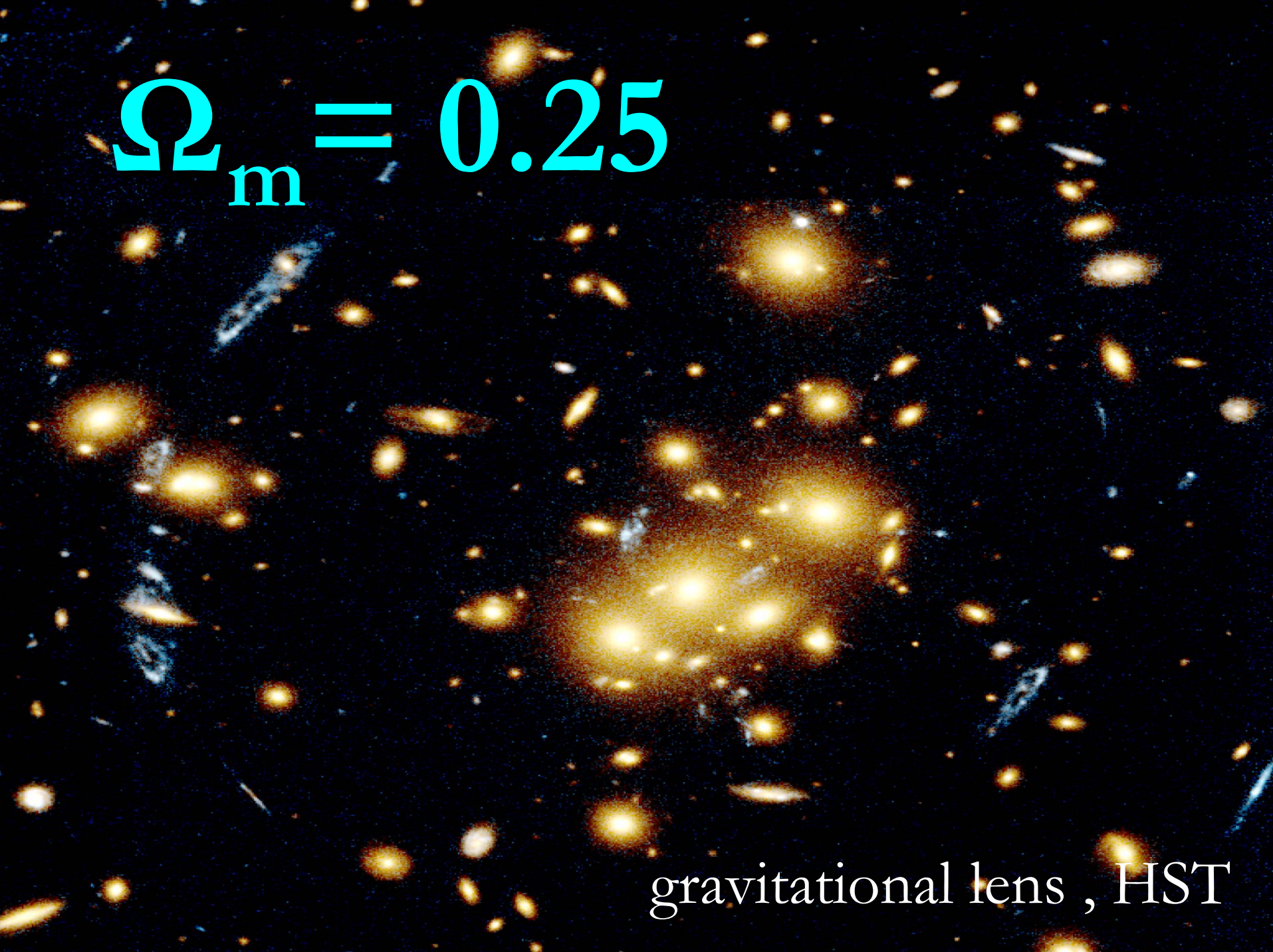
Matter + Dark Energy

25 % + 75 %

Matter : Everything that clumps

A deep-field astronomical image of the Abell 2255 galaxy cluster. The field is filled with hundreds of galaxies of various sizes, shapes, and colors, including yellow, white, blue, and red. Some galaxies are bright and prominent, while others are faint and distant. The background is a dark, starry space.

Abell 2255 Cluster
~300 Mpc

A deep-field astronomical image showing a large number of galaxies. In the center, there is a prominent, bright, yellowish-white galaxy cluster acting as a gravitational lens. This lens causes the light from background galaxies to be distorted, creating multiple images and arcs of light. The background galaxies are mostly yellow and orange, with some blue galaxies scattered throughout. The overall scene is set against a dark, starry background.
$$\Omega_m = 0.25$$

gravitational lens , HST

Wilkinson Microwave Anisotropy Probe

A partnership between
NASA/GSFC and Princeton

Science Team:

NASA/GSFC

Chak Doreani (PI)
Michael Gresson
Bob Hill
Gary Hinshaw
Al Kogut
Michelle Linton
Nils Odgers
Janet Weiland
Ed Willard

Brown

Greg Tucker

UCLA

Neil Wright

UBC

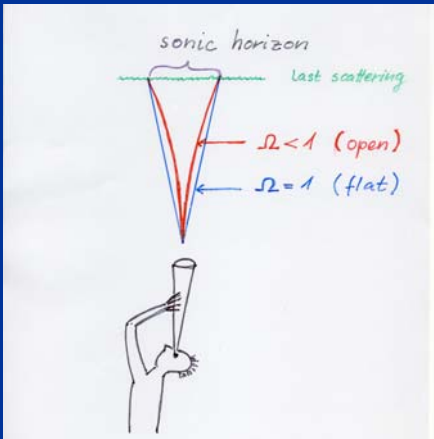
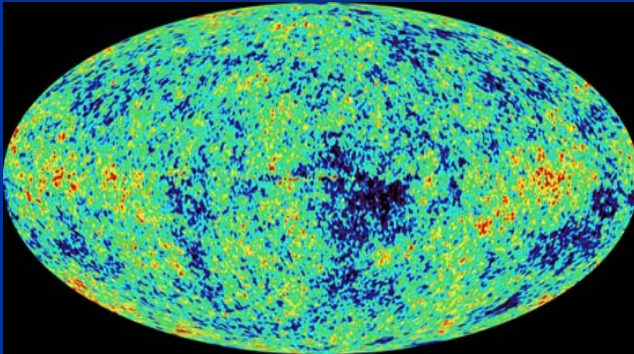
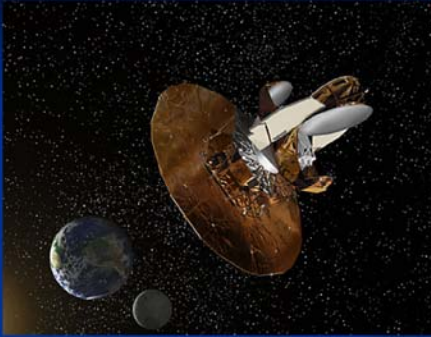
Mark Halpern

Chicago

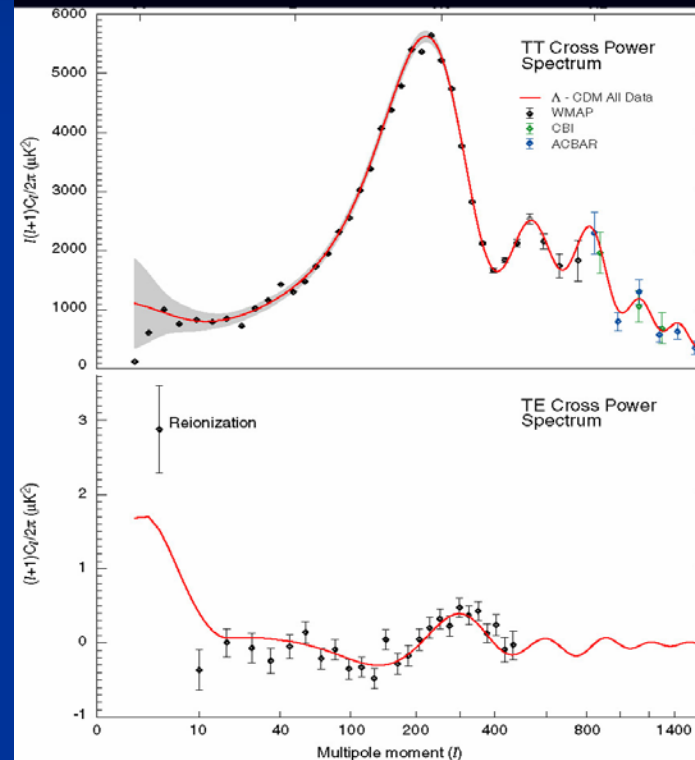
Stephan Meyer

Princeton

Chris Barnes
Norm Janosi
Eiichiro Komatsu
Michael Nolte
Lyman Page
Hiranya Peiris
David Spergel
Licia Verde



$$\Omega_{\text{tot}} = 1$$



mean values

$$\Omega_{\text{tot}} = 1.02$$

$$\Omega_{\text{m}} = 0.27$$

$$\Omega_{\text{b}} = 0.045$$

$$\Omega_{\text{dm}} = 0.225$$

Dark Energy

$$\Omega_m + X = 1$$

$$\Omega_m : 25\%$$

$$\Omega_h : 75\% \quad \text{Dark Energy}$$

h : homogenous , often Ω_Λ instead of Ω_h

**Space between clumps
is not empty :**

Dark Energy !

