A curvature bound from gravitational catalysis.

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Based on a joint work with H. Gies: [arXiv:1802.02865]

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Outline

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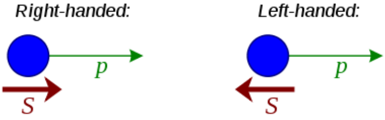
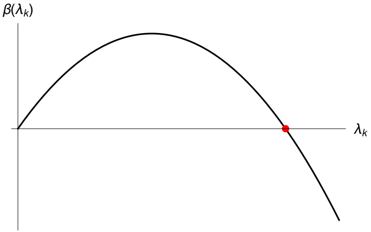
Chiral symmetry



Chiral transformation acts indipendently on the right and left components of fermions:

$$U(N_{\rm f})_R \times U(N_{\rm f})_L \Rightarrow \begin{cases} \psi_L \longrightarrow e^{i\theta_L}\psi_L \\ \psi_R \longrightarrow e^{i\theta_R}\psi_R \end{cases}$$



- A mass term is not chiral invariant
- It can be used to define a chiral condensate

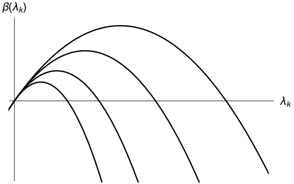
$$m_{eff} \simeq \langle\bar{\psi}\psi\rangle$$

- The condensate represents the order parameter

Gravitational catalysis

- Gravitational catalysis indicates the breaking of chiral symmetry due to the presence of a curved background [Buchbinder and Kirillova, 1989; Sachs and Wipf, 1994; Elizalde, Leseduarte, Odinstov and Sil'nov, 1996].

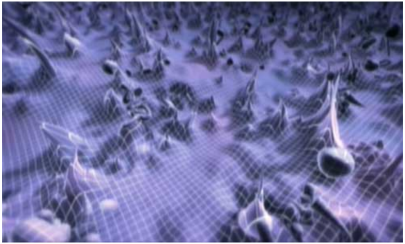
- In negatively curved spacetimes it can be understood as an effective dimensional reduction of the long range dynamics of fermionic modes from $D+1$ to $1+1$ dimensions [Gorbar, 2009].



- The fixed point structure of systems undergoing gravitational catalysis was studied [Scherer and Gies, 2012; Gies and Lippoldt, 2013].

Light fermions

Under the assumption of chiral symmetry breaking being triggered by quantum gravity one would expect a consequent mass gap comparable to the Planck mass [Eichhorn and Gies, 2011].



$\Downarrow$

Can we use gravitational catalysis to constrain quantum gravity?

Outline

Bosonization

The action for our model reads:

$$S[\bar{\psi},\psi] \;=\; \int_x \left\{ \bar{\psi}\nabla\!\!\!\!/\,\psi + \frac{\bar{\lambda}_+}{2}\left[\left(\bar{\psi}^a\gamma_\mu\psi^a\right)^2 - \left(\bar{\psi}^a\gamma_\mu\gamma_5\psi^a\right)^2\right]\right\}. \qquad (1)$$

By means of Fierz identities we re organize the interaction as

$$(V) + (A) = -2[(S^{\mathrm{N}}) - (P^{\mathrm{N}})] \qquad (2)$$

where

$$\begin{aligned} (S^{\mathrm{N}}) &= (\bar{\psi}^a\psi^b)^2 = (\bar{\psi}^a\psi^b)(\bar{\psi}^b\psi^a), \\ (P^{\mathrm{N}}) &= (\bar{\psi}^a\gamma_5\psi^b)^2 = (\bar{\psi}^a\gamma_5\psi^b)(\bar{\psi}^b\gamma_5\psi^a)\,, \end{aligned} \qquad (3)$$

obtaining a NJL-type of action:

$$S[\bar{\psi},\psi] \;=\; \int_x \left\{ \bar{\psi}\nabla\!\!\!\!/\,\psi - \bar{\lambda}_+\left[\left(\bar{\psi}^a\psi^b\right)^2 - \left(\bar{\psi}^a\gamma_5\psi^b\right)^2\right]\right\}. \qquad (4)$$

