Spinor Gravity

A.Hebecker,C.Wetterich

Unified Theory of fermions and bosons

Fermions fundamental

Bosons composite

- Alternative to supersymmetry
- Composite bosons look fundamental at large distances, e.g. hydrogen atom, helium nucleus, pions
- Characteristic scale for compositeness : Planck mass
- Graviton, photon, gluons, W-,Z-bosons , Higgs scalar : all composite

massless bound states –
familiar if dictated by symmetries

In chiral QCD :

Pions are massless bound states of massless quarks !

Gauge bosons, scalars …

from vielbein components
in higher dimensions
(Kaluza, Klein)

→

concentrate first on gravity

Geometrical degrees of freedom

- Ψ(x) : spinor field (Grassmann variable)
- vielbein : fermion bilinear

$$\tilde{E}_\mu^m = i\bar{\psi}\gamma^m\partial_\mu\psi$$

$$E_\mu^m(x) = \langle \tilde{E}_\mu^m(x) \rangle$$

Action

$$S_E \sim \int d^d x \det\left(\tilde{E}_\mu^m(x)\right)$$

$$\tilde{E} = \frac{1}{d!}\epsilon^{\mu_1\ldots\mu_d}\epsilon_{m_1\ldots m_d}\tilde{E}_{\mu_1}^{m_1}\ldots\tilde{E}_{\mu_d}^{m_d} = \det(\tilde{E}_\mu^m)$$

contains 2d powers of spinors

d derivatives contracted with ε - tensor

$$\tilde{E}_\mu^m = i\bar{\psi}\gamma^m\partial_\mu\psi$$

Symmetries

- General coordinate transformations (diffeomorphisms)
- Spinor ψ(x) : transforms as scalar
- Vielbein $\tilde{E}_\mu^m = i\bar{\psi}\gamma^m\partial_\mu\psi$: transforms as vector
- Action S : invariant

K.Akama,Y.Chikashige,T.Matsuki,H.Terazawa (1978)
K.Akama (1978)
D.Amati, G.Veneziano (1981)
G.Denardo,E.Spallucci (1987)

Lorentz- transformations

Global Lorentz transformations:

- spinor ψ
- vielbein transforms as vector
- action invariant

Local Lorentz transformations:

- vielbein does **not** transform as vector
- inhomogeneous piece, missing covariant derivative

$$\tilde{E}^m_\mu = i\bar{\psi}\gamma^m\partial_\mu\psi$$

Two alternatives :

1) Gravity with global and not local Lorentz symmetry ?
Compatible with observation !

2) Action with local Lorentz symmetry ?
Can be constructed !

How to get gravitational field equations ?

How to determine geometry of space-time, vielbein and metric ?

Functional integral formulation of gravity

- Calculability
  (at least in principle)
- Quantum gravity
- Non-perturbative formulation

Vielbein and metric

$$E^m_\mu(x) = \langle \tilde{E}^m_\mu(x) \rangle$$

$$g_{\mu\nu}(x) = E^m_\mu(x) E_{\nu m}(x)$$

Generating functional

$$Z[J] = \int \mathcal{D}\psi \exp\left\{ - (S + S_J) \right\}$$

$$S_J = - \int d^d x J^\mu_m \tilde{E}^m_\mu$$

$$E^m_\mu(x) = \langle \tilde{E}^m_\mu(x) \rangle = \frac{\delta \ln Z}{\delta J^\mu_m(x)}$$

If regularized functional measure can be defined (consistent with diffeomorphisms)

Non- perturbative definition of quantum gravity

$$Z[J] = \int \underline{\mathcal{D}\psi} \exp\left\{ -\left(S + S_J\right)\right\}$$

Effective action

$$\Gamma[E_\mu^m] = -W[J_m^\mu] + \int d^dx J_m^\mu E_\mu^m$$

W=ln Z

Gravitational field equation

$$\frac{\delta \Gamma}{\delta E_\mu^m} = J_m^\mu$$

Symmetries dictate general form of effective action and gravitational field equation

diffeomorphisms !