**Dark Energy density is
the same at every point of space**

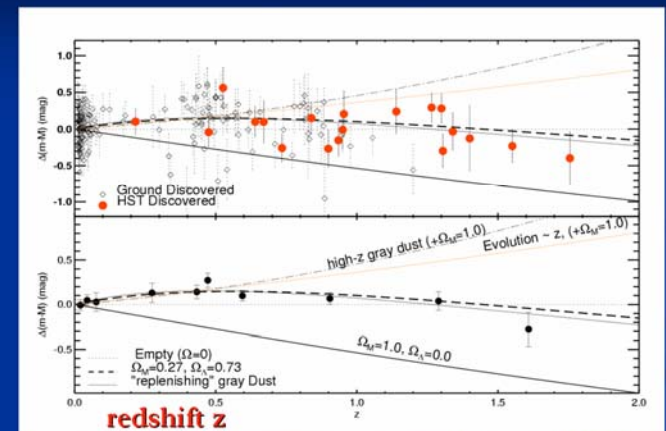
“ homogeneous “

$$\Omega_h$$

Predictions for dark energy cosmologies

*The expansion of the Universe
accelerates today !*

Supernovae 1a Hubble diagram



What is Dark Energy ?

Cosmological Constant
or
Quintessence ?

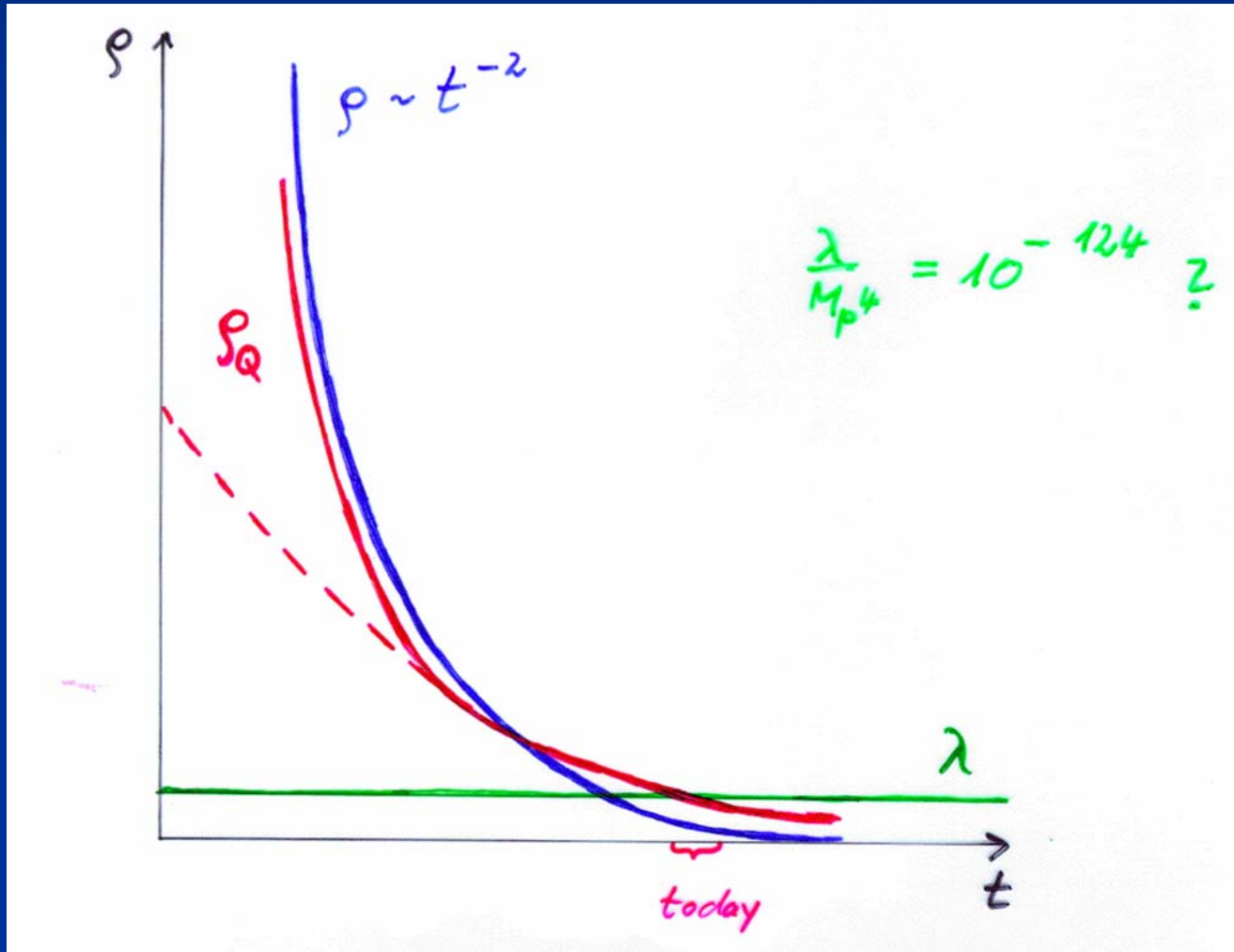
Cosmological Constant

- Einstein -

- Constant λ compatible with all symmetries
- No time variation in contribution to energy density
- Why so small ? $\lambda/M^4 = 10^{-120}$
- Why important just today ?

Cosm. Const.
static

Quintessence
dynamical



**Quintessence and solution of
cosmological constant
problem should be related !**

Cosmological mass scales

- Energy density

$$\rho \sim (2.4 \times 10^{-3} \text{ eV})^{-4}$$

- Reduced Planck mass

$$M = 2.44 \times 10^{18} \text{ GeV}$$

- Newton's constant

$$G_N = (8\pi M^2)$$

Only ratios of mass scales are observable !

homogeneous dark energy: $\rho_h/M^4 = 6.5 \cdot 10^{-121}$

matter: $\rho_m/M^4 = 3.5 \cdot 10^{-121}$

Time evolution

- $\rho_m/M^4 \sim a^{-3} \sim t^{-2}$ matter dominated universe
- $\rho_r/M^4 \sim a^{-4} \sim t^{-3/2}$ radiation dominated universe
- $\rho_r/M^4 \sim a^{-4} \sim t^{-2}$ radiation dominated universe

Huge age \Rightarrow small ratio

Same explanation for small dark energy ?

Time dependent Dark Energy : Quintessence

- What changes in time ?
- **Only dimensionless ratios of mass scales are observable !**
- V : potential energy of scalar field or cosmological constant
- V/M^4 is observable
- **Imagine the Planck mass M increases ...**

Quintessence

from time evolution of
fundamental mass scale

Fundamental mass scale

- Unification fixes parameters with dimensions

- Special relativity : c

- Quantum theory : h

- Unification with gravity :

fundamental mass scale

(Planck mass , string tension , ...)

Fundamental mass scale

- Fixed parameter or dynamical scale ?
- Dynamical scale \longleftrightarrow Field
- Dynamical scale compared to what ?

momentum versus mass

(or other parameter with dimension)

Cosmon and fundamental mass scale

- Assume all mass parameters are proportional to scalar field χ (GUTs, superstrings,...)
- $M_p \sim \chi$, $m_{\text{proton}} \sim \chi$, $\Lambda_{\text{QCD}} \sim \chi$, $M_W \sim \chi$, ...
- χ may evolve with time : **cosmon**
- m_n/M : (almost) constant - observation!

Only ratios of mass scales are observable

Example :

Field χ denotes scale of transition
from higher dimensional physics
to effective four dimensional description
in theory without fundamental mass parameter

(except for running of dimensionless couplings...)

Dilatation symmetry

- Lagrange density:

$$L = \sqrt{g} \left(-\frac{1}{2} \chi^2 R + \frac{1}{2} (\delta - 6) \partial^\mu \chi \partial_\mu \chi + V(\chi) + h \chi \bar{\psi} \psi \right)$$

- Dilatation symmetry for

$$V = \lambda \chi^4, \quad \lambda = \text{const.}, \quad \delta = \text{const.}, \quad h = \text{const.}$$

- Conformal symmetry for $\delta=0$

Dilatation anomaly

- Quantum fluctuations responsible for dilatation anomaly
- Running couplings: **hypothesis**

$$\partial\lambda/\partial\ln\chi = -A\lambda, \quad \partial\delta/\partial\ln\chi = E\delta^2$$

- Renormalization scale μ : (momentum scale)
- $\lambda \sim (\chi/\mu)^{-A}$
- $E > 0$: crossover Quintessence

Dilatation anomaly and quantum fluctuations

- Computation of running couplings (beta functions) needs unified theory !
- Dominant contribution from modes with momenta $\sim \chi$!
- No prejudice on “natural value “ of anomalous dimension should be inferred from tiny contributions at QCD- momentum scale !

