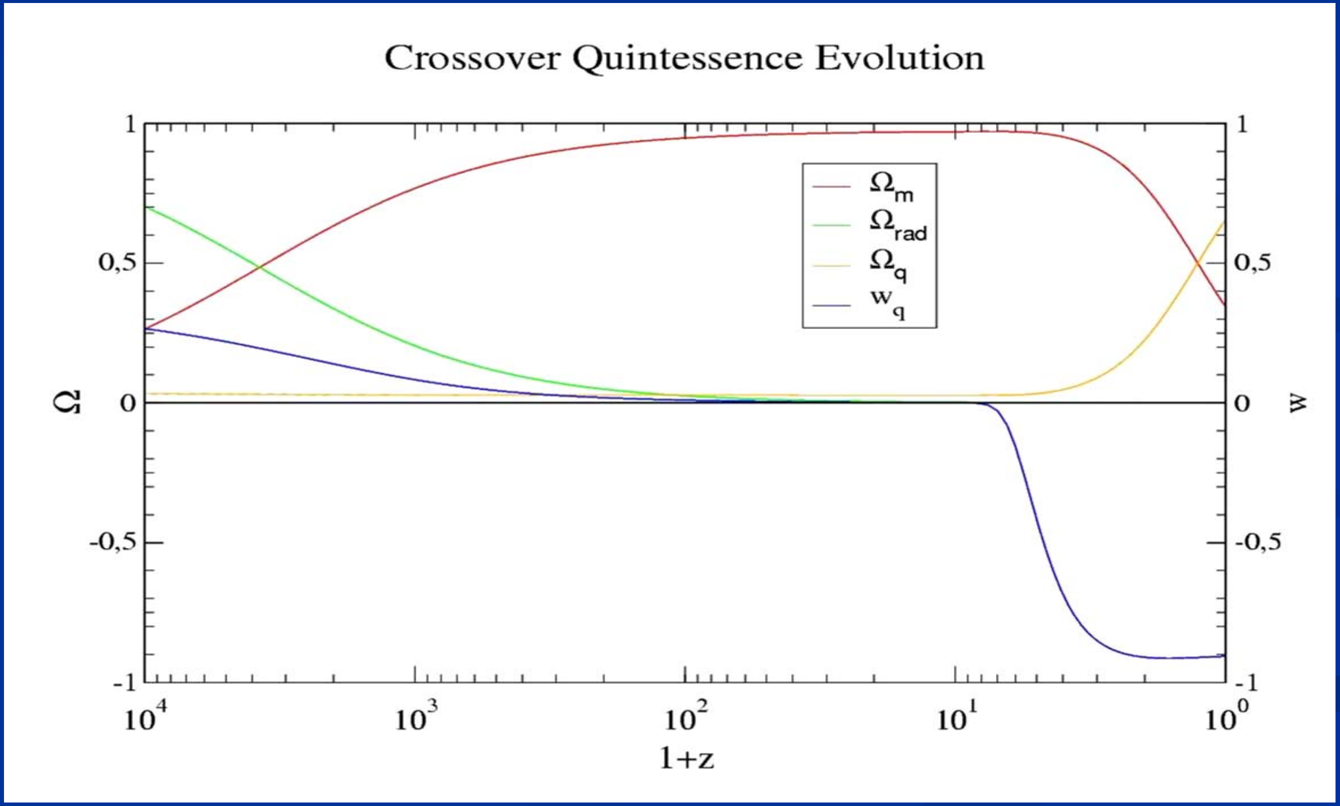
Quintessence – Phenomenology

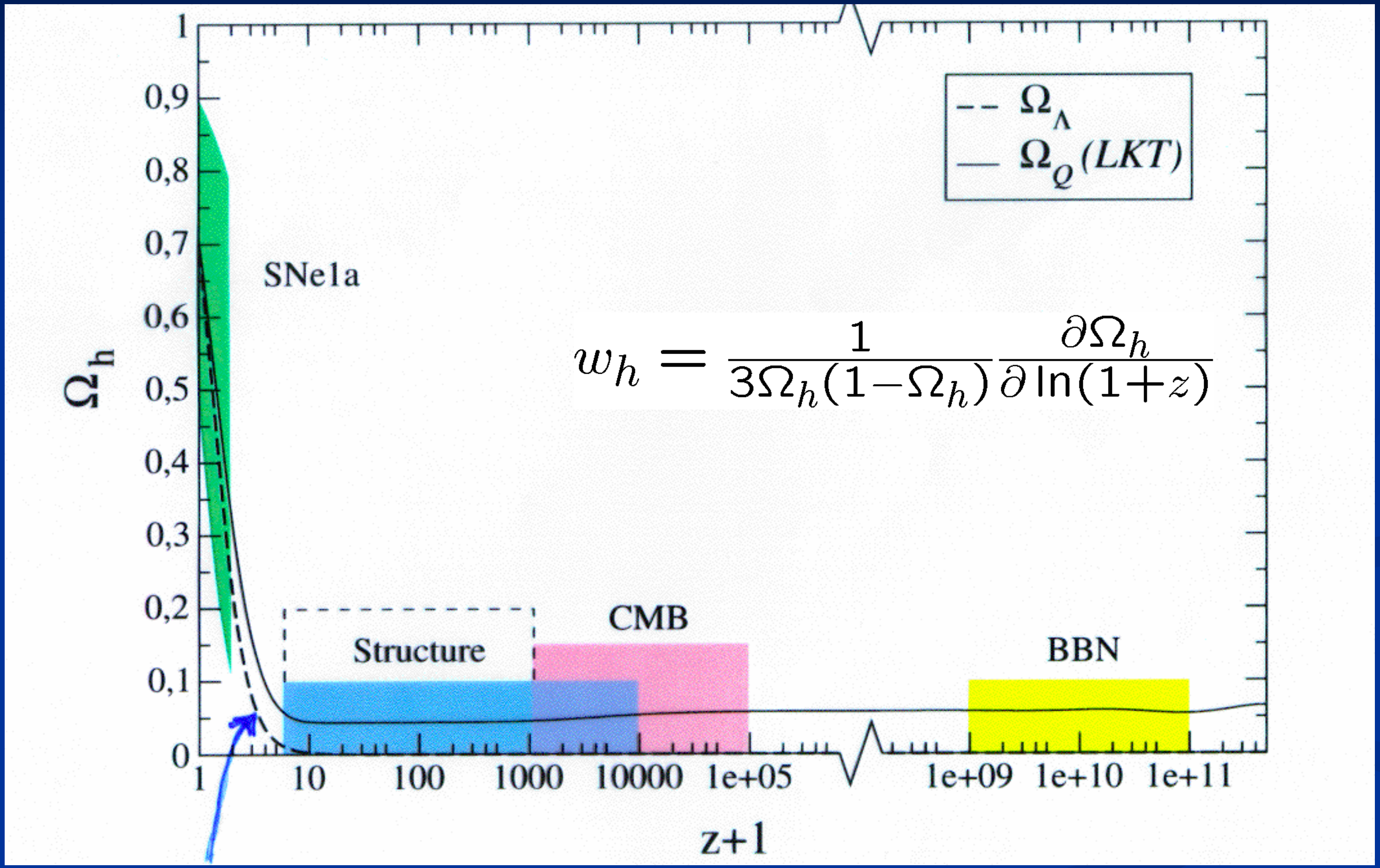
How can quintessence be distinguished from a cosmological constant ?

Early dark energy

…predicted in models where
dark energy is naturally of the same order as matter



Time dependence of dark energy



$$w_h = \frac{1}{3\Omega_h(1-\Omega_h)} \frac{\partial \Omega_h}{\partial \ln(1+z)}$$