Effective action : curvature scalar R + additional terms

Gravitational field equation and energy momentum tensor

$$\frac{\delta \Gamma}{\delta E_\mu^m} = J_m^\mu$$

$$T^{\mu\nu} = E^{-1} E^{m\mu} J_m^\nu$$

Special case : effective action depends only on metric

$$\Gamma_0'[E_\mu^m] = \Gamma_0'\Big[g_{\nu\rho}[E_\mu^m]\Big]$$

$$g_{\mu\nu} = E_\mu^m E_{\nu m}$$

$$T_{(g)}^{\mu\nu} = -\frac{2}{\sqrt{g}} \frac{\delta \Gamma_0'}{\delta g_{\mu\nu}}$$

$$T^{\mu\nu} = -E^{-1} E^{m\mu} \frac{\delta \Gamma_0'}{\delta g_{\rho\sigma}} \frac{\delta g_{\rho\sigma}}{\delta E_\nu^m} = T_{(g)}^{\mu\nu}$$

Unified theory in higher dimensions and energy momentum tensor

- Only spinors , no additional fields – no genuine source
- $J^{\mu}{}_{m}$: expectation values different from vielbein

  and incoherent fluctuations

- Can account for matter or radiation in effective four dimensional theory (including gauge fields as higher dimensional vielbein-components)

Approximative computation of field equation

for action $$S_E \sim \int d^d x \, \det\left(\tilde{E}_\mu^m(x)\right)$$

Loop- and Schwinger-Dyson- equations

Terms with two derivatives

$$\Gamma_{(2)} = \frac{\mu}{2}\int d^d x E\Big\{ - R + \tau\big[D^\mu E^\nu_m D_\mu E^m_\nu - 2D^\mu E^\nu_m D_\nu E^m_\mu\big]\Big\}$$

expected

new !

covariant derivative

$$D_\mu E^m_\nu = \partial_\mu E^m_\nu - \Gamma_{\mu\nu}{}^\lambda E^m_\lambda$$

has no spin connection !

Fermion determinant in background field

$$\Gamma_{(1l)} = -\frac{1}{2} Tr \ \ln(E\mathcal{D}) \ ,$$
$$\mathcal{D} = \gamma^\mu \partial_\mu + \frac{1}{2E}\gamma^m \partial_\mu (E E^\mu_m) = \gamma^\mu \hat{D}_\mu,$$
$$\gamma^\mu = E^\mu_m \gamma^m$$

$$\mathcal{D} = \gamma^m (E^\mu_m \partial_\mu - \Omega_m) \ , \ \Omega_m = -\frac{1}{2E}\partial_\mu (E E^\mu_m)$$

Comparison with Einstein gravity :
totally antisymmetric part of
spin connection is missing !

$$\mathcal{D} = \mathcal{D}_E[E] + \frac{1}{4}\Omega_{[mnp]}[E]\gamma^{mnp}_{(3)}$$

$$\mathcal{D}_E[e] = \gamma^m e^\mu_m \partial_\mu - \frac{1}{4}\Omega_{[mnp]}[e]\gamma^{mnp}_{(3)}$$

Ultraviolet divergence

new piece from missing totally antisymmetric spin connection :

$$\Gamma_K = \frac{\rho^2}{64} Tr\{\mathcal{D}_E^{-1} A_K \mathcal{D}_E^{-1} A_K\} = \tilde{\tau} \int d^d x e K_{[mnp]} K^{[mnp]}$$

$$A_K = K_{[mnp]} \gamma_{(3)}^{mnp}$$

$\Omega \rightarrow K$

$$\Gamma_K = -\frac{\rho^2}{64} \Omega_d \int \frac{d^d p}{(2\pi)^d} \frac{p_\mu p_\nu}{p^4} K_{[\rho_1 \rho_2 \rho_3]} K^{[\sigma_1 \sigma_2 \sigma_3]} tr\{\gamma^\mu \gamma_{(3)}^{\rho_1 \rho_2 \rho_3} \gamma^\nu \gamma_{(3) \sigma_1 \sigma_2 \sigma_3}\}$$

naïve momentum cutoff Λ :

$$\tilde{\tau} = \frac{v_d (d-6) q_d}{24 d (d-2)} A_{d-2} \Lambda^{d-2}$$

Functional measure needs regularization !

Assume diffeomorphism symmetry preserved :

relative coefficients become calculable

B. De Witt

$$\Gamma_{(2)} = \frac{\mu}{2}\int d^d x E\Big\{-R + \tau\big[D^\mu E^\nu_m D_\mu E^m_\nu - 2D^\mu E^\nu_m D_\nu E^m_\mu\big]\Big\}$$

d=4 :

$\tau=3$

New piece reflects violation of local Lorentz – symmetry !

Gravity with global and not local Lorentz symmetry :

Compatible with observation !

No observation constrains additional term in effective action that violates local Lorentz symmetry ($\sim \tau$)

Action with local Lorentz symmetry can be constructed !

Time space asymmetry

unified treatment of time and space –

but important difference between time and space due to **signature**

Origin ?