Cosmology

Cosmology : χ increases with time !
(due to coupling of χ to curvature scalar)

for large χ the ratio V/M^4 decreases to zero



Effective cosmological constant vanishes
asymptotically for large t !

Asymptotically vanishing effective “cosmological constant”

- Effective cosmological constant $\sim V/M^4$
- $\lambda \sim (\chi/\mu)^{-A}$
- $V \sim (\chi/\mu)^{-A} \chi^4$
- $M = \chi$

$$V/M^4 \sim (\chi/\mu)^{-A}$$

Weyl scaling

$$\text{Weyl scaling : } g_{\mu\nu} \rightarrow (M/\chi)^2 g_{\mu\nu},$$
$$\varphi/M = \ln (\chi^4/V(\chi))$$

$$L = \sqrt{g} \left(-\frac{1}{2} M^2 R + \frac{1}{2} k^2(\phi) \partial^\mu \phi \partial_\mu \phi \right. \\ \left. + V(\phi) + m(\phi) \bar{\psi} \psi \right)$$

Exponential potential : $V = M^4 \exp(-\varphi/M)$

No additional constant !

Without dilatation – anomaly :

$V = \text{const.}$

Massless Goldstone boson = dilaton

Dilatation – anomaly :

$V(\varphi)$

Scalar with tiny time dependent mass :

cosmon

Crossover Quintessence

$$\partial\delta/\partial \ln \chi = E\delta^2$$

(like QCD gauge coupling)

critical χ where δ grows large
critical φ where k grows large

$$k^2(\varphi) = \delta(\chi)/4$$

$$k^2(\varphi) = "1/(2E(\varphi_c - \varphi)/M)"$$

if $\varphi_c \approx 276/M$ (tuning !) :

this will be responsible for relative increase of dark energy in present cosmological epoch

Realistic cosmology

*Hypothesis on running couplings
yields realistic cosmology
for suitable values of A , E , φ_c*

Quintessence cosmology

Quintessence

Dynamical dark energy ,
generated by scalar field

(cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87
P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

**homogeneous dark energy
influences recent cosmology**

- of same order as dark matter -

Original models do not fit the present observations
.... Modifications (i.e. $E > 0$)

Quintessence

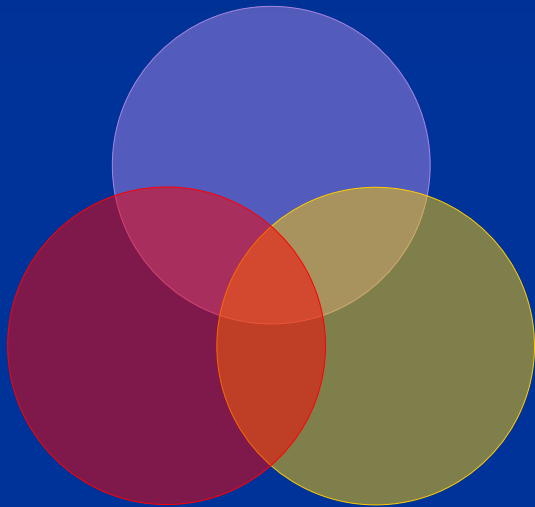
Cosmon – Field $\varphi(\mathbf{x},y,z,t)$

Homogeneous und isotropic Universe : $\varphi(\mathbf{x},y,z,t)=\varphi(t)$

Potential und kinetic energy of the cosmon -field
contribute to a dynamical energy density of the Universe !

“Fundamental” Interactions

Strong, electromagnetic, weak interactions



gravitation

cosmodynamics

On astronomical length scales:

graviton

+

cosmon

Dynamics of quintessence

- **Cosmon** ϕ : scalar singlet field
- Lagrange density $L = V + \frac{1}{2} k(\phi) \partial\phi \partial\phi$
(units: reduced Planck mass $M=1$)
- Potential : $V = \exp[-\phi]$
- “Natural initial value” in Planck era $\phi=0$
- today: $\phi=276$

Quintessence models

- Kinetic function $k(\varphi)$: parameterizes the details of the model - “kinetial”
 - $k(\varphi) = k = \text{const.}$ Exponential Q.
 - $k(\varphi) = \exp((\varphi - \varphi_1)/\alpha)$ Inverse power law Q.
 - $k^2(\varphi) = “1/(2E(\varphi_c - \varphi))”$ Crossover Q.
- possible naturalness criterion:

$k(\varphi=0) / k(\varphi_{\text{today}})$: not tiny or huge !

- else: explanation needed -

Cosmon

- *Scalar field changes its value even in the **present** cosmological epoch*
- *Potential und kinetic energy of cosmon contribute to the energy density of the Universe*
- *Time - variable dark energy :
 $\rho_b(t)$ decreases with time !*

Cosmon

- *Tiny mass*
- $m_c \sim H$
- *New long - range interaction*

cosmon mass changes with time !

