Ultraviolet Dynamics of Fermions and Gravity

Cold Quantum Coffee on June 26, 2018

Marc Schiffer, Heidelberg University

with A. Eichhorn and S. Lippoldt arXiv 18xx.xxxxx

with A. Eichhorn, S. Lippoldt, J. M. Pawlowski and M. Reichert arXiv 1807.xxxxx

Emmy Noether-Programm

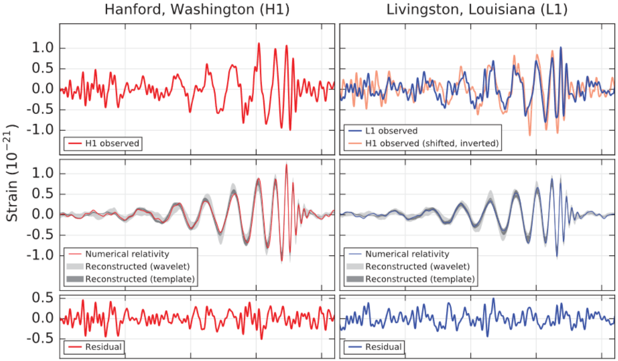
DFG Deutsche Forschungsgemeinschaft

Motivation for Quantum Gravity

Why quantum gravity?

General Relativity:

- several high precision tests
- latest confirmation:
  Existence of black holes by LIGO collaboration



[LIGO Collaboration, 2016]

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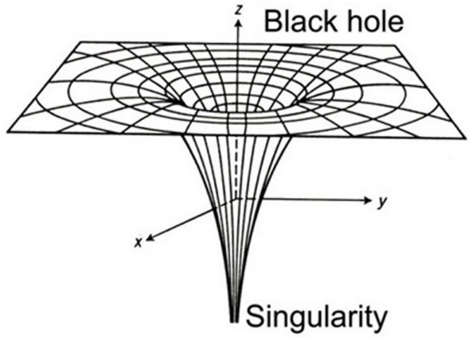
  physical singularity at $r = 0$.

  $\Rightarrow$ GR is not a fundamental theory



Source: Northern Arizona University

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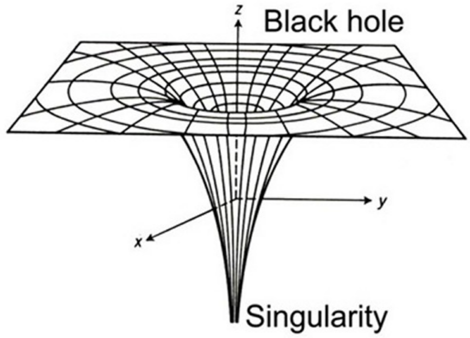
- For $E > M_{\mathsf{PL}} = \sqrt{\frac{\hbar\, c}{G_{\mathsf{N}}}}$:
  QG effects are expected



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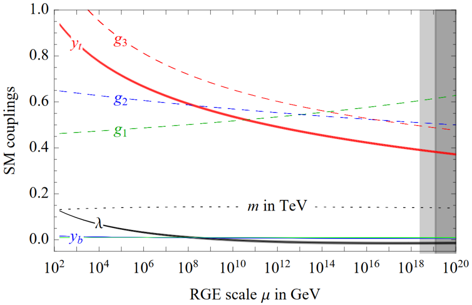
Standard Model:

- well tested low energy model
- formulated as QFT



[D. Buttazzo, 2013]

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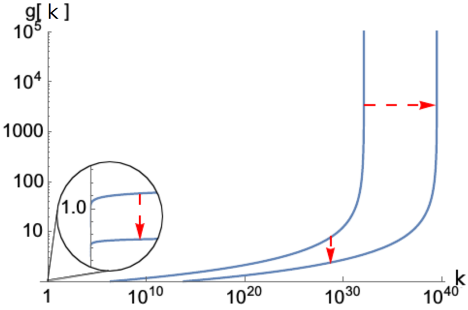

Image credits: Fleur Versteegen

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- Singularities might carry over to SM

  $\rightarrow$ SM is only effective theory

  $\rightarrow$ breakdown beyond Planck scale

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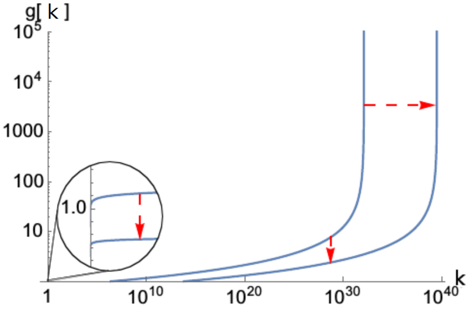

