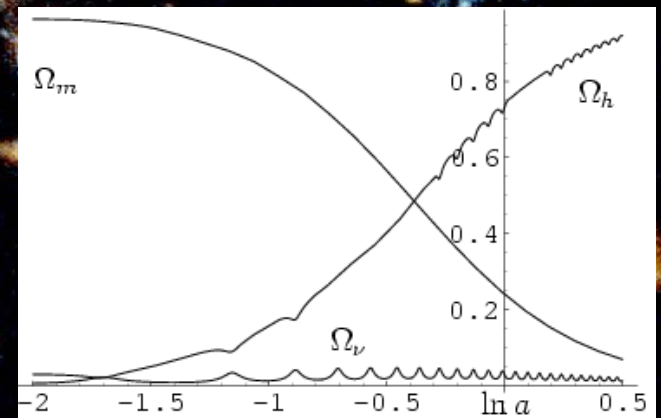
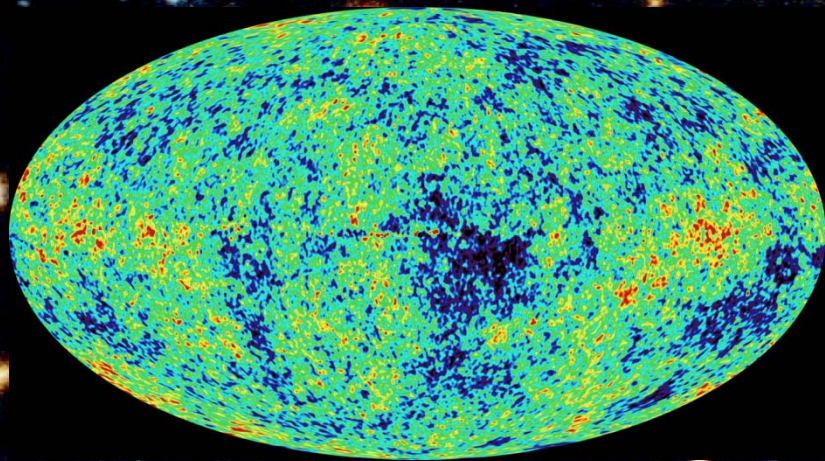


Have neutrinos to do with Dark Energy ?



Why neutrinos may play a role

Mass scales :

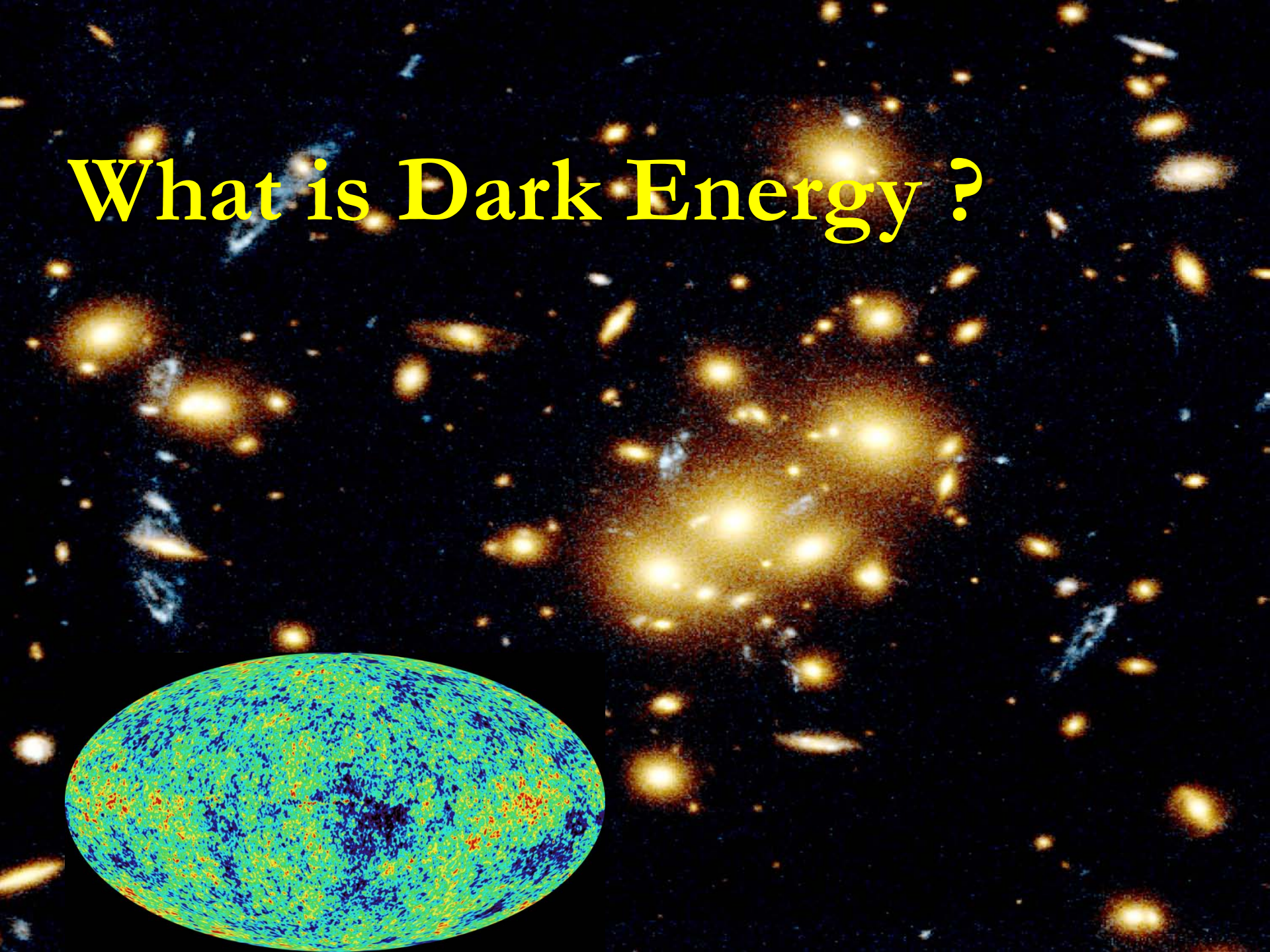
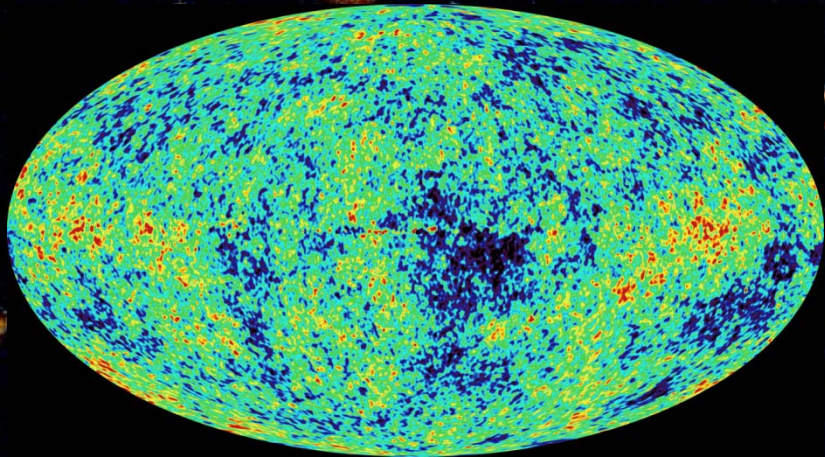
Dark Energy density : $\rho \sim (2 \times 10^{-3} \text{ eV})^{-4}$.

Neutrino mass : eV or below.

Cosmological trigger : Neutrinos became non-relativistic only in the late Universe .

Neutrinos can have coupling to cosmon stronger than gravity.

What is Dark Energy ?



Dark Energy dominates the Universe

Energy - density in the Universe

=

Matter + Dark Energy

25 % + 75 %

Composition of the universe

Atoms : $\Omega_b = 0.045$

Dark Matter : $\Omega_{dm} = 0.225$

Dark Energy : $\Omega_h = 0.73$

critical density

- $\rho_c = 3 H^2 M^2$

critical energy density of the universe

(M : reduced Planck-mass , H : Hubble parameter)

- $\Omega_b = \rho_b / \rho_c$

fraction in baryons

energy density in baryons over critical
energy density

$$H = \dot{a}/a$$

Matter : Everything that clumps

The image displays a vast field of galaxies, characteristic of a galaxy cluster. The galaxies are densely packed and exhibit a wide variety of colors, including bright yellow, white, blue, and red. Some galaxies are large and elliptical, while others are smaller and more irregular. The background is a deep black, punctuated by the light of the galaxies. The overall appearance is that of a rich, multi-colored stellar population.

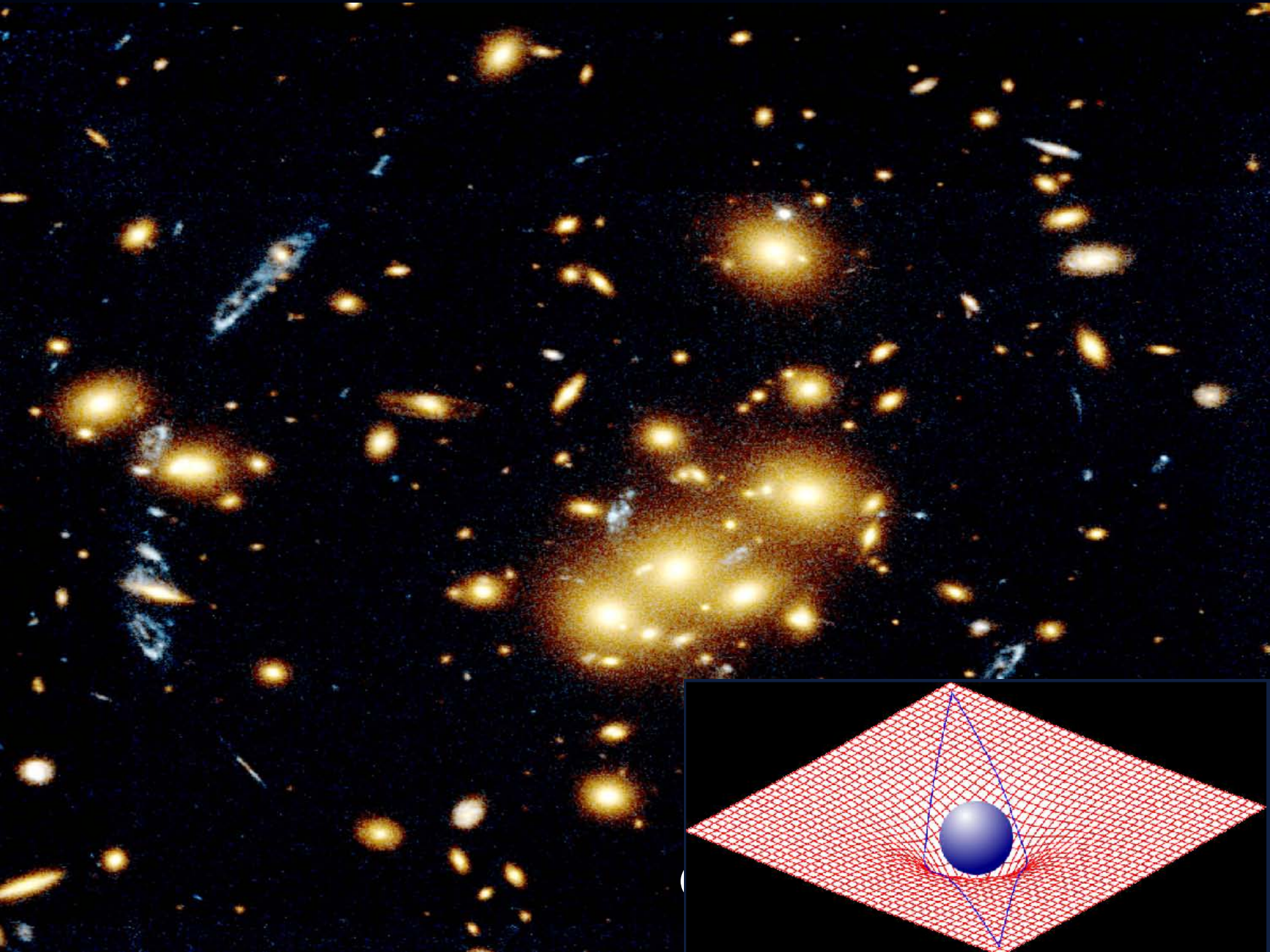
Abell 2255 Cluster
~300 Mpc

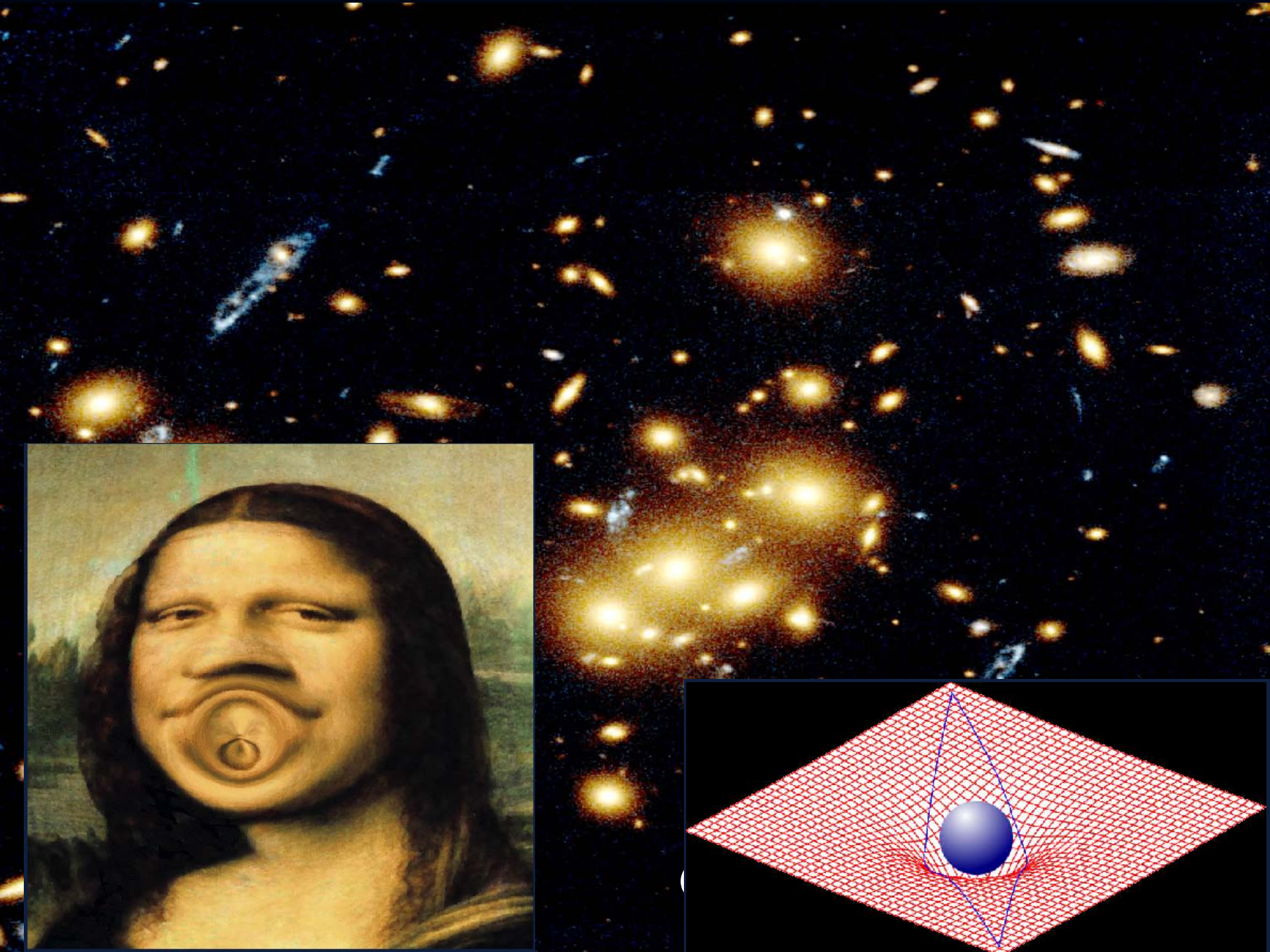
Dark Matter

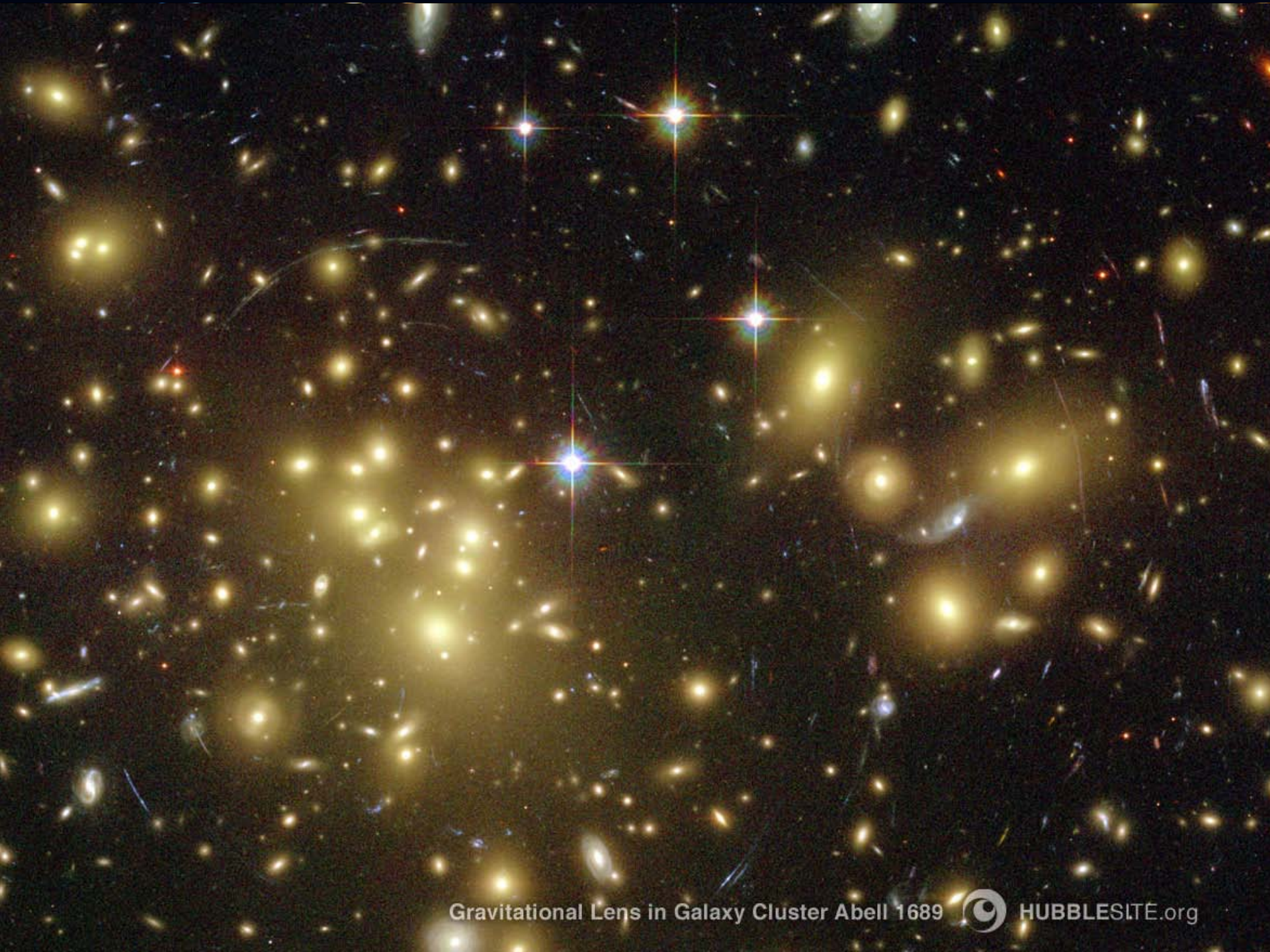
- $\Omega_m = 0.27$ total “matter”
- Most matter is dark !
- So far tested only through gravity
- Every local mass concentration →
gravitational potential
- Orbits and velocities of stars and galaxies →
measurement of gravitational potential
and therefore of local matter distribution