cosmological constant : $\Omega_h \sim t^2 \sim (1+z)^{-3}$

M.Doran,…

Early dark energy

A few percent in the early Universe

Not possible for a cosmological constant

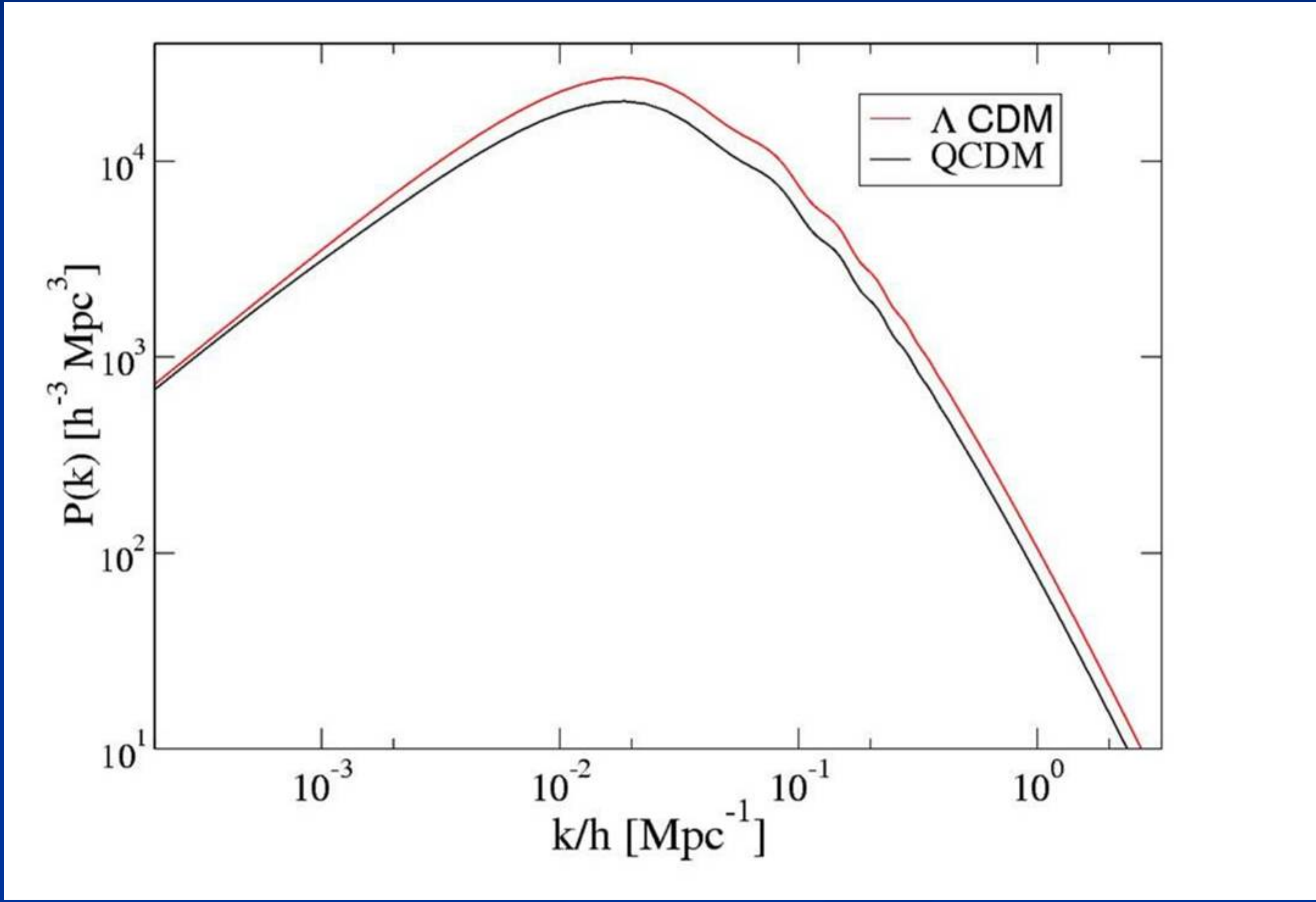
Structure formation

Structures in the Universe grow from tiny fluctuations in density distribution

stars , galaxies, clusters

One primordial fluctuation spectrum describes all correlation functions !

Early quintessence slows down the growth of structure



Growth of density fluctuations

- Matter dominated universe with constant Ω_h :

$$\Delta\rho \sim a^{1-\frac{\epsilon}{2}} \; , \; \epsilon = \frac{5}{2}\left(1 - \sqrt{1 - \frac{24}{25}\Omega_h}\right)$$

P.Ferreira,M.Joyce

- Dark energy slows down structure formation

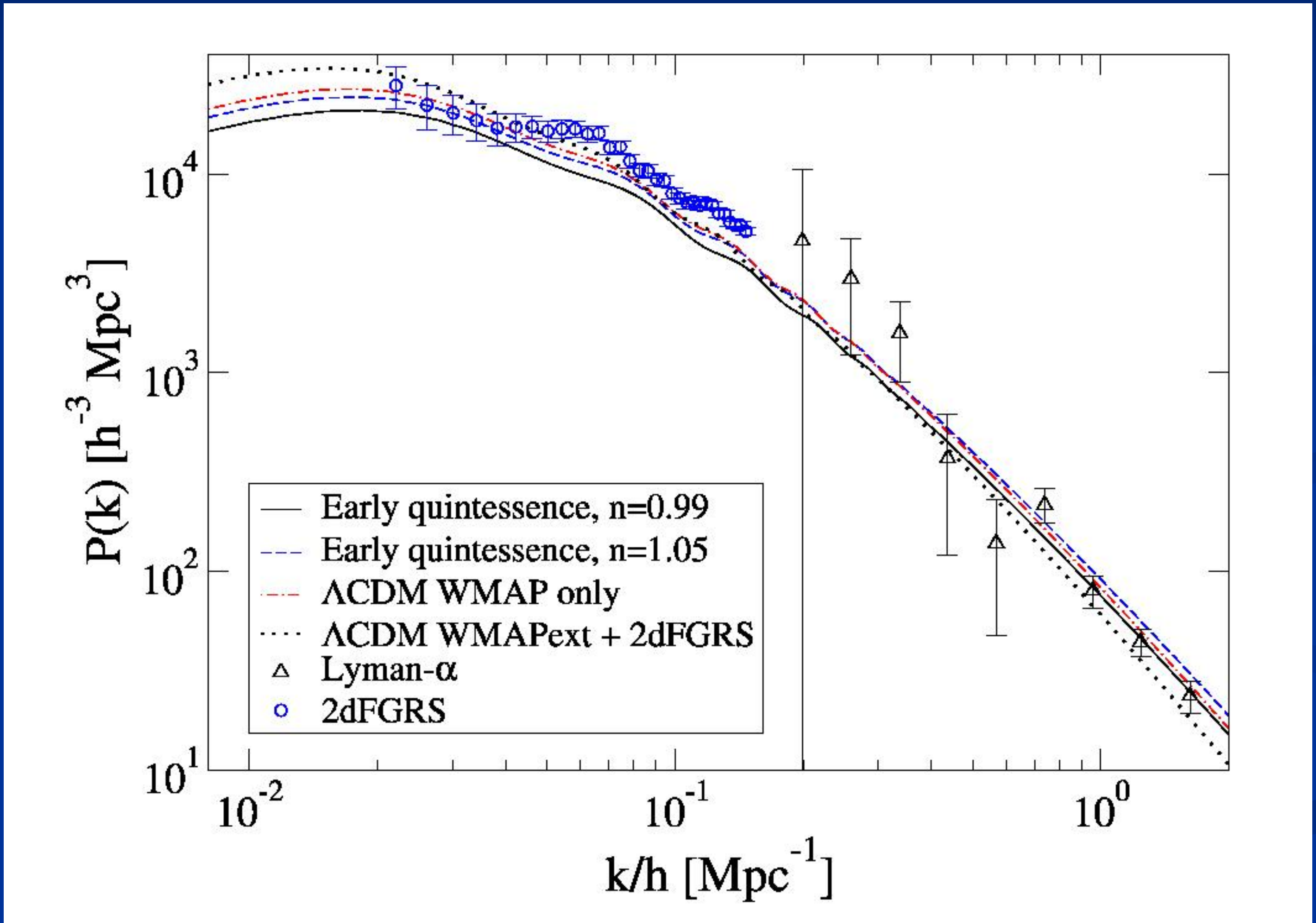
  $\Longrightarrow$ $\Omega_h < 10\%$ during structure formation

- Substantial increase of $\Omega_h(t)$ since structure has formed!

  $\Longrightarrow$ negative w_h

- Question "why now" is back (in mild form)

Fluctuation spectrum



Caldwell,Doran,Müller,Schäfer,…

Models and Parameters

	A	B	C	D
$\Omega_q^{(sf)}$	0.03	0.05	0	0
$\Omega_q^{(ls)}$	0.03	0.05	0	0
$w_q^{(0)}$	-0.91	-0.95	-1	-1
n_s	0.99	1.05	0.97	0.93
h	0.65	0.70	0.68	0.71
$\Omega_m h^2$	0.15	0.16	0.15	0.135
$\Omega_b h^2$	0.024	0.025	0.023	0.0224
τ	0.17	0.26	0.1	0.17
σ_8	0.81	0.87	0.87	0.85
χ^2_{eff}/ν	$\frac{1432}{1342}$	$\frac{1432}{1342}$	$\frac{1430}{1342}$	$\frac{1432}{1342}$

↑ running $n_s(k)$

normalization of matter fluctuations

rms density fluctuation averaged over $8h^{-1}$ Mpc spheres

compare quintessence with cosmological constant

$$\frac{\sigma_8(Q)}{\sigma_8(\Lambda)} \approx (a_{\rm eq})^{3\bar{\Omega}_{\rm d}^{\rm sf}/5}\left(1-\Omega_\Lambda^0\right)^{-\left(1+\bar{w}^{-1}\right)/5}\sqrt{\frac{\tau_0(Q)}{\tau_0(\Lambda)}}$$

Doran, Schwindt,…

Early quintessence and growth of matter fluctuations

$$\frac{\sigma_8(Q)}{\sigma_8(\Lambda)} \approx (a_{eq})^{3\bar{\Omega}_d^{sf}/5} \left(1 - \Omega_\Lambda^0\right)^{-\left(1+\bar{w}^{-1}\right)/5} \sqrt{\frac{\tau_0(Q)}{\tau_0(\Lambda)}}$$

$$a_{eq} = \frac{\Omega_r^0}{\Omega_m^0} = \frac{4.31 \times 10^{-5}}{h^2(1-\Omega_d^0)}$$

early quintessence

$$\Delta\rho \sim a^{1-\frac{\epsilon}{2}} \ , \ \epsilon = \frac{5}{2}\left(1 - \sqrt{1 - \frac{24}{25}\Omega_h}\right)$$

for small Ω_h :

$\varepsilon/2 = 3/5$

$$\frac{\sigma_8(Q)}{\sigma_8(\Lambda)} \approx (a_{\rm eq})^{3\,\bar{\Omega}_{\rm d}^{\rm sf}/5} \left(1-\Omega_\Lambda^0\right)^{-\left(1+\bar{w}^{-1}\right)/5} \sqrt{\frac{\tau_0(Q)}{\tau_0(\Lambda)}}$$

for varying early dark energy : weighted average of Ω

$$\bar{\Omega}_{\rm d}^{\rm sf} \equiv [\ln a_{\rm tr} - \ln\ a_{\rm eq}]^{-1} \int_{\ln a_{\rm eq}}^{\ln a_{\rm tr}} \Omega_{\rm d}(a)\ {\rm d}\ln a$$

influence of late evolution of quintessence through conformal time τ_0 and averaged equation of state

$$\frac{1}{\bar{w}} = \frac{\int_{\ln a_{\rm tr}}^{0} \Omega_{\rm d}(a)/w(a)\ d\ln a}{\int_{\ln a_{\rm tr}}^{0} \Omega_{\rm d}(a)\ d\ln a}$$

a_{tr} : transition from slow early evolution of Ω_h to more rapid late evolution

at most a few percent dark energy in the early universe !

