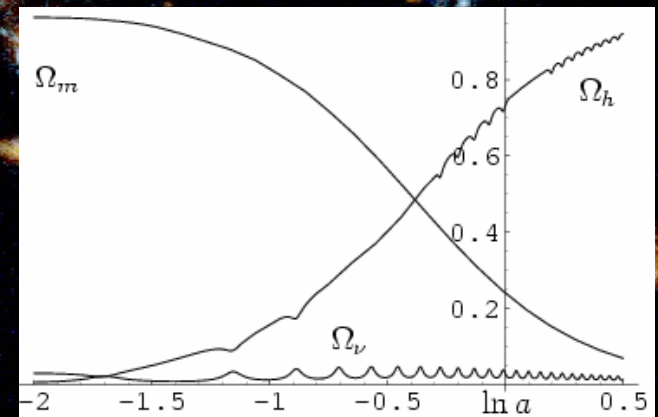
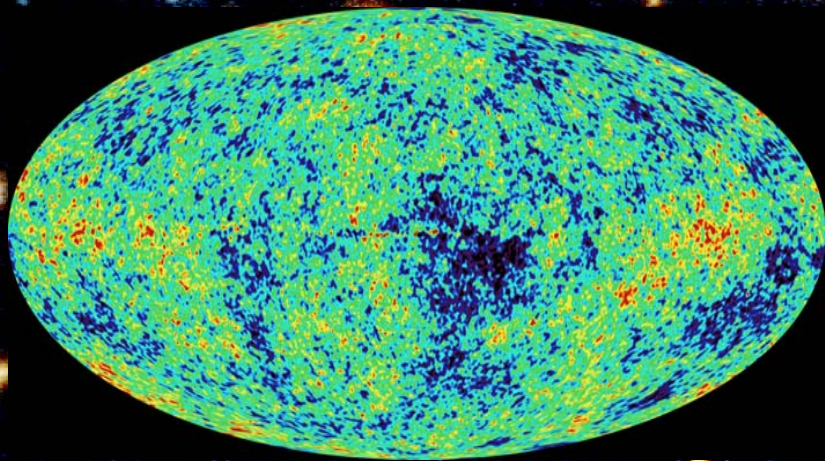


Neutrinos and Dark Energy



Quintessence

C.Wetterich

A.Hebecker, M.Doran, M.Lilley, J.Schwindt,
C.Müller, G.Schäfer, E.Thommes,
R.Caldwell, M.Bartelmann,
K.Kharwan, G.Robbers, T.Dent, S.Steffen,
L.Amendola, M.Baldi

What is our universe made of ?



Dark Energy dominates the Universe

Energy - density in the Universe

=

Matter + Dark Energy

25 % + 75 %

Matter : everything that clumps

Dark Energy density is
the same at every point of space

“ homogeneous “

Space between clumps is not empty

What is Dark Energy ?

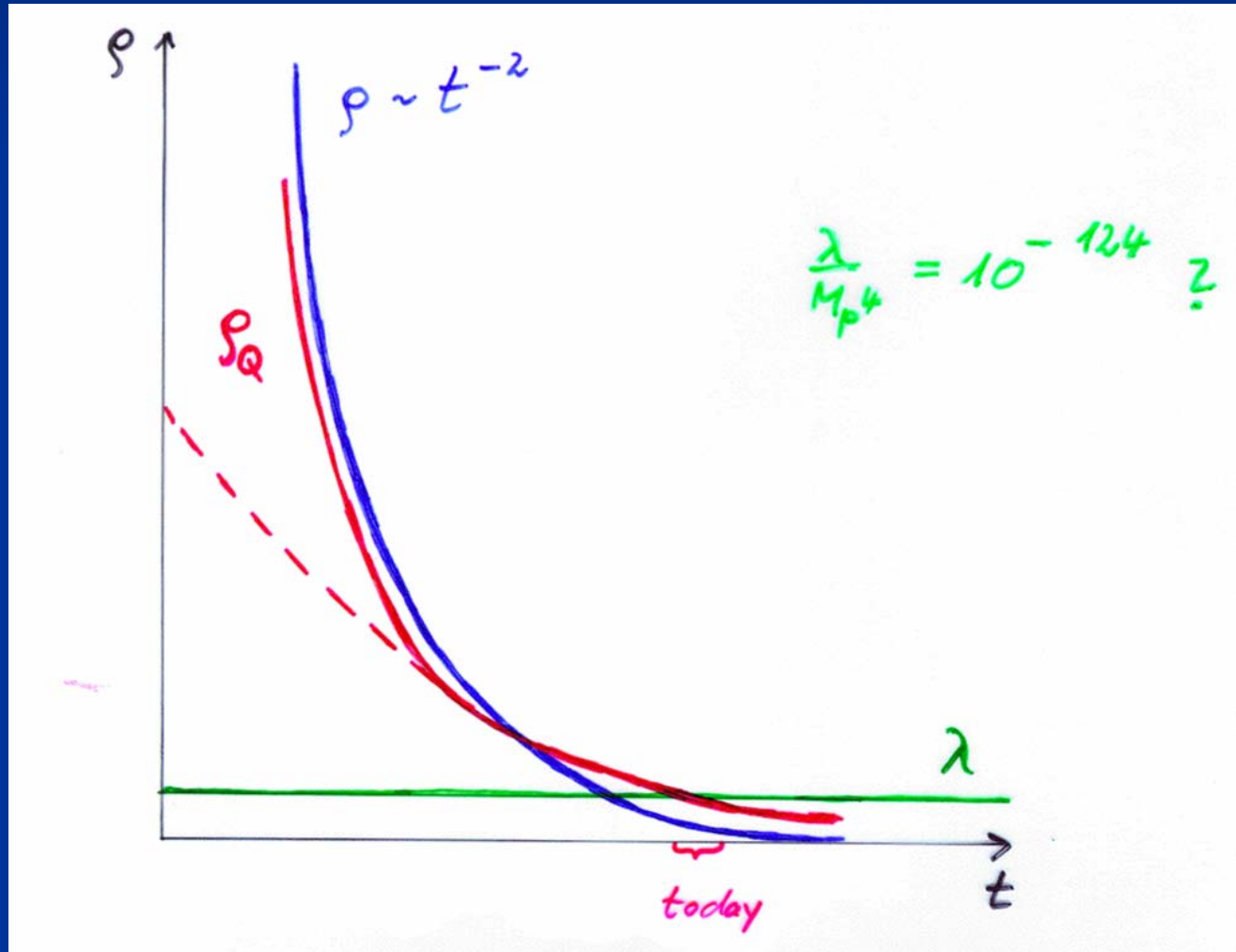
Cosmological Constant

or

Quintessence ?

Cosm. Const
static

Quintessence
dynamical



Cosmological Constant

- Einstein -

- Constant λ compatible with all symmetries
- No time variation in contribution to energy density
- Why so small ? $\lambda/M^4 = 10^{-120}$
- Why important just today ?

Cosmological mass scales

- Energy density

$$\rho \sim (2.4 \times 10^{-3} \text{ eV})^{-4}$$

- Reduced Planck mass

$$M = 2.44 \times 10^{18} \text{ GeV}$$

- Newton's constant

$$G_N = (8\pi M^2)$$

Only ratios of mass scales are observable !

homogeneous dark energy: $\rho_h/M^4 = 6.5 \cdot 10^{-121}$

matter: $\rho_m/M^4 = 3.5 \cdot 10^{-121}$

Quintessence

Dynamical dark energy ,
generated by scalar field

(cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87

P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

**homogeneous dark energy
influences recent cosmology**

- of same order as dark matter -

Original models do not fit the present observations
.... modifications

Cosmon

- *Scalar field changes its value even in the **present** cosmological epoch*
- *Potential und kinetic energy of cosmon contribute to the energy density of the Universe*

$$3M^2H^2 = V + \frac{1}{2}\dot{\phi}^2 + \rho$$

- *Time - variable dark energy :
 $\rho_b(t)$ decreases with time !*

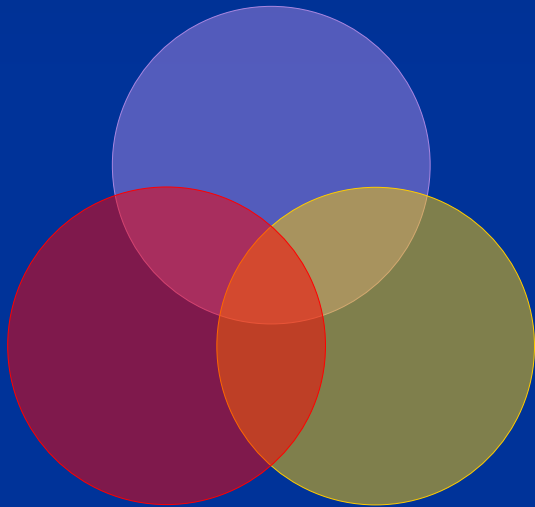
$$V(\varphi) = M^4 \exp(-\alpha\varphi/M)$$

Cosmon

- *Tiny mass*
- $m_c \sim H$
- *New long - range interaction*

“Fundamental” Interactions

Strong, electromagnetic, weak interactions



gravitation

cosmodynamics

On astronomical length scales:

graviton

+

cosmon

Evolution of cosmon field

Field equations

$$\ddot{\phi} + 3H\dot{\phi} = -dV/d\phi$$

$$3M^2H^2 = V + \frac{1}{2}\dot{\phi}^2 + \rho$$

Potential $V(\varphi)$ determines details of the model

$$V(\varphi) = M^4 \exp(-\alpha\varphi/M)$$

for increasing φ the potential decreases
towards zero !

Cosmic Attractors

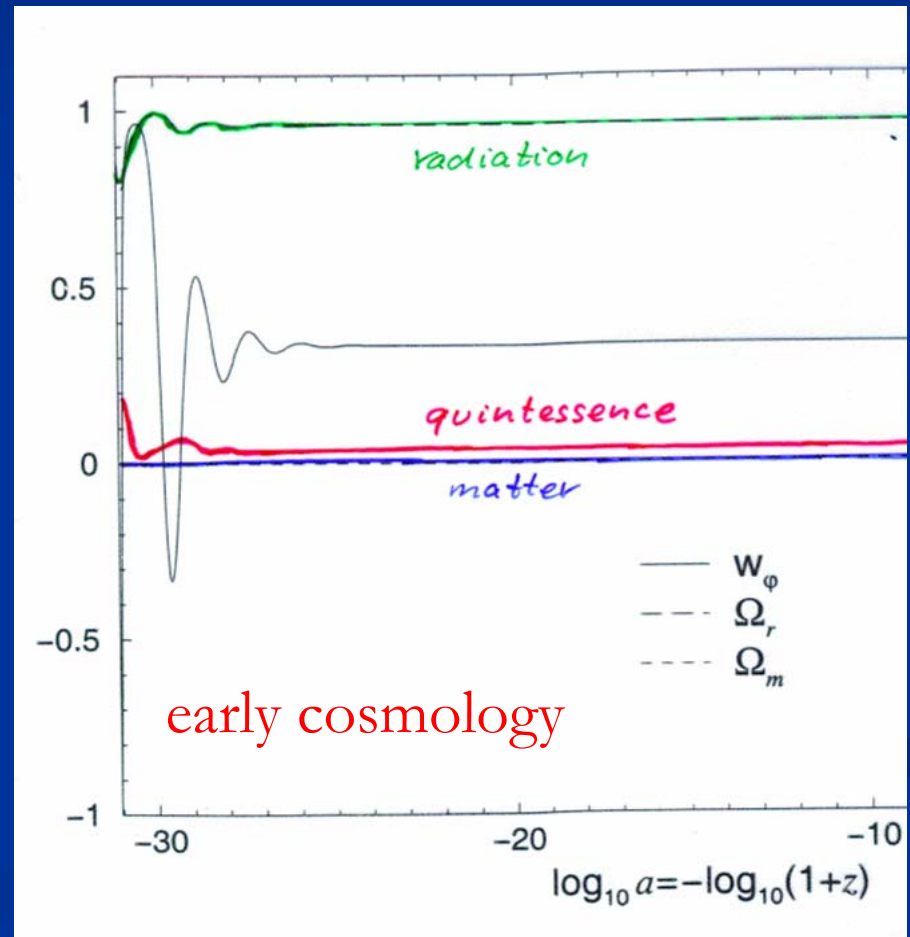
Solutions independent
of initial conditions

typically $V \sim t^{-2}$

$\varphi \sim \ln(t)$

$\Omega_h \sim \text{const.}$

details depend on $V(\varphi)$
or kinetic term



exponential potential →
constant fraction in dark energy

$$\Omega_h = 3(4)/\alpha^2$$

can explain order of magnitude
of dark energy !