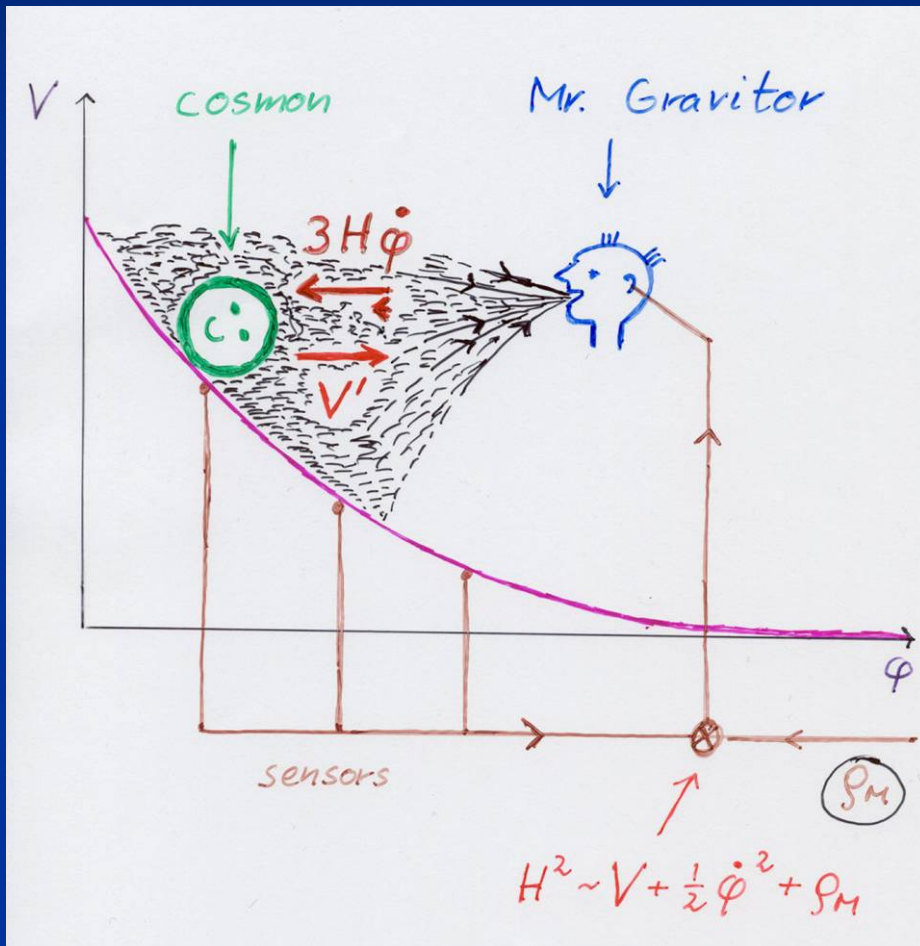
for standard kinetic term

- $m_c^2 = V''$

for standard exponential potential , $k \approx \text{const.}$

- $m_c^2 = V'' / k^2 = V / (k^2 M^2)$
 $= 3 \Omega_h (1 - w_h) H^2 / (2 k^2)$

Cosmological equations



$$\ddot{\phi} + 3H\dot{\phi} = -dV/d\phi$$

$$3M^2H^2 = V + \frac{1}{2}\dot{\phi}^2 + \rho$$

Cosmic Attractors

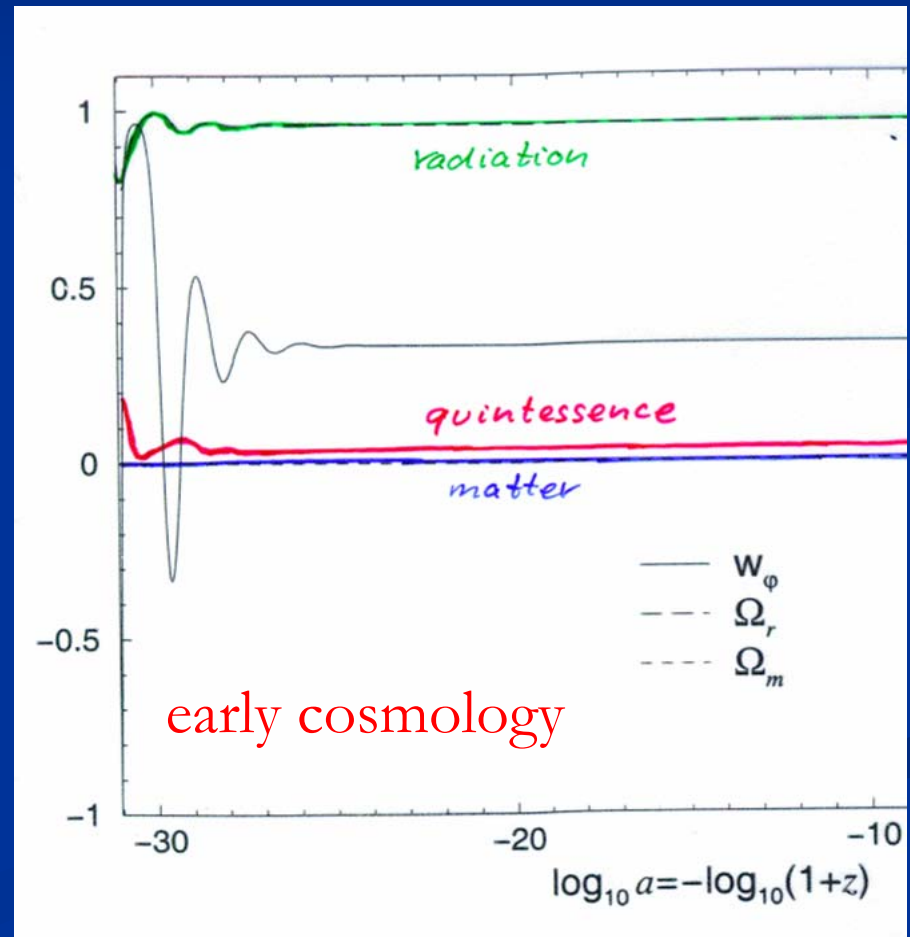
Solutions independent
of initial conditions

typically $V \sim t^{-2}$

$\varphi \sim \ln(t)$

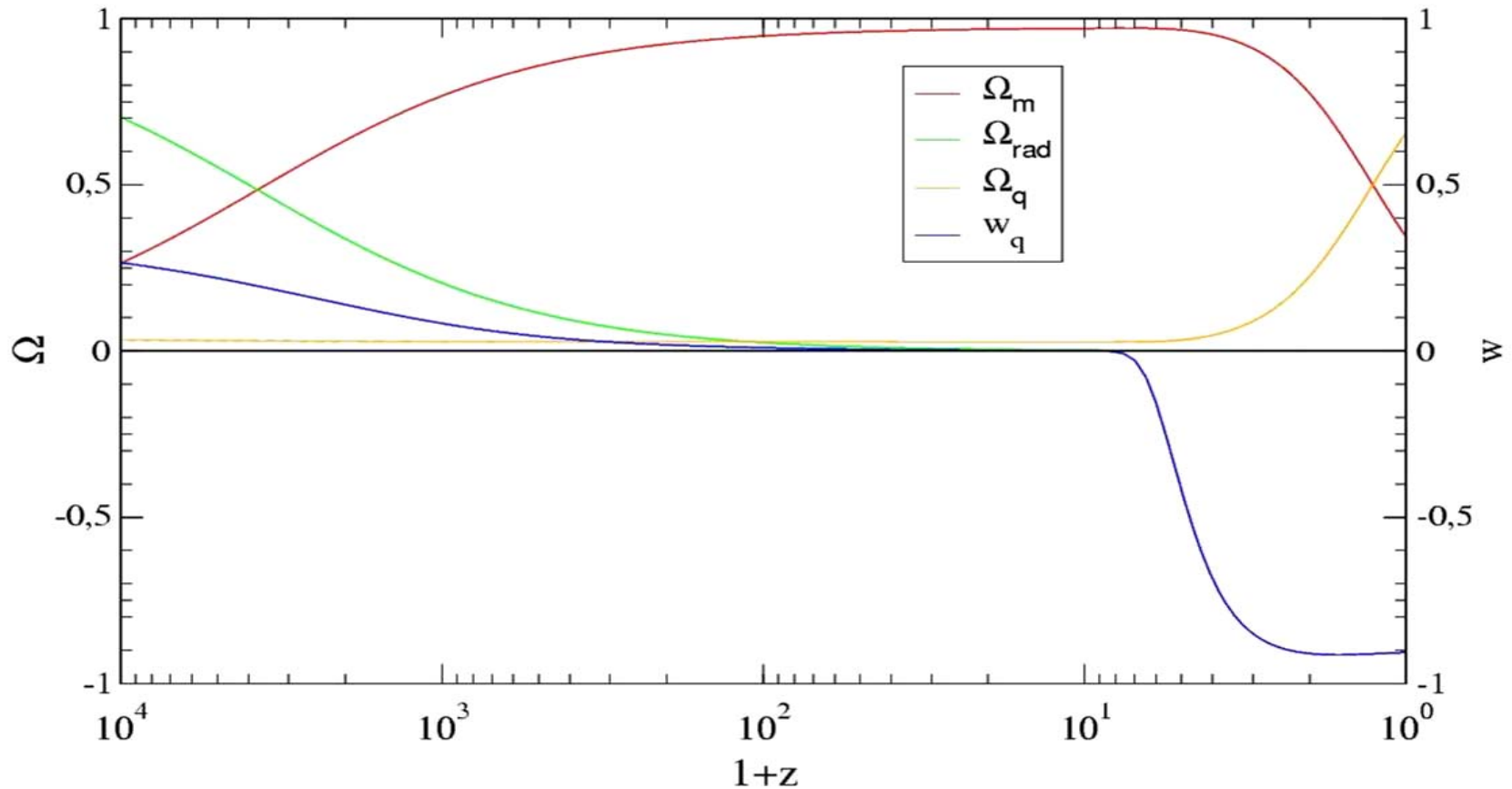
$\Omega_h \sim \text{const.}$

details depend on $V(\varphi)$
or kinetic term



Quintessence becomes important “today”

Crossover Quintessence Evolution



Equation of state

$$p = T - V$$

pressure

kinetic energy

$$\rho = T + V$$

energy density

$$T = \frac{1}{2} \dot{\phi}^2$$

Equation of state

$$w = \frac{p}{\rho} = \frac{T - V}{T + V}$$

Depends on specific evolution of the scalar field

Negative pressure

- $w < 0$ Ω_h increases (with decreasing z)

late universe with
small radiation component :

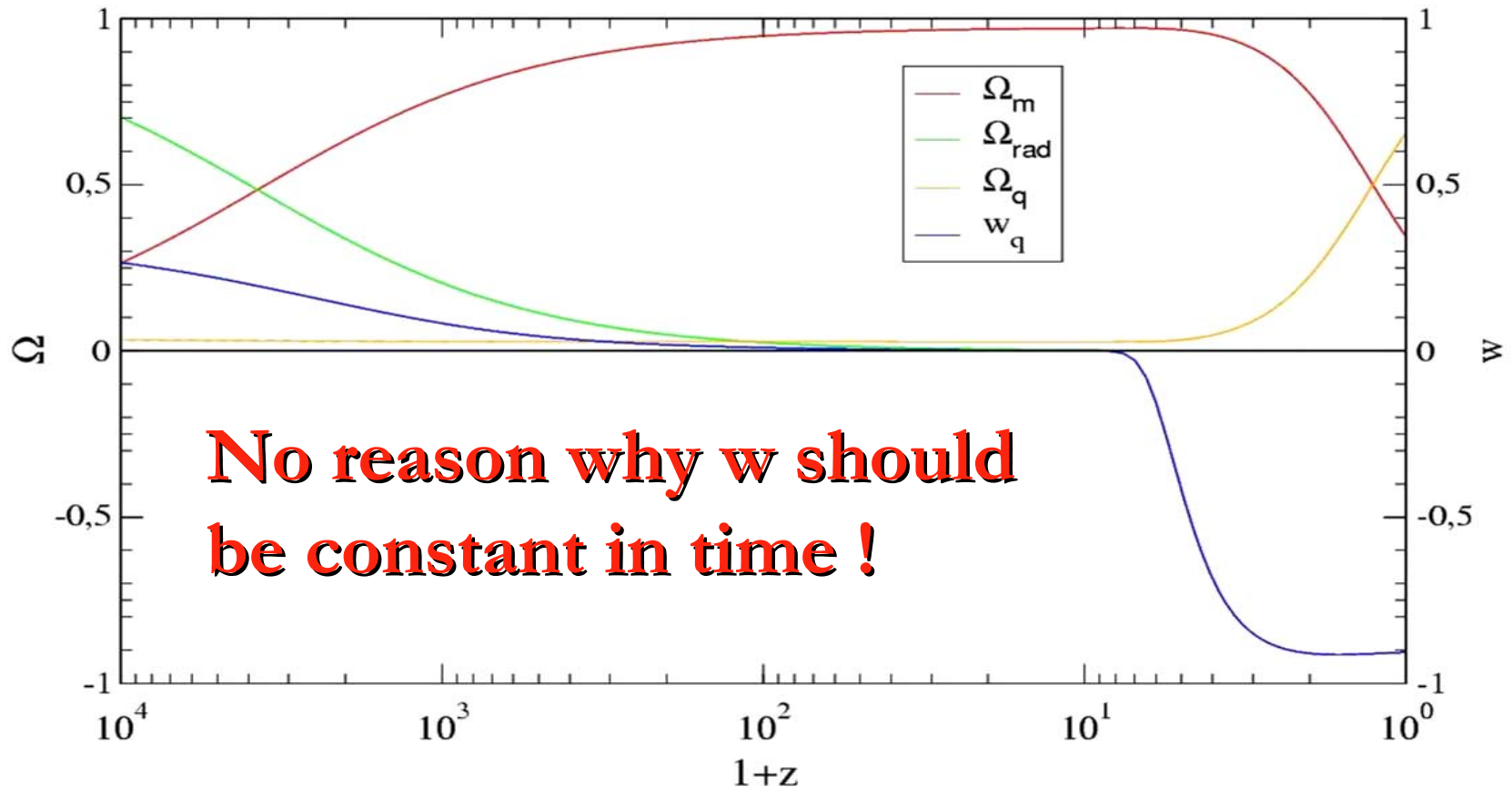
$$w_h = \frac{1}{3\Omega_h(1-\Omega_h)} \frac{\partial\Omega_h}{\partial \ln(1+z)}$$

