Dimensional crossover in ultracold Fermi gases

from Functional Renormalisation

Bruno Faigle-Cedzich

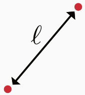
8th May 2018

Cold Quantum Coffee
Heidelberg University

Table of contents

Physics of ultracold atoms

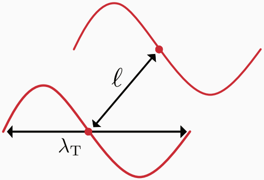
Scales



- interparticle spacing $n = \ell^{-d}$

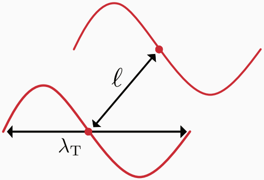
Scales

- interparticle spacing $n = \ell^{-d}$
- thermal wavelength λ_{th}



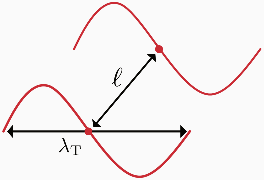
Scales

- interparticle spacing $n = \ell^{-d}$
- thermal wavelength λ_{th}



Scales

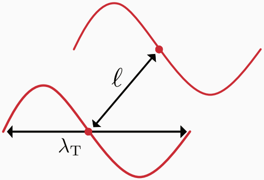
- interparticle spacing $n = \ell^{-d}$
- thermal wavelength λ_{th}



Ultracold: $\ell/\lambda_{\text{th}} \lesssim 1$

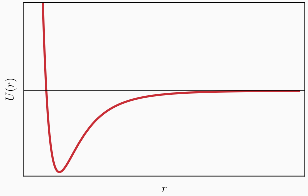
Scales

- interparticle spacing $n = \ell^{-d}$
- thermal wavelength λ_{th}



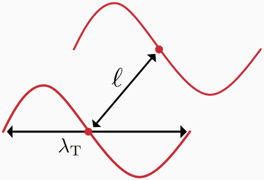
Ultracold: $\ell/\lambda_{\text{th}} \lesssim 1$

- van der Waals length λ_{vdW}



Scales

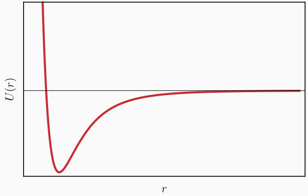
- interparticle spacing $n = \ell^{-d}$
- thermal wavelength λ_{th}



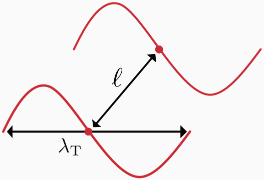
Ultracold: $\ell/\lambda_{\text{th}} \lesssim 1$

- van der Waals length λ_{vdW}
- oscillator length ℓ_{osc}

$$V_{\text{ext}} = \frac{\hbar\,\omega}{2}\,(r/\ell_{\text{osc}})^2$$



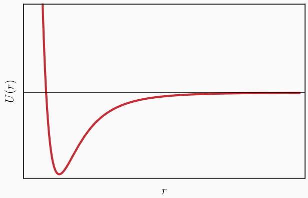
Scales



- interparticle spacing $n = \ell^{-d}$
- thermal wavelength λ_{th}

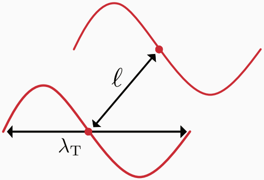
Ultracold: $\ell/\lambda_{\text{th}} \lesssim 1$

- van der Waals length λ_{vdW}
- oscillator length ℓ_{osc}
- scattering length a



Scales

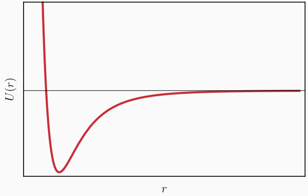
- interparticle spacing $n = \ell^{-d}$
- thermal wavelength λ_{th}