A deep-field astronomical image showing a large number of galaxies. In the center, there is a prominent, bright, yellowish-white region, likely a gravitational lensing event. The surrounding galaxies are mostly yellow and orange, with some blue galaxies scattered throughout. The background is dark with some faint, diffuse light.
$$\Omega_m = 0.27$$

gravitational lens , HST







Gravitational Lens in Galaxy Cluster Abell 1689



HUBBLESITE.org



Dark Matter in collision

bullet cluster

Matter : Everything that clumps

$$\Omega_m = 0.27$$

Abell 2255 Cluster
~300 Mpc



Dark Energy :

Energy density that does not clump

Photons , gravitons : insignificant

spatially flat universe

$$\Omega_{\text{tot}} = 1$$

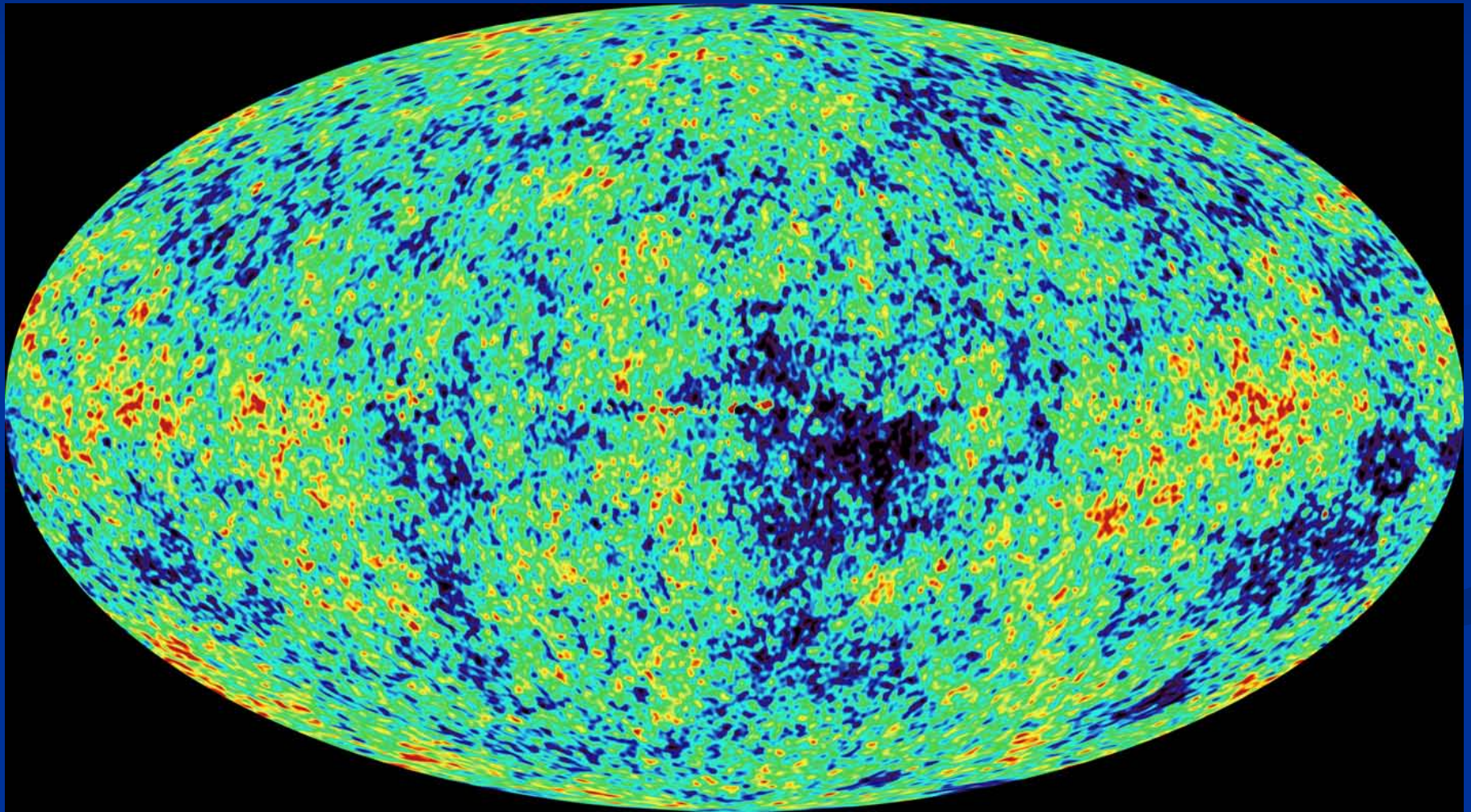
- theory (inflationary universe)

$$\Omega_{\text{tot}} = 1.0000\dots\dots\dots x$$

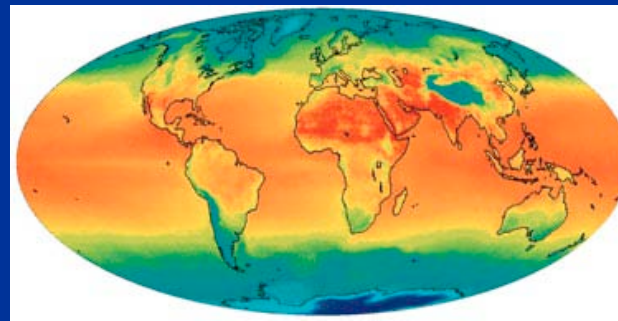
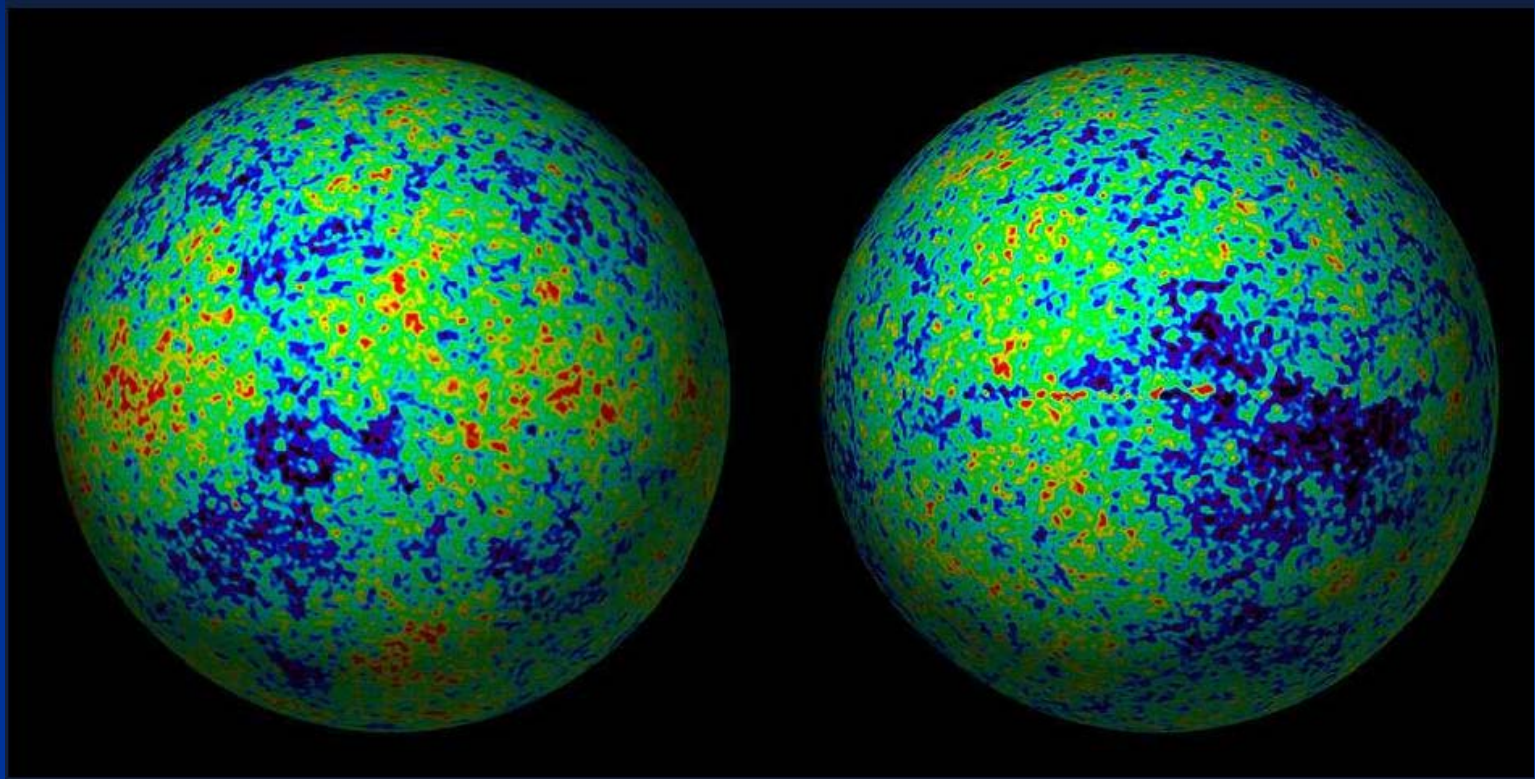
- observation (WMAP)

$$\Omega_{\text{tot}} = 1.02 (0.02)$$

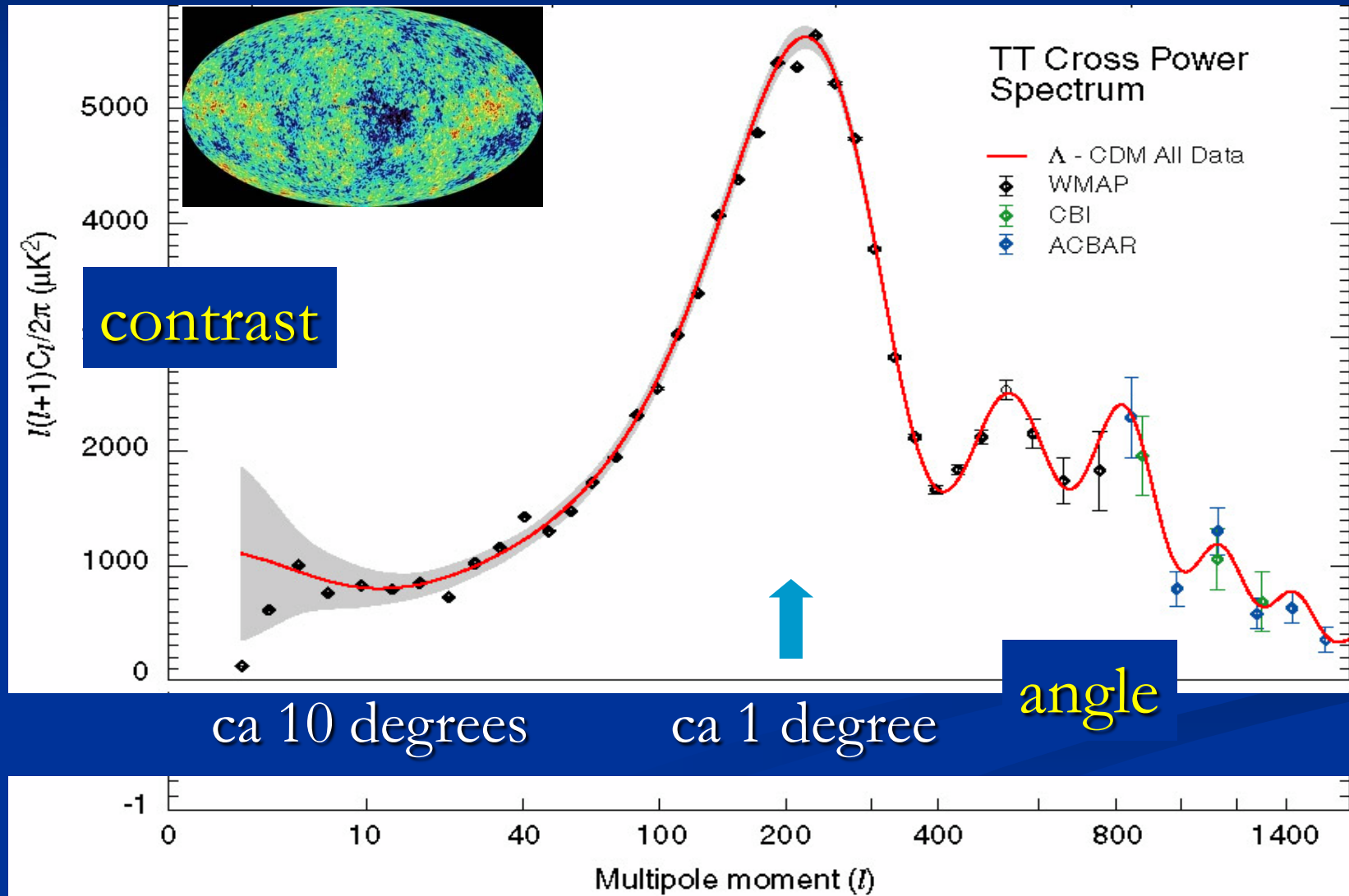
Picture of the big bang



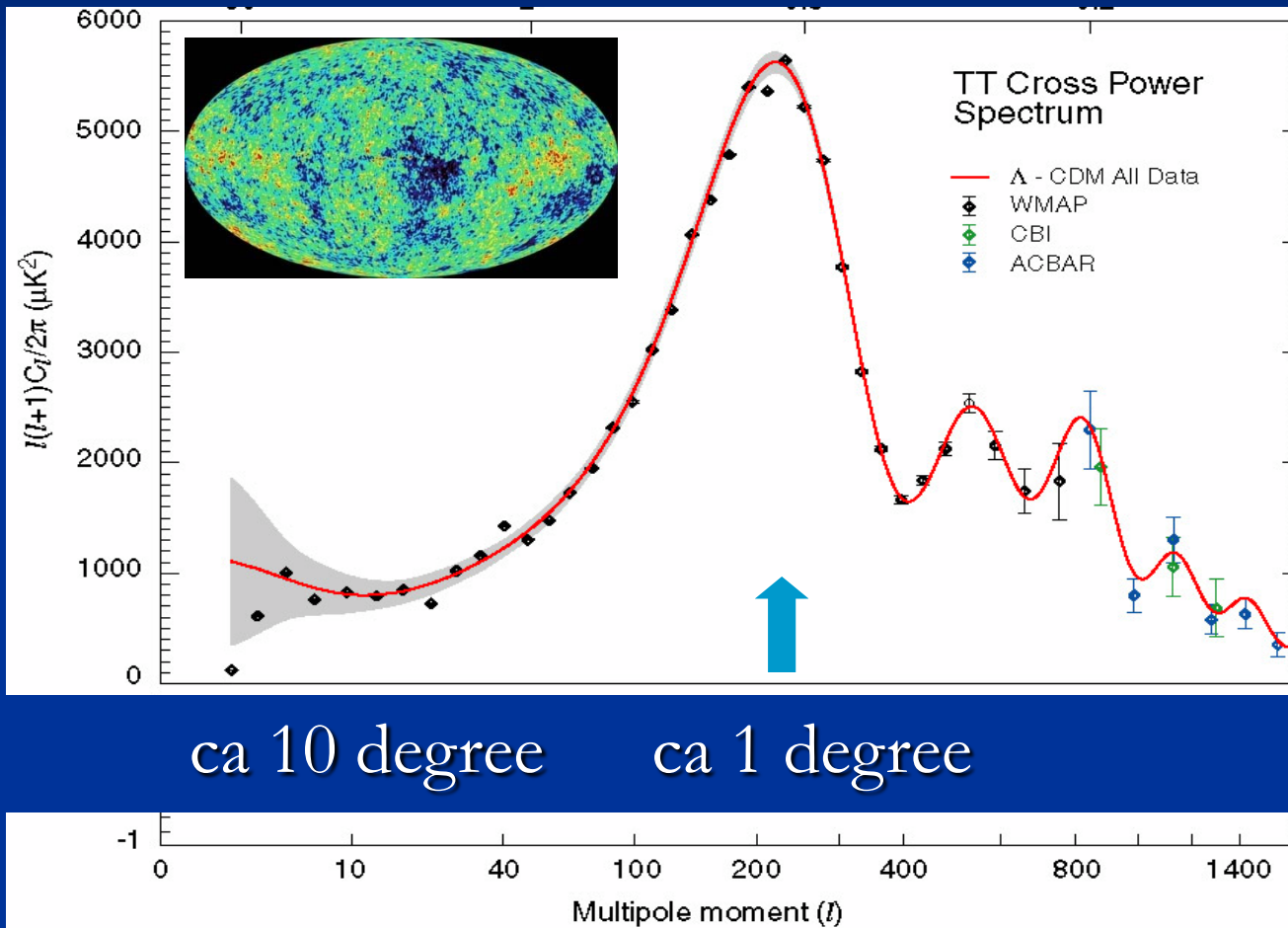
Anisotropy of background radiation : size of hot and cold spots