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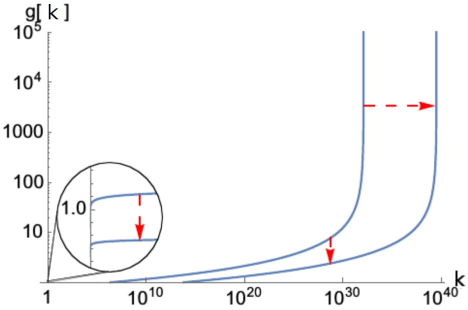

Image credits: Fleur Versteegen

compatibility with SM in IR provides test for quantum theory of gravity

Outline

Motivation for Quantum Gravity

Asymptotically Safe Quantum Gravity

Effective Universality for Gravity and Matter

Induced Couplings at UV Fixed Point

Asymptotically Safe Quantum Gravity

How to quantize Gravity?

- $[G_\mathsf{N}] = 2 - d$

  $\Rightarrow$ GR is *perturbatively non-renormalizable* in $d = 4$

  [G. 't Hooft and M. J. G. Veltman, 1974]

  [M. H. Goroff and A. Sagnotti, 1986]

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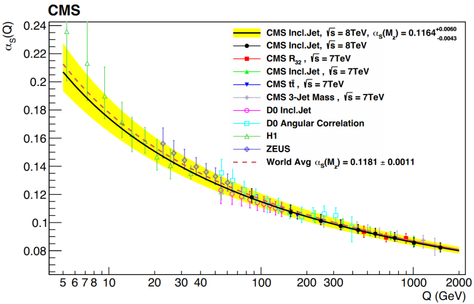
- Effective field theory approach: Loss of predictivity at M_PL

  [J. F. Donoghue and B. R. Holstein, 2015]

Asymptotic safety

- Asymptotic freedom
  - all couplings vanish in the UV
  - perturbative renormalizability



CMS Collaboration, 2017

Asymptotic safety

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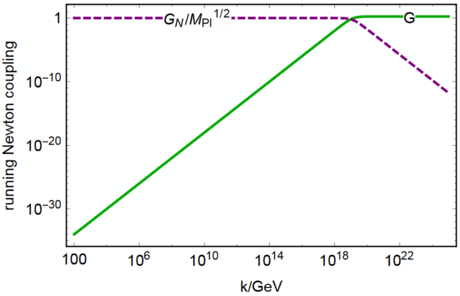
- Asymptotic safety
  [S. Weinberg, 1979]
  - all dimensionless couplings enter a scale invariant regime
  - interacting theory in the UV
  - non-perturbative renormalizability



A. Eichhorn, 2017

Search for Asymptotic safety

Study RG-flow of dimensionless couplings $g_i = \bar{g}_i \, k^{-d_{\bar{g}_i}}$

$$\beta_{g_i}(\vec{g}) = k\,\partial_k g_i = \boxed{-d_{\bar{g}_i}\, g_i} + \boxed{f_i(\vec{g})}$$

dimensional   quantum

$\Rightarrow$ balancing of dimensional term with quantum correction can lead to AS

Critical Exponents

- Linearized β-functions

$$\beta_{g_i} = \beta_{g_i}\Big|_{\mathbf{g}=\mathbf{g}^*} + \sum_j \left(\frac{\partial \beta_{g_i}}{\partial g_j}\right)\Big|_{\mathbf{g}=\mathbf{g}^*} (g_j - g_j^*) + \mathcal{O}\left((g_j - g_j^*)^2\right)$$

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- Solution to linearized flow equations

$$g_i(k) = g_i^* + \sum_j c_j V_j^i \left(\frac{k}{k_0}\right)^{-\Theta_j} \quad \text{with} \quad -\,\text{eig}\,(M) = \Theta_i \,.$$

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- irrelevant direction
- $g_i(k) \xrightarrow{k\to\infty} g_i^*$
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- adjust c_j to reach g_i^*
- one **free parameter** for each relevant direction