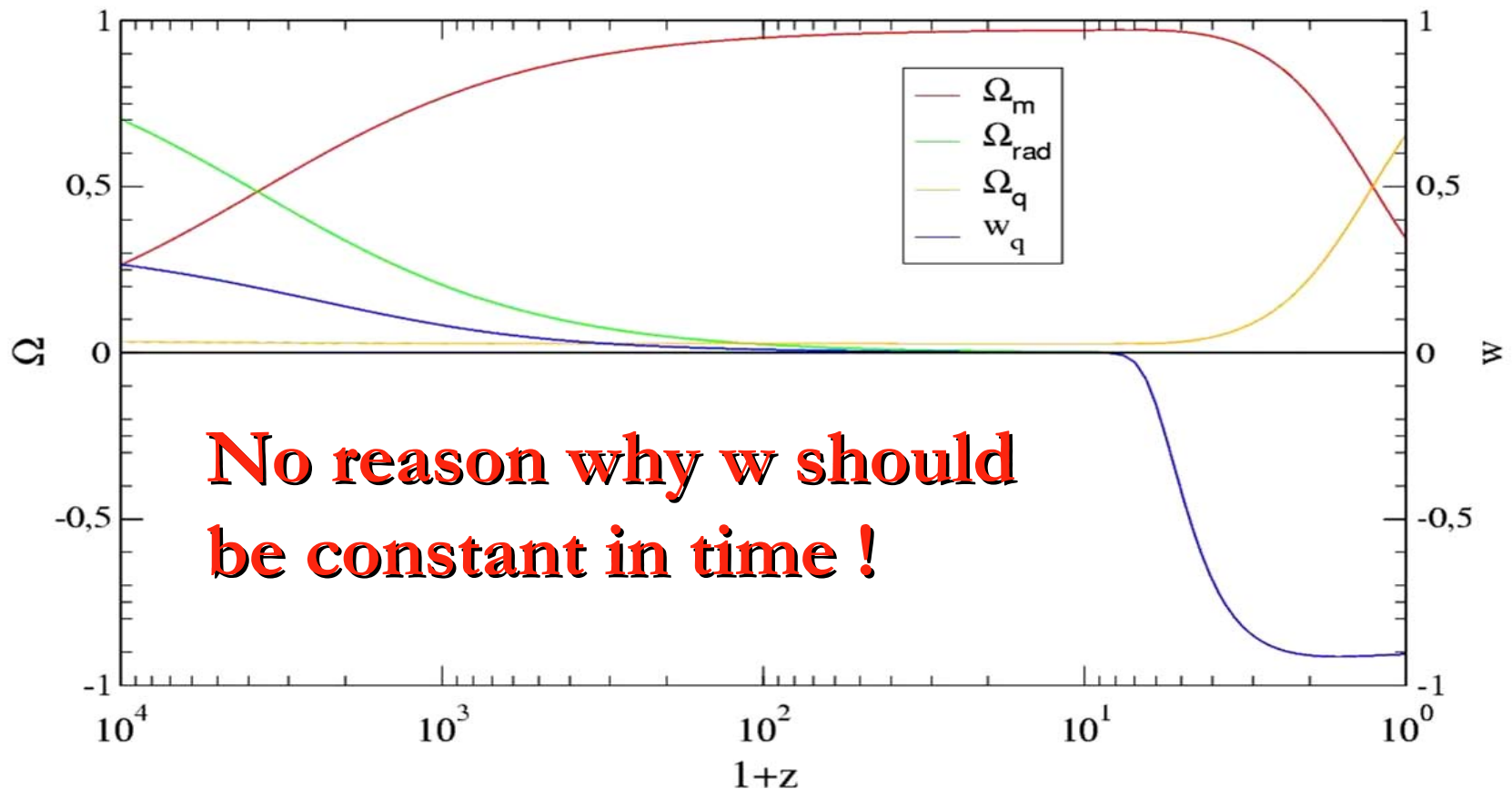
realistic quintessence

fraction in dark energy has to
increase in “recent time” !

cosmic coincidence

Quintessence becomes important “today”

Crossover Quintessence Evolution



coincidence problem

What is responsible for increase of Ω_h for $z < 6$?

a) Properties of cosmon potential or kinetic term

Late quintessence

- w close to -1
- Ω_h negligible in early cosmology
- needs tiny parameter, similar to cosmological constant

Early quintessence

- Ω_h changes only modestly
- w changes in time

transition

- special feature in cosmon potential or kinetic term becomes important “now”
- tuning at ‰ level

b) Quintessence reacts to some special event in cosmology

- Onset of matter dominance

K-essence

Amendariz-Picon, Mukhanov,
Steinhardt

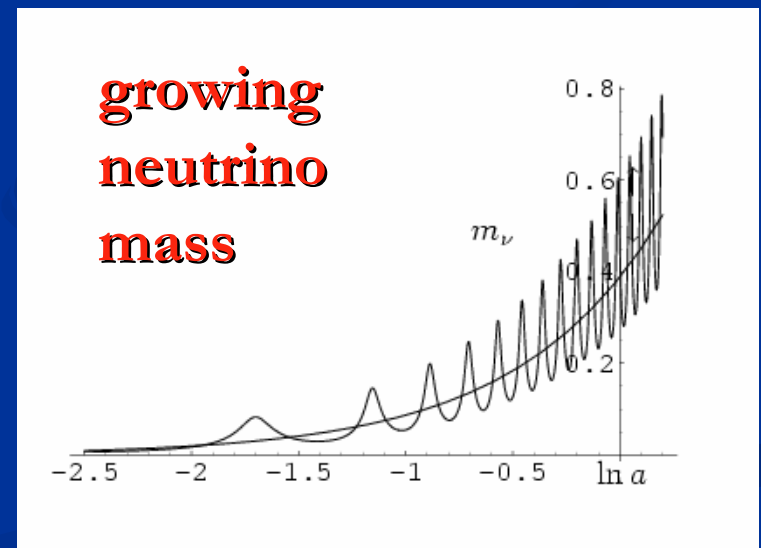
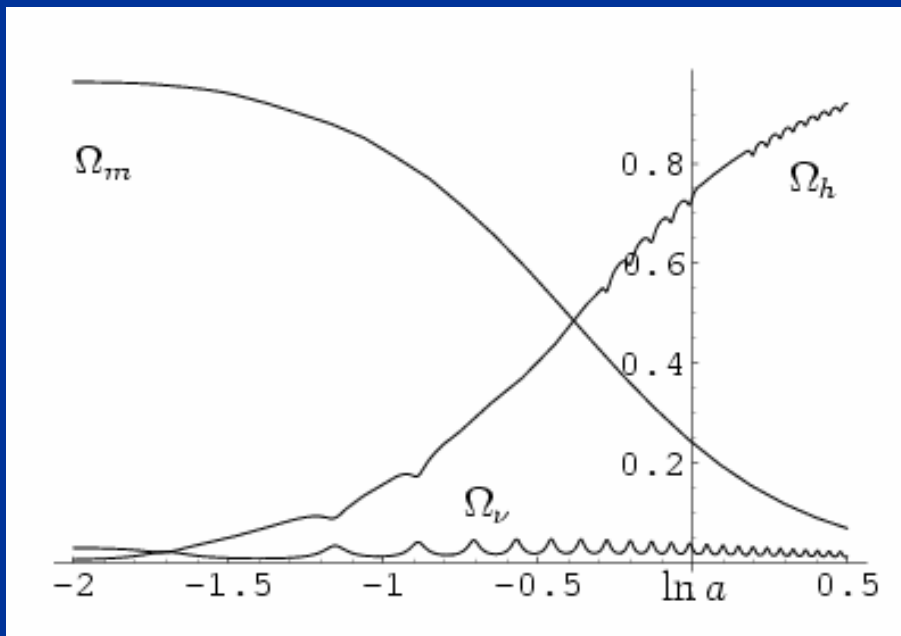
needs higher derivative
kinetic term

- Appearance of non-linear structure

Back-reaction effect

needs coupling between
Dark Matter and
Dark Energy

growing neutrino mass triggers transition to almost static dark energy



basic ingredient :

cosmon coupling to neutrinos

Cosmon coupling to atoms

- Tiny !!!
- Substantially weaker than gravity.
- Non-universal couplings bounded by tests of equivalence principle.
- Universal coupling bounded by tests of Brans-Dicke parameter ω in solar system.
- Only very small influence on cosmology.

Cosmon coupling to Dark Matter

- Only bounded by cosmology
- Substantial coupling possible
- Can modify scaling solution and late cosmology
- Role in clustering of extended objects ?

L. Amendola

Cosmon coupling to neutrinos

- can be large !