- $w < -1/3$ expansion of the Universe is
accelerating

- $w = -1$ cosmological constant

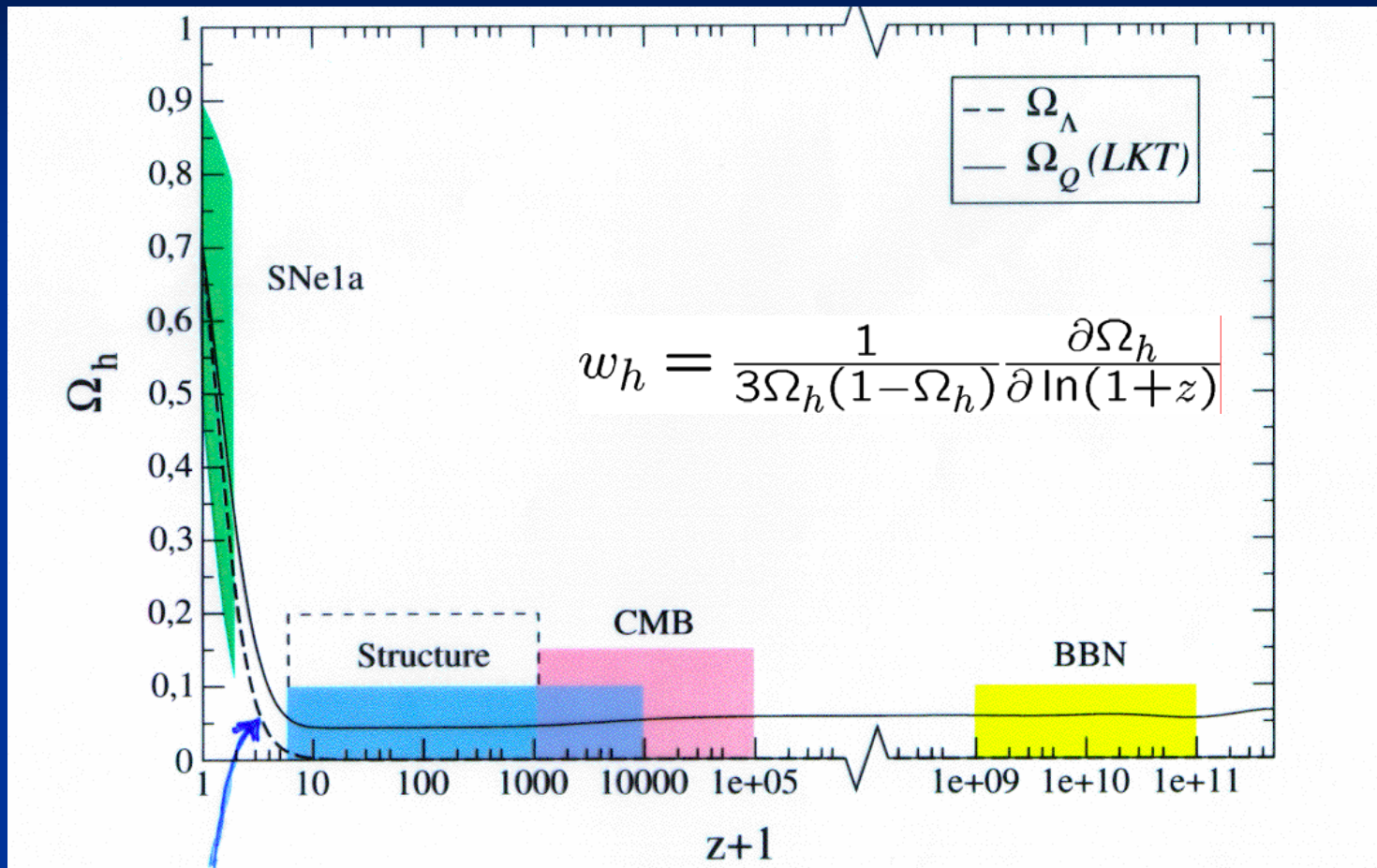
Quintessence becomes important “today”

Crossover Quintessence Evolution



How can quintessence be distinguished from a cosmological constant ?

Time dependence of dark energy



cosmological constant : $\Omega_h \sim t^2 \sim (1+z)^{-3}$

small early and large present dark energy

fraction in dark energy has substantially
increased since end of structure formation



expansion of universe accelerates in present
epoch

$$w_h = \frac{1}{3\Omega_h(1-\Omega_h)} \frac{\partial \Omega_h}{\partial \ln(1+z)}$$

Early dark energy

A few percent in the early Universe

Not possible for a cosmological constant

A few percent Early Dark Energy

If linear power spectrum fixed today (σ_8) :

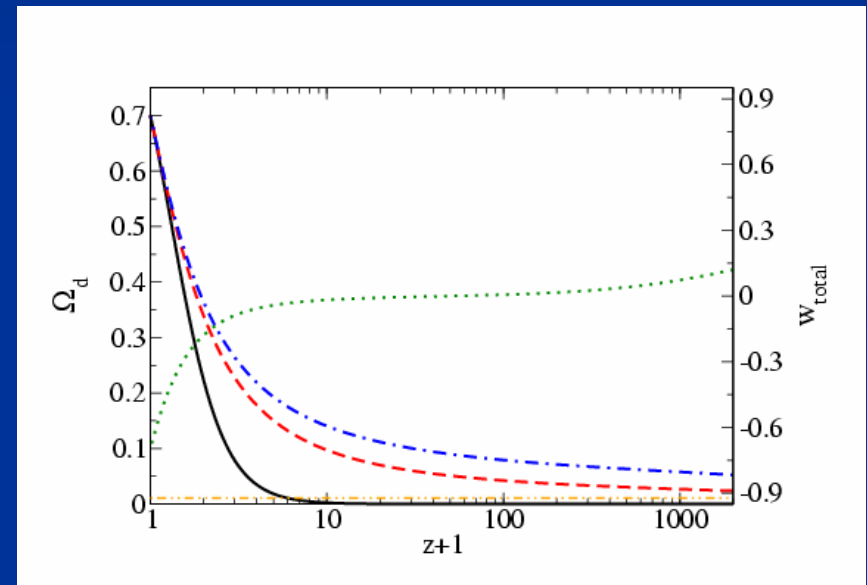
More Structure at high z !

Bartelmann, Doran, ...