Jeans analysis for fluctuations inside the horizon

$k_{phys} \gg H \quad ; \quad k_{phys} = \frac{k}{a}$

pressure effects $(\vec{\nabla} p)$

v_s : velocity of sound

$$v_s^2 = \partial p / \partial \rho$$

$$p = \bar{p} + \delta p = \bar{p} + v_s^2 \, \delta\rho$$

$$\vec{\nabla} p = v_s^2 \, \vec{\nabla} \rho = v_s^2 \, \vec{\nabla} \delta\rho$$

photons + baryons + electrons (in equilibrium)

$$v_s^2 = \frac{1}{3} \frac{\rho_\gamma}{\rho_\gamma + \rho_B + \rho_e}$$

Small fluctuations in Fourier space

$\vec{k}$: comoving wave number

$\vec{k}_{phys} = \vec{k} / a$

δ_k : density contrast

$\vec{v}_k$: comoving pecular velocity

ψ_k : Newtonian potential

$$\dot{\delta}_k + i\vec{k}\vec{v}_k = -i\int\frac{d^3k'}{(2\pi)^3}(\vec{k}\,\vec{v}_{k-k'})\,\delta_{k'}$$

$$\dot{\vec{v}}_k + 2H\vec{v}_k + i\frac{v_s^2}{a^2}\vec{k}\,\delta_k + \frac{i}{a^2}\vec{k}\,\psi_k = -i\int\frac{d^3k'}{(2\pi)^3}(\vec{k}'\,\vec{v}_{k-k'})\,\vec{v}_{k'}$$

$$-\vec{k}^2\psi_k = 4\pi G\bar{\rho}\,a^2\,\delta_k$$

background cosmological solution

$H(t)\ , \quad \bar{\rho}(t)$

$$H(t) = \tilde{h}\, t^{-1}$$

$$4\pi G \bar{\rho}(t) = \tilde{\rho}\, t^{-2}$$

matter domination $\tilde{h} = \frac{2}{3}$

radiation domination $\tilde{h} = \frac{1}{2}$

—

$\bar{\rho}$ dominant component : $(\Omega = 1)$

$$\frac{8\pi G}{3}\rho = H^2 = \frac{2}{3}\tilde{\rho}\, t^{-2} = \tilde{h}^2 t^{-2}$$

$$\tilde{\rho} = \frac{3}{2}\tilde{h}^2$$

matter domination $\tilde{\rho} = \frac{2}{3}$

radiation domination $\tilde{\rho} = \frac{3}{8}$

Linear analysis

$$\dot{\delta}_k = -i\vec{k}\,\vec{v}_k$$

$$\ddot{\delta}_k = -i\vec{k}\,\dot{\vec{v}}_k$$

$$= i\vec{k}\left\{\frac{2\tilde{h}}{t}\vec{v}_k + i\frac{v_s^2}{a^2}\vec{k}\,\delta_k + i\vec{k}\left(-\frac{1}{\vec{k}^2}\frac{\tilde{\rho}}{t^2}\delta_k\right)\right\}$$

$$= -\frac{2\tilde{h}}{t}\dot{\delta}_k - \frac{v_s^2}{a^2}\vec{k}^2\delta_k + \frac{\tilde{\rho}}{t^2}\delta_k$$

$$\boxed{\ddot{\delta}_k + \frac{2\tilde{h}}{t}\dot{\delta}_k + \left(v_s^2\frac{\vec{k}^2}{a^2} - \frac{\tilde{\rho}}{t^2}\right)\delta_k = 0}$$

damping — pressure — gravitational attraction

Jeans wave number k_J

separates pressure dominated from gravity dominated behavior

$$k_J^2 = \frac{\tilde{\rho}}{v_s^2}\frac{a^2}{t^2}$$

effect of early quintessence

$$\ddot{\delta}_k + \frac{2\tilde{h}}{t}\dot{\delta}_k + \left(v_s^2 \frac{\vec{k}^2}{a^2} - \frac{\tilde{\rho}}{t^2}\right)\delta_k = 0$$

damping   pressure   gravitational attraction

$$4\pi G \bar{\rho}(t) = \tilde{\rho}\, t^{-2}$$

presence of early dark energy decreases ϱ for given t
slower growth of perturbation

Jeans wave number

$$k_J^2 = \frac{\tilde{\rho}}{v_s^2} \frac{a^2}{t^2}$$

k_J depends on time

$$\frac{k_J^2}{a^2} = \frac{\tilde{\rho}}{\tilde{h}^2} v_s^{-2} H^2$$

$k \gg k_J$: oscillations

(pressure dominated)

$k \ll k_J$: modes can grow

(gravity dominated)

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radiation dominated epoch ($v_s^2 = \frac{1}{3}$)

$$\frac{k_J^2}{a^2} = \frac{9}{2} H^2$$

no growing modes !

condition for growing modes inside horizon :

$$H^2 \lessapprox \frac{k^2}{a^2} \lessapprox \frac{k_J^2}{a^2} = \frac{\tilde{\rho}}{\tilde{h}^2} v_s^{-2} H^2$$

$$v_s^2 \ll \frac{\tilde{h}^2}{\tilde{\rho}} \approx \frac{2}{3}$$

$$\boxed{v_s \ll 1}$$

not possible for radiation

possible for dark matter

(or baryons) where $v_s^2 \ll 1$

oscillations

$$k^2 \gg k_J^2$$

(CMB fluctuations inside horizon)

$$\ddot{\delta}_k + 2H\dot{\delta}_k + \frac{v_s^2 k^2}{a^2}\delta_k = 0$$

conformal time $\quad d\tau = dt/a$

$$\partial_t = \frac{1}{a}\partial_\tau \ , \quad \partial_t^2 = -\frac{H}{a}\partial_\tau + \frac{1}{a^2}\partial_\tau^2$$

$$\frac{1}{a^2}\left\{\partial_\tau^2 + aH\partial_\tau + v_s^2 k^2\right\}\delta_k = 0$$

$$\delta_k = A_k \, e^{i\omega_k \tau}$$

$$\omega_k = v_s k$$

check: $aH \ll \omega_k \ \hat{=} \ k \gg \frac{a}{v_s} H$

$$\hat{=} \ k \gg \frac{\tilde{h}}{\sqrt{\tilde{\rho}}} k_J$$

CMB - spectrum

at last scattering

* photons decouple rapidely at time of last scattering τ_{ls}

* fluctuation spectrum in CMB $\hat{=}$ fluctuation spectrum at τ_{ls}

„ snapshot"

$$P(k,\tau) = A_k \{ \sin^2(v_s k \tau + \varphi_k) + c_k \}$$

$P(k, \tau_{ls})$:

maxima at $k_m \approx \frac{m\pi}{v_s \tau_{ls}}$

k_m can be calculated precisely!

How do we see peaks in $P(k, \tau_{ls})$?

after last scattering:

photons travel freely over distance $\tau_0 - \tau_{ls} \approx \tau_0$

plane waves ~ $e^{ik\tau_0} \left(P(k, \tau_{ls}) \right)^{1/2}$

multipole analysis:

Legendre expansion of $e^{ik\tau_0}$ ⇒ Bessel fcts $j_\ell(k\tau_0)$

- are peaked at $\ell = k\tau_0$

Peaks for $\ell_m = k_m \tau_0$

$$\ell_m = \frac{m\pi}{v_s} \frac{\tau_0}{\tau_{ls}}$$

τ_0 test geometry of late universe!

⇒ $\Omega \approx 1$

Equation of state

p=T-V pressure

ϱ=T+V energy density

kinetic energy

$$T = \frac{1}{2}\dot{\phi}^2$$

Equation of state

$$w = \frac{p}{\rho} = \frac{T-V}{T+V}$$

Depends on specific evolution of the scalar field

Negative pressure

- w < 0 Ω_h increases (with decreasing z)

late universe with
small radiation component :

$$w_h = \frac{1}{3\Omega_h(1-\Omega_h)} \frac{\partial \Omega_h}{\partial \ln(1+z)}$$

- w < -1/3 expansion of the Universe is accelerating

- w = -1 cosmological constant

small early and large present dark energy

- fraction in dark energy has substantially increased since end of structure formation
- expansion of universe accelerates in present epoch

$$w_h = \frac{1}{3\Omega_h(1-\Omega_h)} \frac{\partial \Omega_h}{\partial \ln(1+z)}$$

exact relation between w_h and change in Ω_h

$$\frac{d\Omega_h}{dy} = \Omega_h (1-\Omega_h) \left\{ 3 w_h - \left(1 + \frac{e^{-y}}{a_{eq}}\right)^{-1} \right\}$$