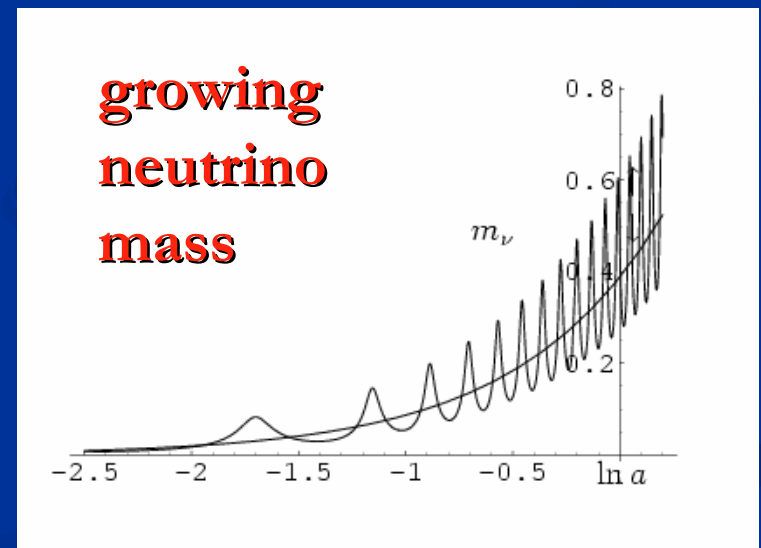
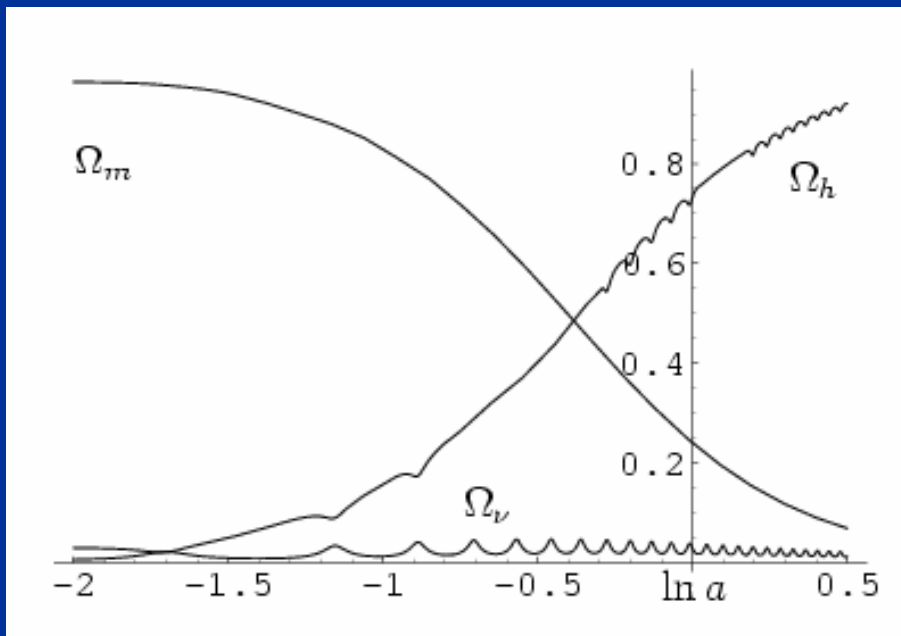
Fardon, Nelson, Weiner

- interesting effects for cosmology if neutrino mass is growing
- growing neutrinos can stop the evolution of the cosmon
- transition from early scaling solution to cosmological constant dominated cosmology

L. Amendola, M. Baldi, ...

growing neutrinos

crossover due to non-relativistic neutrinos



end of matter domination

- growing mass of neutrinos



- at some moment energy density of neutrinos becomes more important than energy density of dark matter



- end of matter dominated period
- similar to transition from radiation domination to matter domination
- this transition happens in the recent past
- cosmology plays crucial role

cosmological selection

- present value of dark energy density set by cosmological event
(neutrinos become non – relativistic)
- not given by ground state properties !

connection between dark energy and neutrino properties

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.07 \left(\frac{\gamma m_\nu(t_0)}{eV} \right)^{\frac{1}{4}} 10^{-3} eV$$

present dark energy density given by neutrino mass

**present equation
of state given by
neutrino mass !**

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12eV}$$

dark energy fraction determined by neutrino mass

$$\Omega_h(t_0) \approx \frac{\gamma m_\nu(t_0)}{16eV}$$

$$\gamma = -\frac{\beta}{\alpha}$$

constant neutrino - cosmon coupling β

$$\Omega_h(t_0) \approx -\frac{\epsilon}{\alpha} \frac{m_\nu(t_0)}{\bar{m}_\nu} \frac{m_\nu(t_0)}{16eV}$$

variable neutrino - cosmon coupling

varying neutrino – cosmon coupling

- specific model
- can naturally explain why neutrino – cosmon coupling is much larger than atom – cosmon coupling

neutrino mass

$$M_\nu = M_D M_R^{-1} M_D^T + M_L$$

$$M_L = h_L \gamma \frac{d^2}{M_t^2}$$

seesaw and
cascade
mechanism

triplet expectation value \sim doublet squared

$$m_\nu = \frac{h_\nu^2 d^2}{m_R} + \frac{h_L \gamma d^2}{M_t^2}$$

omit generation
structure

cascade mechanism

$$U = U_0(\varphi) + \frac{\lambda}{2}(d^2 - d_0^2)^2 + \frac{1}{2}M_t^2(\varphi)t^2 - \gamma d^2 t$$

triplet expectation value \sim

$$\gamma \frac{d^2}{M_t^2}$$

M.Magg , ...

G.Lazarides , Q.Shafi , ...

$$M_t^2(\varphi) = \bar{M}_t^2 \left[1 - \exp\left(-\frac{\epsilon}{M}(\varphi - \varphi_t)\right) \right]$$

varying neutrino mass

$$M_t^2 = c_t M_{GUT}^2 \left[1 - \frac{1}{\tau} \exp\left(-\epsilon \frac{\varphi}{M}\right) \right] \quad \epsilon \approx -0.05$$

triplet mass depends on cosmon field φ

$$m_\nu(\varphi) = \bar{m}_\nu \left\{ 1 - \exp\left[-\frac{\epsilon}{M}(\varphi - \varphi_t)\right] \right\}^{-1}$$

→ neutrino mass depends on φ

“singular” neutrino mass

$$M_t^2 = c_t M_{GUT}^2 \left[1 - \frac{1}{\tau} \exp\left(-\epsilon \frac{\varphi}{M}\right) \right]$$

triplet mass vanishes for $\varphi \rightarrow \varphi_t$

$$\frac{\varphi_t}{M} = -\frac{\ln \tau}{\epsilon}$$

$$m_\nu(\varphi) = \frac{\bar{m}_\nu M}{\epsilon(\varphi - \varphi_t)}$$

➔ neutrino mass diverges for $\varphi \rightarrow \varphi_t$

strong effective
neutrino – cosmon coupling
for $\varphi \rightarrow \varphi_t$

$$\beta(\varphi) = -M \frac{\partial}{\partial \varphi} \ln m_\nu(\varphi) = \frac{M}{\varphi - \varphi_t}$$

**crossover from
early scaling solution to
effective cosmological constant**

early scaling solution (tracker solution)

$$V(\varphi) = M^4 \exp\left(-\alpha \frac{\varphi}{M}\right)$$

$$\varphi = \varphi_0 + (2M/\alpha) \ln(t/t_0)$$

$$\Omega_{h,e} = \frac{n}{\alpha^2}$$

neutrino mass unimportant in early cosmology

growing neutrinos change cosmological evolution

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial V}{\partial \varphi} + \frac{\beta(\varphi)}{M}(\rho_\nu - 3p_\nu),$$
$$\beta(\varphi) = -M \frac{\partial}{\partial \varphi} \ln m_\nu(\varphi) = \frac{M}{\varphi - \varphi_t}$$

modification of conservation equation for neutrinos

$$\begin{aligned} \dot{\rho}_\nu + 3H(\rho_\nu + p_\nu) &= -\frac{\beta(\varphi)}{M}(\rho_\nu - 3p_\nu)\dot{\varphi} \\ &= -\frac{\dot{\varphi}}{\varphi - \varphi_t}(\rho_\nu - 3p_\nu) \end{aligned}$$

effective stop of cosmon evolution

cosmon evolution almost stops once

- neutrinos get non-relativistic
- β gets large

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial V}{\partial \varphi} + \frac{\beta(\varphi)}{M}(\rho_\nu - 3p_\nu)$$

$$\beta(\varphi) = -M \frac{\partial}{\partial \varphi} \ln m_\nu(\varphi) = \frac{M}{\varphi - \varphi_t}$$

$$m_\nu(\varphi) = \frac{\beta(\varphi)}{\epsilon} \bar{m}_\nu$$

**This always
happens
for $\varphi \rightarrow \varphi_t$!**

effective cosmological trigger
for stop of cosmon evolution :
neutrinos get non-relativistic

- this has happened recently !
- sets scales for dark energy !

effective cosmological constant

$$V_t = M^4 \exp\left(-\alpha \frac{\varphi_t}{M}\right)$$

realistic value

for

$$\alpha \varphi_t / M \approx 276$$



$$\epsilon = -\frac{\alpha \ln \tau}{276}$$

effective cosmological constant linked to neutrino mass

realistic value $\propto \varphi_t / M \approx 276$:

needed for neutrinos to become non-relativistic
in recent past -

as required for observed mass range of neutrino
masses

adjustment of one dimensionless parameter
in order to obtain for the present time the
correct ratio between dark energy and neutrino
energy density

dark energy fraction determined by neutrino mass

$$\Omega_h(t_0) \approx \frac{\gamma m_\nu(t_0)}{16eV}$$

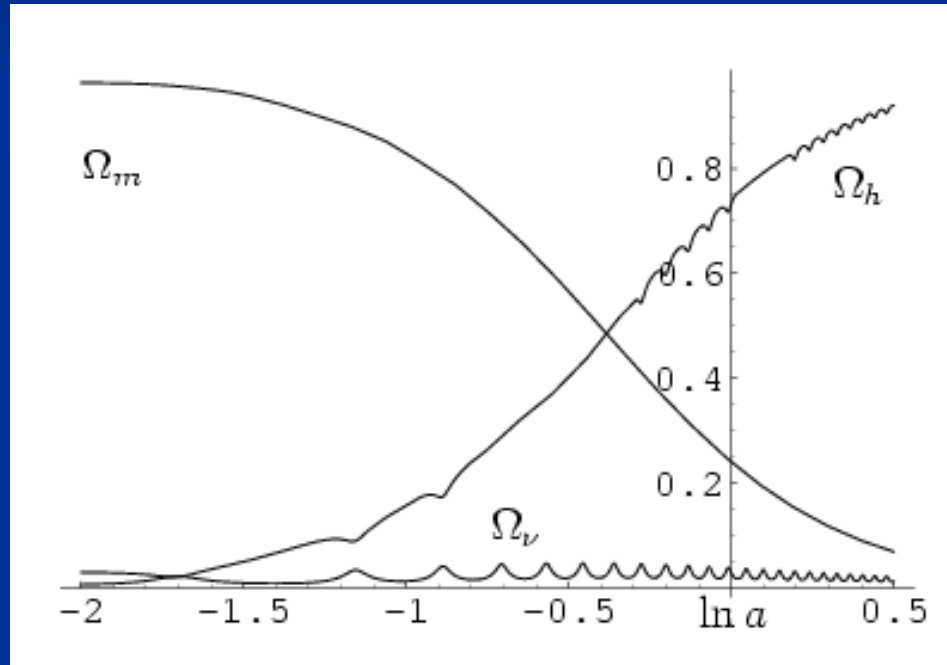
$$\gamma = -\frac{\beta}{\alpha}$$

constant neutrino - cosmon coupling β

$$\Omega_h(t_0) \approx -\frac{\epsilon}{\alpha} \frac{m_\nu(t_0)}{\bar{m}_\nu} \frac{m_\nu(t_0)}{16eV}$$