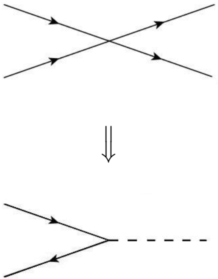
Bosonization

Making use of the chiral projectors

$$P_{\mathrm{L}} = \frac{\mathbb{1} - \gamma_5}{2}\,, \quad P_{\mathrm{R}} = \frac{\mathbb{1} + \gamma_5}{2}\,, \quad \mathbb{1} = P_{\mathrm{L}} + P_{\mathrm{R}}\,, \tag{5}$$

and the following auxiliary fields, satisfying:

$$\phi_{ab} = -2\bar{\lambda}\bar{\psi}^b_{\mathrm{R}}\psi^a_{\mathrm{L}}\,, \qquad (\phi^\dagger)_{ab} = -2\bar{\lambda}\bar{\psi}^b_{\mathrm{L}}\psi^a_{\mathrm{R}}$$
$$\bar{\lambda} = 2\bar{\lambda}_+\,, \tag{6}$$



we can implement the **Hubbard-Stratonovich** trick and map our model to a Yukawa-type interaction as:

$$\mathcal{L}(\phi,\bar{\psi},\psi) = \bar{\psi}^a[\nabla\!\!\!/ + P_{\mathrm{L}}(\phi^\dagger)_{ab} + P_{\mathrm{R}}\phi_{ab}]\psi^b + \frac{1}{2\bar{\lambda}}\mathrm{tr}(\phi^\dagger\phi)\,. \tag{7}$$

It is clear that the nonzero components of $\langle\phi_{ab}\rangle$ are connected to the dynamically generated fermion mass.

Mean field analysis

assuming breaking pattern, $\phi_{ab} = \phi_0 \delta_{ab}$ and introducing Schwinger proper time T we finally write:

$$U(\phi) = \frac{N_\mathrm{f}}{2\bar{\lambda}}\phi_0^2 + \frac{N_\mathrm{f}}{2}\int_0^\infty \frac{dT}{T}\, e^{-\phi_0^2 T}\,\mathrm{Tr}\, e^{\nabla\!\!\!\!/^2 T}\,. \tag{8}$$

with:

$$\mathrm{Tr}\, e^{\nabla\!\!\!\!/^2 T} = \mathrm{Tr}\, K(x,x';T) =: K_T\,, \tag{9}$$

and the heat kernel obeying:

$$\frac{\partial}{\partial T}K = \nabla\!\!\!\!/^2 K, \quad \lim_{T\to 0^+} K(x,x';T) = \frac{\delta(x-x')}{\sqrt{g}}. \tag{10}$$

RG analysis

- In order to investigate the flow of the potential we introduce a propertime regulator function f_k:

$$f_k = e^{-(k^2 T)^p} . \tag{11}$$

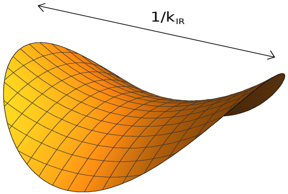
- Thus, at some given average scale $k_{\text{IR}} \sim \frac{1}{\sqrt{T}}$:

$$U_{k_{\text{IR}}} = U_\Lambda - \int_{k_{\text{IR}}}^{\Lambda} dk\, \partial_k U_k \tag{12}$$

together with

$$U_\Lambda = \frac{N_{\text{f}}}{2\bar{\lambda}_\Lambda} \phi_0^2 , \tag{13}$$

$$\partial_k U_k = \frac{N_{\text{f}}}{2} \int_0^\infty \frac{dT}{T}\, e^{-\phi_0^2 T} \partial_k f_k K_T .$$

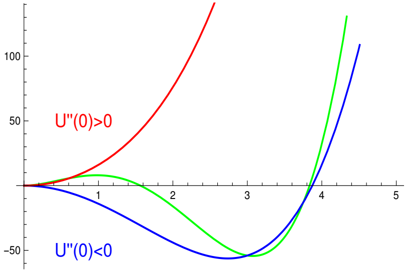


SSB

The fermion mass is generated by breaking a $U(N_f)_R \times U(N_f)_L$ symmetry.