Size of temperature fluctuations in dependence on size of anisotropies (angle)

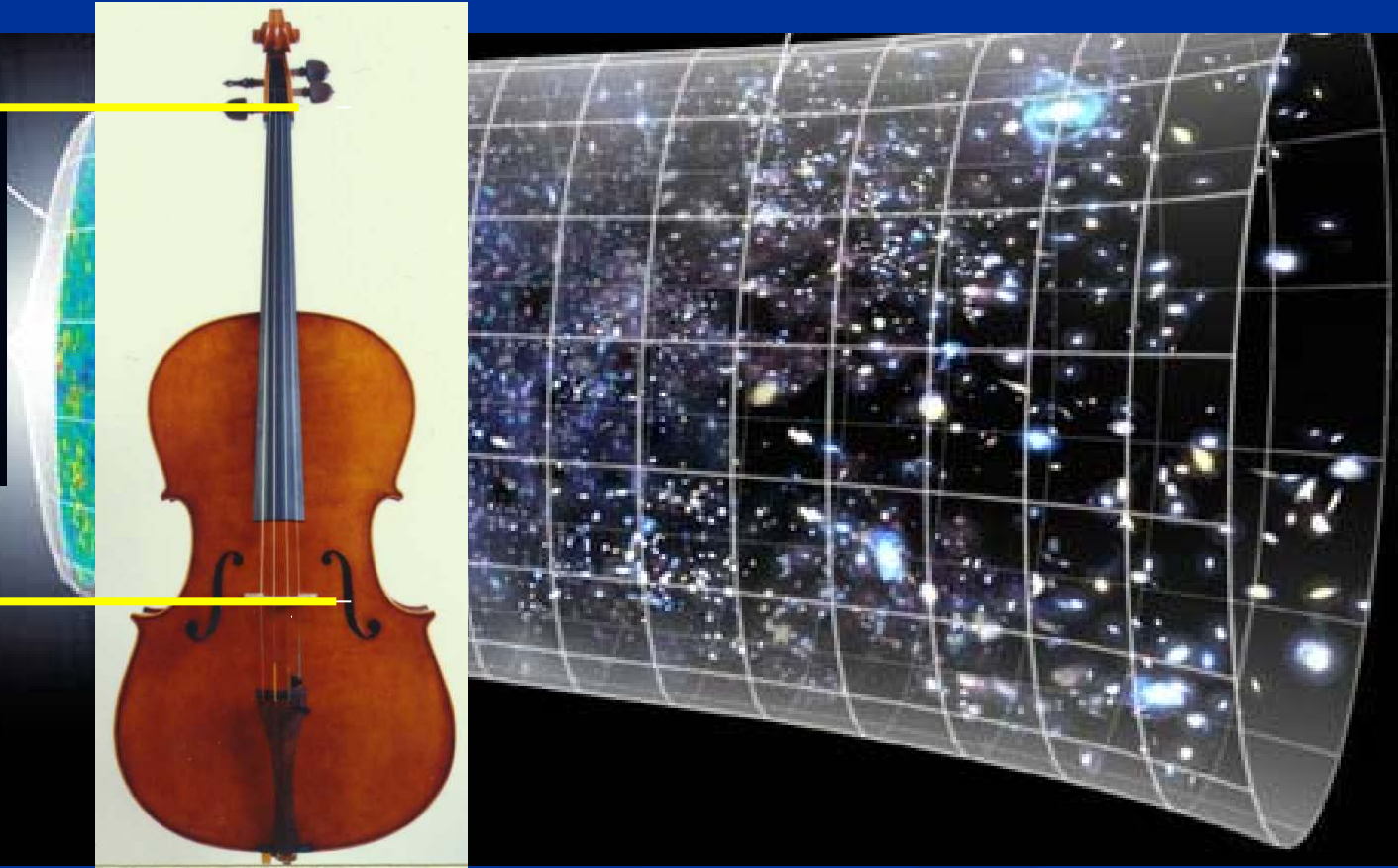


Acoustic oscillations in plasma



acoustic waves in the early Universe

length
compu-
table



$$\Omega_{\text{tot}} = 1$$



last scattering

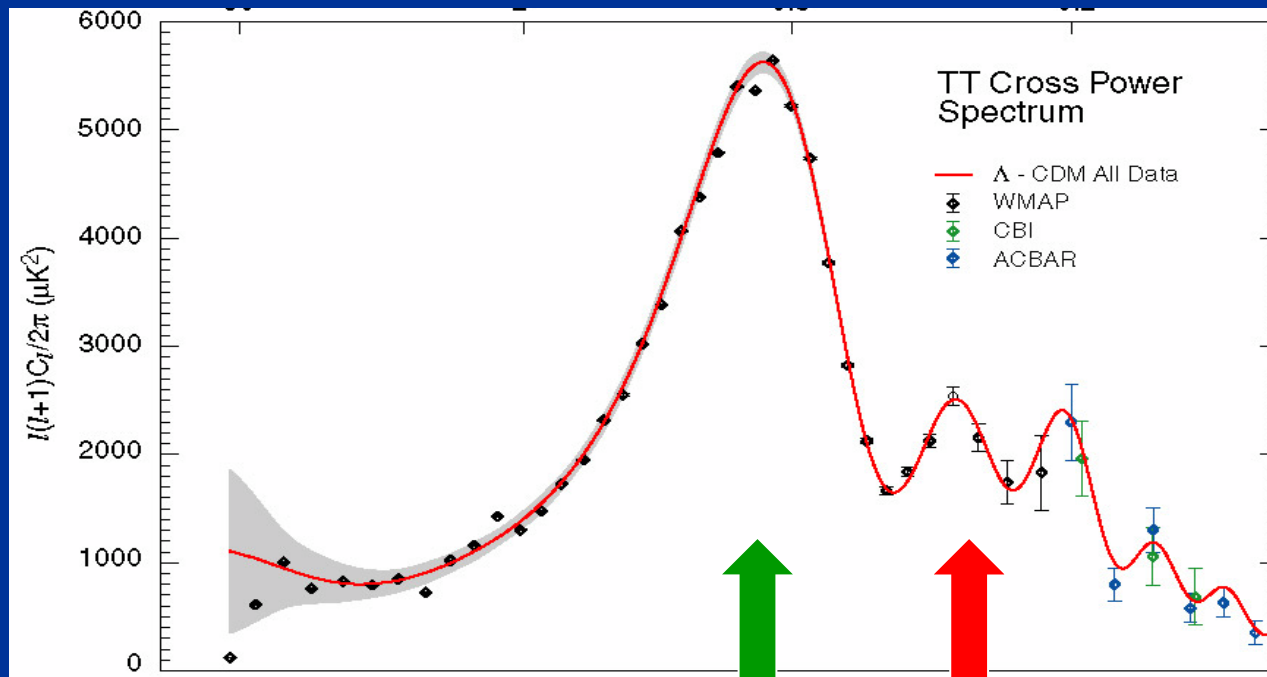
$\Omega < 1$ (open)

$\Omega = 1$ (flat)



spatially flat Universe

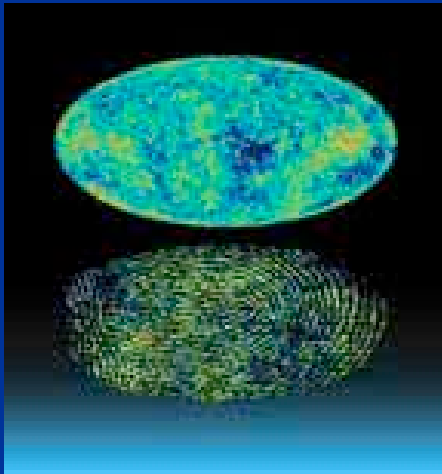
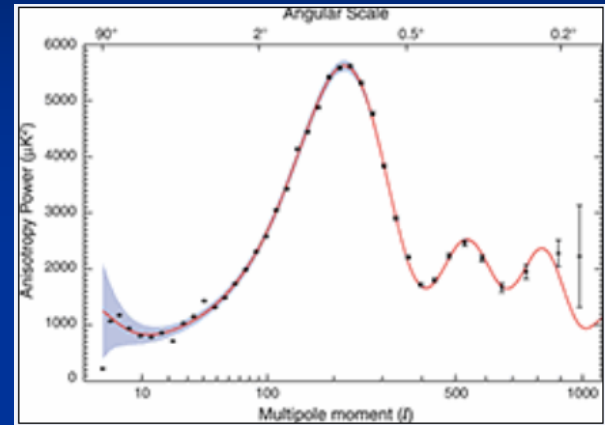
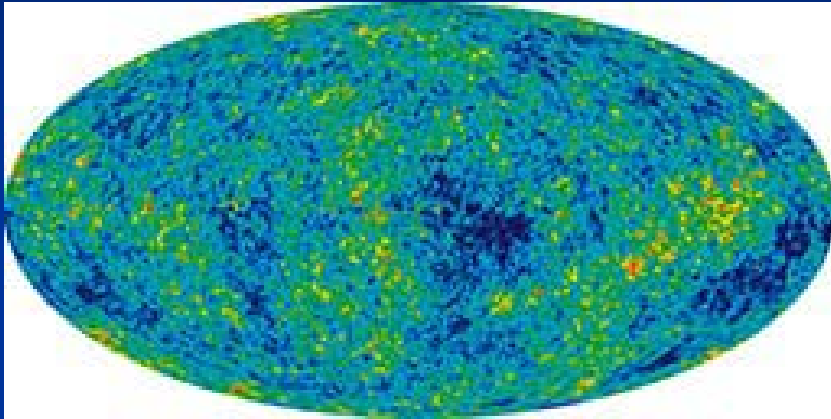
$$\Omega_{\text{tot}} = 1$$



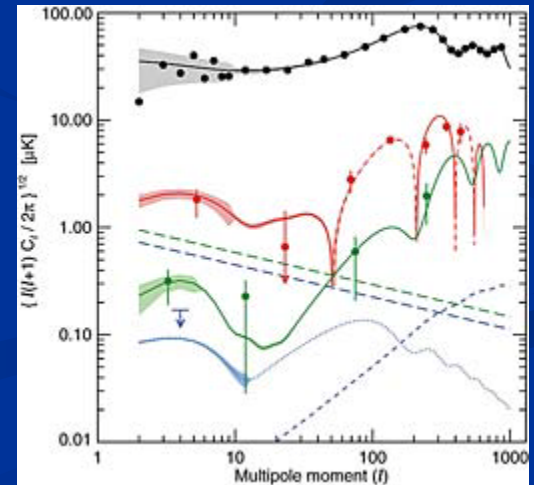
$$\Omega_{\text{tot}} = 1$$

$$\Omega_{\text{tot}} = 0.25$$

WMAP 2006



Polarization



Wilkinson Microwave Anisotropy Probe

A partnership between
NASA/GSFC and Princeton

Science Team:

NASA/GSFC

Chuck Demarest (PI)
Michael Greason
Bob Hill
Gary Hinshaw
Al Kogut
Michele Linnon
Nils Odgaard
Janet Weiland
Ed Weiland

Brown
Greg Tucker

UCLA
Neil Wright

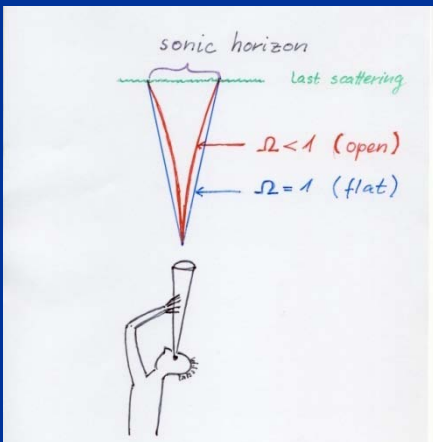
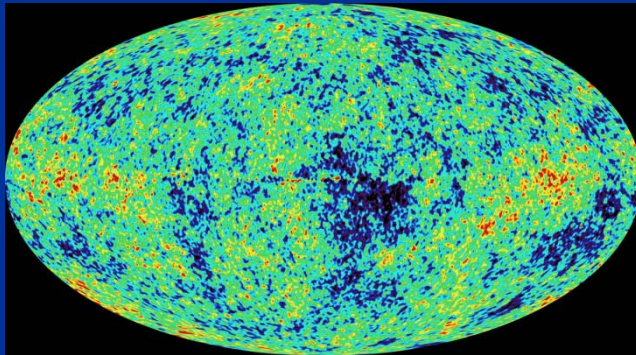


UBC
Mark Halpern

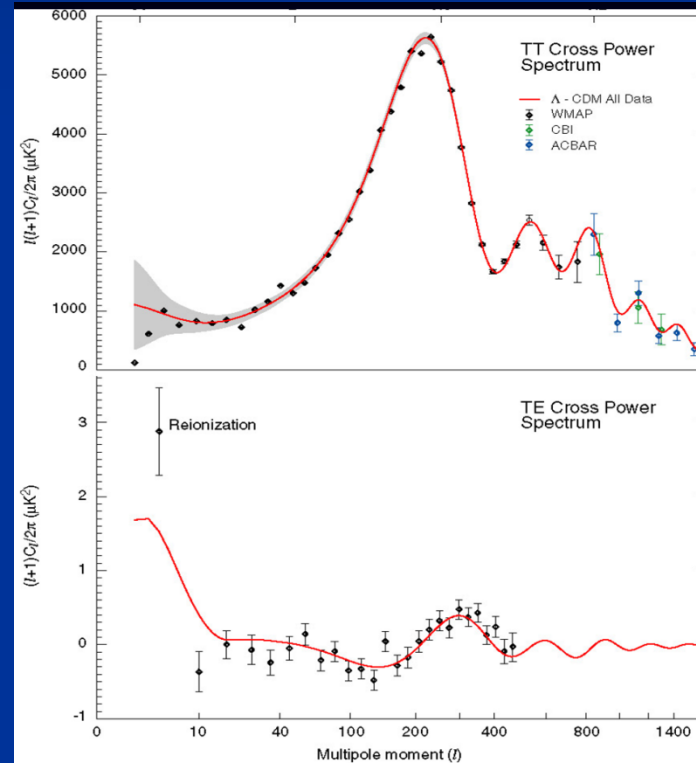
Chicago
Stephan Meyer

Princeton

Chris Barnes
Stefan Jurasik
Eiichiro Komatsu
Michael Nolta
Lynn Page
Hiranya Peiris
David Spergel
Licia Verde



$$\Omega_{\text{tot}} = 1$$



mean values

$$\Omega_{\text{tot}} = 1.02$$

$$\Omega_{\text{m}} = 0.27$$

$$\Omega_{\text{b}} = 0.045$$

$$\Omega_{\text{dm}} = 0.225$$

Dark Energy

$$\Omega_m + X = 1$$

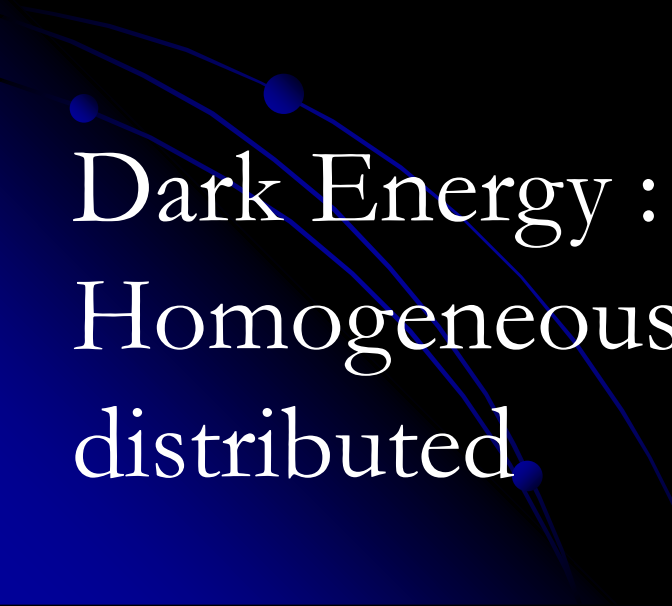
$$\Omega_m : 25\%$$

$$\Omega_h : 75\% \quad \text{Dark Energy}$$

h : homogenous , often Ω_Λ instead of Ω_h

**Space between clumps
is not empty :**

Dark Energy !



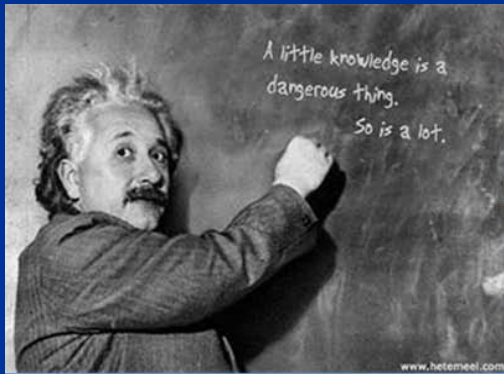
Dark Energy :
Homogeneously
distributed.