Ultracold: $\ell/\lambda_{\text{th}} \lesssim 1$

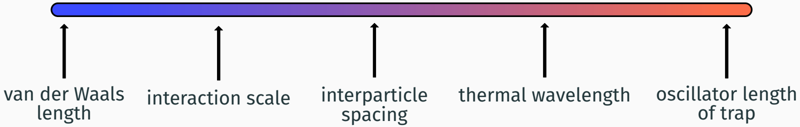
Dilute: $a\, n^{1/d} \ll 1$

- van der Waals length λ_{vdW}
- oscillator length ℓ_{osc}
- scattering length a



Scale hierarchy & Hamiltonian

adapted from Boettcher et al. Nuclear Physics B (2012)

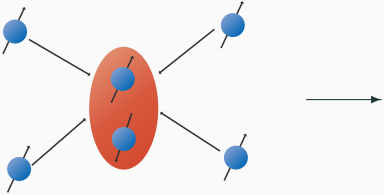


Effective Hamiltonian valid on scales $\gg \ell_{\mathrm{vdW}}$:

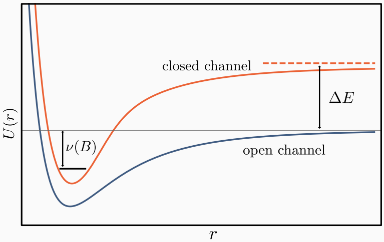
$$\hat{H} = \int_{\vec{x}} \left[\hat{a}^{\dagger}(\vec{x}) \left(-\frac{\hbar\, \nabla^2}{2\, M} + V_{\mathrm{ext}}(\vec{x}) \right) \hat{a}(\vec{x}) + g_{\Lambda}\, \hat{n}(\vec{x})^2 \right]$$

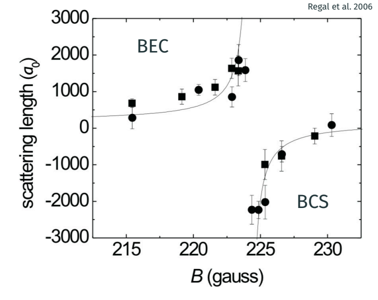
with $\hat{n} = \hat{a}^{\dagger}\, \hat{a}$ and $g_{\Lambda} = \frac{4\,\pi\,\hbar^2}{M}\, a$

Feshbach resonances



2-atom scattering





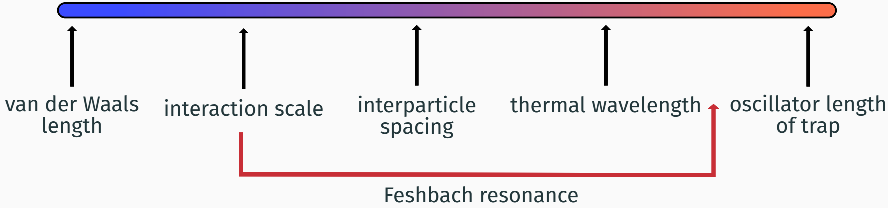
$$\nu(B) = \Delta\mu\,(B - B_0)$$

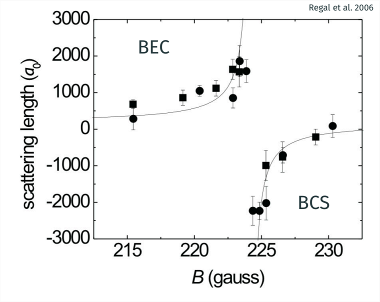
$$a = a_{\text{bg}}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

with $\nu \to 0$ @ resonance

Feshbach resonances

adapted from Boettcher et al. Nuclear Physics B (2012)





Regal et al. 2006

$$\nu(B) = \Delta\mu\,(B - B_0)$$

$$a = a_{\mathrm{bg}}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

with $\nu \to 0$ @ resonance

The BCS-BEC crossover in 3D

two-component fermionic atoms

The BCS-BEC crossover in 3D

two-component fermionic atoms

fermions with attractive interactions

The BCS-BEC crossover in 3D

two-component fermionic atoms

→ fermions with attractive interactions

→ bound molecules of two atoms

The BCS-BEC crossover in 3D

two-component fermionic atoms

fermions with attractive interactions

bound molecules of two atoms

low T

BCS-superfluidity (condensation of Cooper pairs)