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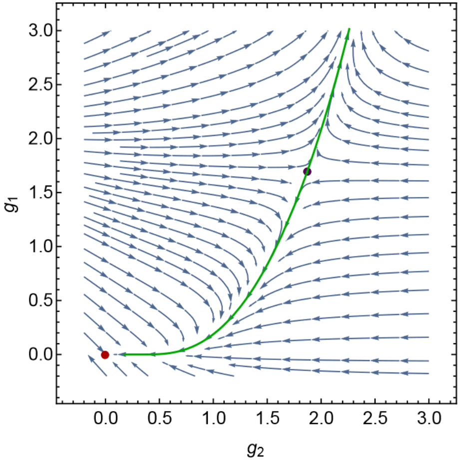
Number of relevant directions determines **predictivity**

Critical Exponents



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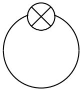
Tool: Functional Renormalization Group

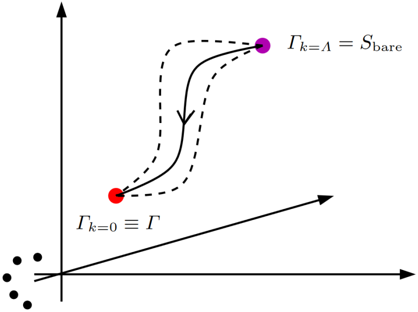
Non-Perturbative Renormalisation Group Equation [Wetterich, 1993], [Reuter, 1996]

$$k\,\partial_k \Gamma_k = \frac{1}{2}\,\mathrm{STr}\left(\left(\Gamma_k^{(2)} + R_k\right)^{-1} k\,\partial_k R_k\right) = \frac{1}{2}$$ 

$\Gamma_k =$ scale dependent effective action

$R_k =$ IR regulator

- exact 1-loop equation
- extract β-functions via projection
- truncation needed $\rightarrow$ not closed



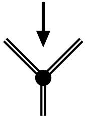
[H. Gies, 2006]

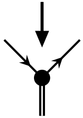
Effective Universality for Gravity and Matter

Avatars of the Newton coupling

- Einstein-Hilbert gravity minimally coupled to fermions, i.e.

$$S = -\frac{1}{16\,\pi\,G_{\mathsf{N}}}\int \mathrm{d}^4x\,\sqrt{g}\,(R-2\Lambda) + \sum_{i=1}^{N_{\mathrm{f}}}\int \mathrm{d}^4x\,\sqrt{g}\,\bar{\psi}^i \not{\nabla}\,\psi^i$$





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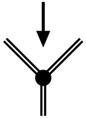
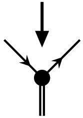


$$\sim \sqrt{G_{3h}}$$



$$\sim \sqrt{G_{h\bar{\psi}\psi}}$$

- two different "avatars" of the Newton coupling

Effective universality

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  - $[G_\mathrm{N}] = -2 \Rightarrow$ 2-loop universality is lost

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  $\Rightarrow$ set of identities (mSTI's) relates avatars
- **Effective universality:**

  Quantitative agreement of different avatars of the Newton coupling

  [A. Eichhorn, P. Labus, J. M. Pawlowski and M. Reichert, 2018]
- Compare both avatars on the level of their β-functions at $(\mu^*, \lambda_3^*, G_{3h} = G_{h\bar{\psi}\psi})$

$$\beta_{G_{3h}} = 2G - 3.4G^2 + 0.37G^3 + \mathcal{O}(G^4)$$
$$\beta_{G_{h\bar{\psi}\psi}} = 2G - 2.8G^2 + 0.42G^3 + \mathcal{O}(G^4)$$

Effective universality for fermions and gravity

Measure of effective universality

$$\varepsilon(G,\mu,\lambda_3) = \left|\frac{\Delta\beta_{G_i} - \Delta\beta_{G_j}}{\Delta\beta_{G_i} + \Delta\beta_{G_j}}\right|_{G_i=G_j}$$

with

$$\Delta\beta_{G_i} = \beta_{G_i} - 2\,G_i$$

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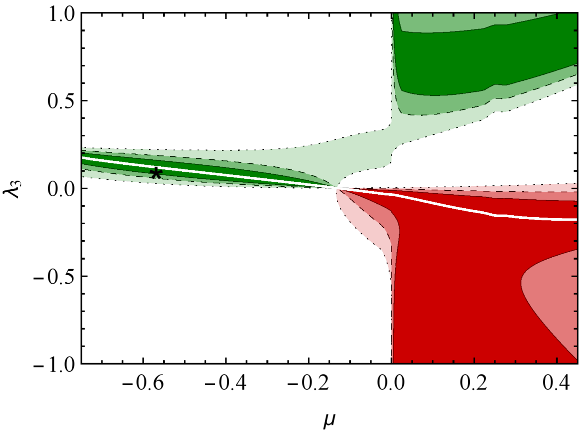