**Dark Energy density is
the same at every point of space**

“ homogeneous “

**No force in absence of matter –
“ In what direction should it draw ? “**

Einstein's equations : almost static Dark Energy predicts accelerated expansion of Universe



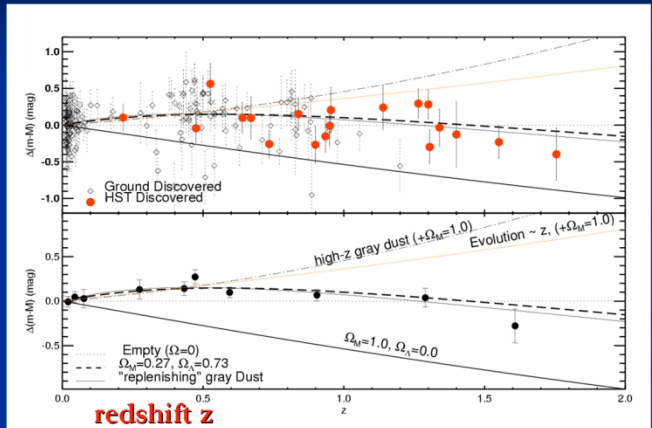
$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$



Predictions for dark energy cosmologies

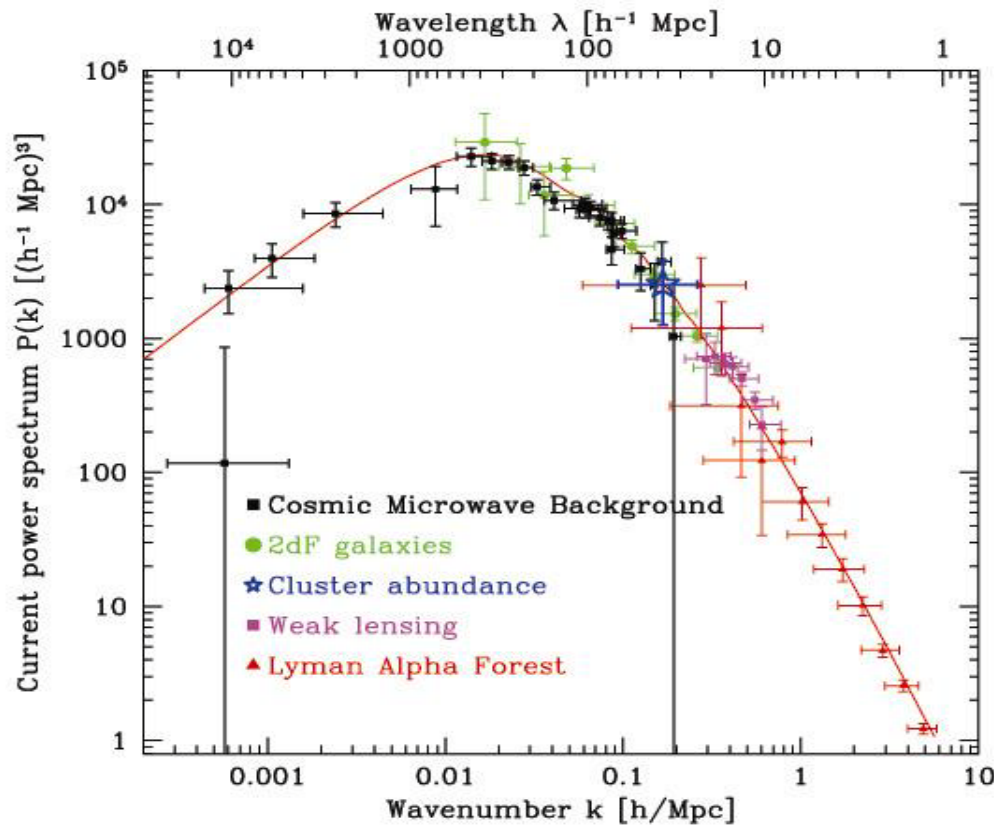
*The expansion of the Universe
accelerates today !*

Supernovae 1a Hubble diagram



Structure formation :

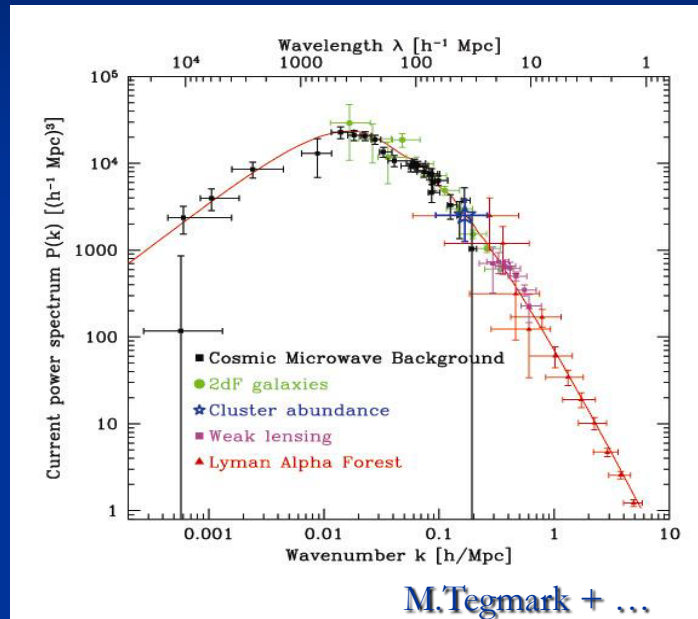
One primordial fluctuation spectrum



Waerbeke

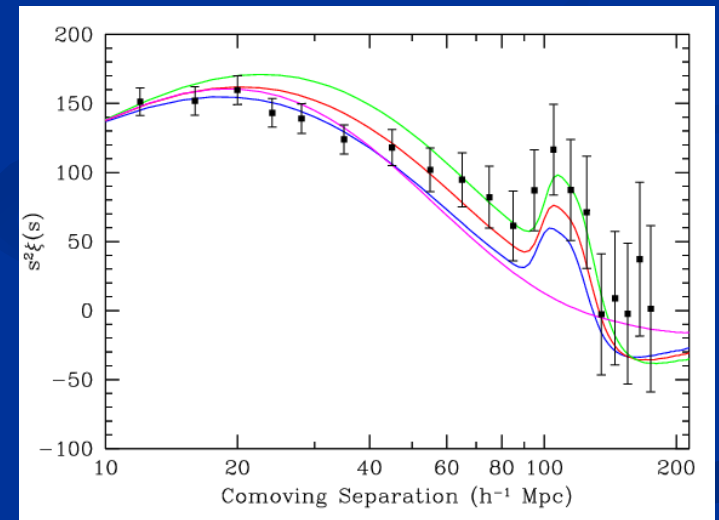
CMB agrees with
Galaxy distribution
Lyman - α
and
Gravitational
Lensing !

Power spectrum



Baryon - Peak

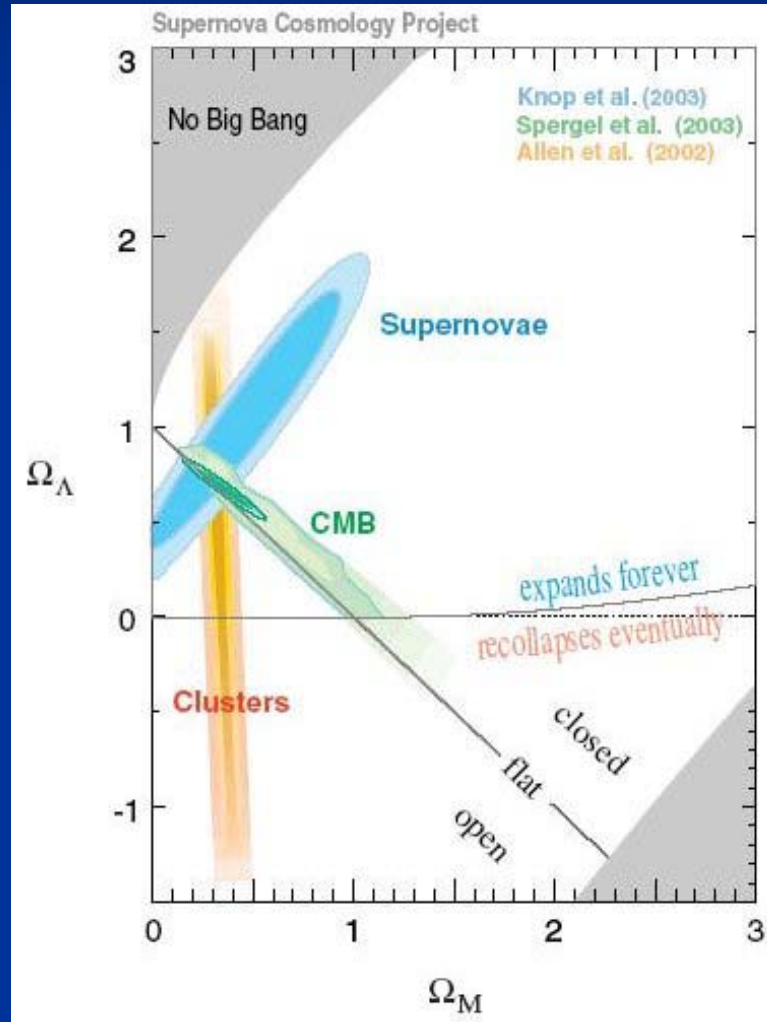
galaxy –
correlation –
function



Structure formation :
One primordial
fluctuation- spectrum

SDSS

Dark Energy : observations fit together !



Composition of the Universe

$$\Omega_b = 0.045$$

visible

clumping

$$\Omega_{dm} = 0.2$$

invisible

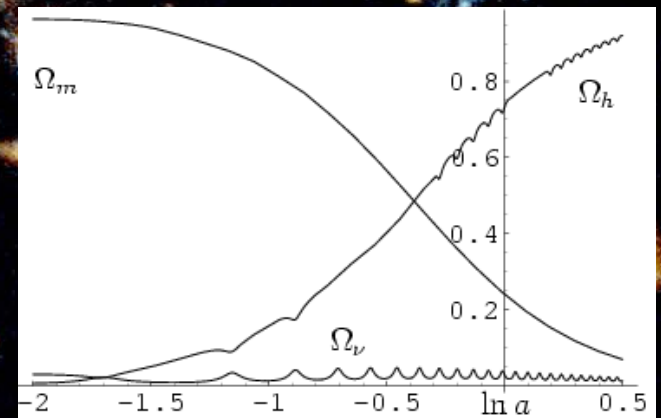
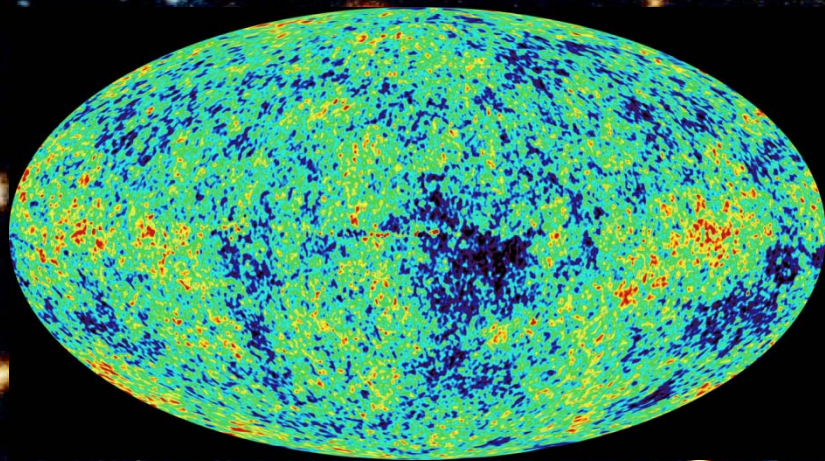
clumping

$$\Omega_h = 0.75$$

invisible

homogeneous

Dark Energy – a cosmic mystery



Cosmological Constant

- Einstein -

- Constant λ compatible with all symmetries
- No time variation in contribution to energy density
- Why so small ? $\lambda/M^4 = 10^{-120}$
- Why important just today ?

Cosmological mass scales

- Energy density

$$\rho \sim (2.4 \times 10^{-3} \text{ eV})^{-4}$$

- Reduced Planck mass

$$M = 2.44 \times 10^{27} \text{ eV}$$

- Newton's constant

$$G_N = (8\pi M^2)$$

Only ratios of mass scales are observable !

homogeneous dark energy: $\rho_h/M^4 = 6.5 \cdot 10^{-121}$

matter: $\rho_m/M^4 = 3.5 \cdot 10^{-121}$

Time evolution

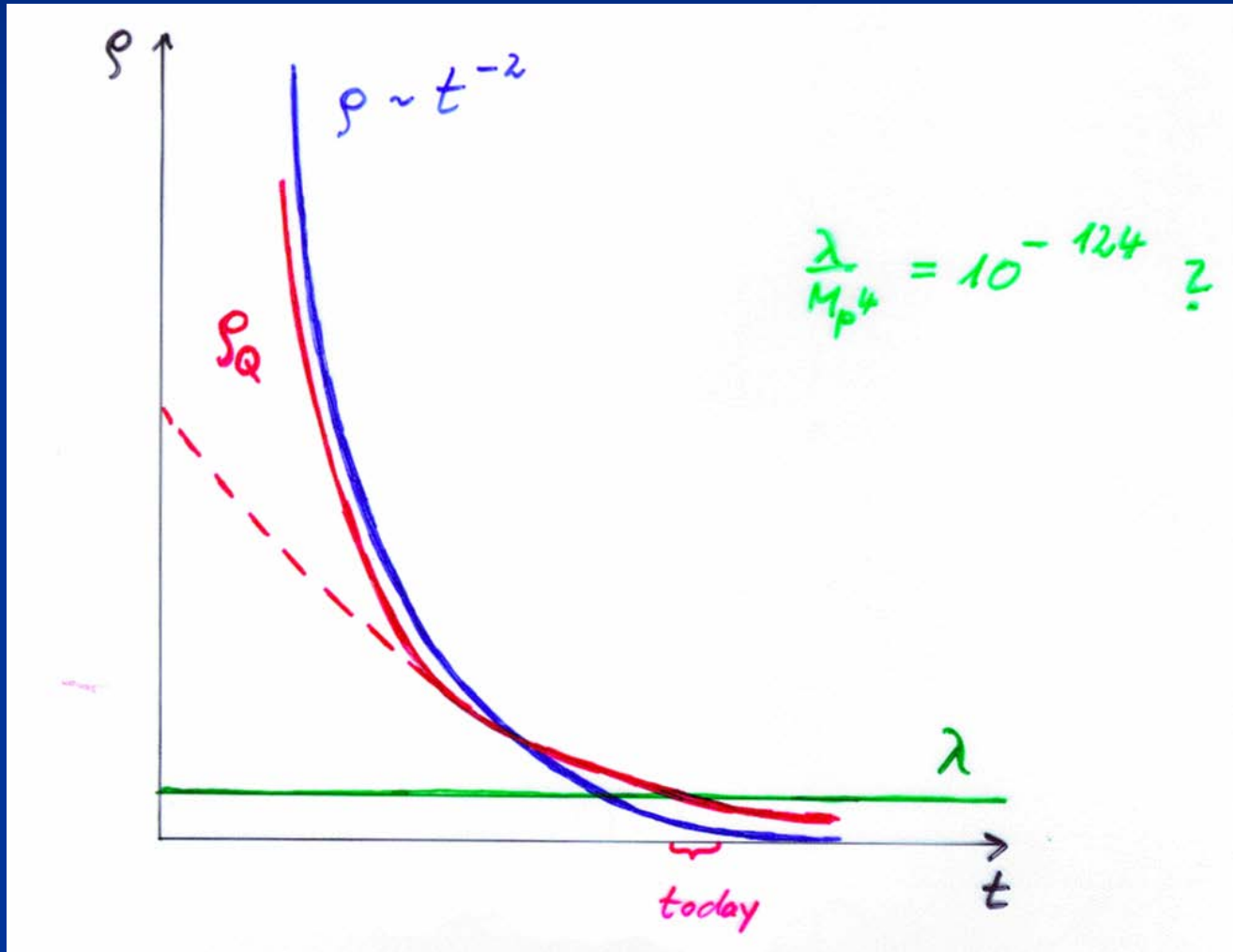
- $\rho_m/M^4 \sim a^{-3} \sim t^{-2}$ matter dominated universe
- $\rho_r/M^4 \sim a^{-4} \sim t^{-3/2}$ radiation dominated universe
- $\rho_r/M^4 \sim a^{-4} \sim t^{-2}$ radiation dominated universe

Huge age \Rightarrow small ratio

Same explanation for small dark energy?

Cosm. Const.
static

Quintessence
dynamical



Quintessence

Dynamical dark energy ,
generated by scalar field
(cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87

P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

**homogeneous dark energy
influences recent cosmology**

- of same order as dark matter -

Original models do not fit the present observations
.... modifications

Quintessence

Cosmon – Field $\varphi(x,y,z,t)$

similar to electric field , but no direction (scalar field)

Homogeneous und isotropic Universe : $\varphi(x,y,z,t) = \varphi(t)$

Potential und kinetic energy of the cosmon -field
contribute to a dynamical energy density of the Universe !

Cosmon

- *Scalar field changes its value even in the **present** cosmological epoch*
- *Potential und kinetic energy of cosmon contribute to the energy density of the Universe*
- *Time - variable dark energy :
 $\rho_b(t)$ decreases with time !*

Evolution of cosmon field

Field equations

$$\ddot{\phi} + 3H\dot{\phi} = -dV/d\phi$$

$$3M^2H^2 = V + \frac{1}{2}\dot{\phi}^2 + \rho$$

Potential $V(\varphi)$ determines details of the model

$$V(\varphi) = M^4 \exp(-\alpha\varphi/M)$$

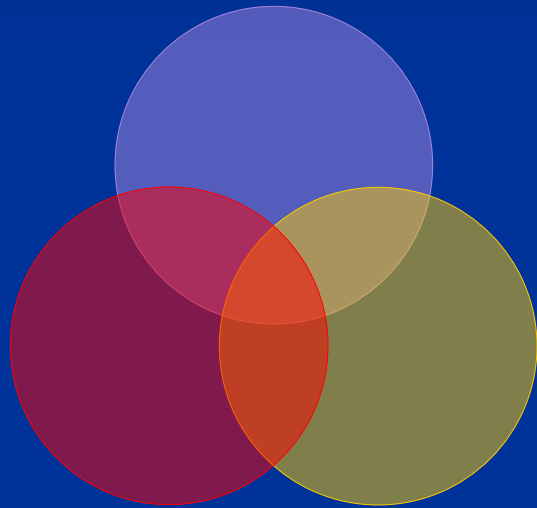
for increasing φ the potential decreases
towards zero !