The BCS-BEC crossover in 3D

two-component fermionic atoms

fermions with attractive interactions

low T

BCS-superfluidity (condensation of Cooper pairs)

bound molecules of two atoms

low T

BEC (Bose-Einstein condensate)

The BCS-BEC crossover in 3D

two-component fermionic atoms

fermions with attractive interactions

↓ low T

BCS-superfluidity (condensation of Cooper pairs)

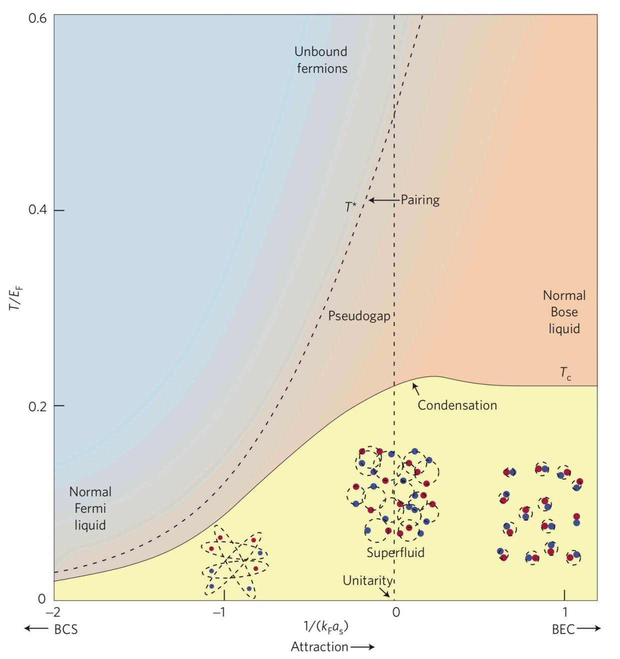
bound molecules of two atoms

↓ low T

BEC (Bose-Einstein condensate)

→ Crossover (magnetic field)

The BCS-BEC crossover in 3D



$k_F = (3\pi^2 n)^{1/3}$

Randeria
Nature (2010)

Features

advantages

- couplings are tunable
- high precision experiments
- microphysics known

challenges

- from microphysic laws to macroscopic observation including fluctuations
- large couplings
- different effective degrees of freedom
    - microphysics: single atoms & molecules
    - macrophysics: bosonic collective degrees of freedom

BCS-BEC physics from Functional Renormalisation

The functional RG

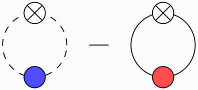
full quantum
effective action

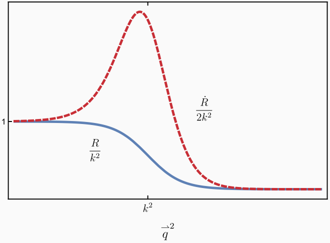
Flow equation (Wetterich 1993)

$$\partial_k \, \Gamma_k = \frac{1}{2} \, \mathrm{STr} \left[(\Gamma^{(2)} + R_k)^{-1} \, \partial_k \, R_k \right]$$

Exact 1-loop equation

$$\partial_t \, \Gamma_k[\phi, \psi] = \frac{1}{2}$$

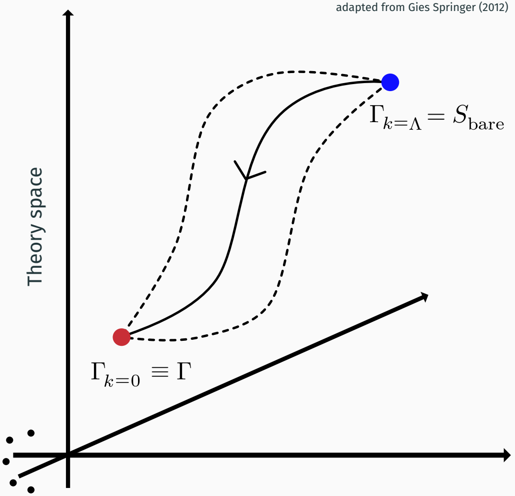




bosons _ _ _ _ _ fermions ______

Regulator and truncation dependence

- flow of Γ_k regulator dependent, yet Γ is not
- during flow all possible interactions may be produced
- $\partial_t \Gamma^{(n)}$ depends on $\Gamma^{(n+1)}$ and $\Gamma^{(n+2)}$ →truncation needed