Early Dark Energy

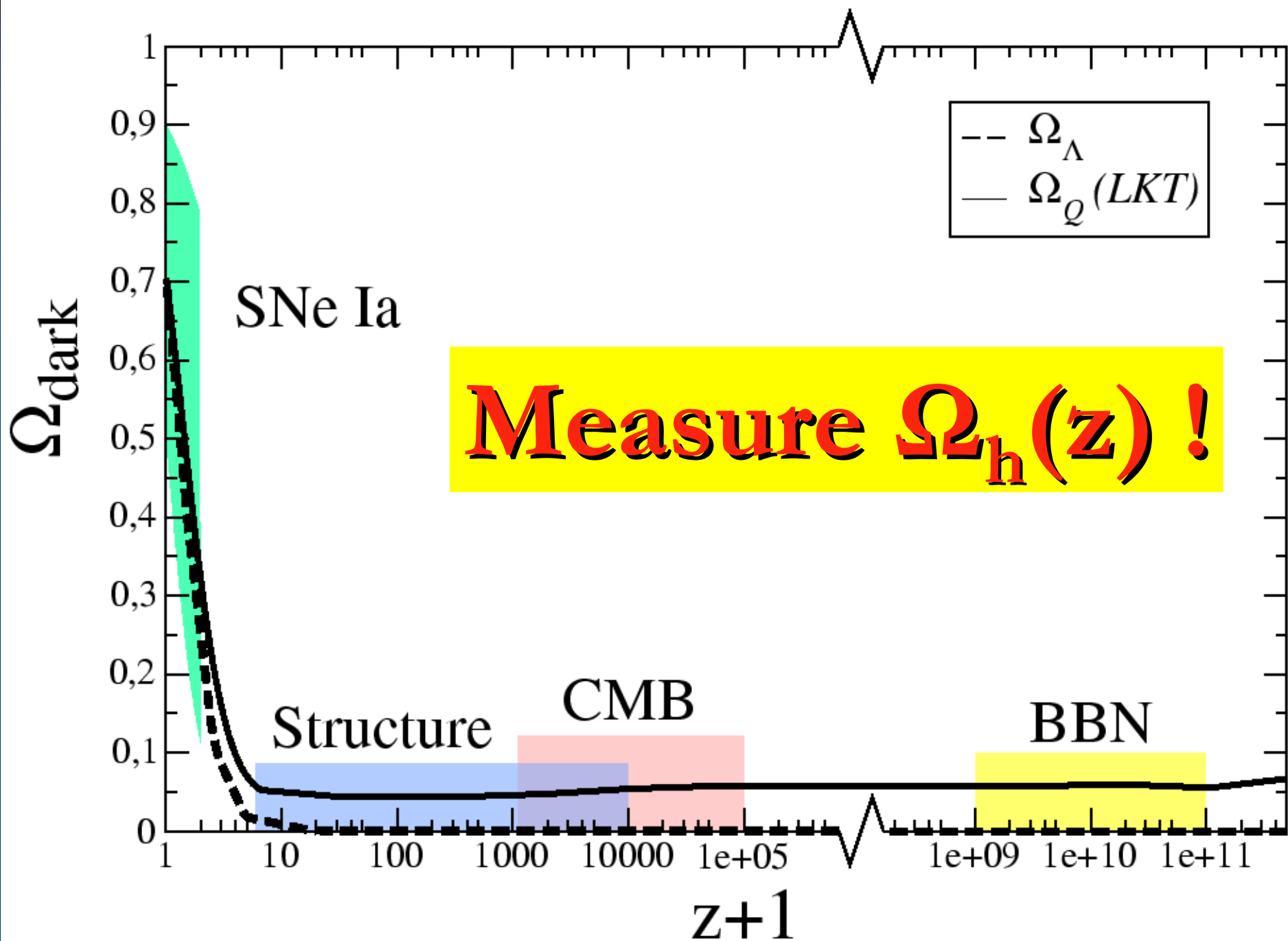
A few percent in the
early Universe

Not possible for a
cosmological
constant



1 σ and 2 σ limits

Doran, Karwan, ..



How to distinguish Q from Λ ?

A) Measurement $\Omega_h(z) \iff H(z)$

i) $\Omega_h(z)$ at the time of structure formation , CMB - emission or nucleosynthesis

ii) equation of state $w_h(\text{today}) > -1$

B) Time variation of fundamental “constants”

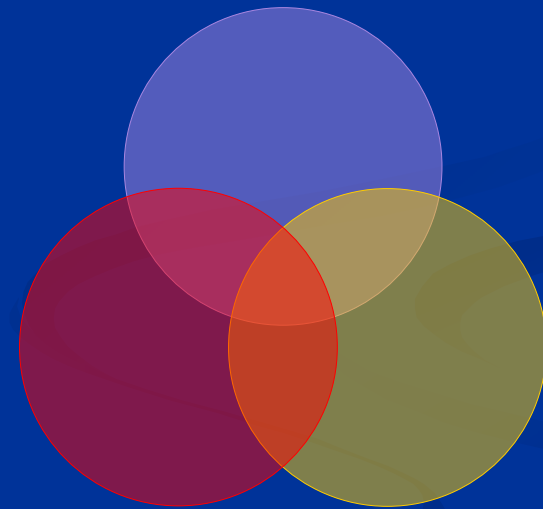
C) Apparent violation of equivalence principle

Quintessence and time variation of fundamental constants

Generic prediction

Strength unknown

Strong, electromagnetic, weak interactions



gravitation

cosmodynamics

C.Wetterich ,
Nucl.Phys.B302,645(1988)

Time varying constants

- It is not difficult to obtain quintessence potentials from higher dimensional or string theories
- Exponential form rather generic
(after Weyl scaling)
- But most models show too strong time dependence of constants !

Quintessence from higher dimensions

work with J. Schwindt

hep-th/0501049

Quintessence from higher dimensions

An instructive example:

Einstein – Maxwell theory in six dimensions

$$S = \int d^6x \sqrt{-g} \left\{ -\frac{M_6^4}{2} R + \lambda_6 + \frac{1}{4} F^{AB} F_{AB} \right\}$$

Warning : not scale - free !

Dilatation anomaly replaced by explicit mass scales.

Metric

Ansatz with particular metric (not most general !)

which is consistent with

d=4 homogeneous and isotropic Universe

and internal $U(1) \times Z_2$ isometry

$$ds^2 = \exp\left(-\frac{\phi(t)}{\bar{M}}\right) \{-dt^2 + a^2(t) d\vec{x}d\vec{x}\}$$

$$+ \exp\left(\frac{\phi(t)}{\bar{M}}\right) r_0^2 \{d\rho^2 + B^2 \sin^2 \rho d\theta^2\}$$

$$r_0^2 = \frac{\bar{M}^2}{4\pi B M_6^4}$$

$B \neq 1$: football shaped internal geometry

Exact solution

$$A_\theta = \frac{m}{2e_6}(1 - \cos \rho)$$

m : monopole number (integer)

$$H^2 = \frac{1}{3\bar{M}^2} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right)$$

cosmology with scalar

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

and potential V :

$$V(\phi) = \bar{M}^4 \left\{ \frac{\lambda_6}{M_6^4 \bar{M}^2} e^{-\frac{\phi}{\bar{M}}} - 4\pi B \frac{M_6^4}{\bar{M}^4} e^{-\frac{2\phi}{\bar{M}}} + 2\pi^2 m^2 \frac{M_6^4}{e_6^2 \bar{M}^6} e^{-\frac{3\phi}{\bar{M}}} \right\}$$

Asymptotic solution for large t

$$H = 2t^{-1}, \quad \phi = 2\bar{M} \ln \frac{t}{\sqrt{10}M_6^2\lambda_6^{-1/2}}$$

$$\Omega_h = \frac{V + \frac{1}{2}\dot{\phi}^2}{3\bar{M}^2 H^2} \rightarrow 1$$

$$V + \frac{1}{2}\dot{\phi}^2 \propto t^{-2}$$

Naturalness

- No tuning of parameters or integration constants
- Radiation and matter can be implemented
- Asymptotic solution depends on details of model, e.g. solutions with constant $\Omega_h \neq 1$

problem :

time variation of fundamental constants

Are fundamental “constants” time dependent ?

Fine structure constant α (electric charge)

Ratio electron to proton mass

Ratio nucleon mass to Planck mass

Quintessence and Time dependence of “fundamental constants”

- Fine structure constant depends on value of
cosmon field : $\alpha(\varphi)$

*(similar in standard model: couplings depend on
value of Higgs scalar field)*

- Time evolution of φ 
Time evolution of α

Jordan,...

Standard – Model of electroweak interactions : Higgs - mechanism

- The masses of all fermions and gauge bosons are proportional to the (vacuum expectation) value of a scalar field φ_H (Higgs scalar)
- For electron, quarks , W- and Z- bosons :

$$m_{\text{electron}} = h_{\text{electron}} * \varphi_H \quad \text{etc.}$$

Restoration of symmetry at high temperature in the early Universe

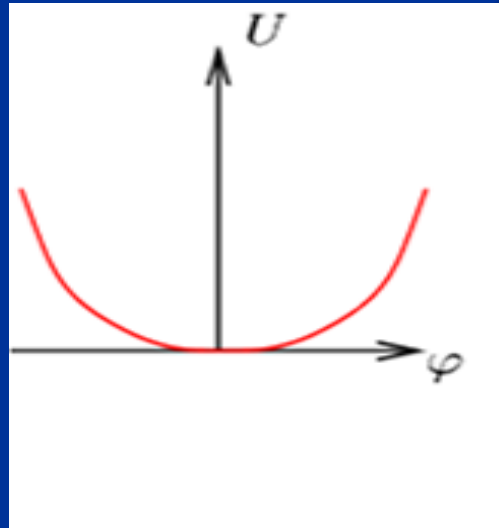
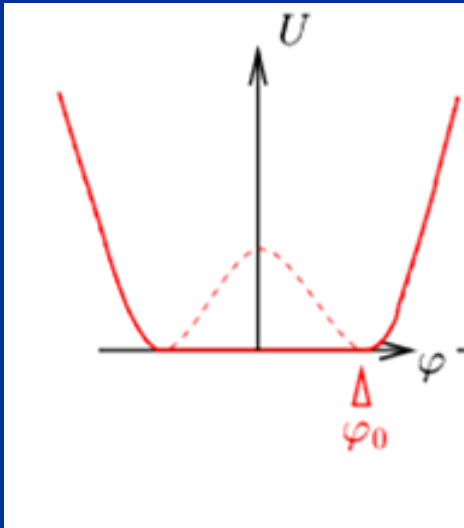
Low T
SSB

$$\langle \phi_H \rangle = \phi_0 \neq 0$$

High T
SYM

$$\langle \phi_H \rangle = 0$$

high T :
less order
more symmetry



example:
magnets

In the hot plasma
of the early Universe :

**No difference in mass for
electron and muon !**

Quintessence :
Couplings are still varying now !