variable neutrino - cosmon coupling

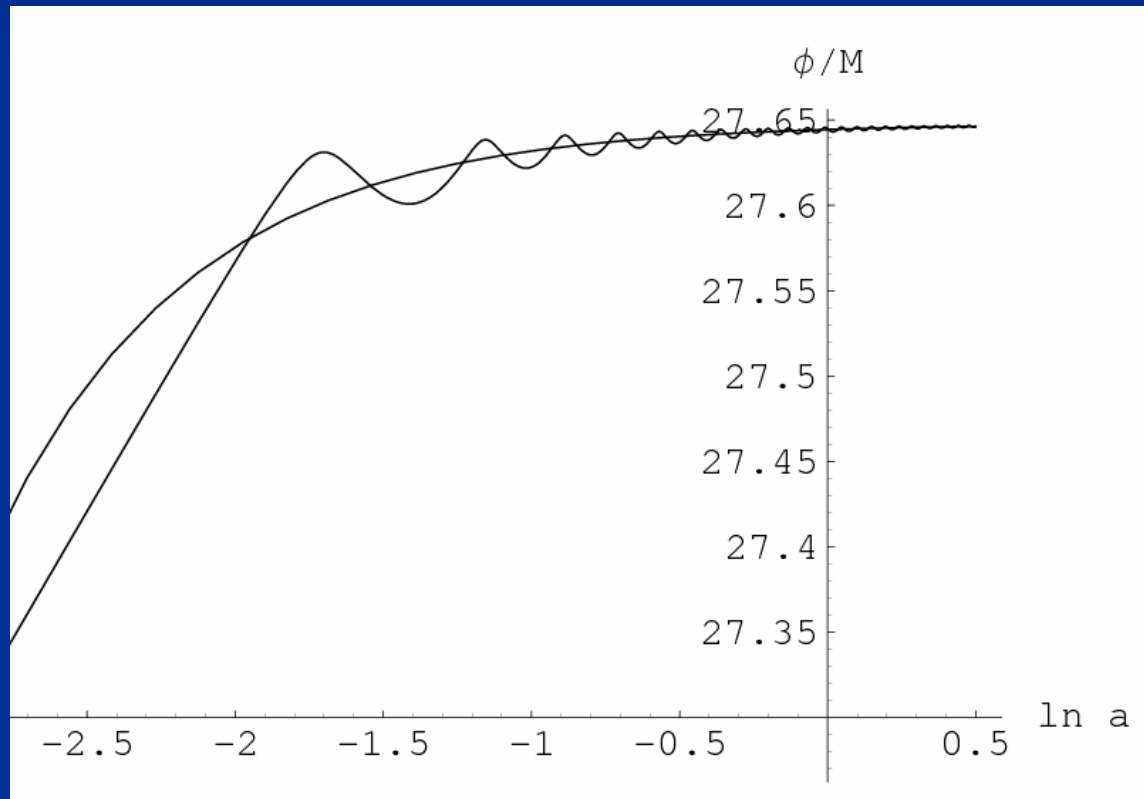
crossover to dark energy dominated universe



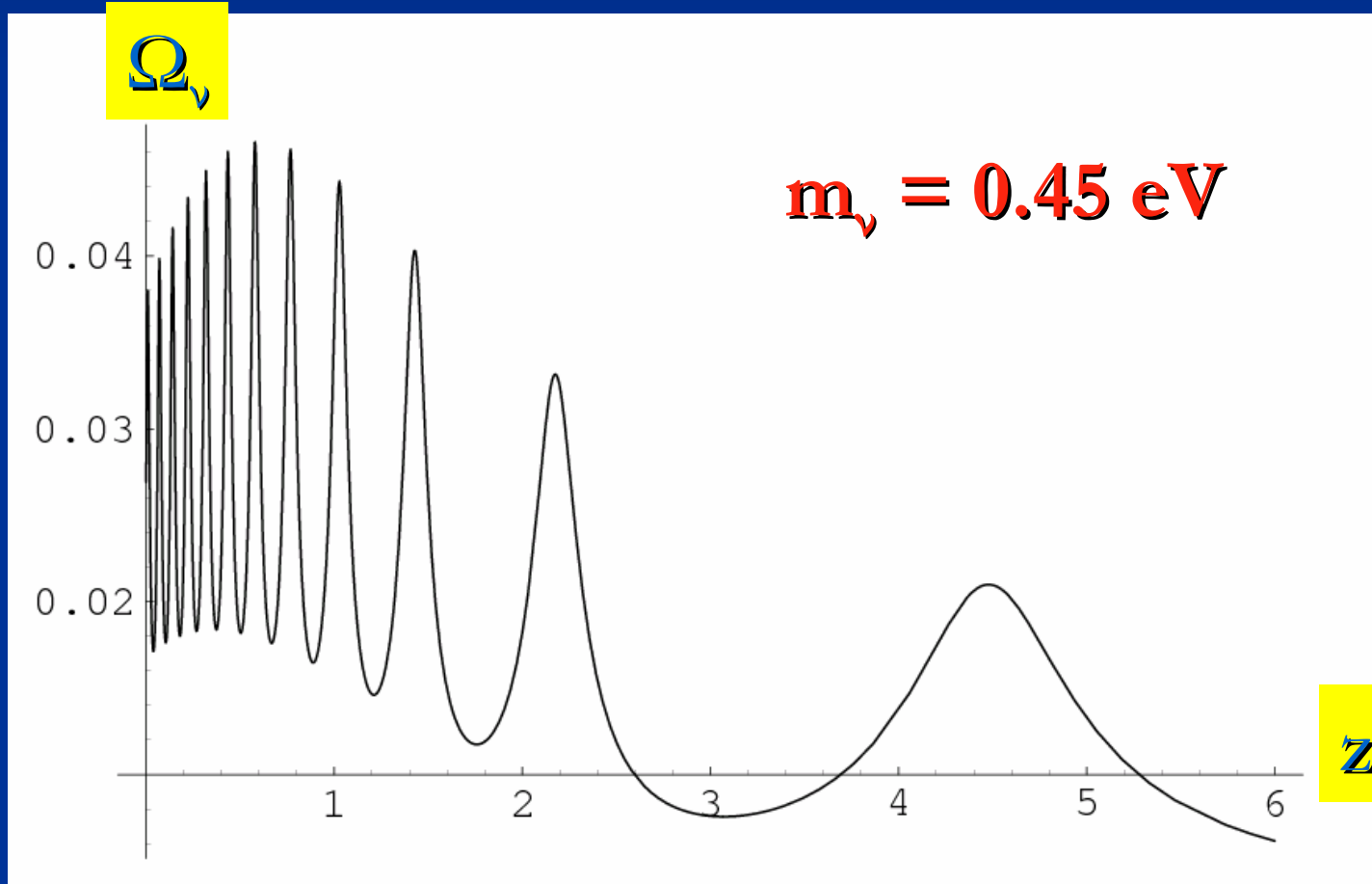
starts at time when “neutrino force” becomes
important for the evolution of the cosmon field

cosmological selection !

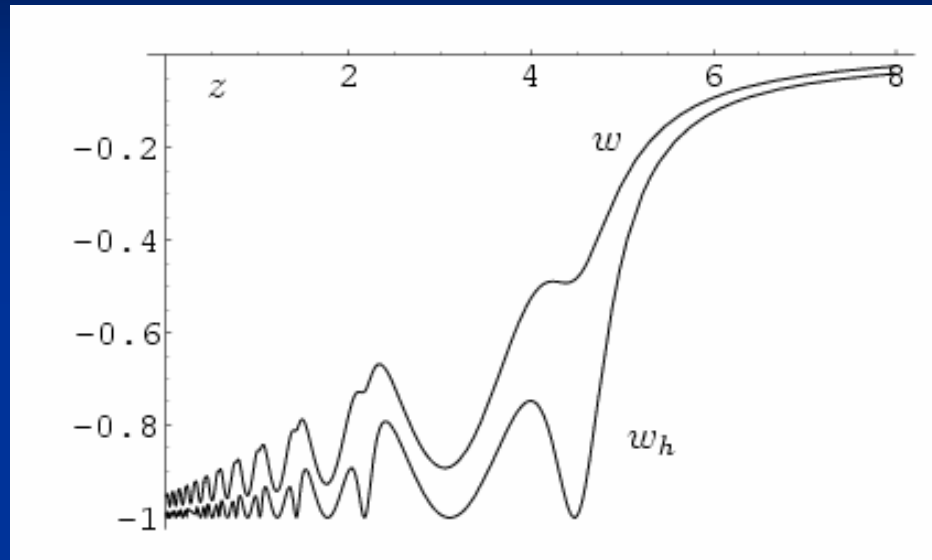
cosmon evolution



neutrino fraction remains small



equation of state

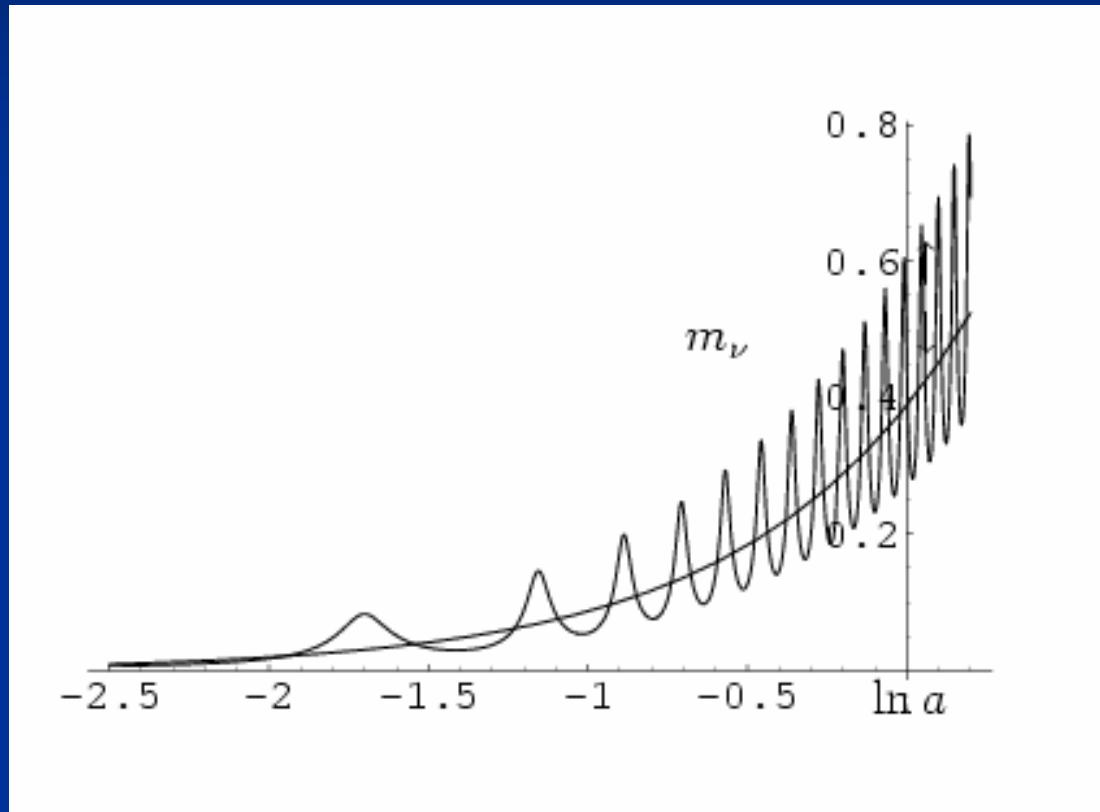


$$w = \frac{T - V + w_\nu \rho_\nu}{T + V + \rho_\nu} \approx -1 + \frac{\rho_\nu}{V} \approx -1 + \frac{\Omega_\nu}{\Omega_h},$$