Action and truncation

Microscopic action

$$S = \int_X \left[\psi^* \left(\partial_\tau - \nabla^2 - \mu \right) \psi + \phi^* \left(\partial_\tau - \frac{\nabla^2}{2} + \nu - 2\mu \right) \phi \right.$$
$$\left. - h \left(\phi^* \psi_1 \psi_2 - \phi \psi_1^* \psi_2^* \right) \right]$$

with

- ψ: Grassmann field
- ϕ: bosonic field consisting of two atoms
- ν: detuning
- τ: Euclidean time on torus with circumference $1/T$
- μ: chemical potential

Units

Chosen such that:

$$\hbar = k_B = 2\,M = 1$$

Consequences:

- $\hbar = 1$: [momentum]=[length]$^{-1}$ with typical momentum unit k_F
- $k_B = 1$: [temperature]=[energy]
- $2\,M = 1$: [momentum]=[energy] (equiv. $c = 1$)

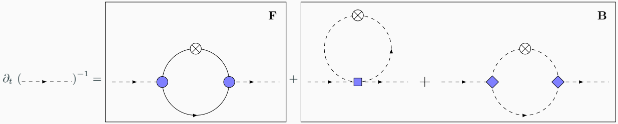
i.e.

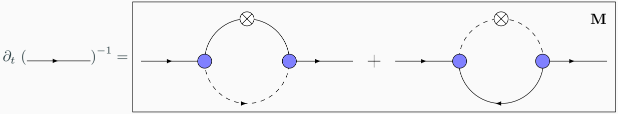
$$[t] = l^2,\ [\vec{p}] = l^{-1},\ [T] = l^{-2},\ [\mu] = l^{-2},\ [n] = l^{-3}\,.$$

$\rightarrow$ canonical dimensions differ from relativistic QFT!

Truncation: derivative expansion

- vertices are expanded in powers of the momenta
- effective average potential U_k @ least to 2nd order in $\rho = \phi^*\phi$ (2nd order phase transition)





Ansatz for effective action $\Gamma_k = \Gamma_{\text{kin}} + \Gamma_{\text{int}}$

$$\Gamma_{\text{kin}}[\psi,\phi] = \int_X \left[\sum_{\sigma=\{1,2\}} \psi_\sigma^* \left(S_\psi \partial_\tau - \nabla^2 - \mu\right) \psi_\sigma + \phi^* \left(S_\phi \partial_\tau - \frac{1}{2}\nabla^2 \right) \phi \right]$$

$$\Gamma_{\text{int}}[\psi,\phi] = \int_X \left[U(\phi^*\,\phi) - h\,\left(\phi^*\,\psi_1\,\psi_2 - \phi\,\psi_1^*\,\psi_2^*\right) \right]$$

with

- renormalised fields: $\psi = A_\psi^{1/2}\,\bar{\psi}$, $\phi = A_\phi^{1/2}\,\bar{\phi}$
- $S_{\psi,\phi} = Z_{\psi,\phi}/A_{\psi,\phi}$
- anomalous dimensions: $\eta_{\psi,\phi} = -\,\partial_t\,\log A_{\psi,\phi}$

and

$$U(\rho) = \sum_{n=1}^{N} \frac{u_n}{n!}\,(\rho - \rho_0)^n - n_k\,(\mu - \mu_0) + \sum_{m=1}^{M} \frac{\alpha_m}{m!}\,(\mu - \mu_0)\,(\rho - \rho_0)^m$$

$$u_1 = m_\phi^2$$

Regularisation scheme

IR-regularisation:

$$\lim_{p^2/k^2 \to 0} R_k(p^2)/k^2 > 0, \qquad \lim_{k^2/p^2 \to 0} R_k(p^2) \to 0$$