Time space asymmetry from spontaneous symmetry breaking

C.W. , PRL , 2004

Idea : difference in signature from spontaneous symmetry breaking

With spinors : signature depends on
signature of Lorentz group

- Unified setting with complex orthogonal group:
- Both **euclidean** orthogonal group and **minkowskian** Lorentz group are subgroups
- Realized signature depends on ground state !

Complex orthogonal group

d=16 , ψ : 256 – component spinor ,
real Grassmann algebra

$$\delta\psi = \begin{pmatrix} \rho, & -\tau \\ \tau, & \rho \end{pmatrix} \psi$$

$$\rho = -\frac{1}{2}\epsilon_{mn}\hat{\Sigma}^{mn} \ , \ \tau = \frac{1}{2}\bar{\epsilon}_{mn}\hat{\Sigma}^{mn}$$

$$\Sigma_E^{mn} = \hat{\Sigma}^{mn}\,\mathbb{1}\ ,\ B^{mn} = -\hat{\Sigma}^{mn} I,$$
$$I = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},\ I^2 = -1$$

SO(16,C)

ϱ ,τ :
antisymmetric
128 x 128 matrices

Compact part : **ϱ**
Non-compact part : **τ**

vielbein

$$\tilde{E}^0_\mu = \psi_\alpha \partial_\mu \psi_\alpha \ , \ \tilde{E}^k_\mu = \psi_\alpha (\hat{a}^k I)_{\alpha\beta} \partial_\mu \psi_\beta$$

$$\{\hat{a}^k, \hat{a}^l\} = -2\delta^{kl} \ , \ k, l = 1 \dots 15$$

$$\hat{\Sigma}^{kl} = \frac{1}{4}[\hat{a}^k, \hat{a}^l] \ , \ \hat{\Sigma}^{0k} = -\frac{1}{2}\hat{a}^k$$

$E_\mu^{\ m} = \delta_\mu^{\ m}$:

SO(1,15) - symmetry

however :

Minkowski signature not singled out in action !

Formulation of action invariant under SO(16,C)

- Even invariant under larger symmetry group

  SO(128,C)

- Local symmetry !

complex formulation

so far real Grassmann algebra

introduce complex structure by

$$\varphi_{\hat{\alpha}} = \psi_{\hat{\alpha}} + i\psi_{128+\hat{\alpha}} \ , \ \varphi^*_{\hat{\alpha}} = \psi_{\hat{\alpha}} - i\psi_{128+\hat{\alpha}}$$

$$\delta\varphi_{\hat{\alpha}} = \sigma_{\hat{\alpha}\hat{\beta}}\varphi_{\hat{\beta}} \ , \ \sigma = \rho + i\tau$$

σ is antisymmetric 128 x 128 matrix , generates SO(128,C)

Invariant action

(complex orthogonal group, diffeomorphisms)

$$S = \alpha \int d^d x W[\varphi] R(\varphi, \varphi^*) + c.c.,$$

$$W[\varphi] = \frac{1}{16!} \epsilon^{\mu_1 \dots \mu_{16}} \partial_{\mu_1} \varphi_{\hat{\alpha}_1} \dots \partial_{\mu_{16}} \varphi_{\hat{\alpha}_{16}} L^{\hat{\alpha}_1 \dots \hat{\alpha}_{16}}$$

$$L^{\hat{\alpha}_1 \dots \hat{\alpha}_{16}} = sym \left\{ \delta^{\hat{\alpha}_1 \hat{\alpha}_2} \delta^{\hat{\alpha}_3 \hat{\alpha}_4} \dots \delta^{\hat{\alpha}_{15} \hat{\alpha}_{16}} \right\}$$

$$R(\varphi, \varphi^*) = T(\varphi) + \tau T(\varphi^*) + \kappa T(\varphi) T(\varphi^*),$$

$$T(\varphi) = \frac{1}{128!} \epsilon^{\hat{\beta}_1 \dots \hat{\beta}_{128}} \varphi_{\hat{\beta}_1} \dots \varphi_{\hat{\beta}_{128}}$$

invariants with respect to
SO(128,C)
and therefore also
with respect to subgroup
SO (16,C)

contractions with
δ and ε – tensors

no mixed terms φ φ*

For τ = 0 : **local** Lorentz-symmetry !!

Generalized Lorentz symmetry

- Example d=16 : SO(128,C) instead of SO(1,15)
- Important for existence of chiral spinors in effective four dimensional theory after dimensional reduction of higher dimensional gravity

S.Weinberg

Unification in d=16 or d=18 ?