Cosmon

- *Tiny mass*
- $m_c \sim H$ (depends on time !)
- *New long - range interaction*

“Fundamental” Interactions

Strong, electromagnetic, weak interactions



gravitation

cosmodynamics

On astronomical length scales:

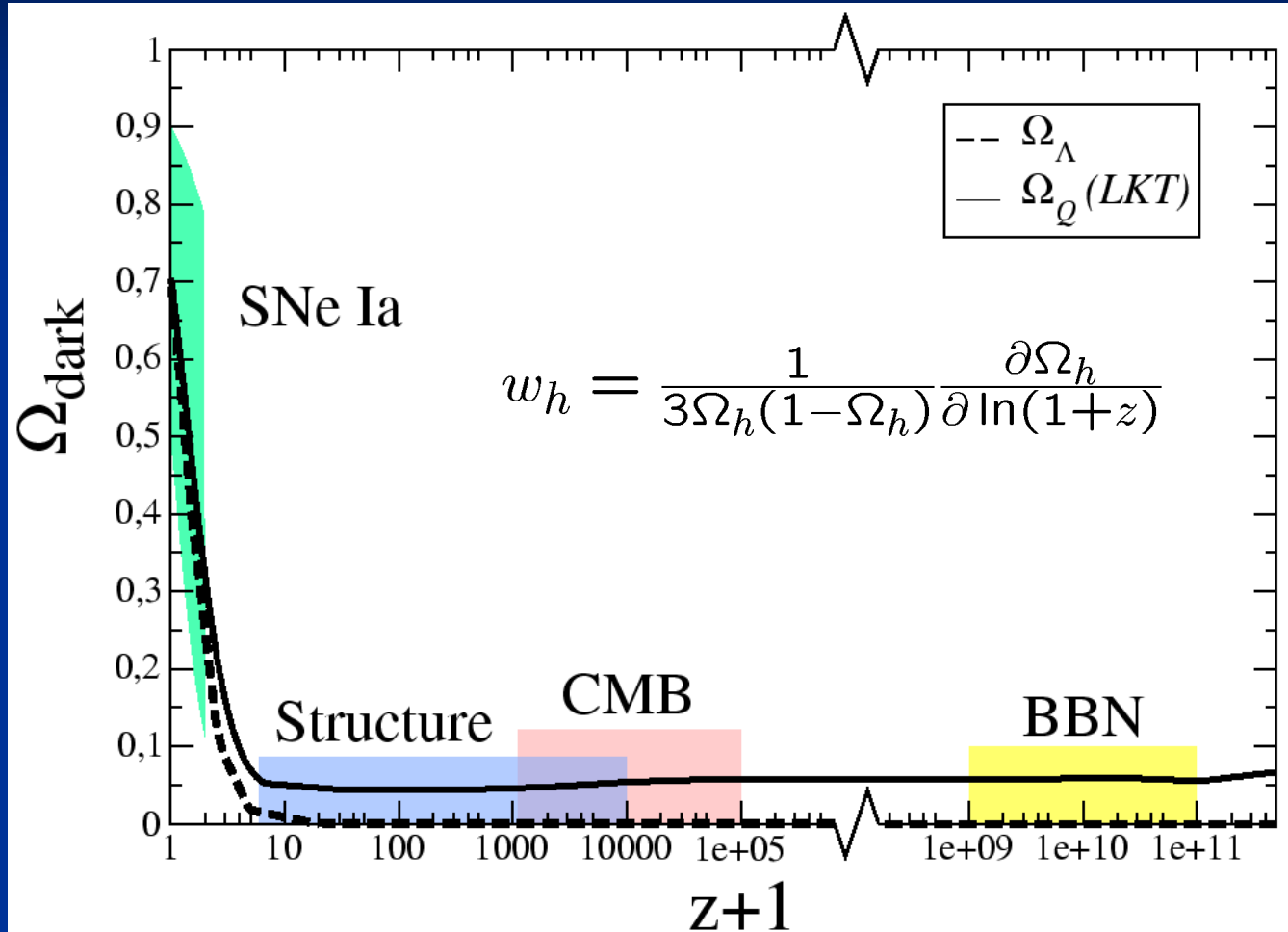
graviton

+

cosmon

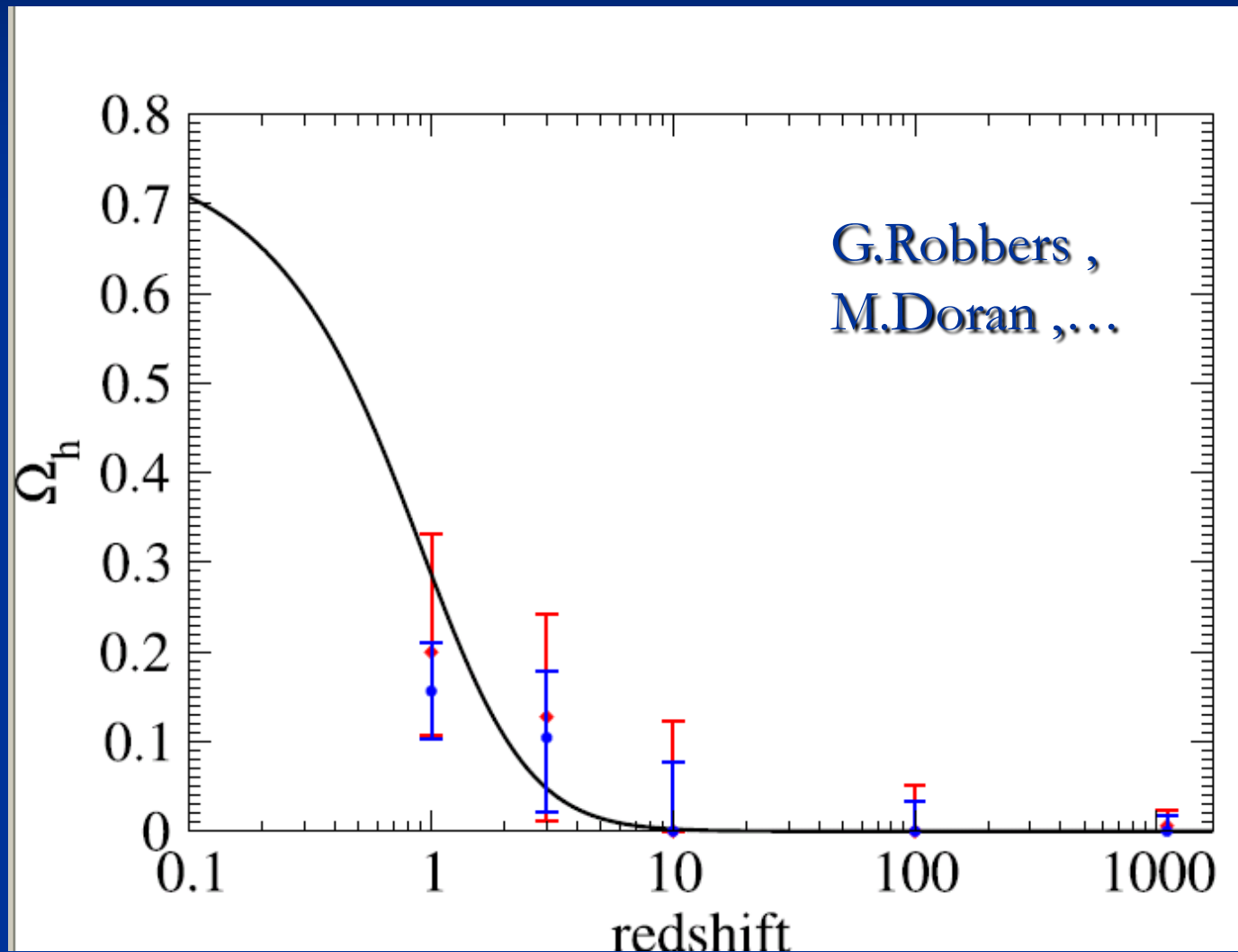
observation will decide !

Early Dark Energy



cosmological constant : $\Omega_h \sim t^2 \sim (1+z)^{-3}$

Observational bounds on Ω_h

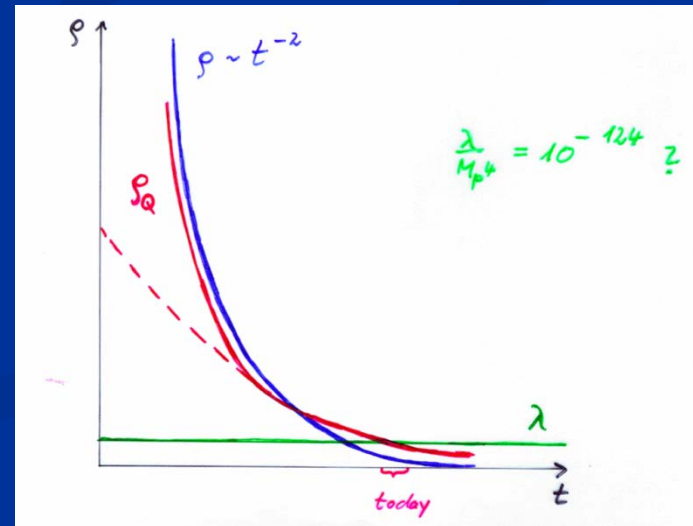


exponential potential \longrightarrow
constant fraction in dark energy

$$\Omega_h = 3/\alpha^2$$

$$V(\varphi) = M^4 \exp(-\alpha\varphi/M)$$

can explain order
of magnitude
of dark energy !



Cosmic Attractors

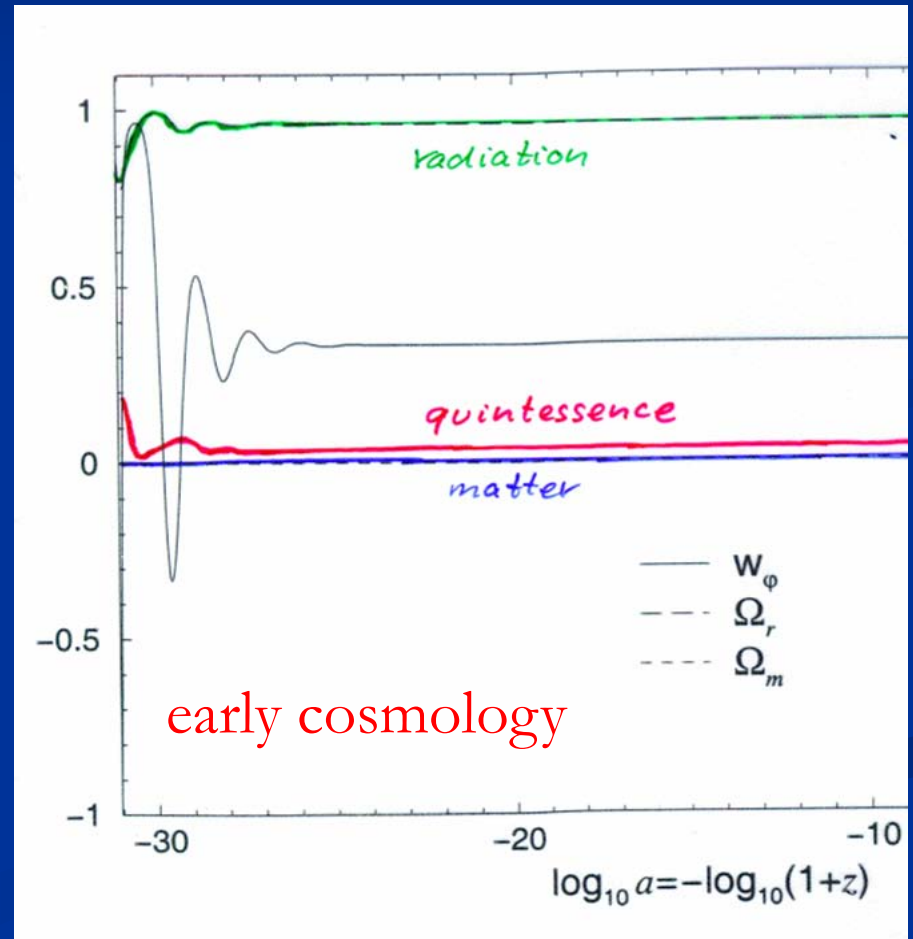
Solutions independent
of initial conditions

typically $V \sim t^{-2}$

$\varphi \sim \ln(t)$

$\Omega_h \sim \text{const.}$

details depend on $V(\varphi)$
or kinetic term

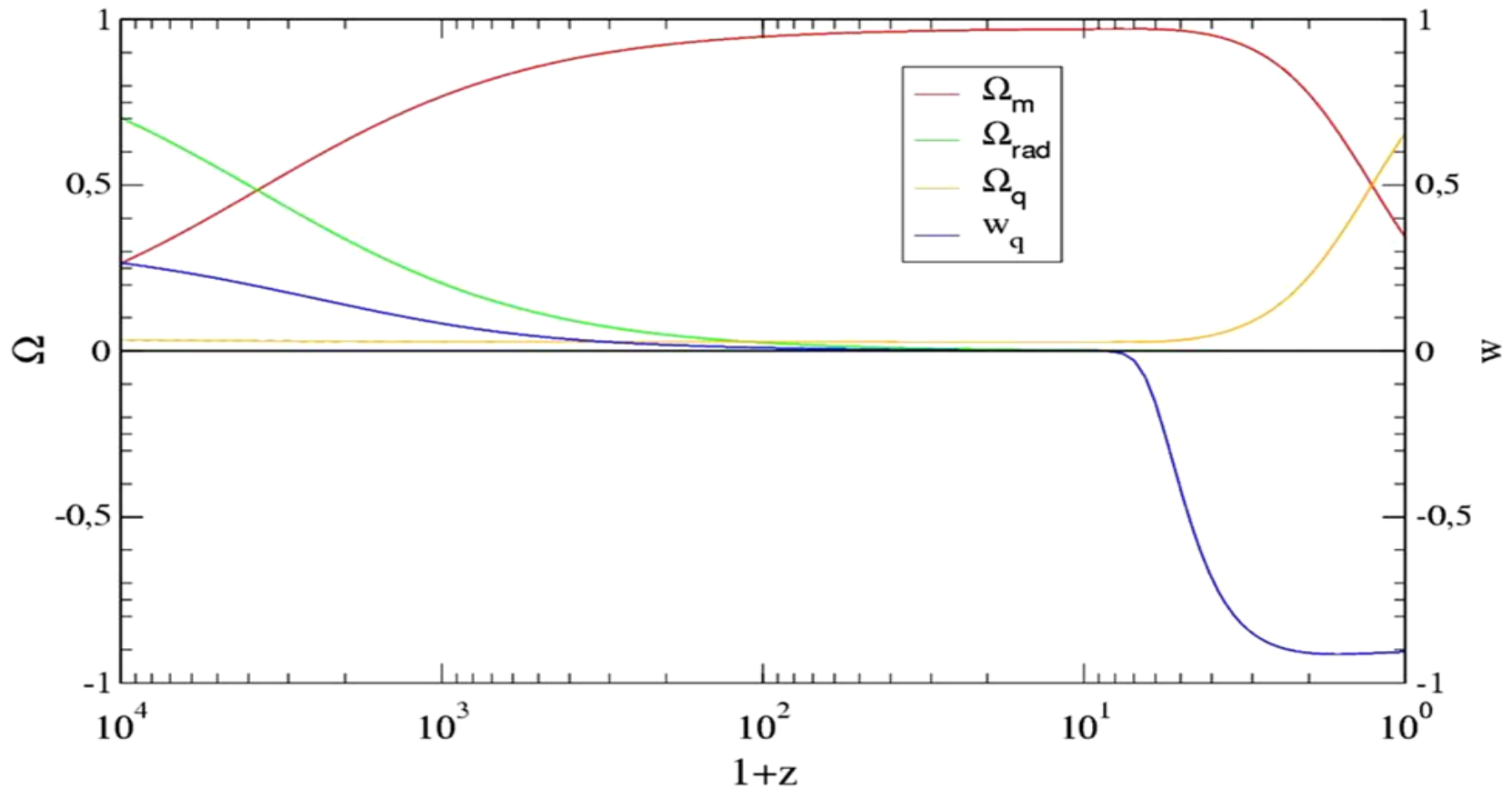


realistic quintessence

fraction in dark energy has to
increase in “recent time” !

Quintessence becomes important “today”

Crossover Quintessence Evolution



Key questions for quintessence

- Why does cosmological potential vanish for infinite time ?
$$V(\varphi) = M^4 \exp(-\alpha\varphi/M)$$
- Why is time variation of fundamental couplings small ? (e.g. fine structure constant , electron-proton mass ratio)
- Why does Dark Energy dominate only in recent cosmology (Why now ? – problem)

Key questions for quintessence

- Why does cosmological potential vanish for infinite time ?

$$V(\varphi) = M^4 \exp(-\alpha\varphi/M)$$

Dilatation symmetry in higher dimensions – not today

- Why is time variation of fundamental couplings small ?
(e.g. fine structure constant , electron-proton mass ratio)

Fixed point behavior – not today

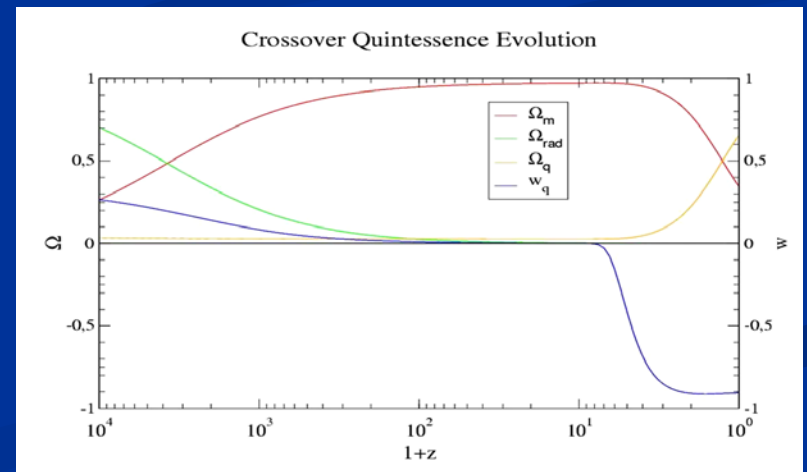
- Why does Dark Energy dominate only in recent cosmology
(Why now ? – problem)

Growing neutrino mass - **today**

coincidence problem

What is responsible for increase of Ω_h for $z < 6$?

Why now ?



Neutrinos in cosmology

only small fraction of energy density



only sub-leading role ?