↑ small after matter-radiation equality

$$y = -\ln a$$

$$\Omega_h = \frac{\rho_h}{\rho_{cr}} = \frac{\rho_h}{\rho_h + \rho_m + \rho_r}$$

$$\rho_r = \frac{a_{eq}}{a} \rho_m = \frac{a_{eq}}{a + a_{eq}} (\rho_m + \rho_r)$$

$$= \left(1 + \frac{a}{a_{eq}}\right)^{-1} (\rho_m + \rho_r)$$

$$= \left(1 + \frac{e^{-y}}{a_{eq}}\right)^{-1} (\rho_m + \rho_r)$$

$$y = -\ln a$$

$$\frac{\rho_m + \rho_r}{\rho_{cr}} = (1 - \Omega_h)$$

$$\frac{d\Omega_h}{dy} = \frac{d}{dy}\left(\frac{\rho_h}{\rho_{cr}}\right)$$

$$= \rho_{cr}^{-1}\,\partial_y \rho_h - \rho_h \rho_{cr}^{-2}\,\partial_y \rho_{cr}$$

$$\partial_y \rho_h = 3(1 + w_h)\rho_h$$

$$\rho_{cr}^{-1}\,\partial_y \rho_{cr} = 3 + 3 w_h \Omega_h + (1-\Omega_h)\left(1 + \frac{e^{-y}}{a_{eq}}\right)^{-1}$$

$$\frac{d\Omega_h}{dy} = 3\Omega_h + 3 w_h \Omega_h$$

$$- 3\Omega_h - 3 w_h \Omega_h^2 - \Omega_h(1-\Omega_h)\left(1 + \frac{e^{-y}}{a_{eq}}\right)^{-1}$$

$$= 3 w_h \Omega_h (1 - \Omega_h)$$

$$- \left(1 + \frac{e^{-y}}{a_{eq}}\right)^{-1} \Omega_h (1 - \Omega_h)$$

$$= \Omega_h (1-\Omega_h)\left(3 w_h - \left(1 + \frac{e^{-y}}{a_{eq}}\right)^{-1}\right)$$

$$\rho_{cr}^{-1} \partial_y \rho_{cr} = \rho_{cr}^{-1} \partial_y \left(\rho_h + \rho_m + \rho_r \right)$$

$$= \rho_{cr}^{-1} \left\{ 3(1+w_h)\rho_h + 3\rho_m + 4\rho_r \right\}$$

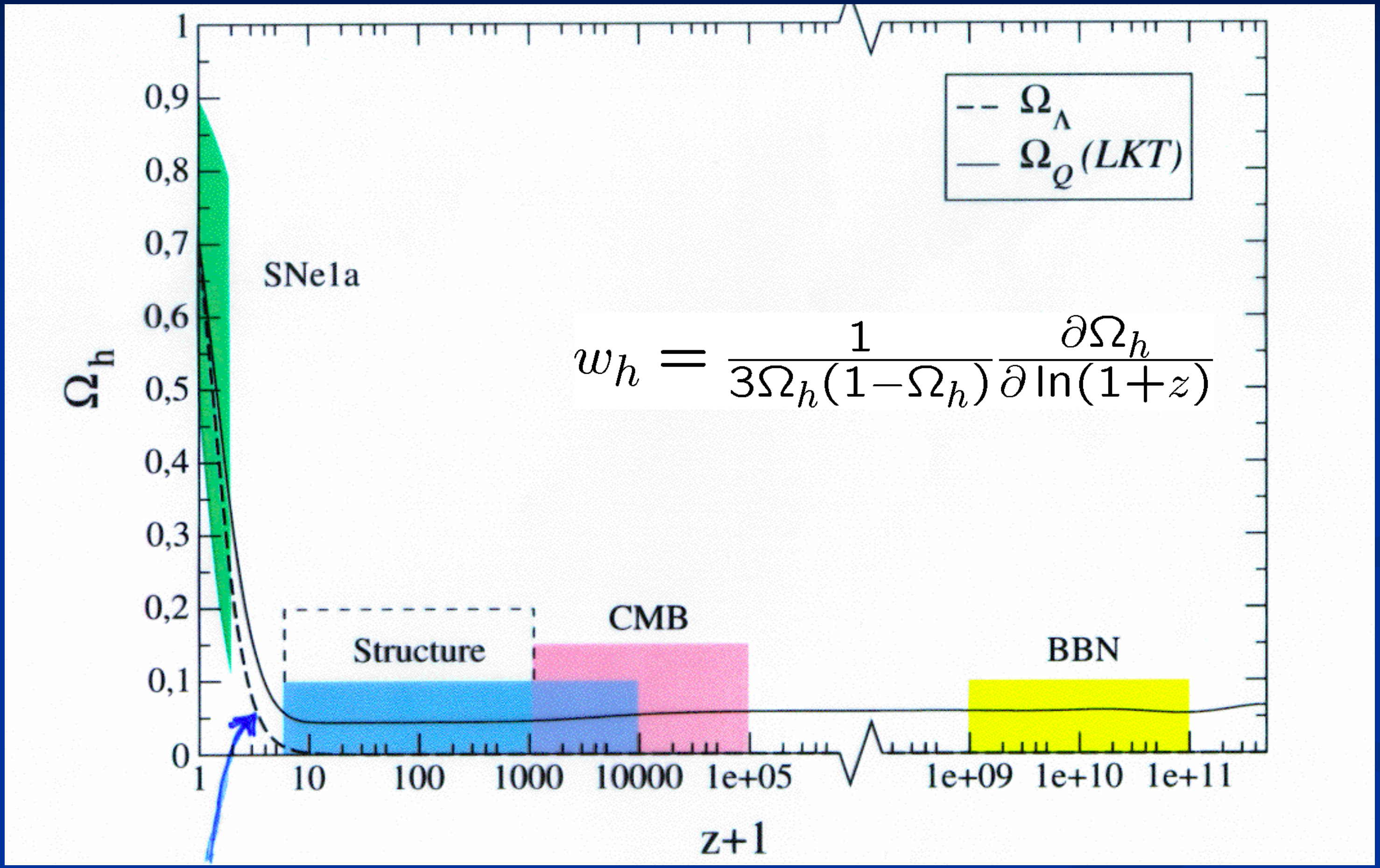
$$= 3(1+w_h)\Omega_h + \frac{3(\rho_m+\rho_r)}{\rho_{cr}}$$

$$+ \left(1 + \frac{e^{-y}}{a_{eq}}\right)^{-1} \frac{\rho_m + \rho_r}{\rho_{cr}}$$

$$= 3(1+w_h)\Omega_h + \left(3 + \left(1 + \frac{e^{-y}}{a_{eq}}\right)^{-1}\right)(1-\Omega_h)$$