**Strong bounds on
the variation of couplings -
interesting perspectives for
observation !**

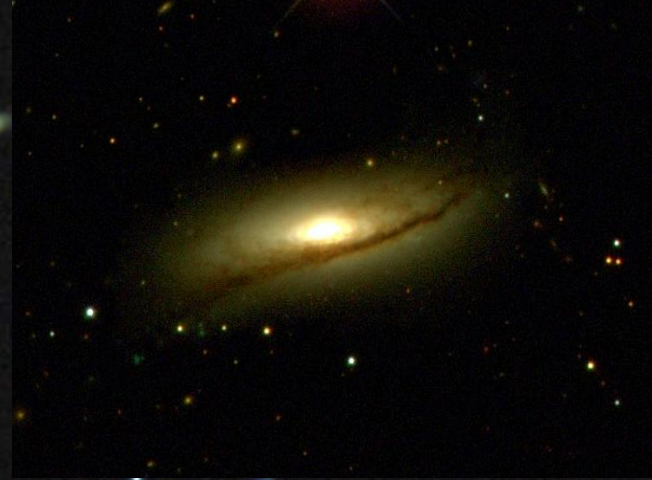
Where to look for time variation of fundamental couplings ?

- Nucleosynthesis
- Molecular absorption lines in the light of distant Quasars
- Oklo natural reactor
- Atomic clocks
- CMB

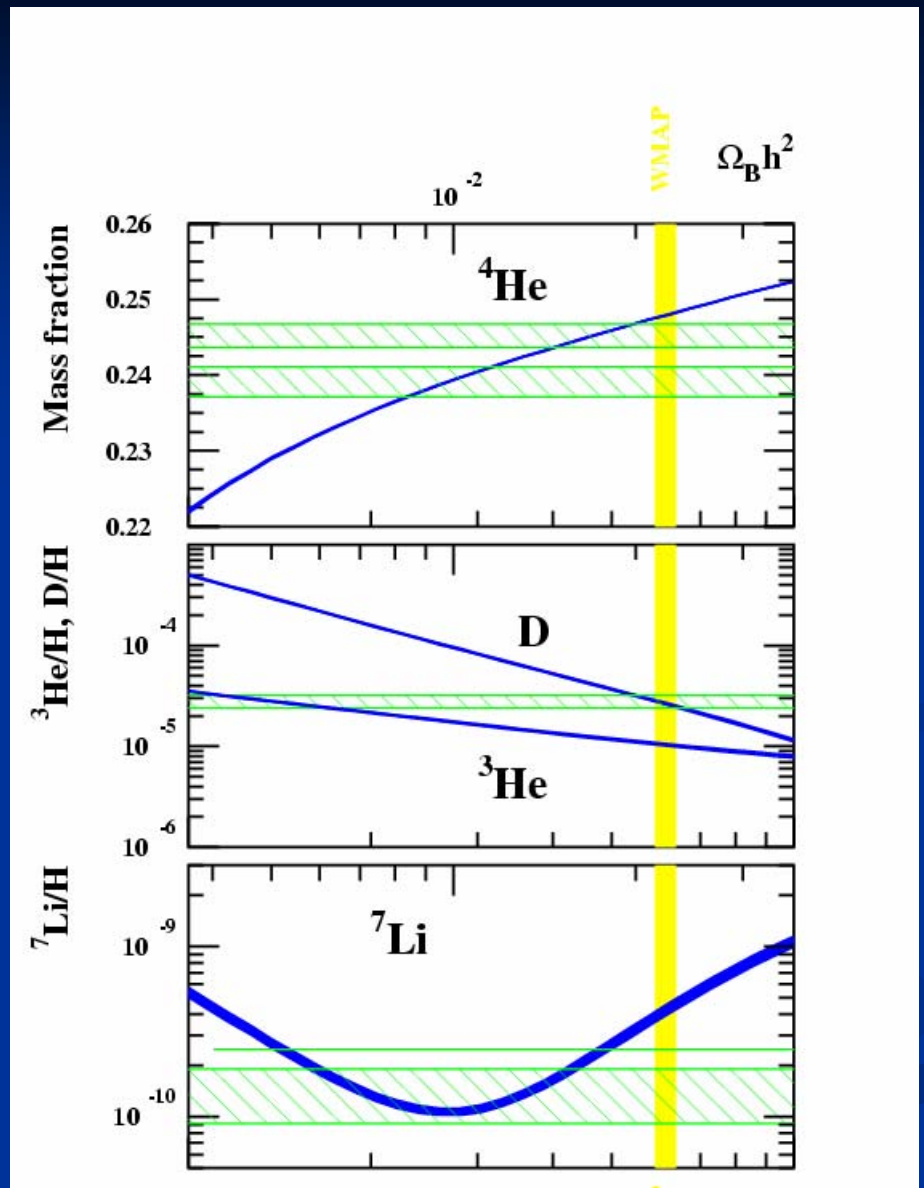
baryons :

the matter of stars and humans

$$\Omega_b = 0.045$$



Abundancies of
primordial
light elements
from
nucleosynthesis



Allowed values for variation of
fine structure constant :

$$\Delta\alpha/\alpha (z=10^{10}) = -1.0 \cdot 10^{-3} \quad \text{GUT 1}$$

$$\Delta\alpha/\alpha (z=10^{10}) = -2.7 \cdot 10^{-4} \quad \text{GUT 2}$$

Time variation of coupling constants
must be tiny –

would be of very high significance !

Possible signal for Quintessence

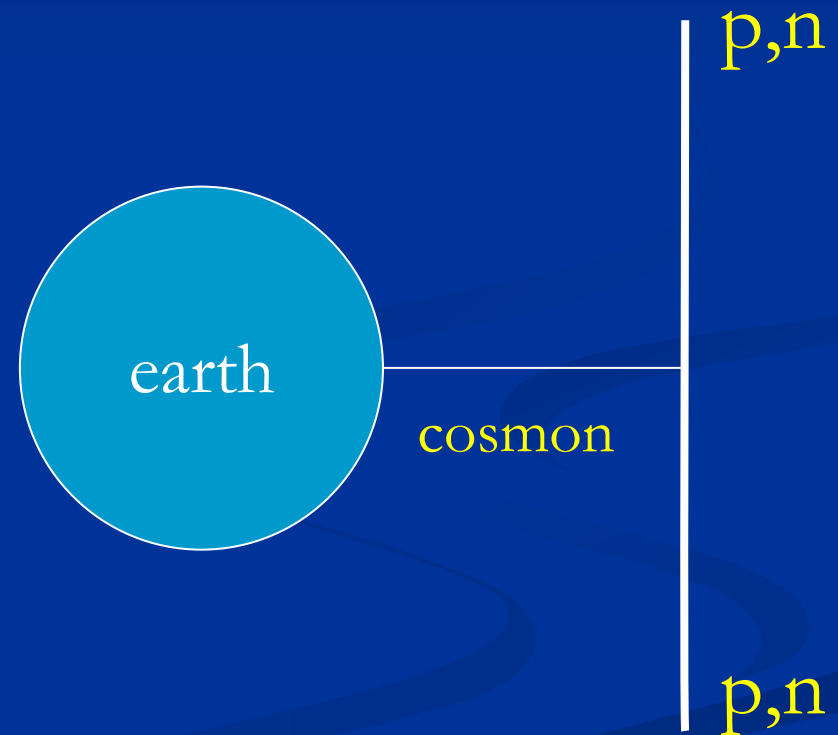
Violation of equivalence principle

Different couplings of
cosmon to proton and
neutron

Differential acceleration

“Violation of
equivalence principle”

only apparent : new “fifth force” !