We focus on a second order phase transition mechanism.

- We check the curvature of the potential in the origin of the field space.
- The condition $U''(0) = 0$ will be a function of the ratio $\frac{k_{\rm IR}^2}{R}$



Outline

$D = 3$

Given $\kappa^2 = \frac{|R|}{D(D-1)} = \frac{|R|}{6}$ the heat kernel can be expressed as:

$$K_T^{D=3} = \frac{1}{8\pi^{\frac{3}{2}} T^{\frac{3}{2}}} \left(1 + \frac{1}{2}\kappa^2 T \right) , \tag{14}$$

and the effective potential can be computed analytically:

$$\begin{aligned} U_{k_{\rm IR}} = &- \frac{N_{\rm f}}{2}\phi_0^2 \left(\frac{1}{\bar{\lambda}_{\rm cr}} - \frac{1}{\bar{\lambda}_{\Lambda}} - \frac{k_{\rm IR}}{4\pi} \right) + \frac{N_{\rm f}}{12\pi} \left((\phi_0^2 + k_{\rm IR}^2)^{\frac{3}{2}} - \frac{3}{2} k_{\rm IR}\phi_0^2 - k_{\rm IR}^3 \right) \\ &- \frac{N_{\rm f}}{16\pi}\kappa^2 \left(\sqrt{\phi_0^2 + k_{\rm IR}^2} - k_{\rm IR} \right) . \end{aligned} \tag{15}$$

At criticality, in order to avoid chiral symmetry breaking the curvature needs to satisfy:

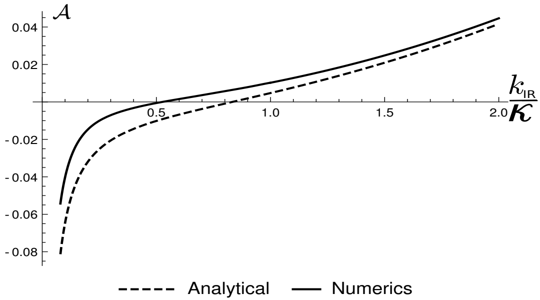
$$\frac{\kappa^2}{k_{\rm IR}^2} \leq 4 \Longrightarrow |R| \leq 24 k_{\rm IR}^2 \tag{16}$$

Outline

$D = 4$

$$U_{k_{\rm IR}}\Big|_{\phi_0^2} = -\frac{N_{\rm f}\phi_0^2}{2}\left[\frac{1}{\bar{\lambda}_{\rm cr}} - \frac{1}{\bar{\lambda}_\Lambda} + \kappa^2\mathcal{A}\left(\frac{\kappa}{k_{\rm IR}};p\right)\right] - 12N_{\rm f}\xi_{k_{\rm IR}}\phi_0^2\kappa^2\,, \qquad (17)$$

$$\mathcal{A} \sim -\Gamma\left(1-\frac{1}{p}\right)\frac{k_{\rm IR}^2}{(4\pi)^2\kappa^2} + 2\frac{\kappa}{k_{\rm IR}}\frac{\Gamma\left(1+\frac{1}{2p}\right)}{\sqrt{\pi}}\,, \qquad \bar{\lambda}_{\rm cr} = \frac{(4\pi)^2}{\Lambda^2\Gamma\left(1-\frac{1}{p}\right)}\,, \qquad (18)$$