$$= 3 + 3w_h\Omega_h + (1-\Omega_h)\left(1 + \frac{e^{-y}}{a_{eq}}\right)^{-1}$$

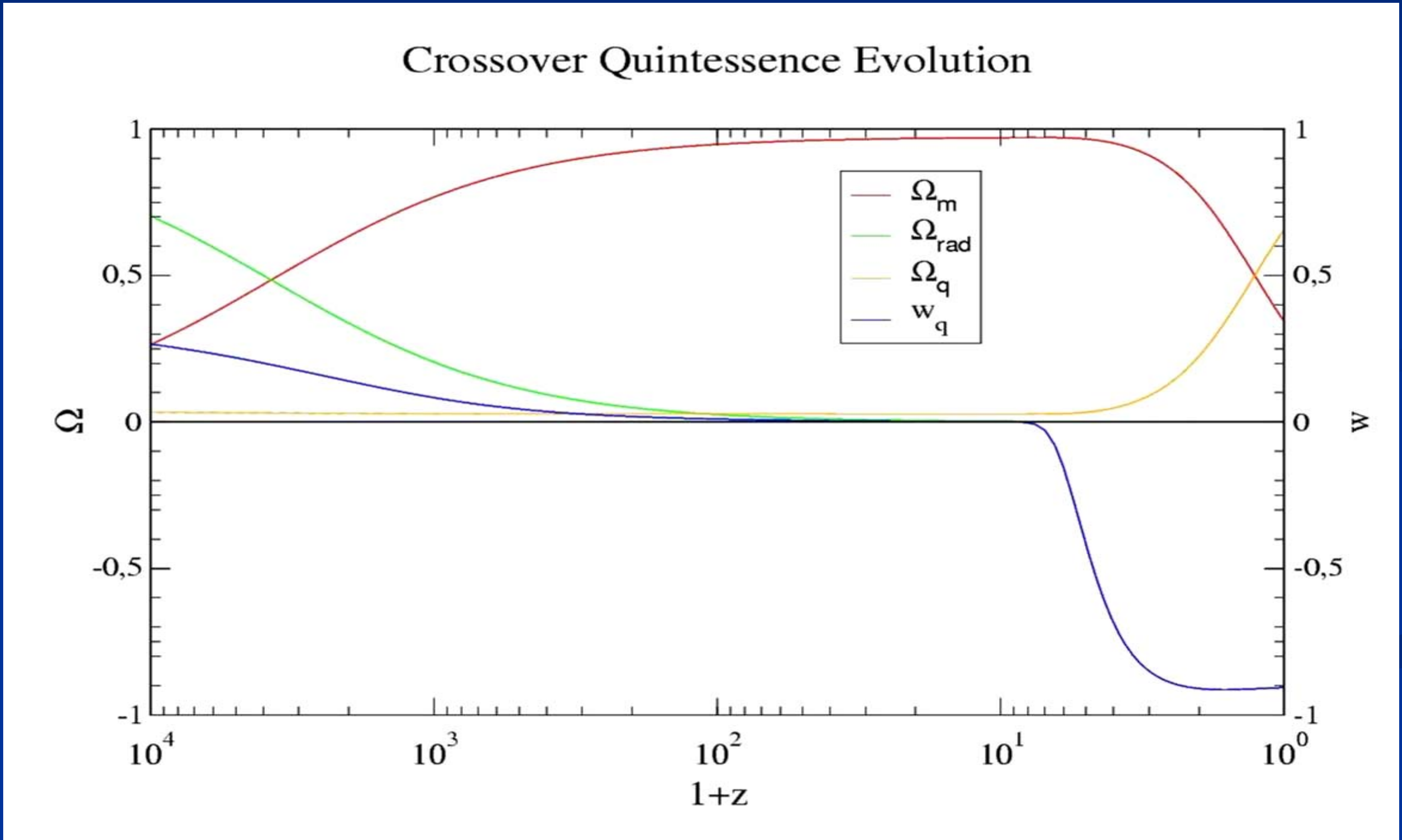
Time dependence of dark energy



cosmological constant : $\Omega_h \sim t^2 \sim (1+z)^{-3}$

M.Doran,…

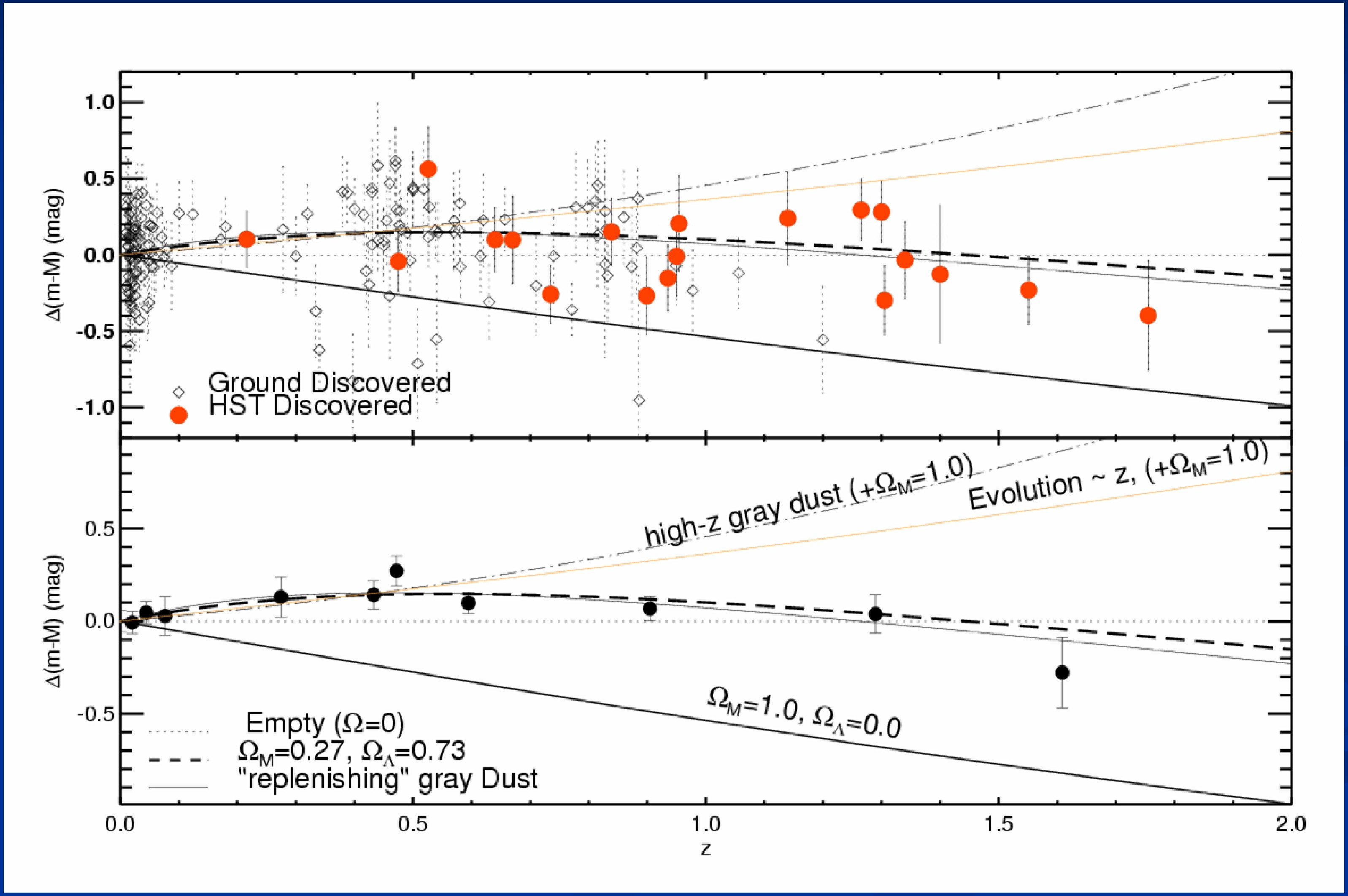
Quintessence becomes important "today"



w_h close to -1

inferred from supernovae and WMAP

Supernova cosmology



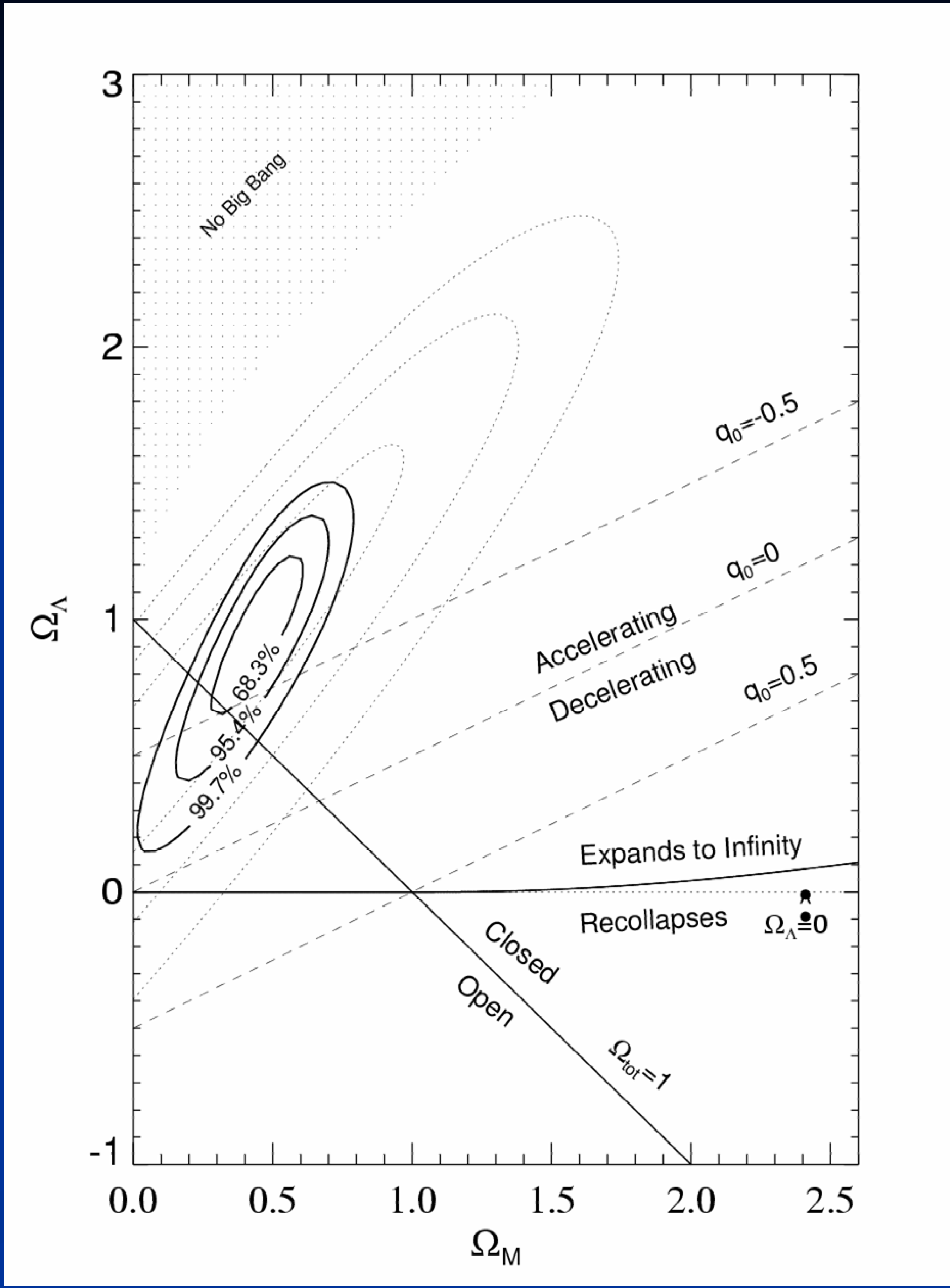
Riess et al. 2004

Dark energy and SN

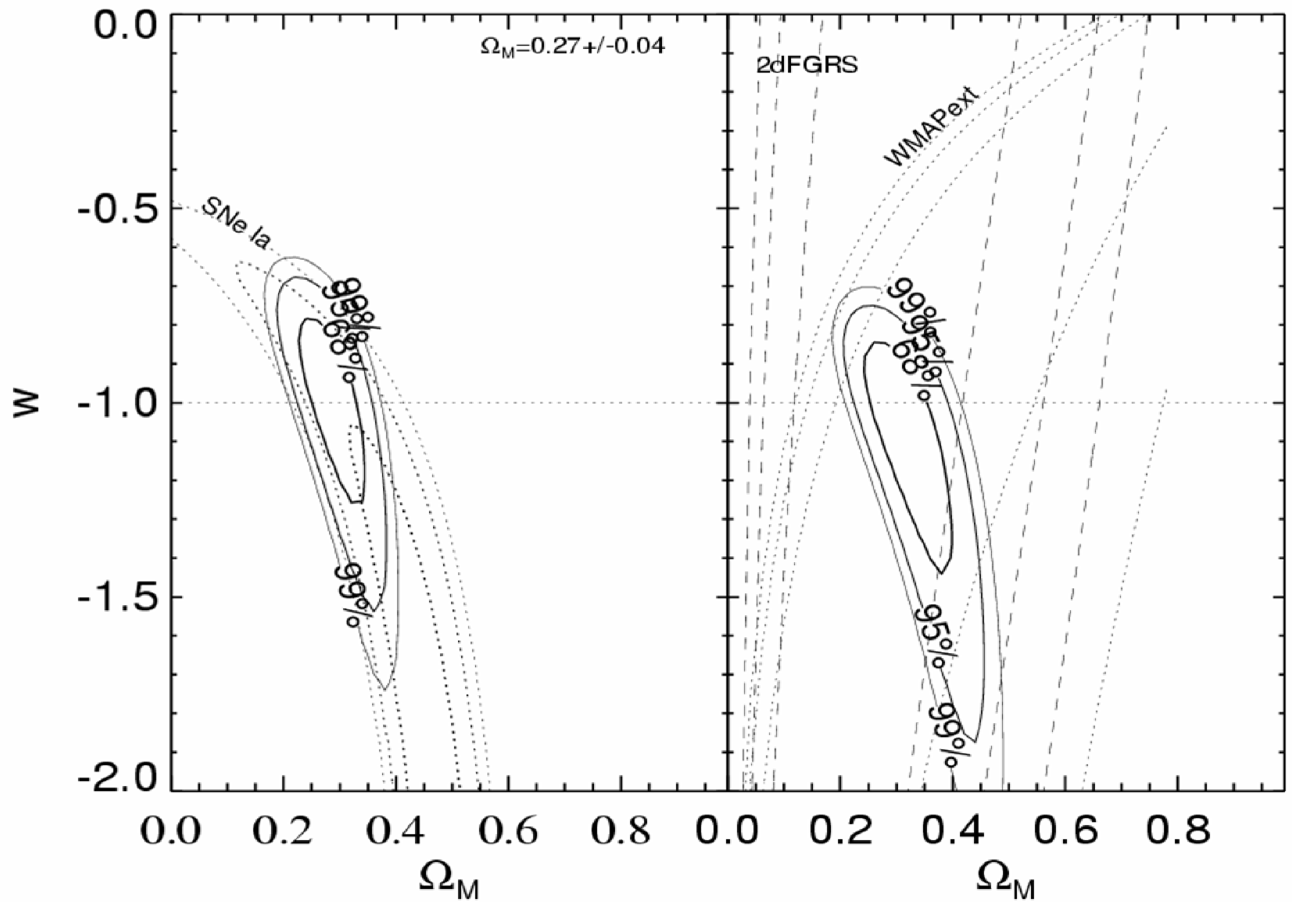
$\Omega_M = 0.29$

$+0.05 - 0.03$

(SN alone, for $\Omega_{tot}=1$)