Apparent violation of equivalence principle

and

time variation of fundamental couplings

measure both the

cosmon – coupling to ordinary matter

Differential acceleration η

For unified theories (GUT) :

$$\eta = -1.75 \cdot 10^{-2} \Delta R_z \left(\frac{\partial \ln \alpha}{\partial z} \right)^2 \frac{1 + \tilde{Q}}{\Omega_h (1 + w_h)}$$

$$\Delta R_z = \frac{\Delta Z}{Z + N} \approx 0.1$$

$$\eta = \Delta a / 2a$$

Q : time dependence of other parameters

Link between time variation of α

and violation of equivalence principle

typically : $\eta = 10^{-14}$

if time variation of α near Oklo upper bound

to be tested by MICROSCOPE

Summary

- o $\Omega_h = 0.75$
- o Q/Λ : dynamical und static dark energy
will be distinguishable
- o Q : time varying fundamental coupling “constants”
violation of equivalence principle

????????????????????????????????

Why becomes Quintessence dominant in the present cosmological epoch ?

Are dark energy and dark matter related ?

Can Quintessence be explained in a fundamental unified theory ?



End

A few references

C.Wetterich , Nucl.Phys.B302,668(1988) , received 24.9.1987

P.J.E.Peebles,B.Ratra , Astrophys.J.Lett.325,L17(1988) , received 20.10.1987

B.Ratra,P.J.E.Peebles , Phys.Rev.D37,3406(1988) , received 16.2.1988

J.Frieman,C.T.Hill,A.Stebbins,I.Waga , Phys.Rev.Lett.75,2077(1995)

P.Ferreira, M.Joyce , Phys.Rev.Lett.79,4740(1997)

C.Wetterich , Astron.Astrophys.301,321(1995)

P.Viana, A.Liddle , Phys.Rev.D57,674(1998)

E.Copeland,A.Liddle,D.Wands , Phys.Rev.D57,4686(1998)

R.Caldwell,R.Dave,P.Steinhardt , Phys.Rev.Lett.80,1582(1998)

P.Steinhardt,L.Wang,I.Zlatev , Phys.Rev.Lett.82,896(1999)

Cosmodynamics

Cosmon mediates new long-range interaction

Range : size of the Universe – horizon

Strength : weaker than gravity

photon

electrodynamics

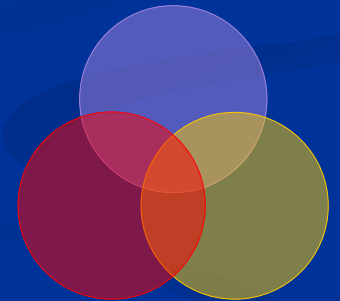
graviton

gravity

cosmon

cosmodynamics

Small correction to Newton's law



Time evolution of fundamental couplings traces time evolution of quintessence

today w_h close to -1 :

- Small kinetic energy
- Slow change of φ
- Slow change of α

Very small $\Delta\alpha/\alpha$ for low z !

Crossover quintessence and time variation of fundamental “constants”

Upper bounds for relative variation of the
fine structure constant

- Oklo natural reactor $\Delta\alpha/\alpha < 10^{-7}$ $z=0.13$
- Meteorites (Re-decay) $\Delta\alpha/\alpha < 3 \cdot 10^{-7}$ $z=0.45$
- Crossover Quintessence compatible with QSO
and upper bounds !

Atomic clocks and OKLO

* Atomic clocks:

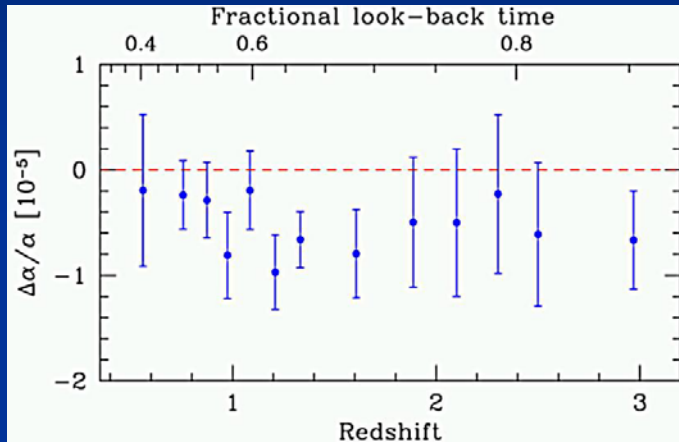
$$\frac{\dot{\alpha}_{em}}{\alpha_{em}} = -5.4 \cdot 10^{-10} \frac{\Delta \alpha_{em}(z=0.13)}{\alpha_{em}} \text{ yr}^{-1}$$

assumes that both effects are dominated by change of fine structure constant

Observation : $|\Delta\alpha/\alpha| < 2 * 10^{-15} / \text{yr}$

Munich group

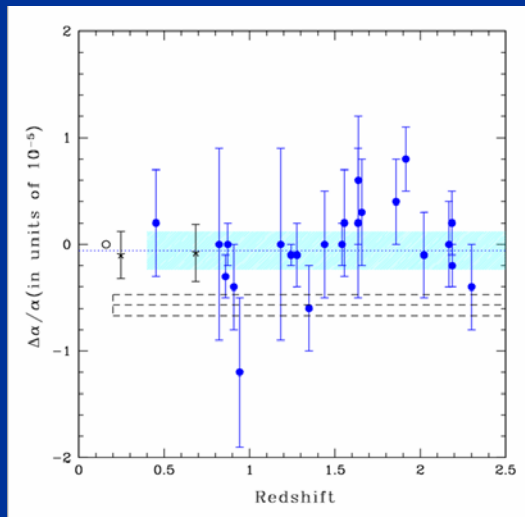
Variation of fine structure constant as function of redshift



Three independent data sets from
Keck/HIRES

$$\Delta\alpha/\alpha = -0.54 (12) 10^{-5}$$

Murphy, Webb, Flammbaum, June
2003



VLT

$$\Delta\alpha/\alpha = -0.06 (6) 10^{-5}$$

Srianand, Chand, Petitjean, Aracil,
Feb. 2004

$z \approx 2$

Cosmon and time variation of couplings : fixed points

- small coupling of cosmon to matter due to fixed points behavior

α_x : unified gauge coupling at unification scale $M_x \sim \chi$

running $\alpha_x(\chi)$:

$$\frac{\partial \alpha_x}{\partial \ln \chi} = b_2 \alpha_x - b_4 \alpha_x^2$$

$b_2, b_4 > 0$

UV-fixed point reached for $\chi \gg m$

$$\alpha_{x*} = \frac{b_2}{b_4} \approx \frac{1}{40}$$

close to fixed point :
small time evolution of couplings
coupling to matter weaker
than gravitational strength

Field equations

$$R_{AB} - \frac{1}{2}Rg_{AB} = M_6^{-4}(T_{AB}^{(F)} + T_{AB}^{(M)} - \lambda_6 g_{AB}),$$

$$\partial_A(\sqrt{-g}F^{AB}) = 0.$$

Energy momentum tensor

$$T_{AB}^{(F)} = F_{AC}F_B{}^C - \frac{1}{4}F_{CD}F^{CD}g_{AB}$$

$$R_{AB} - \frac{1}{2}Rg_{AB} = M_6^{-4}(T_{AB}^{(F)} + T_{AB}^{(M)} - \lambda_6 g_{AB}),$$

$$\partial_A(\sqrt{-g}F^{AB}) = 0.$$

Free integration constants

$M, B, \Phi(t=0), (d\Phi/dt)(t=0)$: continuous

m : discrete

Conical singularities

deficit angle

$$\Delta = 2\pi(1 - B)$$

singularities can be included with
energy momentum tensor on brane

$$(T^{(B)})^\nu{}_\mu = \frac{B - 1}{Br_0^2 e^{\phi/\bar{M}}} M_6^4 \left(\frac{\delta(\rho)}{\rho} + \frac{\delta(\rho - \pi)}{\pi - \rho} \right) \delta^\nu{}_\mu$$

bulk point of view : describe everything in terms of bulk
geometry (no modes on brane without tail in bulk)

Dimensional reduction

$$L^{(4)} = -\frac{\bar{M}^2}{2}R + \frac{Z_1(\phi)}{4}F_{\mu\nu}^{(1)}F^{\mu\nu(1)}$$

$$+ \frac{Z_2(\phi)}{4}F_{\mu\nu}^{(2)}F^{\mu\nu(2)}$$

$$+ i \sum_j \bar{\psi}_j \gamma^\mu (\partial_\mu - iQ_j^{(1)}\bar{e}_1 A_\mu^{(1)} - iQ_j^{(2)}\bar{e}_2 A_\mu^{(2)}) \psi_j$$

$$+ \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi)$$

Time dependent gauge coupling

$$e_{1(2)} = \frac{\bar{e}_{1(2)}}{\sqrt{Z_{1(2)}}}$$

$$Z_1 = e^{\phi/\bar{M}}, \quad Z_2 = e^{2\phi/\bar{M}}$$

Realistic model : Crossover Quintessence

$$\partial\delta/\partial \ln \chi = E\delta^2$$

(like QCD gauge coupling)

critical χ where δ grows large
critical φ where k grows large

$$k^2(\varphi) = \delta(\chi)/4$$

$$k^2(\varphi) = "1/(2E(\varphi_c - \varphi)/M)"$$

if $\varphi_c \approx 276/M$ (tuning !)

Relative increase of dark energy in present
cosmological epoch