[A. Eichhorn, S. Lippoldt, MS, in preparation]

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- Compare fermions, scalars, vector fields and gravity

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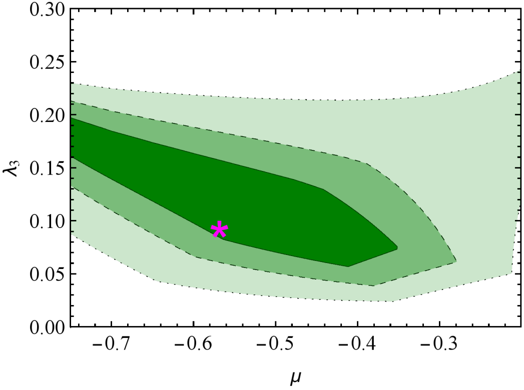

Zoomed in plot region!

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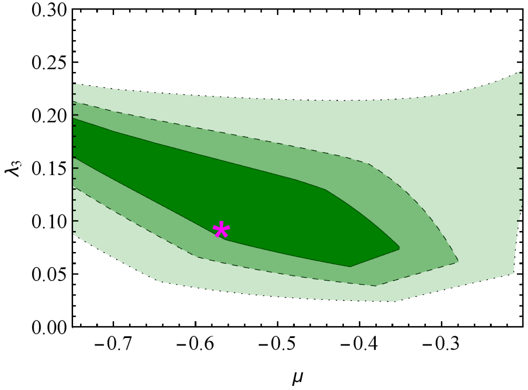

- Compare fermions, scalars, vector fields and gravity
- Effective universality for all avatars of the Newton coupling

<u>Implications</u>

- Highly non-trivial cancellations
- *Not* expected for truncation artifact

$\rightarrow$ strong hint for physical nature of asymptotically safe fixed point

Induced Couplings at UV Fixed Point

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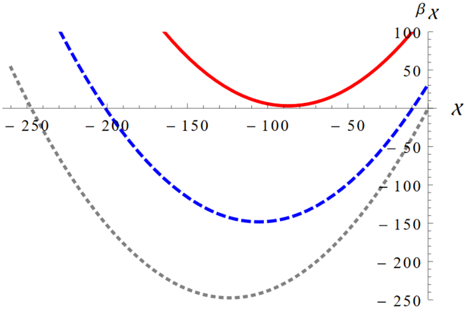

[A. Eichhorn, A. Held and J. M. Pawlowski, 2016]

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- For gravity-matter systems:
  $\exists$ chiral (shift) symmetric matter interactions
- Corresponding coupling does not have GFP if $G_{\text{N}} \neq 0$
- Explicit computations confirm expectation

  [A. Eichhorn and H. Gies, 2011]

  [A. Eichhorn and A. Held, 2017]

	scalars	fermions	gauge fields
scalars	GFP $\lambda_4\,\phi^4$ sGFP $\rho\,(\partial_\mu\phi\partial^\mu\phi)^2$	GFP $y\,\phi\bar{\psi}\psi$ sGFP $\partial_\mu\phi\partial^\mu\phi\bar{\psi}\nabla\!\!\!/\psi$	sGFP $\partial_\mu\phi\partial^\mu\phi\,F^2$
fermions		sGFP $(\bar{\psi}\gamma_\mu\psi)^2$	GFP $e\bar{\psi}A\!\!\!/\psi$ sGFP $\bar{\psi}\nabla\!\!\!/\psi\,F^2$
gauge fields			GFP gluon self-interaction in F^2 sGFP $w_2(F^2)^2\big\vert_{\text{Abelian}}$
complex scalars			GFP $e^2A_\mu\phi^\dagger A^\mu\phi$ sGFP $\partial_\mu\phi^\dagger\partial^\mu\phi\,F^2$

[A. Eichhorn, A. Held and J. M. Pawlowski, 2016]

Induced non-minimal coupling

$$S = S_{\mathsf{EH}} + S_{\mathsf{Dirac}}$$
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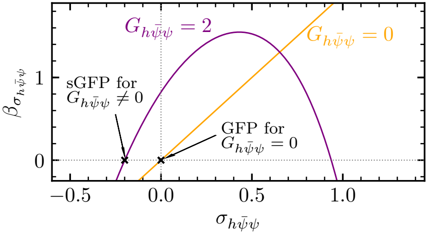