SN and equation of state



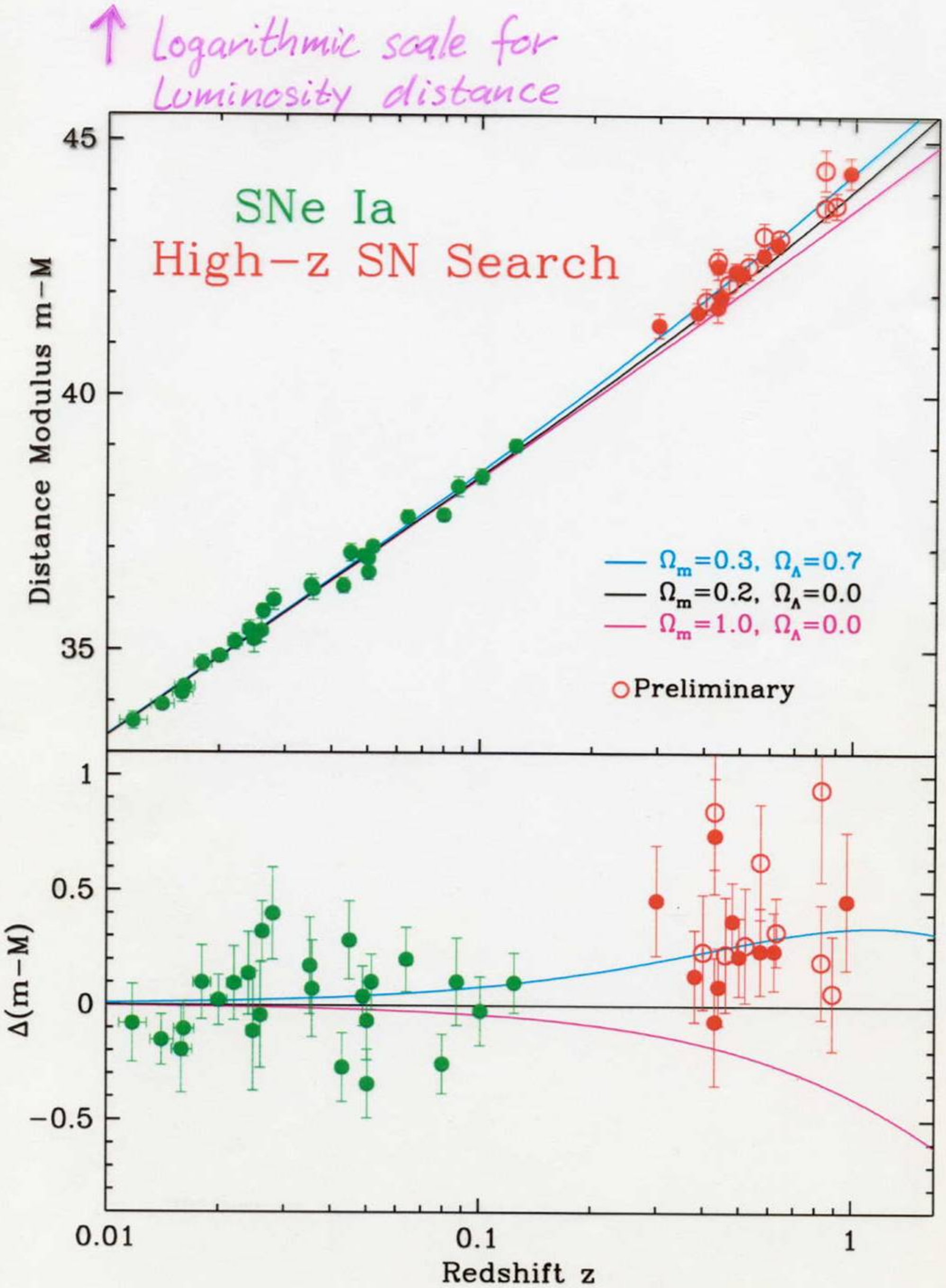
Riess et al. 2004

Hubble diagram

redshift vs. distance

↑

Luminosity distance


↑ Logarithmic scale for
Luminosity distance
SNe Ia
High-z SN Search
Distance Modulus m−M
45
40
35
— Ω_m=0.3, Ω_Λ=0.7
— Ω_m=0.2, Ω_Λ=0.0
— Ω_m=1.0, Ω_Λ=0.0
○ Preliminary
Δ(m−M)
1
0.5
0
−0.5
0.01
0.1
1
Redshift z

Hubble diagram

Luminosity distance d_L

Assume Luminosity $\mathcal{L}$ of object is known.

$$d_L^2 := \frac{\mathcal{L}}{4\pi \mathcal{F}}$$

$\mathcal{F}$: flux $\equiv \frac{\text{energy}}{\text{time} \cdot \text{area}}$

as measured in detector

motivation for definition :

flat space : $\mathcal{F} = \frac{\mathcal{L}}{4\pi d_L^2}$, $d_L = \Delta r$

astronomer's units : magnitudes

$$d_L = 10^{1 + \frac{m-M}{5}} \text{ pc}$$

pc = 3.2615 light years

Cosmological relation between d_L and Δr

earth : $r = 0$

light source: $r = r_1$,

$$\mathcal{F} = \frac{\mathcal{L}}{4\pi a^2(t_0) r_1^2 (1+z)^2}$$

$(1+z)^{-1}$: red shift of energy of each individual photon

$(1+z)^{-1}$: time dilatation - photons arrive less frequently

$4\pi a^2(t_0) r_1^2$: total surface of sphere around light source at t_0

$$\mathcal{F} \sim \frac{\text{energy}}{\Delta A \; \Delta t}$$

$$d_L = a(t_0)\, r_1\, (1+z)$$

$$= (1+z) \int_{t_1}^{t_0} dt' \, \frac{a(t_0)}{a(t')}$$

red shift as time scale : $z(t)$

$$z(t) = \frac{a(t_0)}{a(t)} - 1$$

$$\frac{dz}{dt} = - \frac{a(t_0)}{a^2(t)} \dot{a}(t)$$

$$= - \frac{a(t_0)}{a(t)} H(t)$$

$$\boxed{H = \frac{\dot{a}}{a}}$$

$$d_L = -(1+z) \int_z^0 \frac{dz'}{H(z')}$$

$$\boxed{d_L = (1+z) \int_0^z \frac{dz'}{H(z')}}$$

exact for all cosmologies

(isotropic and homogenous)

expansion for small z

$$H^{-1}(z) = H_0^{-1} + \frac{\partial H^{-1}}{\partial z}\Big|_0 z + \ldots$$

$$d_L = (1+z)\left(H_0^{-1} z + \frac{1}{2}\frac{\partial H^{-1}}{\partial z}\Big|_0 z^2 + \ldots\right)$$

$$= H_0^{-1} z \left(1 + z + \frac{1}{2} H_0 \frac{\partial H^{-1}}{\partial z}\Big|_0 z + \ldots\right)$$

$$H_0 d_L = z + \frac{1}{2}(1 - q_0) z^2 + \ldots$$

$$q_0 = \frac{\partial \ln H}{\partial z}\Big|_0 - 1$$

Lowest order Hubble diagram:

$$d_L = H_0^{-1} z$$ „Hubble's Law"