Cosmon – neutrino coupling

- Can be somewhat stronger than gravitational coupling
- Neutrino mass depends on value of cosmon field

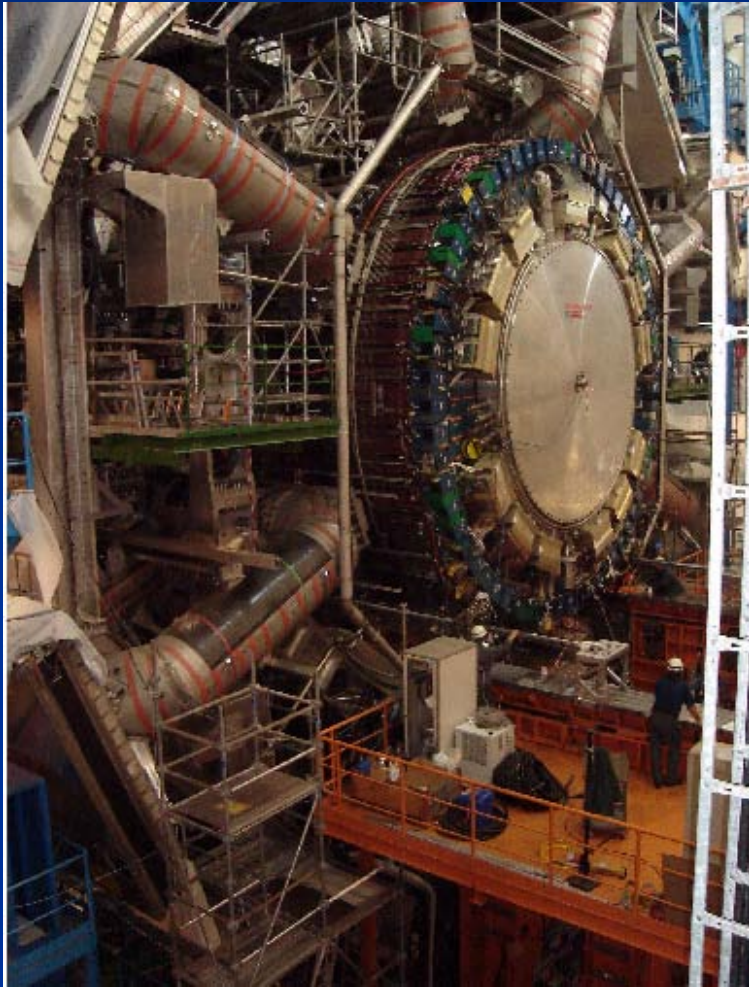
In contrast : cosmon – atom coupling must be weaker than gravity

Fundamental couplings in quantum field theory

*Masses and coupling constants
are determined by properties
of **vacuum** !*

Similar to Maxwell – equations in matter

Spontaneous symmetry breaking to be confirmed at the LHC



*Have coupling constants in the
early Universe
other values than today ?*

Yes !

Restoration of symmetry at high temperature in the early Universe

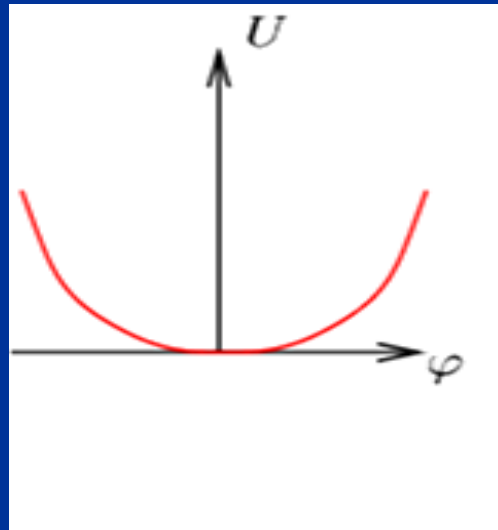
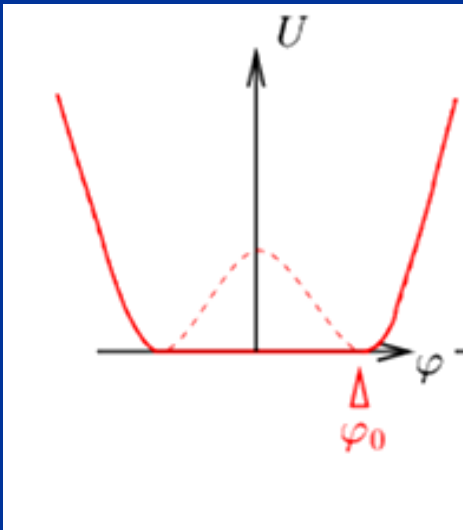
Low T
SSB

$$\langle \varphi \rangle = \varphi_0 \neq 0$$

High T
SYM

$$\langle \varphi \rangle = 0$$

high T :
Less order
More symmetry



Example:
Magnets

In hot plasma
of early Universe :

masses of electron und muon
not different!

similar strength of electromagnetic
and weak interaction

Varying couplings

only question :

How strong is **present** variation of couplings ?

Particle masses in quintessence cosmology

can depend on value of cosmon field

similar to dependence on value of Higgs field

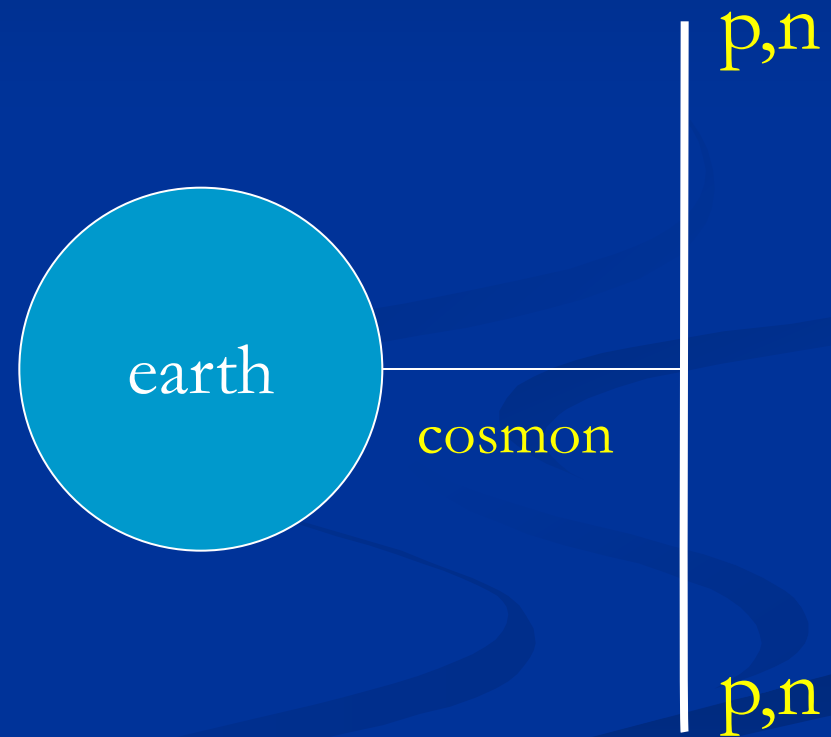
Cosmon – atom coupling induces violation of equivalence principle

Different couplings of
cosmon to proton and
neutron

Differential acceleration

“Violation of
equivalence principle”

only apparent : new “fifth force” !

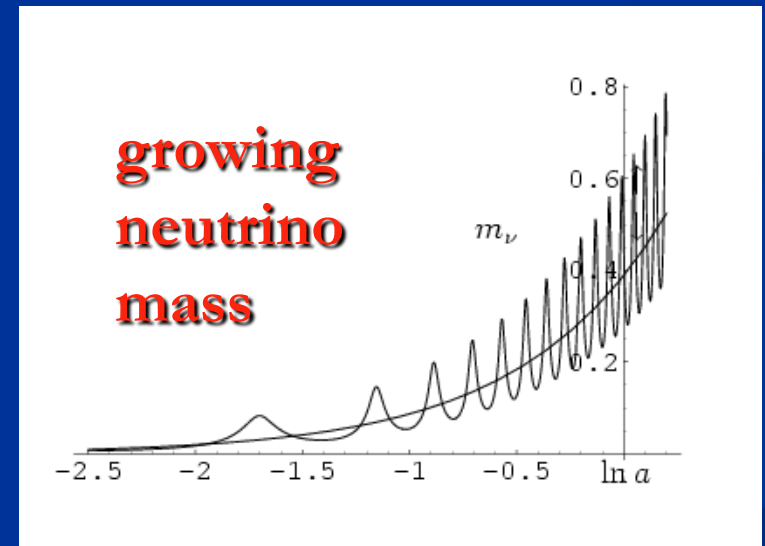
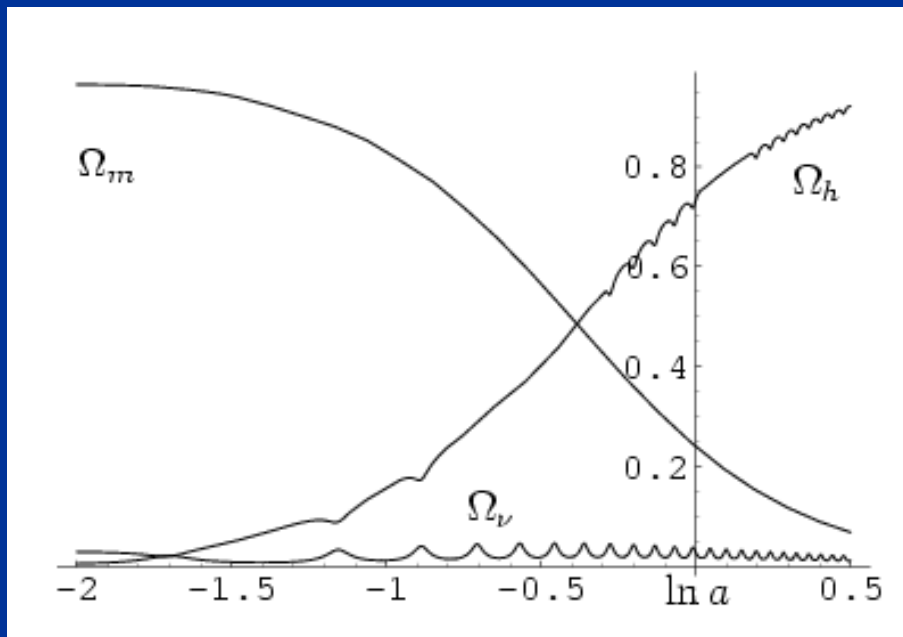


Neutrino cosmon coupling

- Strong bounds on atom-cosmon coupling from tests of equivalence principle or time variation of couplings.
- No such bounds for neutrino-cosmon coupling.
- In particle physics : Mass generation mechanism for neutrinos differs from charged fermions. Seesaw mechanism involves heavy particles whose mass may depend on the value of the cosmon field.

**growing neutrino
quintessence**

growing neutrino mass triggers transition to almost static dark energy



L. Amendola, M. Baldi, ...

effective cosmological trigger
for stop of cosmon evolution :
neutrinos get non-relativistic

- this has happened recently !
- sets scales for dark energy !

connection between dark energy and neutrino properties

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.27 \left(\frac{\gamma m_\nu(t_0)}{eV} \right)^{\frac{1}{4}} 10^{-3} eV$$

present dark energy density given by neutrino mass

present equation
of state given by
neutrino mass !

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12eV}$$

cosmological selection

- present value of dark energy density set by cosmological event :
neutrinos become non – relativistic
- not given by ground state properties !

basic ingredient :

cosmon coupling to neutrinos

Cosmon coupling to neutrinos

- can be large !

Fardon, Nelson, Weiner

- interesting effects for cosmology if neutrino mass is growing
- growing neutrinos can stop the evolution of the cosmon
- transition from early scaling solution to cosmological constant dominated cosmology

L. Amendola, M. Baldi, ...

growing neutrinos change cosmological evolution

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial V}{\partial \varphi} + \frac{\beta(\varphi)}{M}(\rho_\nu - 3p_\nu),$$
$$\beta(\varphi) = -M \frac{\partial}{\partial \varphi} \ln m_\nu(\varphi) = \frac{M}{\varphi - \varphi_t}$$

modification of conservation equation for neutrinos

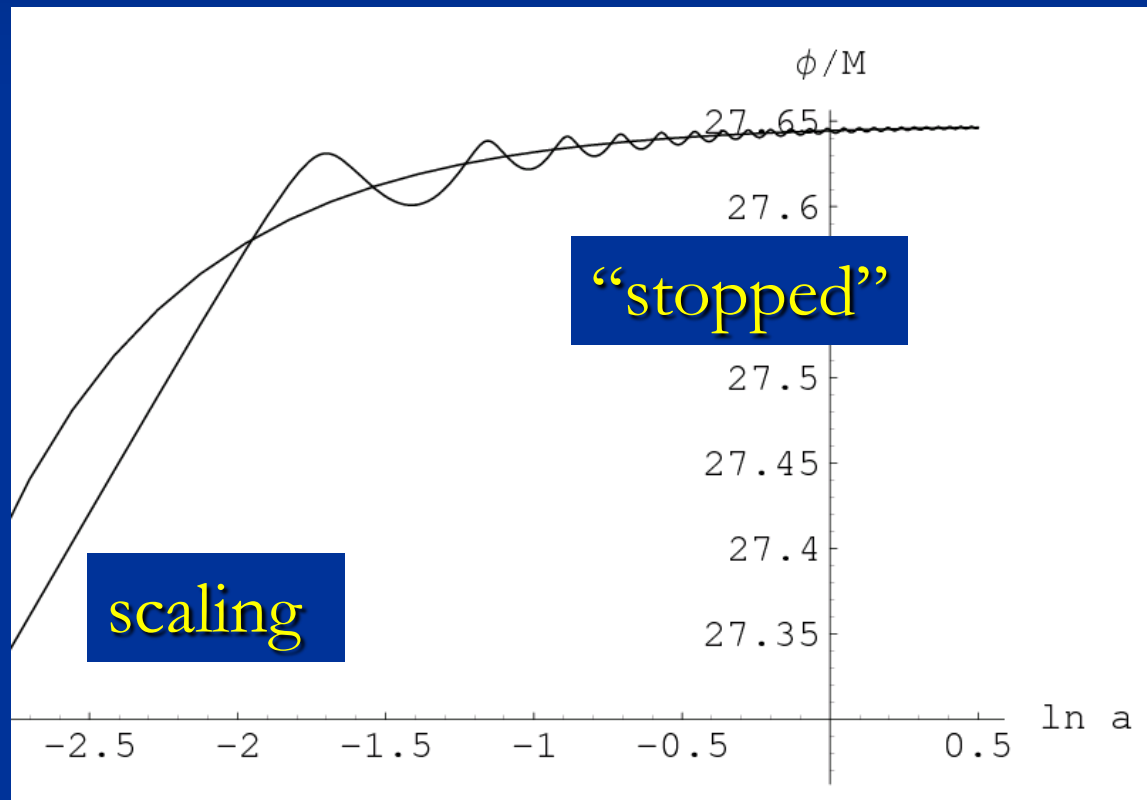
$$\begin{aligned} \dot{\rho}_\nu + 3H(\rho_\nu + p_\nu) &= -\frac{\beta(\varphi)}{M}(\rho_\nu - 3p_\nu)\dot{\varphi} \\ &= -\frac{\dot{\varphi}}{\varphi - \varphi_t}(\rho_\nu - 3p_\nu) \end{aligned}$$

stopped scalar field
mimicks a
cosmological constant
(almost ...)

rough approximation for dark energy :

- before redshift 5-6 : scaling (dynamical)
- after redshift 5-6 : almost static
(cosmological constant)

cosmon evolution



dark energy fraction determined by cosmon – neutrino coupling and neutrino mass

$$\Omega_h(t_0) \approx \frac{\gamma m_\nu(t_0)}{16eV}$$

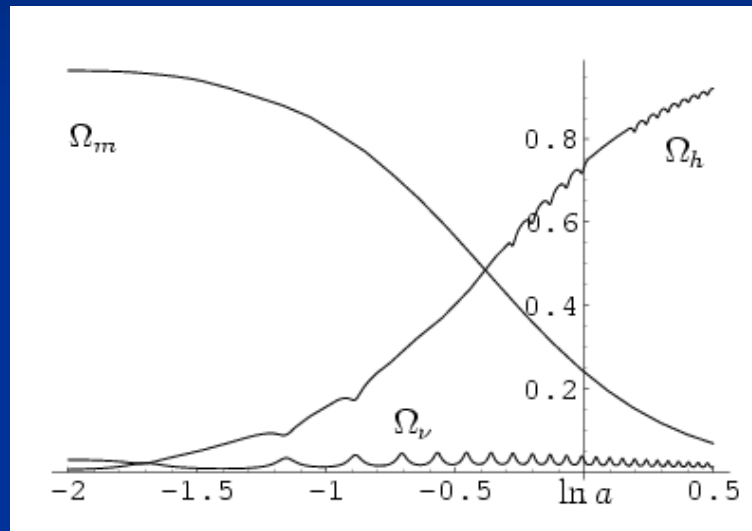
$$\gamma = -\frac{\beta}{\alpha}$$

constant neutrino - cosmon coupling β

$$\Omega_h(t_0) \approx -\frac{\epsilon}{\alpha} \frac{m_\nu(t_0)}{\bar{m}_\nu} \frac{m_\nu(t_0)}{16eV}$$

variable neutrino - cosmon coupling

crossover to dark energy dominated universe



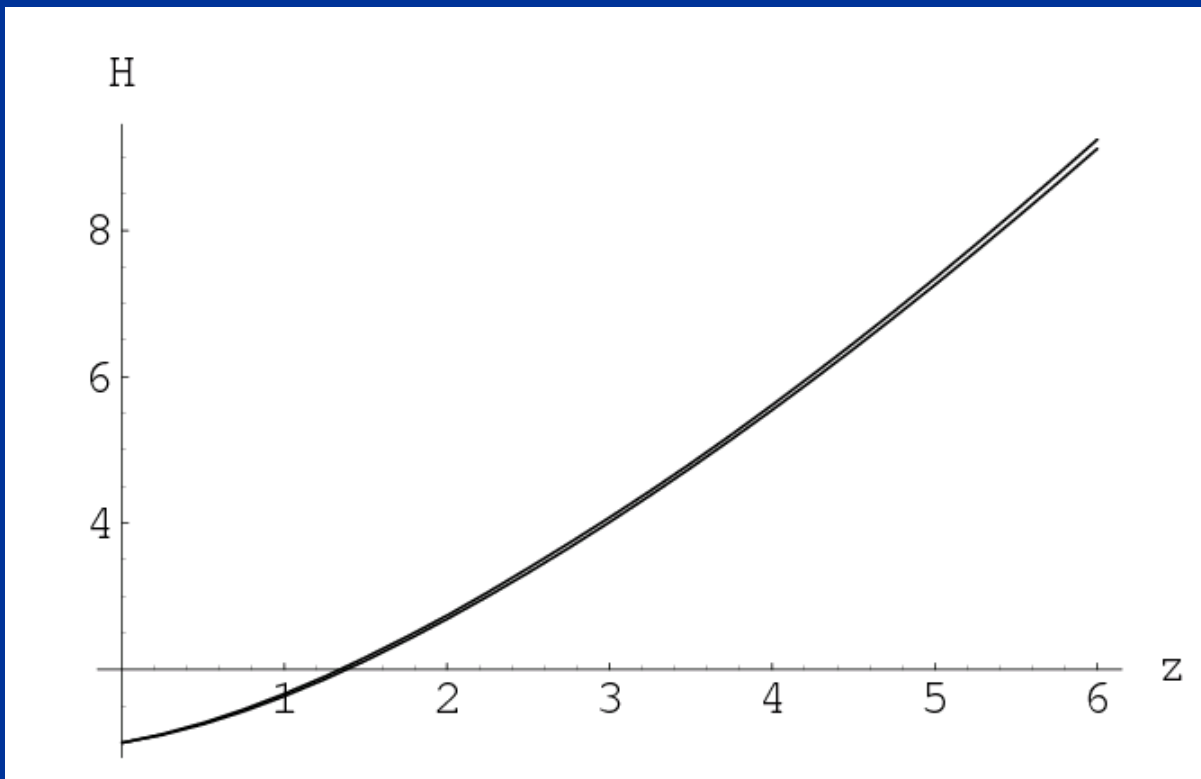
starts at time when “neutrino force” becomes
important for the evolution of the cosmological field

cosmological selection !