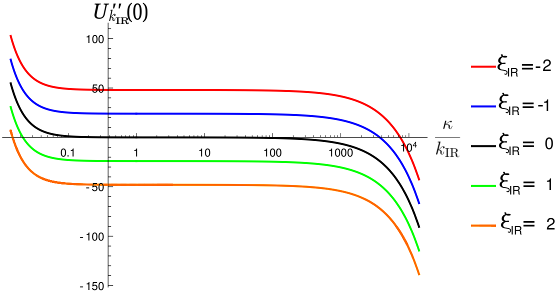


Curvature bound

- For $\xi_{k_{\rm IR}} = 0$, in order to avoid gravitationally catalyzed chiral symmetry breaking we have:

$$\left.\frac{\kappa}{k_{\rm IR}}\right|_{p=2} \leq 1.8998, \qquad \left.\frac{\kappa}{k_{\rm IR}}\right|_{p\to\infty} \leq 1.5757. \tag{19}$$

- Allowing different values for $\xi_{k_{\rm IR}}$, the bound is shifted:



Constraining quantum gravity in asymptotic safety

The background metric is a solution of the semiclassical equations of motion

$$R_{\mu\nu}(\langle g\rangle_k) = \bar{\Lambda}_k \langle g_{\mu\nu}\rangle_k \qquad \underset{\text{at UV f.p.}}{\Longrightarrow} \qquad \frac{R}{k^2} = 4\lambda_* \,, \tag{20}$$

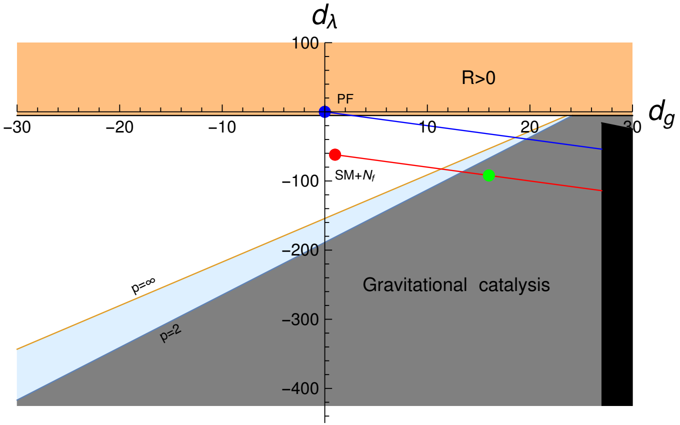
where $\bar{\Lambda}_k$ is the scale dependent cosmological constant and λ_* its UV fixed point value. The presence of **fermionic** d.o.f. drags the cosmological constant UV fixed points towards **negative** values.

Identifying $k_{\rm IR}$ with the coarse graining scale k of asymptotic safety is possible to study a bound on the number of matter d.o.f.:

$$\frac{\kappa^2}{k_{\rm IR}^2} = \frac{|\lambda_*|}{3} \qquad \text{with} \quad \lambda_* = \lambda_*(N_S, N_f, N_V)\,. \tag{21}$$

Constraining quantum gravity in asymptotic safety

Given $d_g = N_S - 4N_V + 2N_f$, $d_\lambda = N_S + 2N_V - 4N_f$ we can parametrize the space of fixed point of a matter-gravity system:



Plot based on the results from [Biemans, Platania and Saueressig, 2017].