„distance vs. redshift plot"

determines H_0

present observation

$$H_0 = \frac{100\ \mathrm{km}}{\mathrm{s\ Mpc}} \cdot h$$

$$h \approx 0.65$$

$$H_0^{-1} = 9.78 \cdot 10^{9}\ \mathrm{yr} \cdot h^{-1}$$

$$H_0 = 2.13 \cdot 10^{-33}\ \mathrm{eV} \cdot h$$

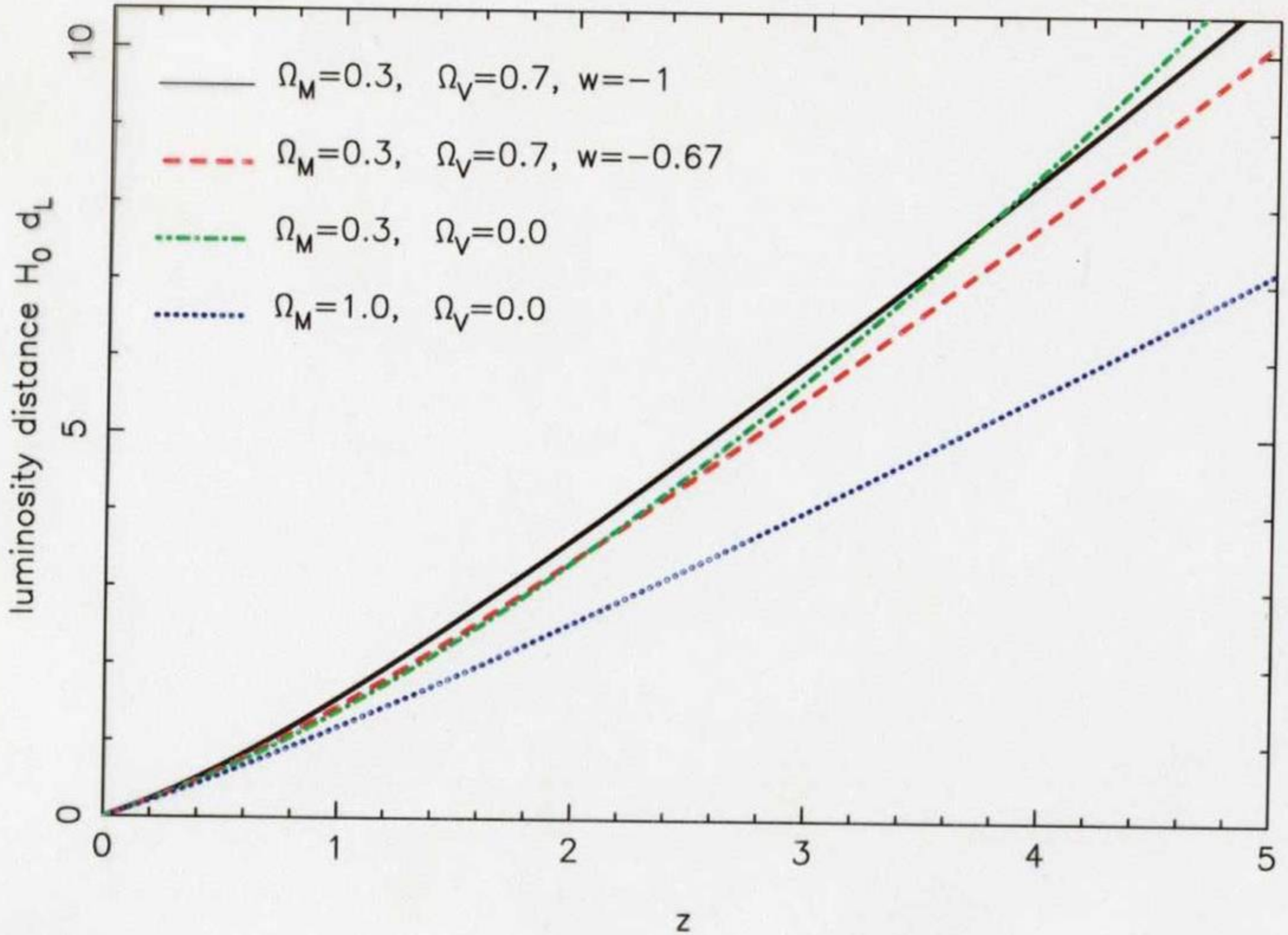
in practice :

$\mathcal{F}$ is measured , not d_L !

$$\mathcal{F}(z) = (4\pi)^{-1} \mathcal{L}(z) (1+z)^{-2} \cdot \left[\int_0^z dz'\, H^{-1}(z') \right]^{-2}$$

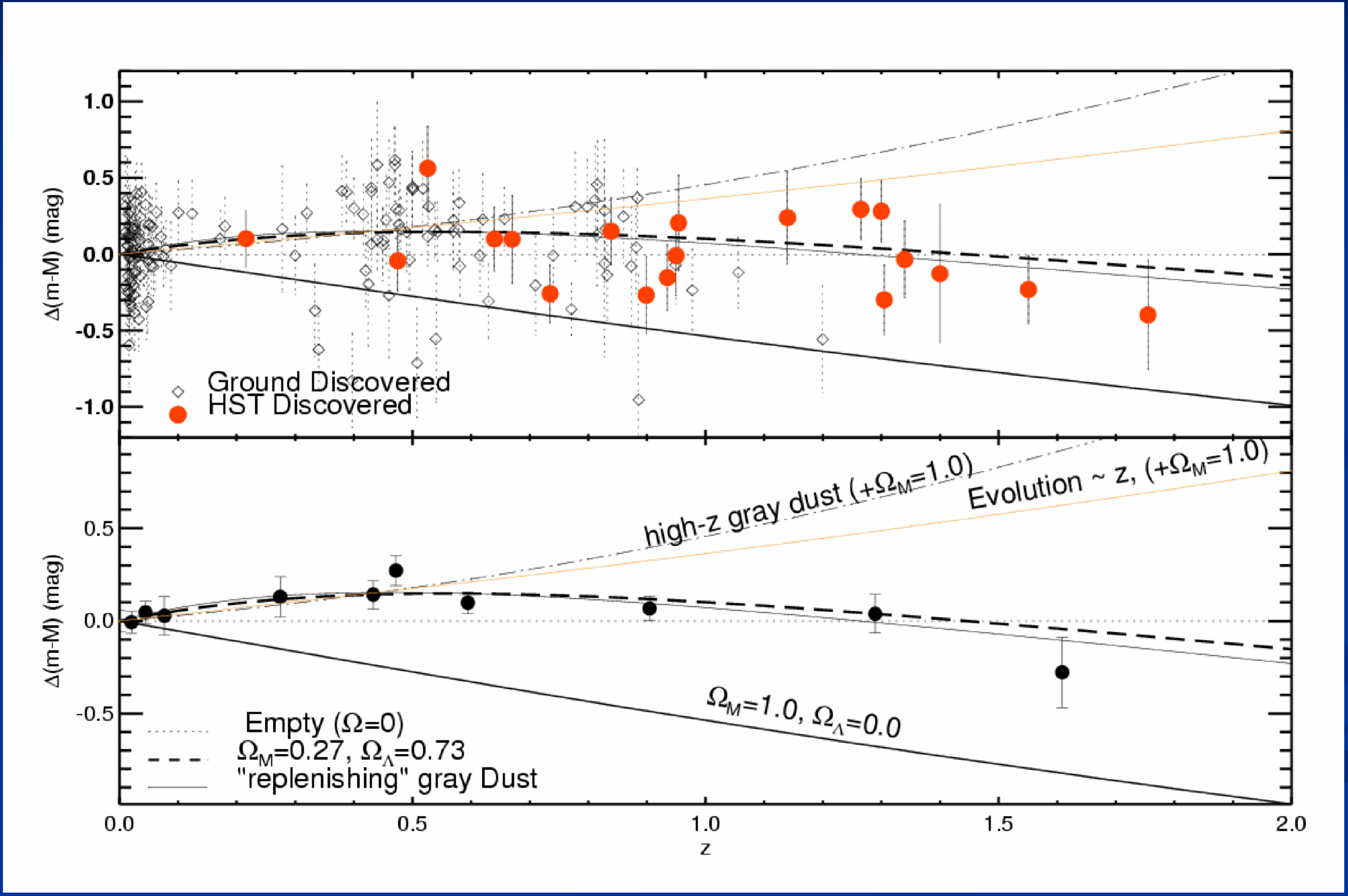
to be extracted

$\mathcal{L}(z)$?



Ω_M=0.3, Ω_V=0.7, w=−1
Ω_M=0.3, Ω_V=0.7, w=−0.67
Ω_M=0.3, Ω_V=0.0
Ω_M=1.0, Ω_V=0.0
luminosity distance H_0 d_L
z
0
5
10
0
1
2
3
4
5

Supernova cosmology



Riess et al. 2004

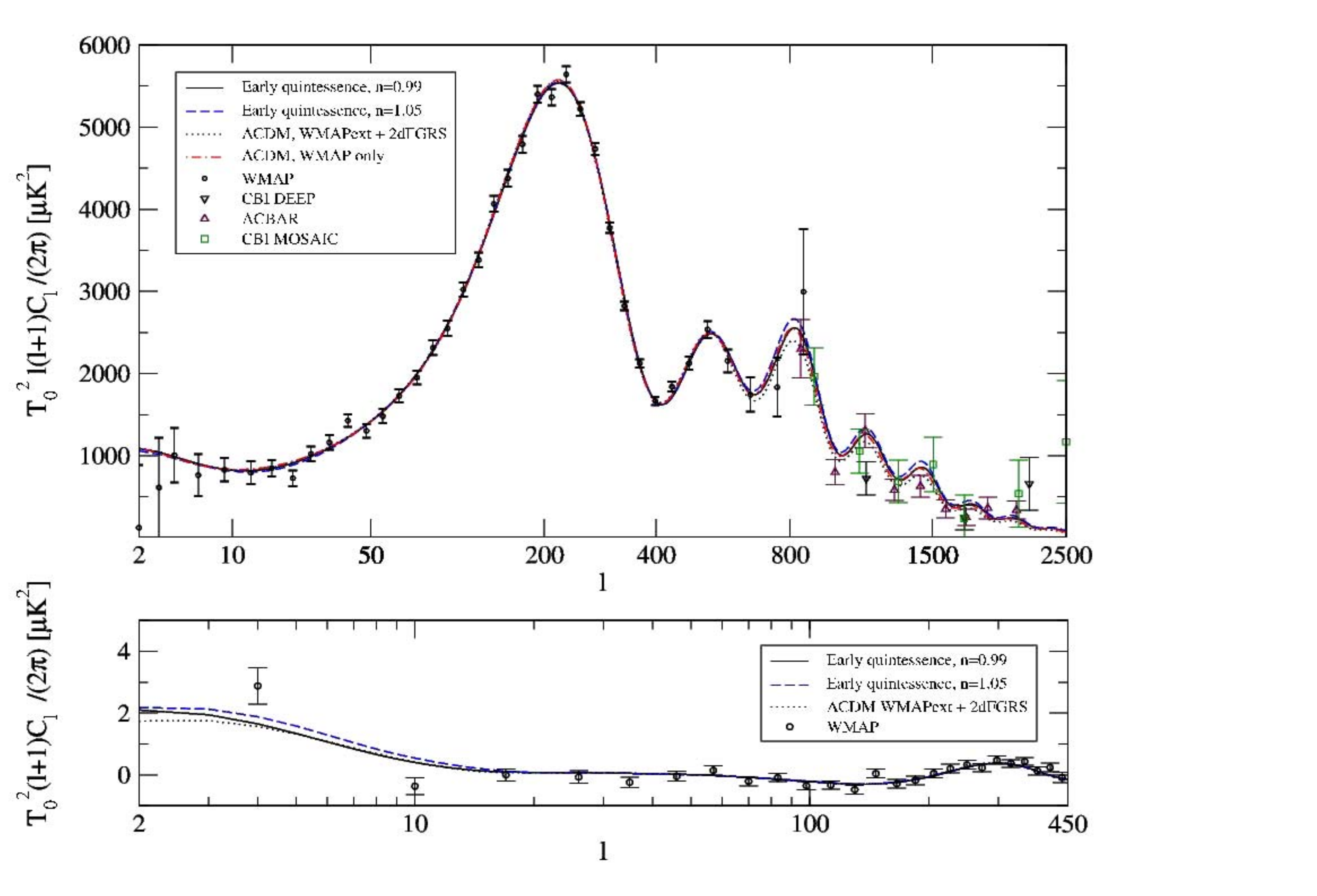
supernovae :
negative equation of state and
recent increase
in fraction of dark energy
are consistent !