- Start with irreducible spinor
- Dimensional reduction of gravity on suitable internal space
- Gauge bosons from Kaluza-Klein-mechanism
- 12 internal dimensions : SO(10) x SO(3) gauge symmetry : unification + generation group
- 14 internal dimensions : more U(1) gener. sym.

(d=18 : anomaly of local Lorentz symmetry)

L.Alvarez-Gaume,E.Witten

Ground state with appropriate isometries:
guarantees massless gauge bosons and graviton in spectrum

Chiral fermion generations

- Chiral fermion generations according to chirality index

  C.W. , Nucl.Phys. B223,109 (1983) ;
  E. Witten , Shelter Island conference,1983
- Nonvanishing index for brane geometries (noncompact internal space)

  C.W. , Nucl.Phys. B242,473 (1984)
- and wharping

  C.W. , Nucl.Phys. B253,366 (1985)
- d=4 mod 4 possible for ‘extended Lorentz symmetry’ (otherwise only d = 2 mod 8)

Rather realistic model known

- d=18 : first step : brane compactifcation

→

- d=6, SO(12) theory : (anomaly free)
- second step : monopole compactification

→

- d=4 with three generations,
  including generation symmetries
- SSB of generation symmetry: realistic mass and mixing hierarchies for quarks and leptons
  (except large Cabibbo angle)

C.W. , Nucl.Phys. B244,359(1984) ; B260,402 (1985) ; B261,461 (1985) ; B279,711 (1987)

Comparison with string theory

	SStrings	Sp.Grav.
Unification of bosons and fermions	ok	ok
Unification of all interactions (d >4)	ok	ok
Non-perturbative (functional integral) formulation	-	ok ?
Manifest invariance under diffeomophisms	-	ok

Comparison with string theory

	SStrings	Sp.Grav.
Finiteness/regularization	ok	-
Uniqueness of ground state/ predictivity	-	?
No dimensionless parameter	ok	?

Conclusions

- Unified theory based only on fermions seems possible
- Quantum gravity –

  if functional measure can be regulated
- Does realistic higher dimensional model exist ?
- Local Lorentz symmetry not verified by observation

Local Lorentz symmetry not verified by observation !

Gravity with global and not local Lorentz symmetry :

Compatible with observation !

No observation constrains additional term in effective action that violates local Lorentz symmetry (~ τ)

Phenomenology, d=4

Most general form of effective action which is consistent with diffeomorphism and global Lorentz symmetry

Derivative expansion

$$\Gamma = \epsilon\Gamma_0 + \mu(I_1 + \tau_A I_2 + \beta_A I_3)$$

$$I_1 = \frac{1}{2}\int d^dx E\{D^\mu E^\nu_m D_\nu E^m_\mu - D_\mu E^\mu_m D^\nu E^m_\nu\}$$
$$I_2 = \frac{1}{2}\int d^dx E\{D^\mu E^\nu_m D_\mu E^m_\nu - 2D^\mu E^\nu_m D_\nu E^m_\mu\}$$
$$I_3 = \frac{1}{2}\int d^dx E D_\mu E^\mu_m D^\nu E^m_\nu$$

$$I_1 = -\frac{1}{2}\int d^dxe\ R[g[e]]$$

new

not in one loop SG

New gravitational degree of freedom

$$E_\mu^{\ n} = e_\mu^{\ m} H_m^{\ \ n}$$

for **local** Lorentz-symmetry:
H is **gauge degree of freedom**

matrix notation :

$$\bar{E} = \bar{e} H \ , \ H \eta H^T = \eta \ , \ \det H = 1$$

$$g = \bar{e} \eta \bar{e}^T \ , \ E = \det \bar{E} = \det \bar{e} = e$$

standard vielbein :

$$D_\mu e_\nu^n = 0$$

new invariants (only global Lorentz symmetry) : derivative terms for H_{mn}

$$
\begin{aligned}
D_\mu E^m_\nu &= e^n_\nu D_\mu H_n{}^m \,, \\
D_\mu H_n{}^m &= \partial_\mu H_n{}^m - \omega_\mu{}^p{}_n[e] H_p{}^m
\end{aligned}
$$

$$
\begin{aligned}
I_2 &= \frac{1}{2}\int d^d x e \{ D^p H^{nm} D_p H_{nm} - 2 D^p H_{nm} D^n H_p{}^m \} \\
I_3 &= \frac{1}{2}\int d^d x e D^n H_{nm} D^p H_p{}^m
\end{aligned}
$$

$$
I_1 = -\frac{1}{2}\int d^d x e \; R[g[e]]
$$

Gravity with global Lorentz symmetry has additional massless field !