**present equation
of state given by
neutrino mass !**

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12\text{eV}}$$

oscillating neutrino mass



crossing time

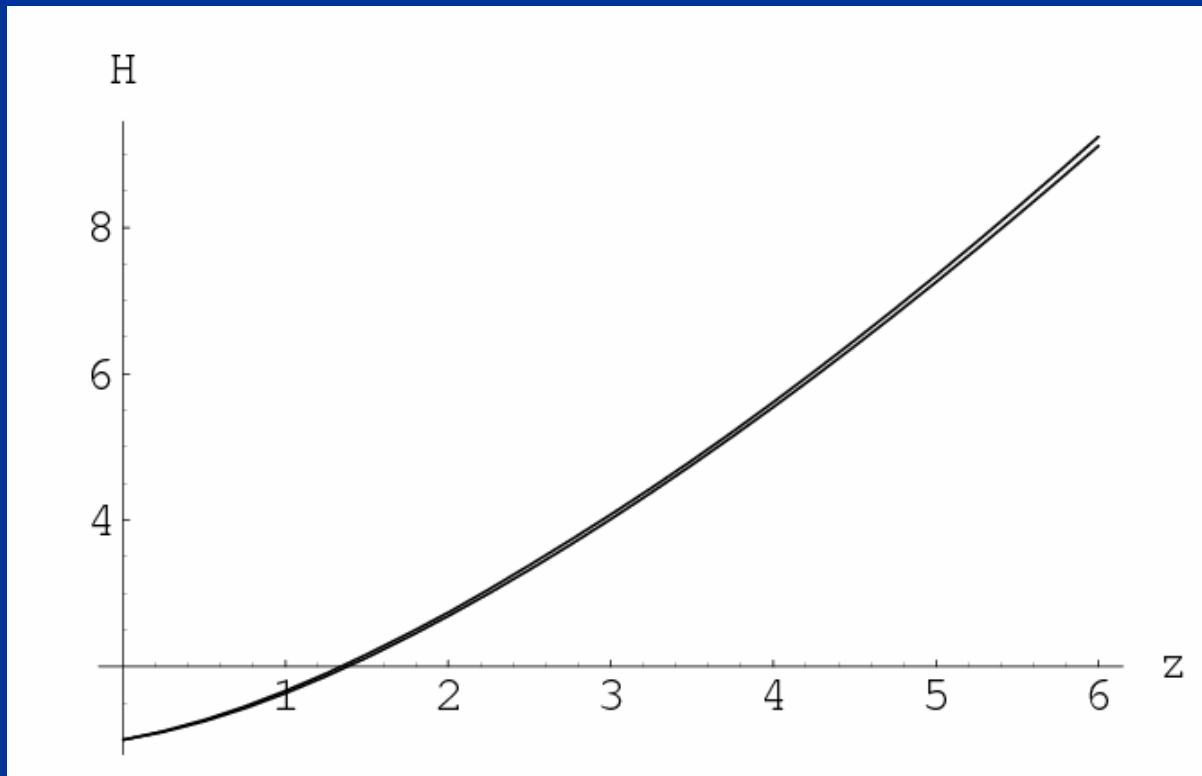
from matching between
early solution and late solution

$$\begin{aligned} V_t \approx V(t_c) &\approx \frac{3}{2} \Omega_{h,e} M^2 H^2(t_c) \\ &= \frac{9}{2\alpha^2} M^2 H^2(t_c) = \frac{2M^2}{\alpha^2 t_c^2} \end{aligned}$$

$$t_c^2 H_0^2 = \frac{2}{3\Omega_{h,0}\alpha^2} \approx \frac{8}{9\alpha^2}$$

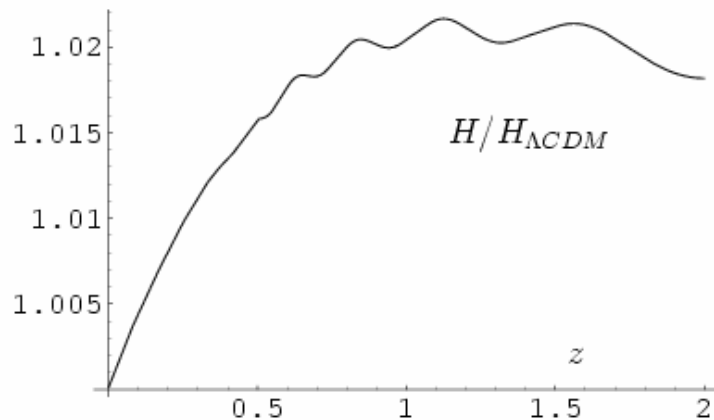
Hubble parameter

as compared to Λ CDM



Hubble parameter ($z < z_c$)

$$H^2 = \frac{1}{3M^2} \left\{ V_t + \rho_{m,0} a^{-3} + 2\tilde{\rho}_\nu,0 a^{-\frac{3}{2}} \right\}$$



only small
difference
from
 Λ CDM!

Can time evolution of neutrino mass be observed ?

- Experimental determination of neutrino mass may turn out higher than upper bound in model for cosmological constant

(KATRIN, neutrinoless double beta decay)

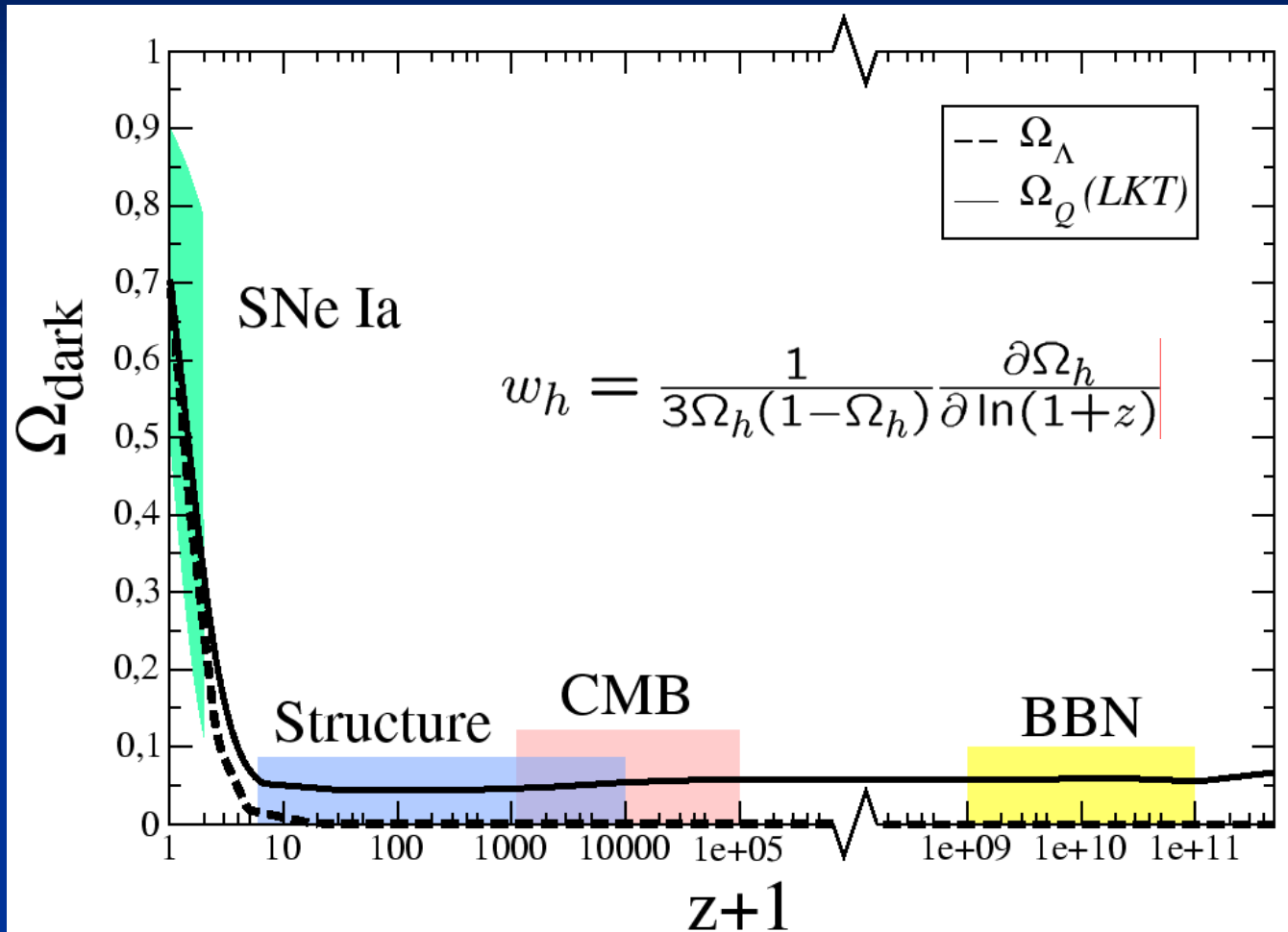
neutrino fluctuations

- time when neutrinos become non – relativistic
- sets free streaming scale

$$a_R = \left(\frac{\tilde{m}_\nu(t_0)}{3T_{\nu,0}} \right)^{-\frac{2}{5}} = 0.05 \left(\frac{\tilde{m}_\nu(t_0)}{eV} \right)^{-2/5}$$

How can quintessence be distinguished from a cosmological constant ?