Litim-type regulator:

$$R_{\phi,k}(Q) = R_{\phi,k}(q^2) = \left(k^2 - \frac{q^2}{2}\right) \theta\left(k^2 - \frac{q^2}{2}\right)$$

$$R_{\psi,k}(Q) = R_{\psi,k}(q^2) = \left[k^2 \,\mathsf{sgn}\left(q^2 - \mu\right) - \left(q^2 - \mu\right)\right] \theta\left(k^2 - |q^2 - \mu|\right)$$

→analytic evaluation of Matsubara sums

UV renormalisation

Connecting to experiment

- initial condition for U_k: $U_\Lambda(\rho) = (\nu_\Lambda - 2\,\mu)\,\rho$
- @ $T = 0$: connect to correct vacuum physics at unitarity ($a^{-1} = 0$), i.e. $m^2_{\phi,k=0} = 0 = \mu$

Thus:

$$a = a(B) = \frac{h_\Lambda^2}{8\,\pi\,\nu(B)}\,, \qquad \nu(B) = \nu_\Lambda - \delta\nu(\Lambda)$$

Universality

- existence of FPs in RG flow: macrophysics independent of the microphysics
- loss of memory of microphysics: start at the FP values

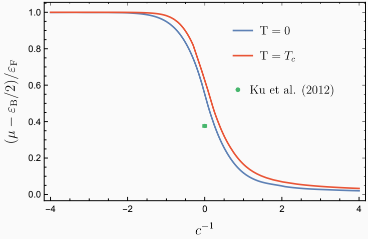
initial values

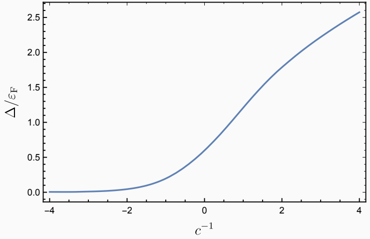
$$h_\Lambda^2 = 6\,\pi^2\,\Lambda, \qquad \lambda_{\phi,\Lambda} = \frac{\tilde{\lambda}_{\phi,*}}{\Lambda}, \qquad m_{\phi,\Lambda}^2 = \nu_\Lambda - 2\,\mu,$$

$$S_{\phi,\Lambda} = 1, \qquad \alpha_\Lambda = -2, \qquad n_\Lambda = \frac{\mu^{3/2}}{3\,\pi^2}\,\Theta(\mu).$$

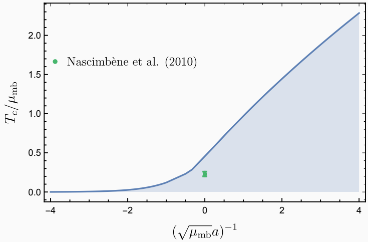
3D BCS-BEC crossover

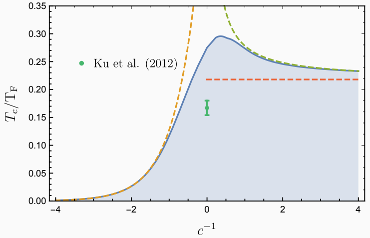
$T = 0$: (with $c = k_F a$)





$T > 0$:



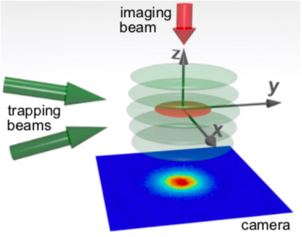


Dimensional crossover

Why are quasi-2D systems interesting?