Local Lorentz symmetry not tested!

loop and SD- approximation : $\beta = 0$

new invariant ~ τ

is compatible with all present tests !

Linear approximation (weak gravity)

$$E^m_\mu = \delta^m_\mu + \frac{1}{2}(h_{\mu\nu} + a_{\mu\nu})\eta^{\nu m}$$

$$h_{\mu\nu} = b_{\mu\nu} + \frac{1}{(d-1)}\left(\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right)\sigma + \frac{\partial_\mu\partial_\nu}{\partial^2}f + \partial_\mu v_\nu + \partial_\nu v_\mu$$

$$a_{\mu\nu} = c_{\mu\nu} + \partial_\mu(v_\nu + w_\nu) - \partial_\nu(v_\mu + w_\mu)$$

for $\beta = 0$: only new massless field $\mathbf{c}_{\mu\nu}$

$$\Gamma_{(2)} = \frac{\mu}{8}\int d^dx \left\{\partial^\mu b^{\nu\rho}\partial_\mu b_{\nu\rho} - \frac{d-2}{d-1}\partial^\mu\sigma\partial_\mu\sigma + \tau\partial^\mu c^{\nu\rho}\partial_\mu c_{\nu\rho}\right\}$$

$c_{\mu\nu}$ couples only to spin

(antisymmetric part of energy momentum tensor)

test would need source with macroscopic spin

and test particle with macroscopic spin

Post-Newtonian gravity

No change in lowest nontrivial order in Post-Newtonian-Gravity !

Schwarzschild and cosmological solutions : not modified !

beyond linear gravity !

Second possible invariant (~β) strongly constrained by observation !

most general bilinear term :

$$\Gamma = \frac{\mu}{8}\int d^d x \left\{ \partial^\mu b^{\nu\rho}\partial_\mu b_{\nu\rho} - \left(\frac{d-2}{d-1} - \beta_A\right)\partial^\mu\sigma\partial_\mu\sigma \right.$$
$$\left. + \tau_A \partial^\mu c^{\nu\rho}\partial_\mu c_{\nu\rho} + \beta_A \partial^2 w^\mu \partial^2 w_\mu \right\}$$

dilatation mode σ is affected !

For β ≠ 0 : linear and Post-Newtonian gravity modified !

Newtonian gravity

$$\Delta\phi = \frac{\rho}{2\mu}\frac{1-2\beta_A}{1-\frac{3}{2}\beta_A} = 4\pi G_N \rho = \frac{\rho}{2\bar{M}^2}$$

$$\bar{M}^2 = M_p^2/8\pi$$

$$\bar{M}^2 = \frac{1-\frac{3}{2}\beta_A}{1-2\beta_A}\mu$$

Schwarzschild solution

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2(d\vartheta^2 + sin^2\vartheta d\varphi^2)$$

no modification for β = 0 !

strong experimental bound on β !

$$B = 1 - \frac{r_s}{r} \;,\; A^{-1} = 1 - \gamma\frac{r_s}{r}$$

$$\gamma - 1 \approx \beta = (2.1 \pm 2.3)10^{-5}$$

Cosmology

general isotropic and homogeneous vielbein :

$$E^0_0 = 1\ ,\ E^i_0 = 0\ ,\ E^0_i = 0\ ,\ E^j_i = a(t)\delta^j_i$$

$$H(t) = \dot{a}(t)/a(t)$$

only the effective Planck mass differs between cosmology and Newtonian gravity if $\beta \neq 0$

$$\frac{\bar{M}_c^2}{\bar{M}^2} = 1 - 2\beta_A$$

Otherwise : same cosmological equations !

Modifications only for $\beta \neq 0$!

Valid theory with global instead of local Lorentz invariance for $\beta = 0$!

General form in one loop / SDE : $\beta = 0$
Can hidden symmetry be responsible?

end

Geometry

One can define new curvature free connection

$$\tilde{\Gamma}_{\mu\nu}{}^{\lambda} = (\partial_\mu E_\nu^m) E_m^\lambda$$

Torsion

$$T_{\mu\nu\rho} = (\partial_\mu E_\nu^m - \partial_\nu E_\mu^m) E_{\rho m}$$

$$\Gamma_{(2)} = \frac{\mu}{2} \int d^d x e \left\{ -R + \tau' T_{[\mu\nu\rho]} T^{[\mu\nu\rho]} \right\}$$

$$\tau' \equiv 3\tau/4 = 9/4$$