quintessence and CMB anisotropies

influence by

- early dark energy
- present equation of state

Anisotropy of cosmic background radiation



Caldwell,Doran,Müller,Schäfer,…

separation of peaks depends on dark energy at last scattering

$$l_m \equiv l_A \left(m - \varphi_m\right)$$

$$l_A = \pi \frac{\tau_0 - \tau_{\rm ls}}{\bar{c}_s \tau_{\rm ls}}$$

$$l_A = \pi \bar{c}_s^{-1} \left[\frac{F(\Omega_d^0, \overline{w}_0)}{(1-\overline{\Omega}_d^{\rm ls})^{1/2}} \left\{ \left(a_{\rm ls} + \frac{\Omega_{\rm r}^0}{1-\Omega_d^0}\right)^{1/2} - \left(\frac{\Omega_{\rm r}^0}{1-\Omega_d^0}\right)^{1/2} \right\}^{-1} - 1 \right]$$

and on conformal time

$$\tau_0 = 2H_0^{-1}(1-\Omega_d^0)^{-1/2} F(\Omega_d^0, \bar{w}_0)$$

involves the integral
(with weighted w)

$$F(\Omega_d^0, \overline{w}_0) = \frac{1}{2} \int_0^1 {\rm d}a \left(a + \frac{\Omega_d^0}{1-\Omega_d^0} a^{(1-3\overline{w}_0)} + \frac{\Omega_{\rm r}^0 (1-a)}{1-\Omega_d^0}\right)^{-1/2}$$

$$\overline{w}_0 = \int_0^{\tau_0} \Omega_d(\tau) w_d(\tau) {\rm d}\tau \times \left(\int_0^{\tau_0} \Omega_d(\tau) {\rm d}\tau\right)^{-1}$$

Peak location in quintessence models for fixed cosmological parameters

M. Doran, M. Lilley, J. Schwindt …

$\overline{\Omega}^{\phi}_{ls}$	$\overline{w}_0$	l_1	l_2	l_2/l_1	$\Delta l^{estim.}$	$\Delta l^{num.}$	σ_8
Leaping kinetic term (A), $\Omega^{\phi}_0 = 0.6$							
8.4×10^{-3}	-0.76	215	518	2.41	292	291	0.864
0.03	-0.69	214	520	2.43	294	293	0.782
0.13	-0.45	211	523	2.48	299	300	0.471
0.22	-0.32	207	524	2.53	302	307	0.286
Inverse power law potential (B), $\Omega^{\phi}_0 = 0.6$							
8.4×10^{-8}	-0.37	199	480	2.41	271	269	0.610
9.9×10^{-2}	-0.13	178	443	2.49	252	252	0.177
0.22	-8.1×10^{-2}	172	444	2.58	257	257	0.089
Pure exponential potential, $\Omega^{\phi}_0 = 0.6$							
0.70	7×10^{-3}	190	573	3.02	368	377	0.011
Pure exponential potential, $\Omega^{\phi}_0 = 0.2$							
0.22	4.7×10^{-3}	194	490	2.53	282	281	0.375
Cosmological constant (C), $\Omega^{\phi}_0 = 0.6$							
0	-1	219	527	2.41	296	295	0.965
Cold Dark Matter - no dark energy, $\Omega^{\phi}_0 = 0$							
0	–	205	496	2.42	269	268	1.493

phenomenological parameterization of quintessence

…based on parameterization of Ω

natural "time" variable

$$y = \ln(1+z) = -\ln a$$

use relation

$$\frac{d\Omega_h}{dy} = 3\Omega_h(1-\Omega_h)w_h$$

(matter domination)

define

$$R(y) = \ln\left(\frac{\Omega_h(y)}{1-\Omega_h(y)}\right)$$

$$\frac{\partial R(y)}{\partial y} = 3w_h(y)$$

three parameter family of models

$$R(y) = R_0 + \frac{3w_0 y}{1 + by}$$

$$R(y) = \ln\left(\frac{\Omega_h(y)}{1 - \Omega_h(y)}\right)$$

$$\frac{\partial R(y)}{\partial y} = 3w_h(y)$$

$$R_0 = \ln\left(\frac{1 - \Omega_M}{\Omega_M}\right)$$

fraction in matter	Ω_M
present equation of state	w_0
bending parameter	b

relation of b to early dark energy

$$\Omega_e = \Omega_h(y \to \infty) = \frac{\exp(R_0 + 3w_0/b)}{1 + \exp(R_0 + 3w_0/b)}$$

$$b = -\frac{3w_0}{\ln\left(\frac{1-\Omega_e}{\Omega_e}\right) + \ln\left(\frac{1-\Omega_M}{\Omega_M}\right)}$$

Taylor expansion

$$w_h(z) = w_0 + w'z + \dots$$

$$w' = -2w_0 b$$

does nor make much sense for large z

average equation of state

$$\bar{w}_h(y) = \frac{1}{y}\int_0^y dy' w_h(y') = \frac{R(y) - R_0}{3y}$$

yields simple formula for H

$$\frac{H^2(z)}{H_0^2} = (1 - \Omega_M)(1+z)^{3+3\bar{w}_h(z)} + \Omega_M(1+z)^3$$

simple relation with b

$$\bar{w}_h(z) = \frac{w_0}{1 + b\ln(1+z)}$$

$$w_h(y) = \frac{w_0}{(1+by)^2}$$

equation of state changes between w_0 and 0

$$w_h(y) = \frac{w_0}{(1+by)^2}$$

$$w_h = \frac{T-V}{T+V}$$

reconstruction of cosmon potential or kinetial

$$w_h = \frac{T-V}{T+V}$$

$$V = \frac{1-w_h}{2}\rho_h = \frac{3\bar{M}^2}{2}(1-w_h)\Omega_h H^2$$

$$\begin{aligned} T &= \frac{3\bar{M}^2}{2}(1+w_h)\Omega_h H^2 \\ &= \frac{1}{2}k^2(\varphi)\dot{\varphi}^2 = \frac{k^2}{2}\left(\frac{\partial\varphi}{\partial y}\right)^2 \dot{y}^2 = \frac{k^2}{2}H^2\left(\frac{\partial\varphi}{\partial y}\right)^2 \end{aligned}$$

Dynamics of quintessence

- Cosmon φ : scalar singlet field
- Lagrange density $L = V + \frac{1}{2}\, \mathbf{k(\varphi)}\, \partial\varphi\, \partial\varphi$

  (units: reduced Planck mass M=1)
- Potential : $V=\exp[-\varphi]$
- "Natural initial value" in Planck era $\varphi=0$
- today: $\varphi=276$

for " standard " exponential potential :

$$V = \bar{M}^4 \exp\left(-\frac{\phi}{\bar{M}}\right)$$

construction of kinetial from equation of state

$$k^{-1} = [3(1+w_h)\Omega_h]^{-1/2} \left\{ 3(1+w_h) - \frac{\partial w_h}{\partial y} \frac{1}{1-w_h} \right\}$$

$$\frac{\phi}{\bar{M}} = -\ln\left(\frac{3H^2}{2\bar{M}^2}(1-w_h)\Omega_h\right)$$

$$\frac{\partial w_h}{\partial y} \neq 3(1-w_h^2)$$

$$\frac{\partial w_h}{\partial y}_{|y=0} < 3(1-w_0^2)$$

How to distinguish Q from Λ ?

A) Measurement $\Omega_h(z)$ ⟺ $H(z)$

i) $\Omega_h(z)$ at the time of
structure formation , CMB - emission
or nucleosynthesis

ii) equation of state $w_h(\text{today}) > -1$

B) Time variation of fundamental “constants” end

cosmological equations

$$\frac{d\ln\rho_\varphi}{d\ln a} = -3(1+w_\varphi)\ , \qquad \frac{d\varphi}{d\ln a} = \sqrt{6\Omega_T/k^2(\varphi)}$$

$$\frac{d\ln\rho_m}{d\ln a} = -3\,(1+w_m)\ , \qquad \frac{d\ln\rho_r}{d\ln a} = -3\,(1+w_r)\ ,$$

$$\frac{d\ln\rho_\varphi}{d\ln a} = -6\left(1-\frac{V(\varphi)}{\rho_\varphi}\right)\ , \qquad \frac{d\varphi}{d\ln a} = \sqrt{\frac{6\,(\rho_\varphi - V(\varphi))}{k^2\,(\varphi)(\rho_m+\rho_r+\rho_\varphi)}}$$

$$\frac{d\ln V}{d\ln a} = -\sqrt{\frac{6\,(\rho_\varphi - V)}{k^2\,(-\ln V)(\rho_m+\rho_r+\rho_\varphi)}}$$