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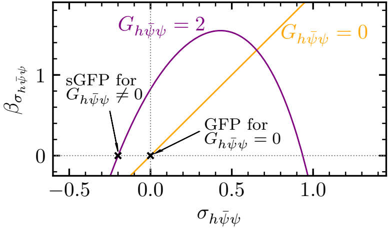

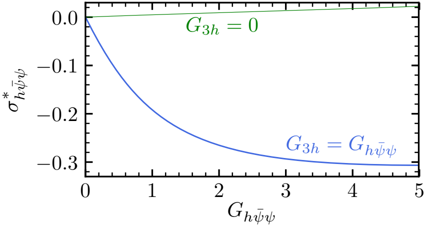

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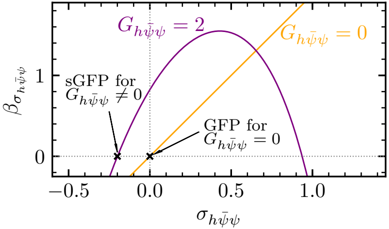

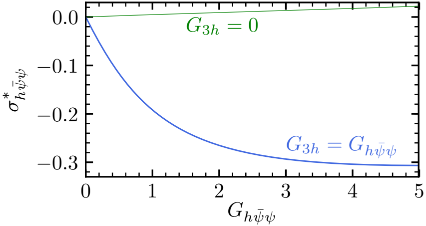

Asymptotic safety passes non-minimal test for a UV complete theory of gravity and matter

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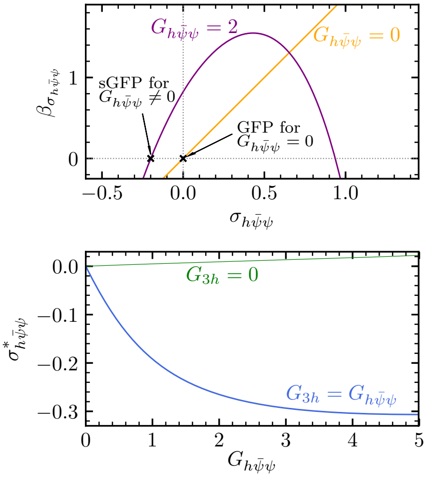
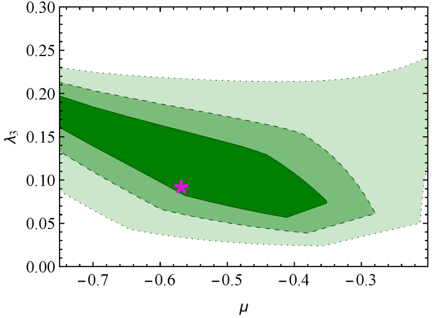


Interacting nature of asymptotically safe fixed point percolates into chiral symmetry-protected matter sector

Summary and Outlook

- Effective universality
  - hint for physical nature of asymptotically safe fixed point
  - guideline/ justification for future/past truncations



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- Explore restrictions of the gravitational parameter space under the inclusion of non-minimal couplings (weak gravity bound [A. Eichhorn and A. Held, 2017])

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Thank you for your attention!

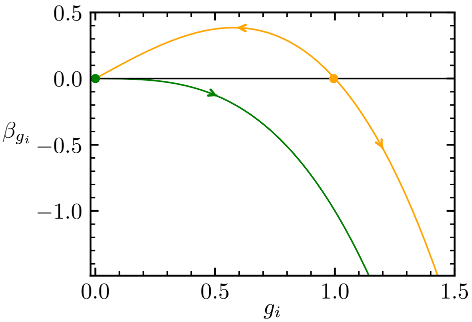
Example for Asymptotic Safety

Example: Yang-Mills Theory in $\mathrm{d} = 4 + \epsilon$ at 1-loop

[M. E. Peskin, 1980], [M. Creutz, 1979], [H. Gies, 2003]

$$[\bar{g}] = -\frac{\epsilon}{2}$$

$$\beta_g = \frac{\epsilon}{2} g \quad - b_0\, g^3$$
$$= g\left(\frac{\epsilon}{2} - b_0\, g^2\right).$$



Nontrivial cancellations