- promising materials: graphene, high T_c-superconductors, layered semiconductors
- pronounced influence of quantum fluctuations
- experimental accessibility via highly anisotropic trapping potentials
- for insufficient anisotropy →dimensional crossover

→disentangle dimensionality from many-body physics



Zürn et al. PRL (2015)

Boundary conditions

- delimit z-direction by potential well of length L

$$V_{\text{box}}(z) = \begin{cases} 0 & 0 \leq z \leq L \\ \infty & \text{else} \end{cases}$$

- impose **periodic** boundary conditions: $\Psi = \{\psi, \phi\}$

$$\Psi(\tau, x, y, z = 0) = \Psi(\tau, x, y, z = L)$$

→quantisation of momentum in z-direction:

$$q_z \to k_n = \frac{2\,\pi\,n}{L}, \qquad n \in \mathbb{N}$$

- spatial Matsubara sum

$$\int \frac{d^d q}{(2\,\pi)^d} = \frac{1}{L} \sum_{k_n} \int \frac{d^{d-1} q}{(2\,\pi)^{d-1}}$$

Regulator and method

Litim-type regulator (as before) with $\vec{q} = \hat{\vec{q}} + q_z \to \hat{\vec{q}} + k_n$

Regulator and method

Litim-type regulator (as before) with $\vec{q} = \hat{\vec{q}} + q_z \to \hat{\vec{q}} + k_n$

Idea:

- initialise RG flow at UV scale $k = \Lambda$ where Γ_Λ coincides with S of 3D gas
- L introduces new length scale to 3D system
- following the RG flow we successively integrate out the 3rd dimension
  →@ $k \to 0$: system @ given confinement length scale L

Regulator and method

Litim-type regulator (as before) with $\vec{q} = \hat{\vec{q}} + q_z \to \hat{\vec{q}} + k_n$

Idea:

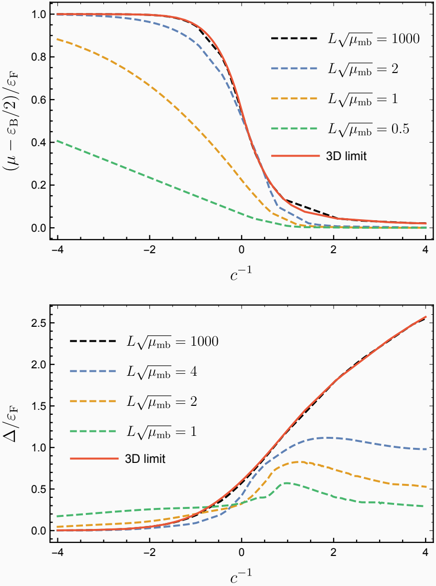
- initialise RG flow at UV scale $k = \Lambda$ where Γ_Λ coincides with S of 3D gas
- L introduces new length scale to 3D system
- following the RG flow we successively integrate out the 3rd dimension
  →@ $k \to 0$: system @ given confinement length scale L

Note

- UV scale is always chosen such that $\Lambda \gg (L^{-1}, \mu^{1/2}, T^{1/2})$
- system is effectively 2D if $L^{-1} \gg$ all other many-body scales

Zero temperature

- correct 3D-limit reached
- qualitatively correct behaviour of equation of state (except very small confinements)

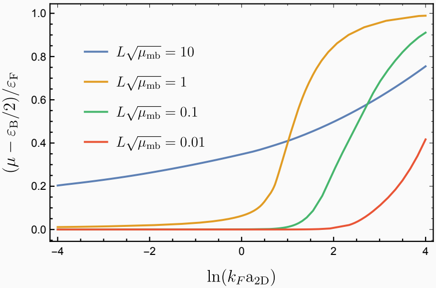


Connecting to experiment

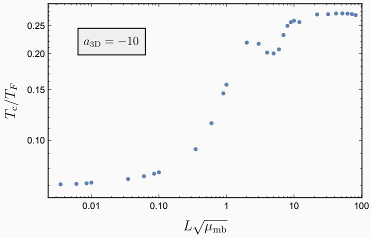
quasi-2d scattering length:

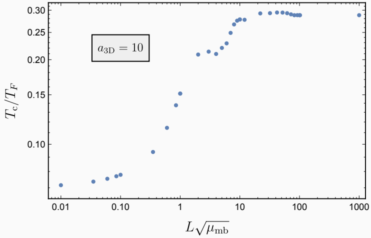
$$a_{2\text{D}}^{(\text{pbc})} = L\,\exp\left\{-\frac{1}{2}\,\frac{L}{a_{3\text{D}}}\right\}, \qquad a_{2\text{D}}^{(\text{trap})} = \ell_z\sqrt{\frac{\mu}{A}}\,\exp\left\{-\sqrt{\frac{\mu}{2}}\,\frac{\ell_z}{a_{3\text{D}}}\right\}$$

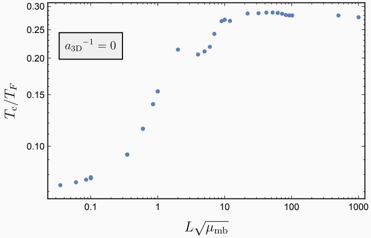
quasi-2d crossover parameter: $\ln(k_F a_{2\text{D}})$



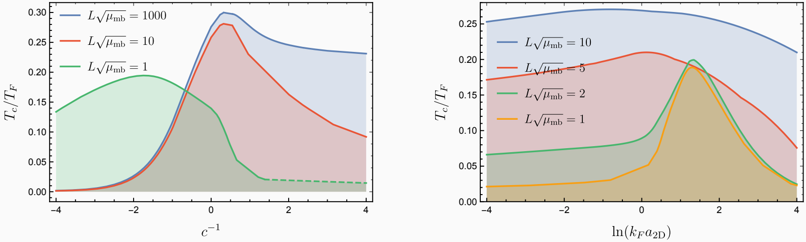
Finite temperature





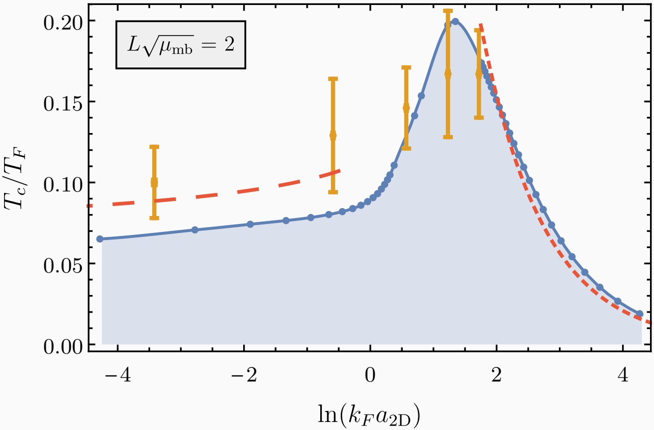


Phase diagram



- 3D phase diagram reproduced for large L
- increased T_c/T_F around $\ln(k_F a_{\rm 2D}) \sim 1$

Comparison to experiment



experimental data from Ries et al. PRL (2015)

Conclusion

Summary and Outlook

Conclusion

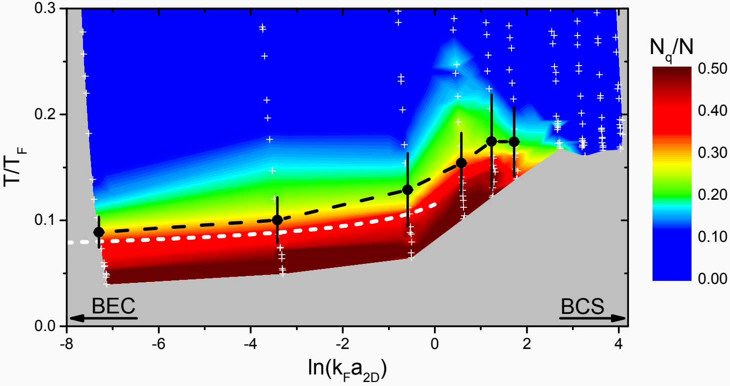
- dimensional crossover in Fermi gas with FRG
- qualitatively comparable to experiments

Outlook

- employ harmonic trapping potential
- quantitative precision
  - particle-hole fluctuations
  - frequency dependent regulator
- explore thermodynamics

Thank you for your attention!

Phase diagram from experiment



Ries et al.
PRL (2015)