Tests for growing neutrino quintessence

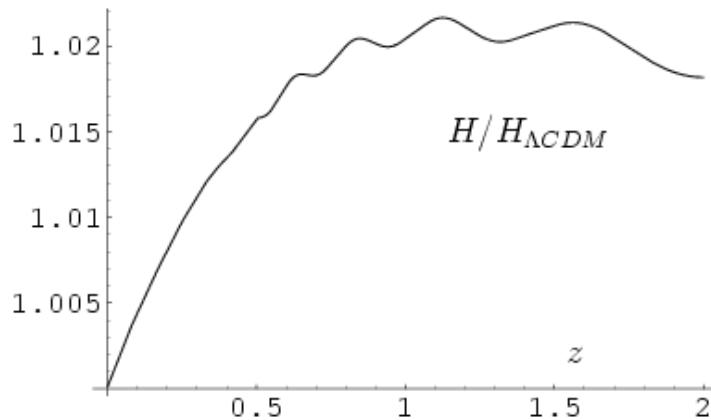
Hubble parameter

as compared to Λ CDM



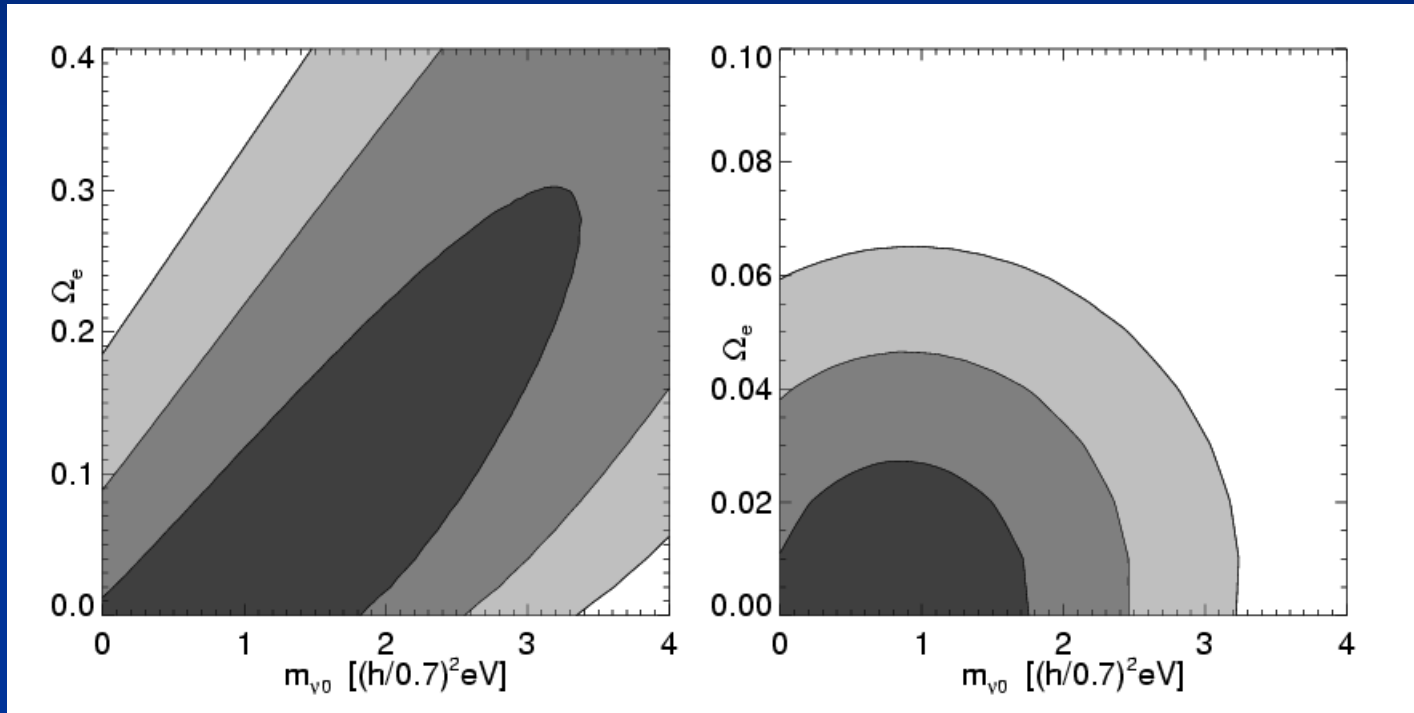
Hubble parameter ($z < z_c$)

$$H^2 = \frac{1}{3M^2} \left\{ V_t + \rho_{m,0} a^{-3} + 2\tilde{\rho}_\nu,0 a^{-\frac{3}{2}} \right\}$$



only small
difference
from
 Λ CDM!

bounds on average neutrino mass



Looking Beyond Lambda with the Union Supernova Compilation

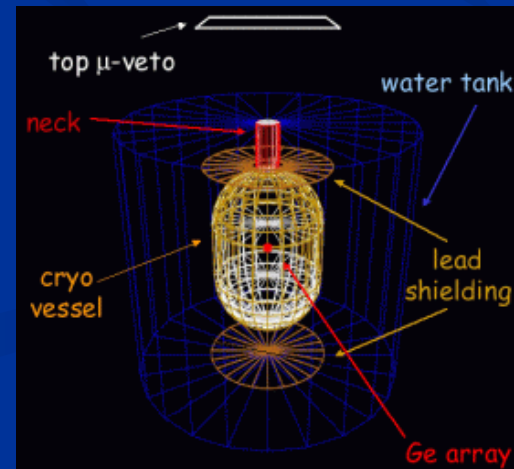
D. Rubin^{1,2}, E. V. Linder^{1,3}, M. Kowalski⁴, G. Aldering¹, R. Amanullah^{1,3}, K. Barbary^{1,2},
N. V. Connolly⁵, K. S. Dawson¹, L. Faccioli^{1,3}, V. Fadeyev⁶, G. Goldhaber^{1,2}, A. Goobar⁷,
I. Hook⁸, C. Lidman⁹, J. Meyers^{1,2}, S. Nobili⁷, P. E. Nugent¹, R. Pain¹⁰, S. Perlmutter^{1,2},
P. Ruiz-Lapuente¹¹, A. L. Spadafora¹, M. Strovink^{1,2}, N. Suzuki¹, and H. Swift^{1,2}

(Supernova Cosmology Project)

Can time evolution of neutrino mass be observed ?

Experimental determination of neutrino mass may turn out higher than cosmological upper bound in model with constant neutrino mass

(KATRIN, neutrino-less double beta decay)

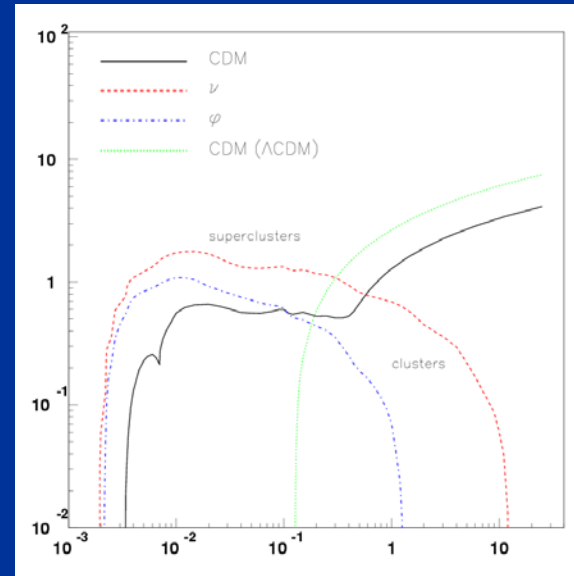
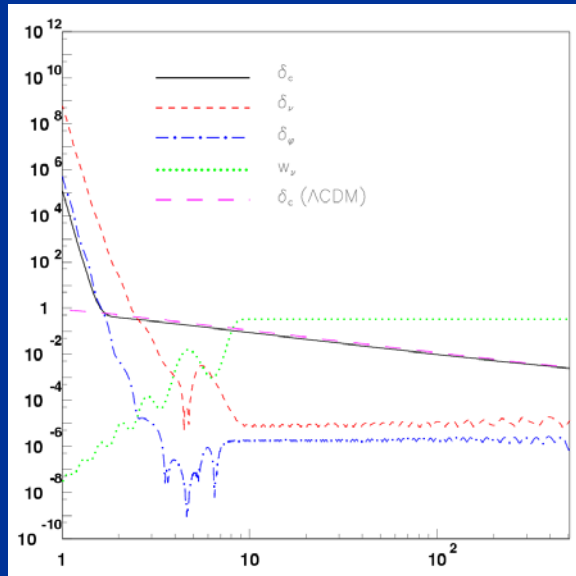


GERDA

neutrino fluctuations

neutrino structures become nonlinear at $z \sim 1$ for
supercluster scales

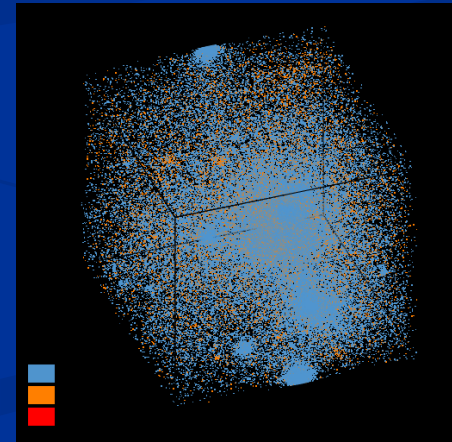
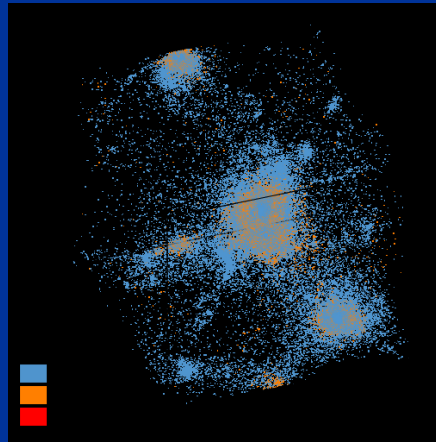
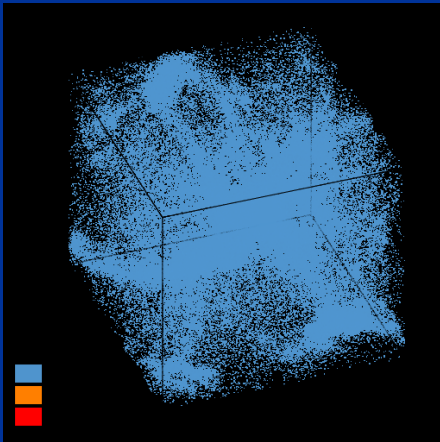
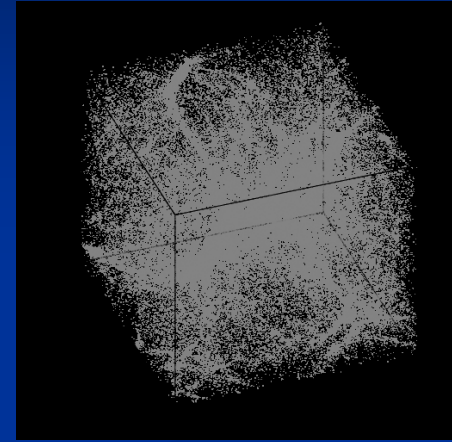
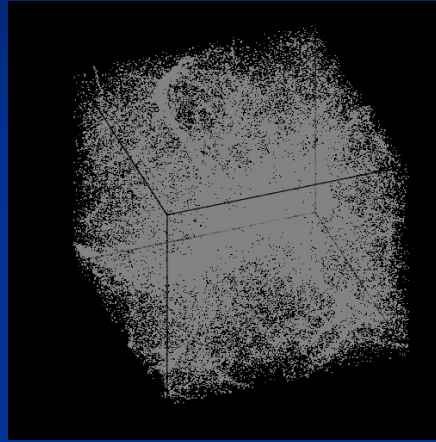
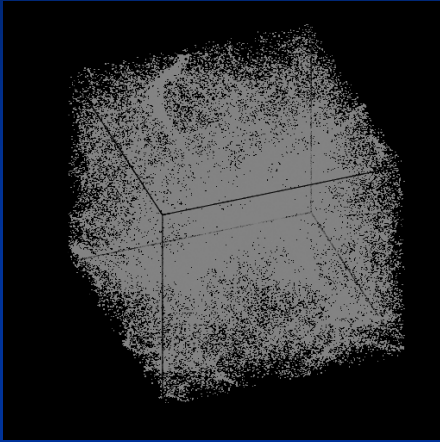
D.Mota , G.Robbers , V.Pettorino , ...



stable neutrino-cosmon lumps exist

N.Brouzakis , N.Tetradis , ...