Time dependence of dark energy



cosmological constant : $\Omega_h \sim t^2 \sim (1+z)^{-3}$

small early and large present dark energy

fraction in dark energy has substantially
increased since end of structure formation



expansion of universe accelerates in present
epoch

$$w_h = \frac{1}{3\Omega_h(1-\Omega_h)} \frac{\partial \Omega_h}{\partial \ln(1+z)}$$

effects of early dark energy

- modifies cosmological evolution (CMB)
- slows down the growth of structure

Growth of density fluctuations

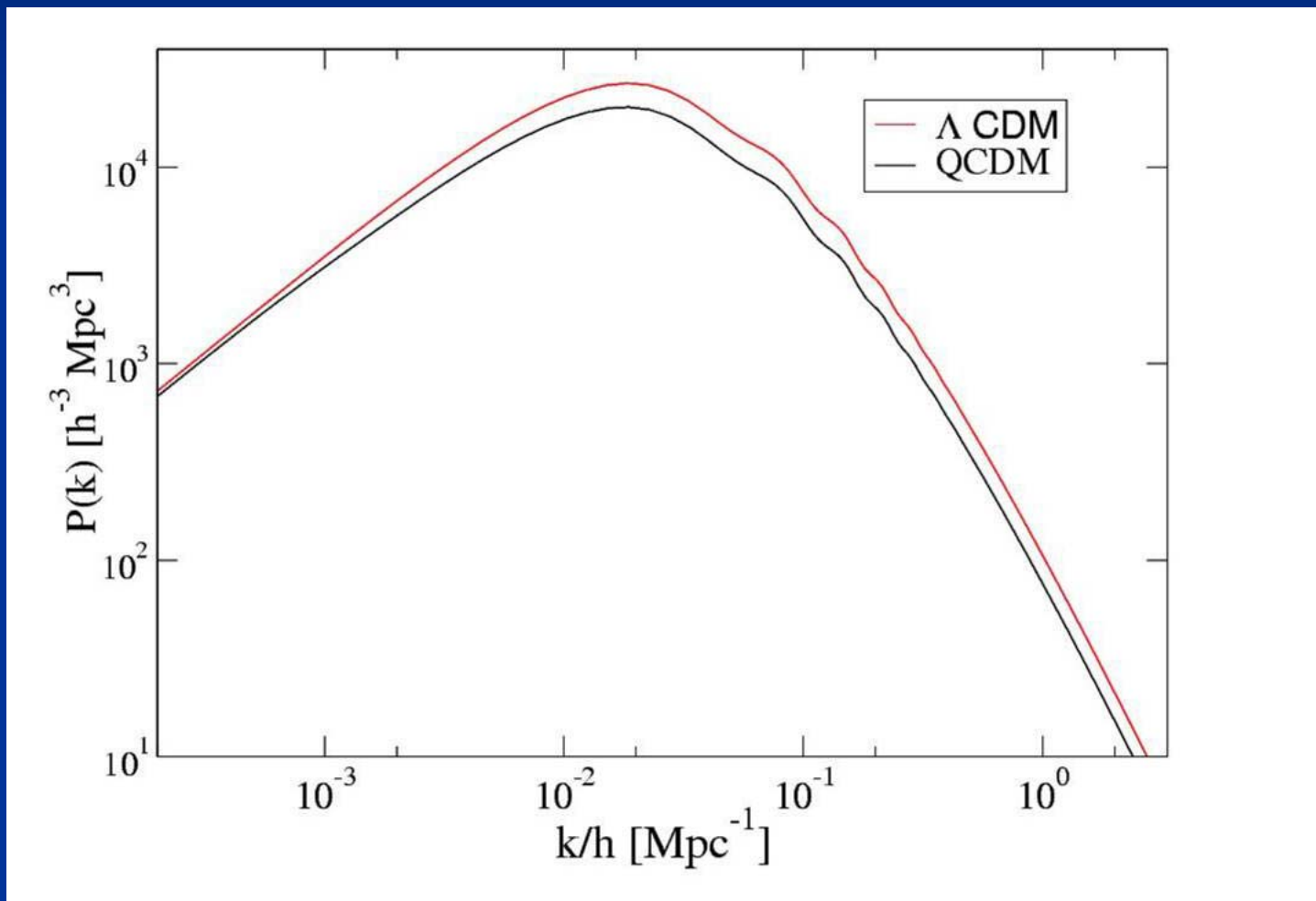
- Matter dominated universe with constant Ω_h :

$$\Delta\rho \sim a^{1-\frac{\epsilon}{2}}, \quad \epsilon = \frac{5}{2}\left(1 - \sqrt{1 - \frac{24}{25}\Omega_h}\right)$$

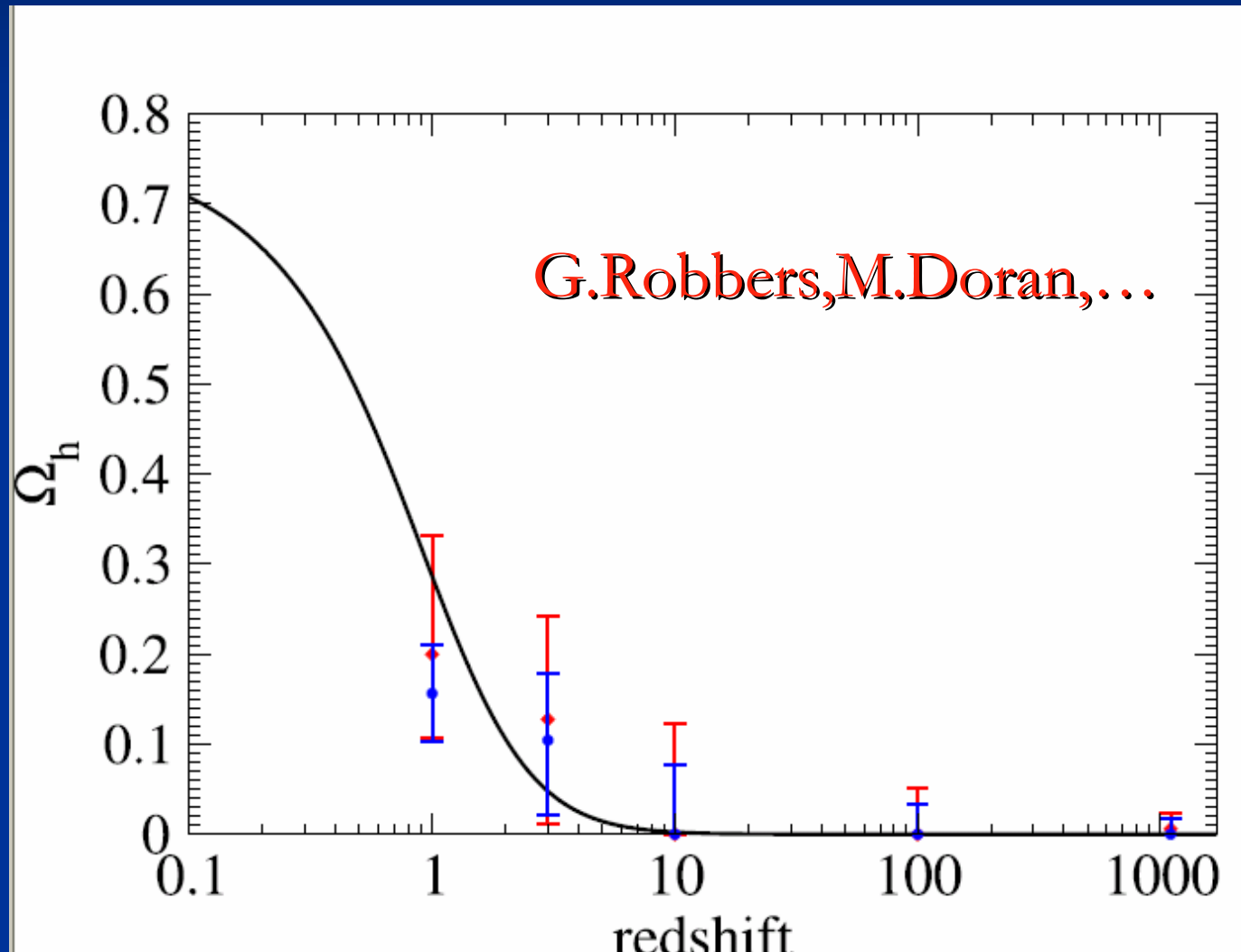
P.Ferreira,M.Joyce

- Dark energy slows down structure formation
→ $\Omega_h < 10\%$ during structure formation

Early quintessence slows down the growth of structure



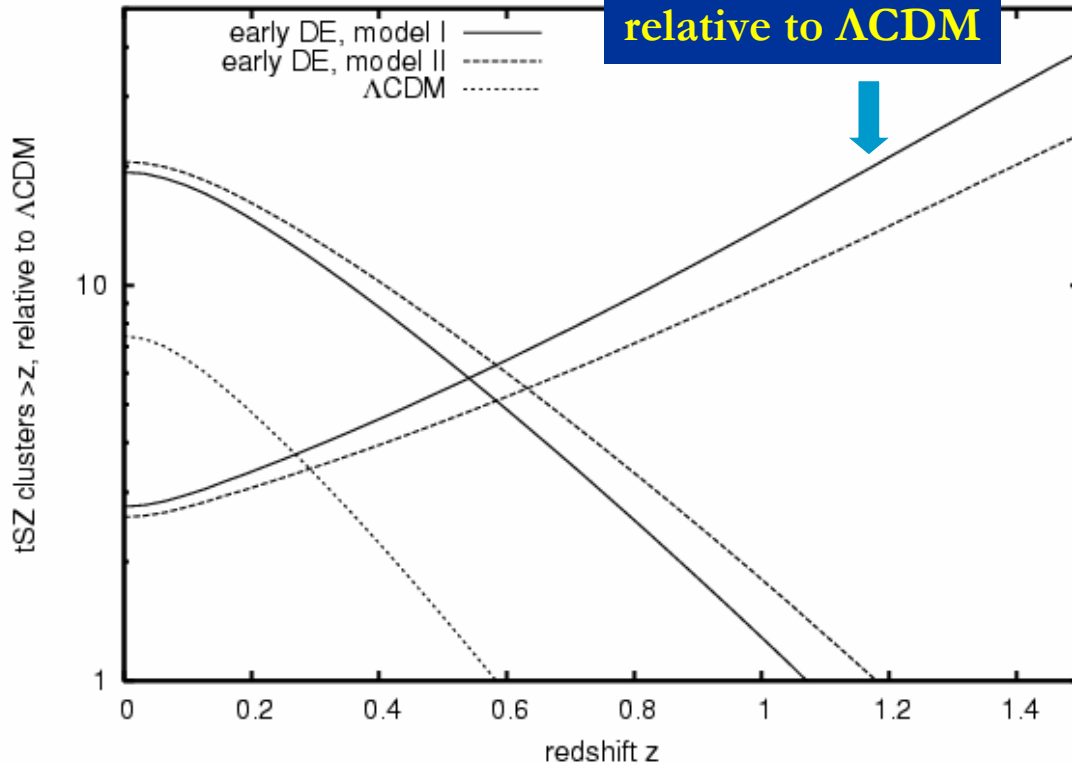
interpolation of Ω_h



Little Early Dark Energy can make large effect !

Non – linear enhancement

Cluster number
relative to Λ CDM



Two models with
4% Dark Energy
during structure
formation

Fixed σ_8
(normalization
dependence !)

More clusters at high redshift !

Bartelmann, Doran, ...

Conclusions

- Cosmic event triggers qualitative change in evolution of cosmon
- Cosmon stops changing after neutrinos become non-relativistic
- Explains why now
- Cosmological selection
- Model can be distinguished from cosmological constant

How to distinguish Q from Λ ?