Comparing different techniques

	$N_{f,gc}$		
	PF	SM+N_f	MSSM+N_f
one-loop approx. (type IIa) [Codello, Percacci and Rahmede, 2009]	17.58	35.97	20.3
background-field approximation [Doná, Eichhorn and Percacci, 2014]	8.21	26.5	no FP
RG flow on foliated spacetimes [Biemans, Platania and Saueressig, 2017]	9.27	27.67	10.01
dynamical FRG [Meybohm, Pawlowski and Reichert, 2016]	48.7		

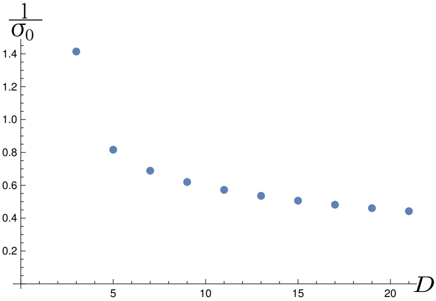
Higher dimensions

In absence of new operators and in the limit $p \to \infty$, the bound in $D = 6$ results in

$$\left.\frac{\kappa}{k_{\rm IR}}\right|_{p\to\infty} \leq 1.0561 \,. \tag{22}$$

For D odd the decreasing behavior is clear:

$$\frac{\kappa}{k_{\rm IR}} \leq \frac{1}{\sigma_0} \equiv \left(\frac{\sqrt{\pi}}{\Gamma\left(\frac{D}{2}\right)(D-2)}\right)^{\frac{1}{D-1}}$$



Summary and Conclusions

- We investigated the scale dependence of gravitational catalysis showing how the curvature of local patches of spacetime may trigger chiral symmetry breaking when being competitive with respect to the energy scale of the process involved.

- It is always possible to find a set of values for the parameters k_{IR} and R preventing the generation of massive fermionic matter.

- We showed that gravitational catalysis can be used as a tool to test quantum gravity theories. The requirements result in a bound for the average curvature of the background spacetime measured in units of the energy scale k_{IR}.

- Even if the formulation of the bound is scheme-dependent, we expect the result to have a scheme-independent meaning.

Thank you!

Heat Kernel

It is possible to give an integral representation of the Heat Kernel [Camporesi, 1991]:

- D odd:

$$K_T^{\text{odd}} = \frac{2}{(4\pi T)^{\frac{D}{2}}\Gamma\left(\frac{D}{2}\right)} \int_0^\infty du\; e^{-u^2} \prod_{j=\frac{1}{2}}^{\frac{D}{2}-1} (u^2 + j^2\kappa^2 T)\,, \tag{23}$$

- D even:

$$K_T^{\text{even}} = \frac{2}{(4\pi T)^{\frac{D}{2}}\Gamma\left(\frac{D}{2}\right)} \int_0^\infty du\; e^{-u^2} u \coth(\pi \frac{u}{\kappa\sqrt{T}}) \prod_{j=1}^{\frac{D}{2}-1} (u^2 + j^2\kappa^2 T)\,. \tag{24}$$

Operator structure

The operator structure can be investigated by expressing the product inside the heat kernel as a sum:

$$\prod_{j=j_0}^{\frac{D}{2}-1}(u^2+j^2\kappa^2 T)=\sum_{m=0}^{\frac{D-1}{2}} C_m u^{2m}(\kappa^2 T)^{\frac{D-1}{2}-m}, \qquad j_0=\begin{cases}\frac{1}{2} \text{ for } D \text{ odd}\\ 1 \text{ for } D \text{ even}\end{cases} \tag{25}$$

- κ-independent term $\longrightarrow$ its UV behavior defines $\bar{\lambda}_{cr}$
- u-independent term $\longrightarrow$ UV-regular, most relevant contribution to g.c.

The competition of their IR behaviors leads to the bound.

- other terms will be relevant/marginal and require regularization via counter-terms.

In $D=4$ the only new operator is proportional to $\phi_0^2 R$.

Scheme dependence

The general choice of regulator can be performed in terms of a parameter p in the following way:

$$f_k = e^{-(k^2 T)^{p}} \tag{26}$$

p specifies the details of the regulator function:

- $p = 1$ we have the Callan-Symanzik regulator

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- $p \to 0 \implies f_k$ is a constant,
- $p \to \infty \implies f_k \to \theta(\frac{1}{k^2} - T)$.