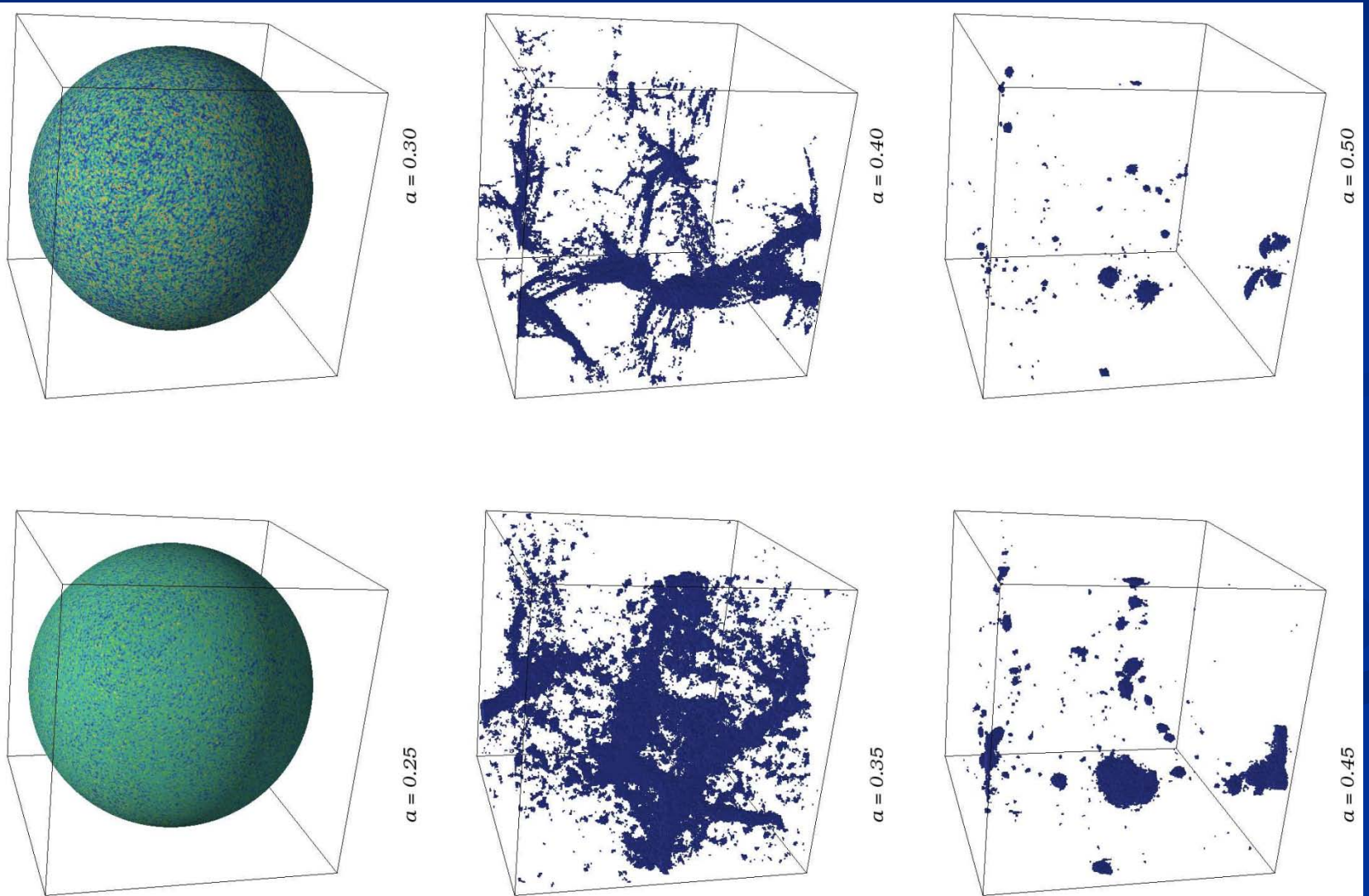
Formation of neutrino lumps



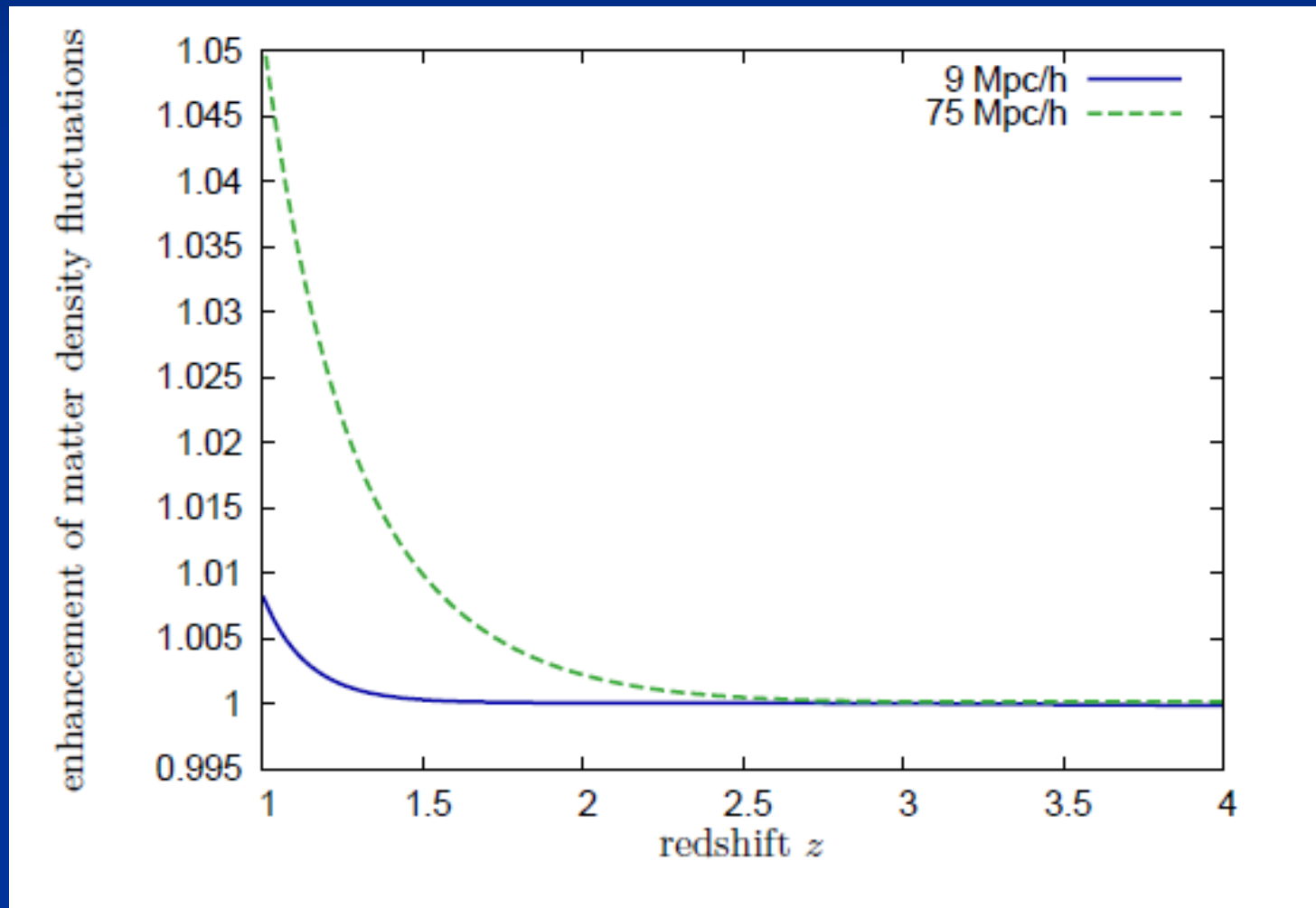
N- body simulation M.Baldi et al

Formation of neutrino lumps

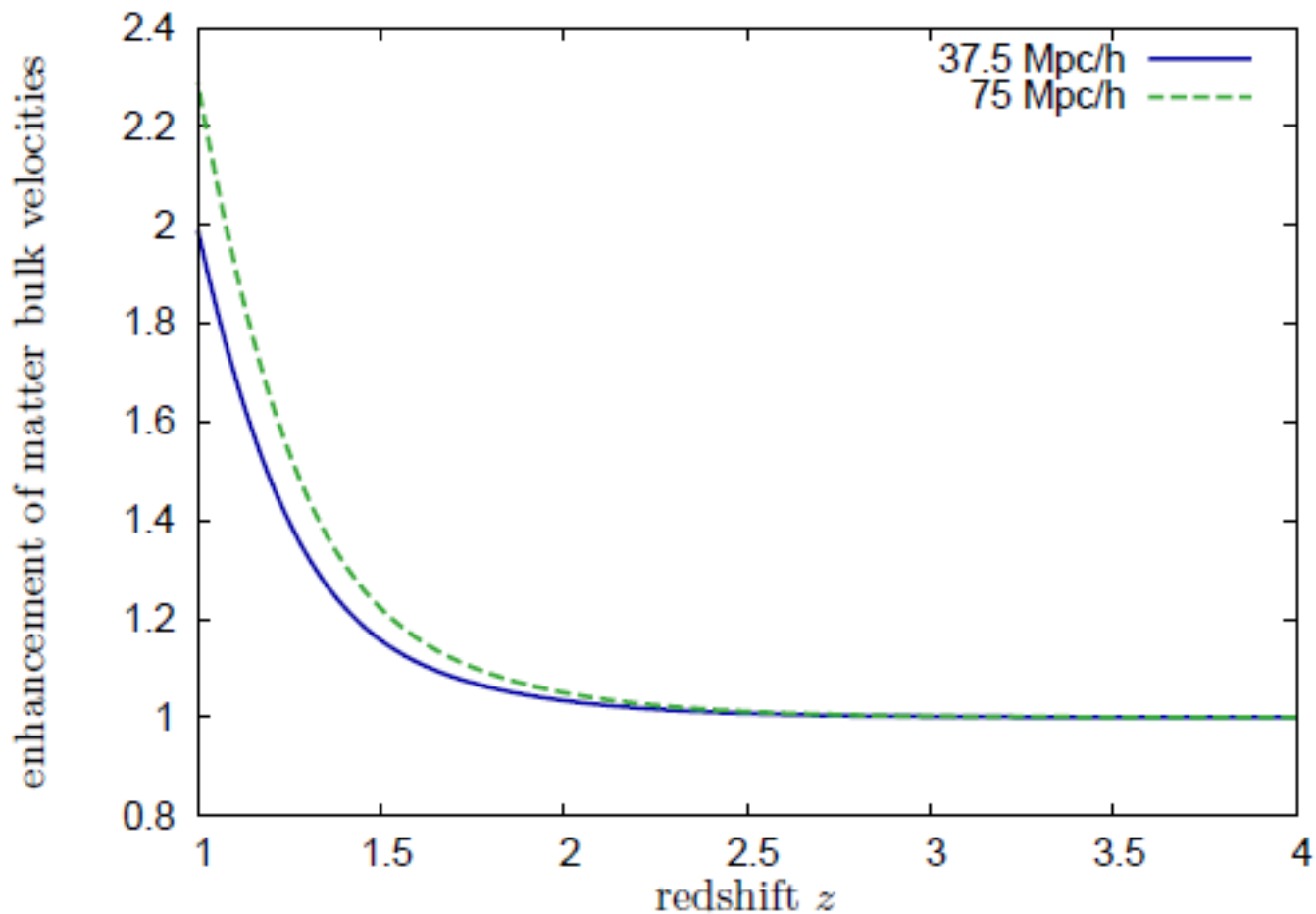
Y. Ayaita, M. Weber, ...



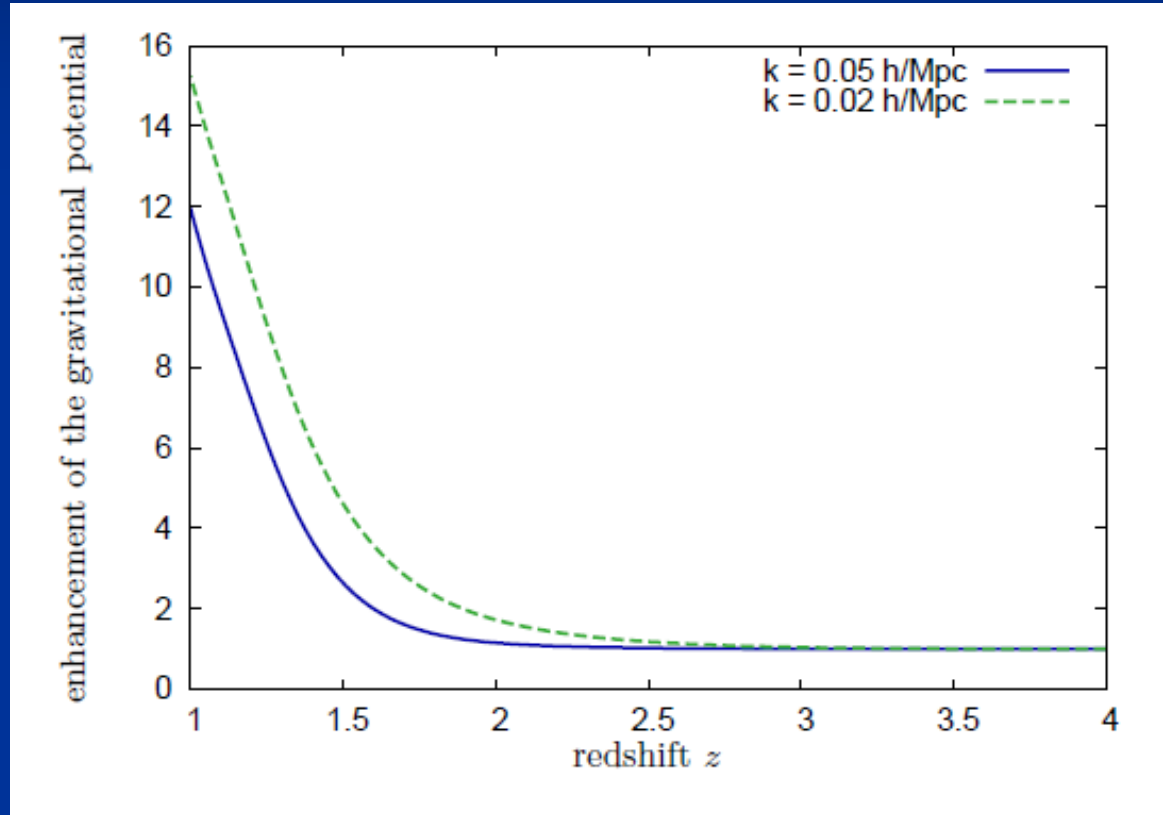
Small induced enhancement of dark matter power spectrum at large scales



Enhanced bulk velocities



Enhancement of gravitational potential



Test of allowed parameter space by ISW effect

Conclusions

- Cosmic event triggers qualitative change in evolution of cosmon
- Cosmon stops changing after neutrinos become non-relativistic
- Explains why now
- Cosmological selection
- Model can be distinguished from cosmological constant

Summary

- o $\Omega_h = 0.73$
- o Q/Λ : dynamical und static dark energy will be distinguishable
- o growing neutrino mass can explain why now problem
- o Q : time varying fundamental coupling “constants”
violation of equivalence principle



End

varying neutrino – cosmon coupling

- specific model
- can naturally explain why neutrino – cosmon coupling is much larger than atom – cosmon coupling

neutrino mass

$$M_\nu = M_D M_R^{-1} M_D^T + M_L$$

$$M_L = h_L \gamma \frac{d^2}{M_t^2}$$

seesaw and
cascade
mechanism

triplet expectation value \sim doublet squared

$$m_\nu = \frac{h_\nu^2 d^2}{m_R} + \frac{h_L \gamma d^2}{M_t^2}$$

omit generation
structure

cascade mechanism

$$U = U_0(\varphi) + \frac{\lambda}{2}(d^2 - d_0^2)^2 + \frac{1}{2}M_t^2(\varphi)t^2 - \gamma d^2 t$$

triplet expectation value \sim

$$\gamma \frac{d^2}{M_t^2}$$

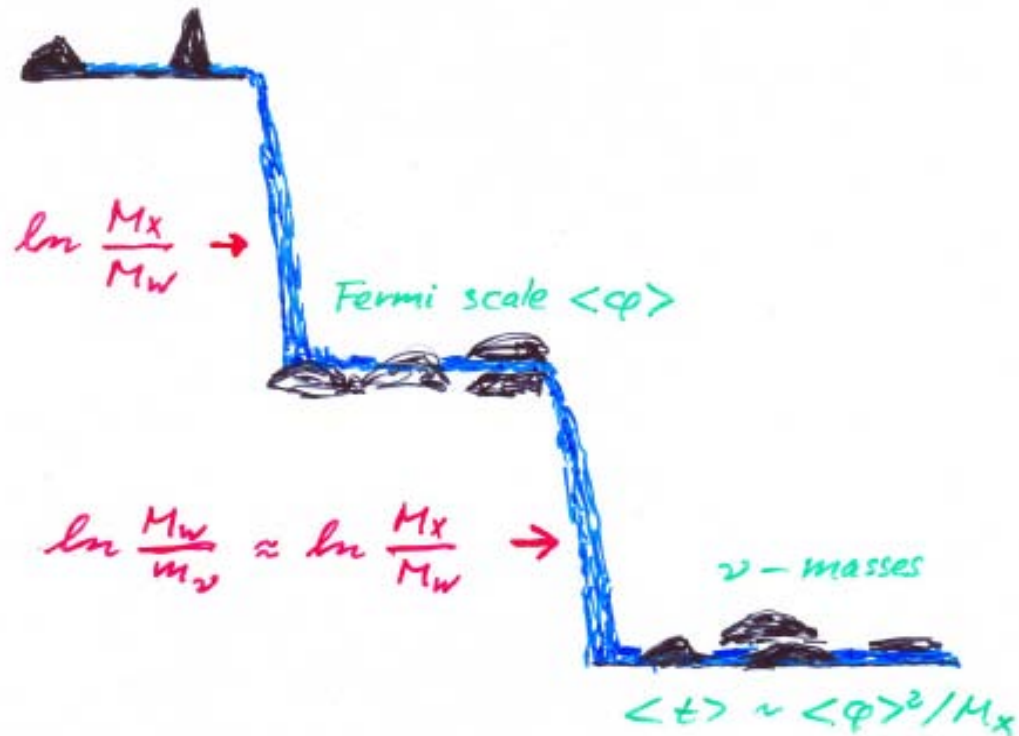
M.Magg , ...

G.Lazarides , Q.Shafi , ...

cascade

Cascade mechanism

unification (M_x)



varying neutrino mass

$$M_t^2 = c_t M_{GUT}^2 \left[1 - \frac{1}{\tau} \exp\left(-\epsilon \frac{\varphi}{M}\right) \right] \quad \epsilon \approx -0.05$$

triplet mass depends on cosmon field φ

$$m_\nu(\varphi) = \bar{m}_\nu \left\{ 1 - \exp\left[-\frac{\epsilon}{M}(\varphi - \varphi_t)\right] \right\}^{-1}$$

→ neutrino mass depends on φ

cascade mechanism

$$U = U_0(\varphi) + \frac{\lambda}{2}(d^2 - d_0^2)^2 + \frac{1}{2}M_t^2(\varphi)t^2 - \gamma d^2 t$$

triplet expectation value \sim

$$\gamma \frac{d^2}{M_t^2}$$

$$M_t^2(\varphi) = \bar{M}_t^2 \left[1 - \exp \left(-\frac{\epsilon}{M}(\varphi - \varphi_t) \right) \right]$$

“singular” neutrino mass

$$M_t^2 = c_t M_{GUT}^2 \left[1 - \frac{1}{\tau} \exp\left(-\epsilon \frac{\varphi}{M}\right) \right]$$

triplet mass vanishes for $\varphi \rightarrow \varphi_t$

$$\frac{\varphi_t}{M} = -\frac{\ln \tau}{\epsilon}$$

$$m_\nu(\varphi) = \frac{\bar{m}_\nu M}{\epsilon(\varphi - \varphi_t)}$$

➔ neutrino mass diverges for $\varphi \rightarrow \varphi_t$

strong effective
neutrino – cosmon coupling
for $\varphi \rightarrow \varphi_t$

$$\beta(\varphi) = -M \frac{\partial}{\partial \varphi} \ln m_\nu(\varphi) = \frac{M}{\varphi - \varphi_t}$$

typical present value : $\beta \approx 50$ \rightarrow
cosmon mediated attraction between neutrinos
is about 50^2 stronger than gravitational attraction

**crossover from
early scaling solution to
effective cosmological constant**

early scaling solution (tracker solution)

$$V(\varphi) = M^4 \exp\left(-\alpha \frac{\varphi}{M}\right)$$

$$\varphi = \varphi_0 + (2M/\alpha) \ln(t/t_0)$$

$$\Omega_{h,e} = \frac{n}{\alpha^2}$$

neutrino mass unimportant in early cosmology

effective cosmological trigger
for stop of cosmon evolution :
neutrinos get non-relativistic

- this has happened recently !
- sets scales for dark energy !

dark energy fraction determined by neutrino mass

$$\Omega_h(t_0) \approx \frac{\gamma m_\nu(t_0)}{16eV}$$

$$\gamma = -\frac{\beta}{\alpha}$$

constant neutrino - cosmon coupling β

$$\Omega_h(t_0) \approx -\frac{\epsilon}{\alpha} \frac{m_\nu(t_0)}{\bar{m}_\nu} \frac{m_\nu(t_0)}{16eV}$$

variable neutrino - cosmon coupling

effective stop of cosmon evolution

cosmon evolution almost stops once

- neutrinos get non-relativistic
- β gets large

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial V}{\partial \varphi} + \frac{\beta(\varphi)}{M}(\rho_\nu - 3p_\nu)$$

$$\beta(\varphi) = -M \frac{\partial}{\partial \varphi} \ln m_\nu(\varphi) = \frac{M}{\varphi - \varphi_t}$$

$$m_\nu(\varphi) = \frac{\beta(\varphi)}{\epsilon} \bar{m}_\nu$$

This always happens for $\varphi \rightarrow \varphi_t$!

Equation of state

$$p = T - V$$

pressure

kinetic energy

$$\rho = T + V$$

energy density

$$T = \frac{1}{2} \dot{\phi}^2$$

Equation of state

$$w = \frac{p}{\rho} = \frac{T - V}{T + V}$$

Depends on specific evolution of the scalar field

Negative pressure

- $w < 0$ Ω_h increases (with decreasing z)

late universe with
small radiation component :

$$w_h = \frac{1}{3\Omega_h(1-\Omega_h)} \frac{\partial \Omega_h}{\partial \ln(1+z)}$$

- $w < -1/3$ expansion of the Universe is
accelerating

- $w = -1$ cosmological constant

A few references

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