A) Measurement $\Omega_h(z) \iff H(z)$

i) $\Omega_h(z)$ at the time of structure formation, CMB - emission or nucleosynthesis

ii) equation of state $w_h(\text{today}) > -1$

B) Time variation of fundamental “constants”

C) Apparent violation of equivalence principle

D) Possible coupling between Dark Energy and Dark Mater

Cosmodynamics

Cosmon mediates new long-range interaction

Range : size of the Universe – horizon

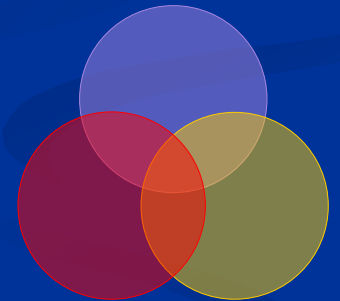
Strength : weaker than gravity

photon	electrodynamics
--------	-----------------

graviton	gravity
----------	---------

cosmon	cosmodynamics
--------	---------------

Small correction to Newton's law

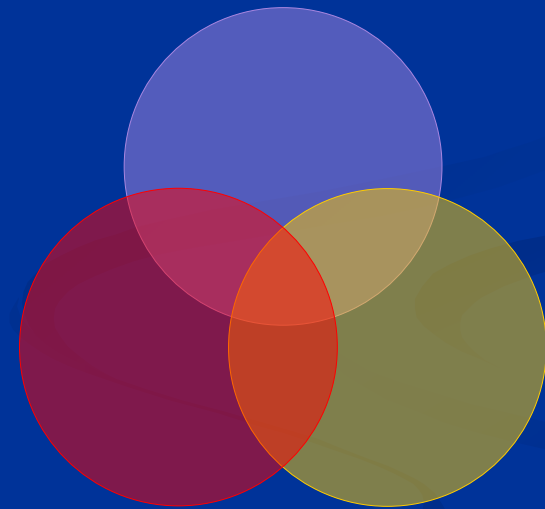


Quintessence and time variation of fundamental constants

Generic prediction

Strength unknown

Strong, electromagnetic, weak interactions



gravitation

cosmodynamics

C.Wetterich ,
Nucl.Phys.B302,645(1988)

Time varying constants

- It is not difficult to obtain quintessence potentials from higher dimensional or string theories
- Exponential form rather generic
(after Weyl scaling)
- But most models show too strong time dependence of constants !

Are fundamental “constants” time dependent ?

Fine structure constant α (electric charge)

Ratio electron mass to proton mass

Ratio nucleon mass to Planck mass

Quintessence and Time dependence of “fundamental constants”

- Fine structure constant depends on value of
cosmon field : $\alpha(\varphi)$


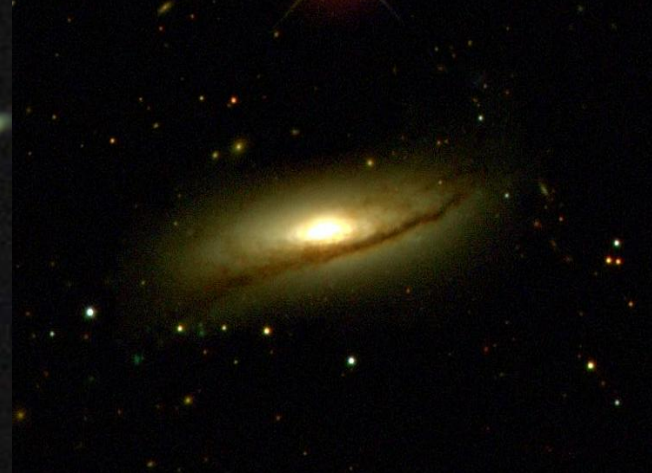
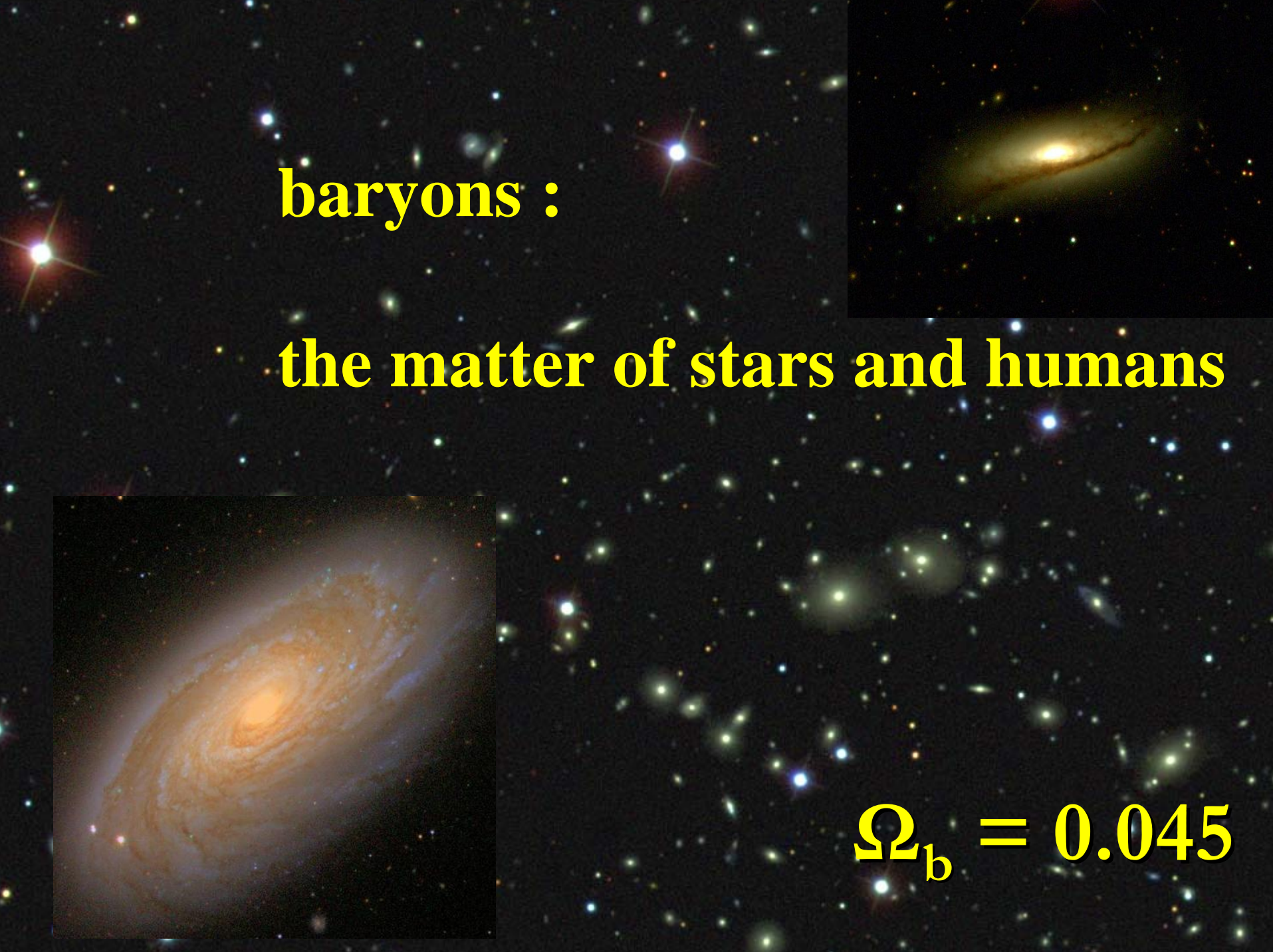
*(similar in standard model: couplings depend on
value of Higgs scalar field)*

- Time evolution of φ 
Time evolution of α

Jordan,...

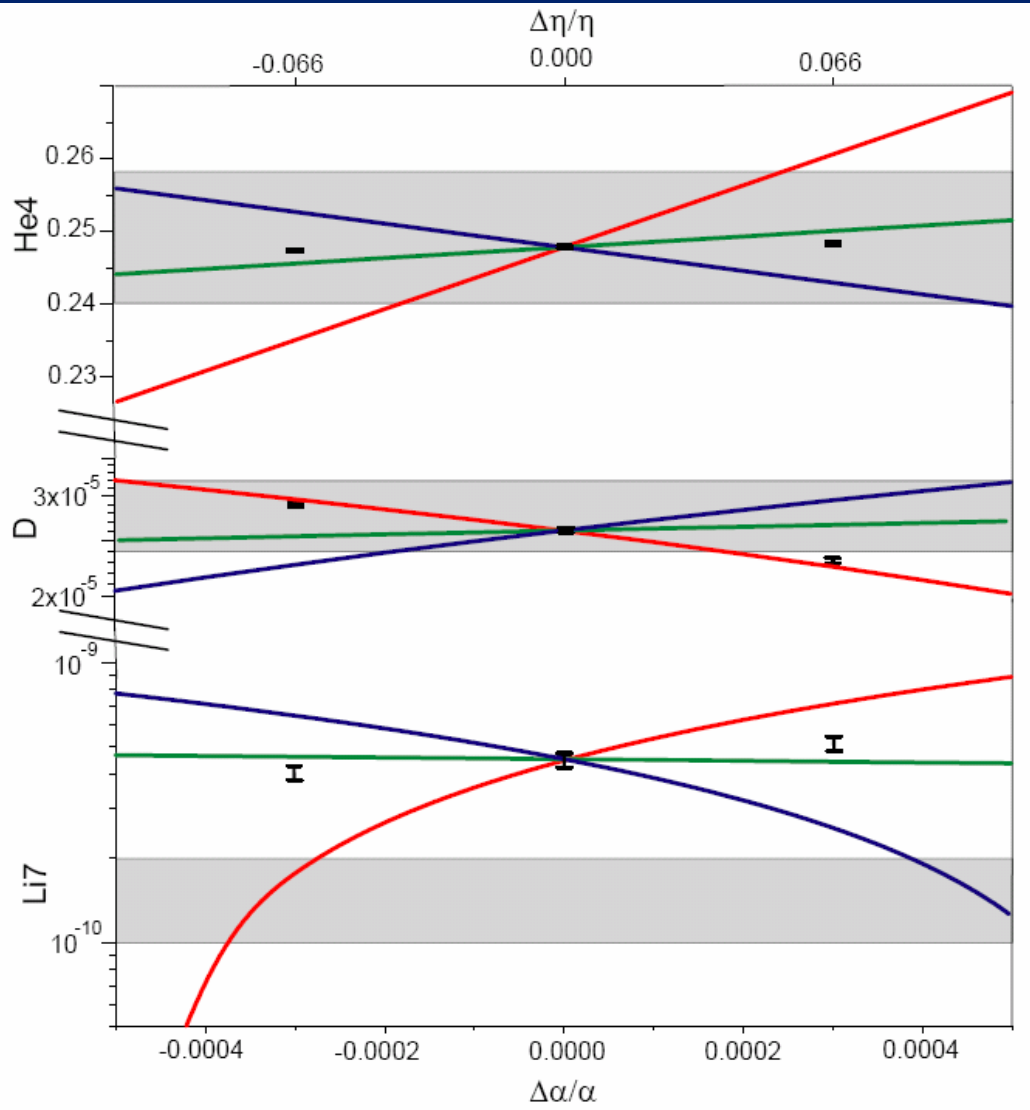
baryons :

the matter of stars and humans


$$\Omega_b = 0.045$$

primordial abundances for three GUT models

He



D

Li

present
observations :
 1σ

T.Dent,
S.Stern,...

three GUT models

- unification scale \sim Planck scale
- 1) All particle physics scales $\sim \Lambda_{\text{QCD}}$
- 2) Fermi scale and fermion masses \sim unification scale
- 3) Fermi scale varies more rapidly than Λ_{QCD}

$\Delta\alpha/\alpha \approx 4 \cdot 10^{-4}$ allowed for GUT 1 and 3 , larger for GUT 2

$\Delta\ln(M_n/M_p) \approx 40 \Delta\alpha/\alpha \approx 0.015$ allowed

time varying Fermi scale

$$U = U_0(\varphi) + \frac{\lambda}{2}(d^2 - d_0^2)^2 + \frac{1}{2}M_t^2(\varphi)t^2 - \gamma d^2 t$$

$$M_t^2(\varphi) = \bar{M}_t^2 \left[1 - \exp\left(-\frac{\epsilon}{M}(\varphi - \varphi_t)\right) \right]$$

yields triplet expectation value
as function of doublet

$$t = \gamma \frac{d^2}{M_t^2}$$

insert :

$$U(\varphi, d, t(d, \varphi)) = U_0(\varphi) + \frac{\lambda}{2}(d^2 - d_0^2)^2 - \frac{\gamma^2 d^4}{2M_t^2(\varphi)}$$

$$d^2(\varphi) = d_0^2 \left(1 - \frac{\gamma^2}{\lambda M_t^2(\varphi)} \right)^{-1}$$

time varying electron mass

$$\partial_t \ln m_e \approx -\frac{R}{2} \partial_t \ln s \approx -\frac{R}{2} \partial_t \ln \rho_\nu \approx \frac{3R}{4} H$$

$$R = \gamma^2 / (\lambda M_t^2)$$

time variation of quantities
not related to triplet

$$\frac{\delta X}{X} = -\frac{m_\nu(t_0)}{12\text{eV}} \frac{\delta}{\alpha} ((1+z)^{3/2} - 1)$$

Time variation of coupling constants
must be tiny –

would be of very high significance !

Possible signal for Quintessence

Summary

- o $\Omega_h = 0.75$
- o Q/Λ : dynamical und static dark energy will be distinguishable
- o growing neutrino mass can explain why now problem
- o Q : time varying fundamental coupling “constants”
violation of equivalence principle

????????????????????????????????

Are dark energy and dark matter related ?

Can Quintessence be explained in a fundamental unified theory ?

**Quintessence and solution of
cosmological constant
problem should be related !**

End

A few references

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R.Caldwell,R.Dave,P.Steinhardt , Phys.Rev.Lett.80,1582(1998)

P.Steinhardt,L.Wang,I.Zlatev , Phys.Rev.Lett.82,896(1999)

approximate late solution

variables :

$$s = -\alpha(\varphi - \varphi_t)/M,$$
$$x = \ln a$$

$$\partial_x \ln \rho_\nu + \partial_x \ln s = -3, \quad \partial_x \ln \rho_m = -3$$

$$\rho_\nu = \frac{c_\nu}{sa^3}, \quad \rho_m = \frac{\rho_{m,0}}{a^3}$$

approximate smooth solution
(averaged over oscillations)

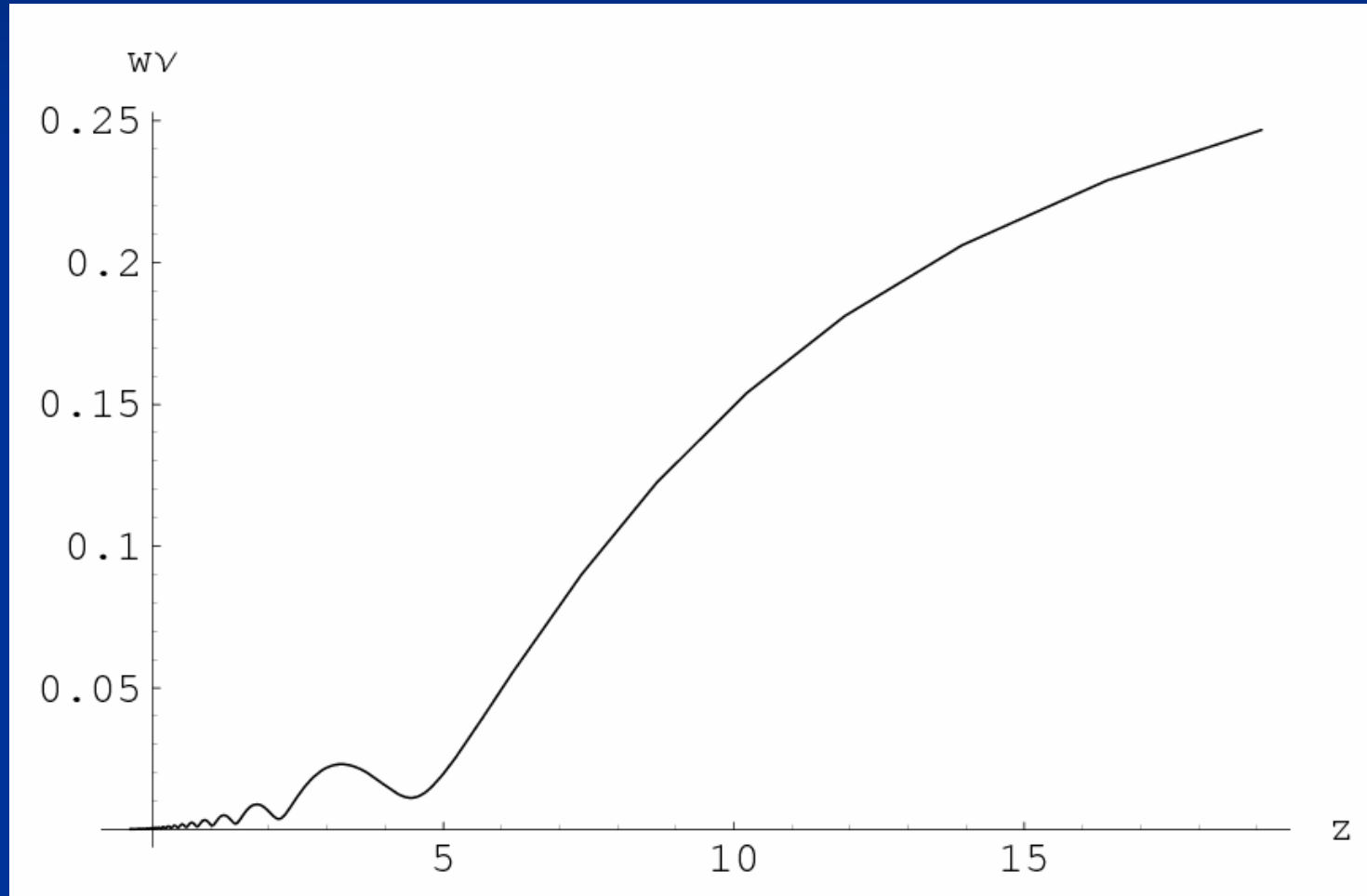
$$s^{(0)}(x) = \left(\frac{c_\nu}{V_t}\right)^{1/2} e^{-\frac{3x}{2}} = \frac{\tilde{\rho}_\nu(x)}{V_t}$$

$$s_0^{(0)} = \left(\frac{c_\nu}{V_t}\right)^{1/2} = \frac{\tilde{\rho}_{\nu,0}}{V_t} \approx \frac{\Omega_\nu(t_0)}{\Omega_h(t_0)}$$

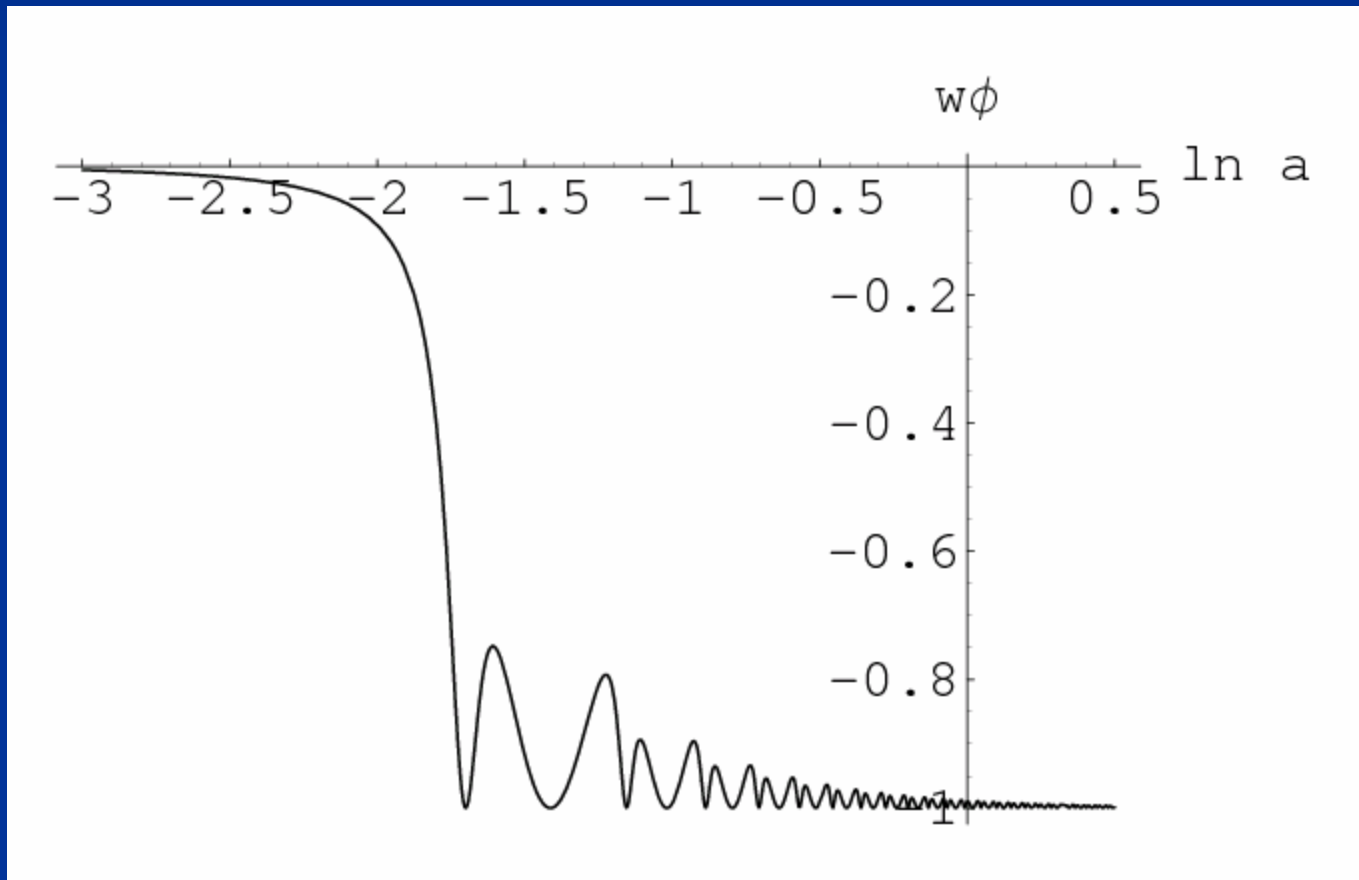
dark energy fraction

$$\tilde{\Omega}_h(a) = \begin{cases} \frac{\tilde{\Omega}_{h,0}a^3 + 2\Omega_{\nu,0}(a^{3/2} - a^3)}{1 - \tilde{\Omega}_{h,0}(1 - a^3) + 2\Omega_{\nu,0}(a^{3/2} - a^3)} & \text{for } a > a_c \\ \frac{3}{a^2} & \text{for } a < a_c \end{cases}$$

neutrino equation of state



cosmon equation of state



fixed point behaviour : apparent tuning

$$V(\varphi) = U_0(\varphi) - \frac{\lambda d_0^4 \gamma^2}{2(\lambda M_t^2(\varphi) - \gamma^2)}$$

$$V(\varphi) = U_0(\varphi) - \frac{m_\nu(\varphi) d^2 \gamma}{2h_L}$$