Generic features of the heavy quark QCD phase diagram in 1-loop models and 2-loop Curci-Ferrari

Jan Maelger[1,2]

In collaboration with: U.Reinosa[1] and J.Serreau[2]

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1. Centre de Physique Theorique, Ecole Polytechnique

2. AstroParticule et Cosmologie, Univ. Paris 7 Diderot

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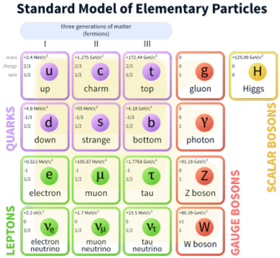
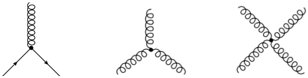
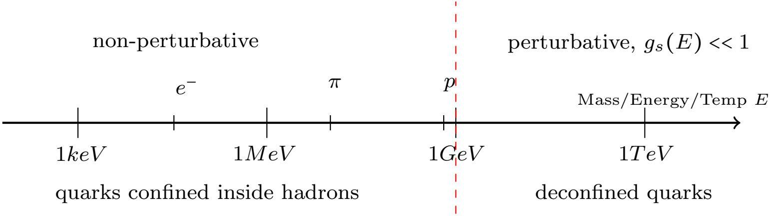
What is QCD?



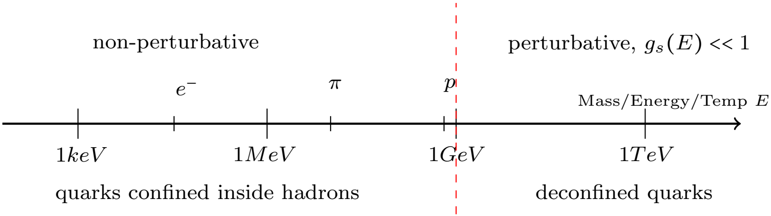


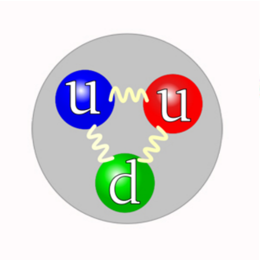
$g_s(E)$ $g_s(E)$ $g_s^2(E)$

A celebrated property:
Asymptotic freedom
$g_s(E) << 1$ for $E >> 1\text{GeV}$



What is QCD?


non-perturbative
perturbative, $g_s(E) << 1$
e^-
π
p
Mass/Energy/Temp E
$1keV$
$1MeV$
$1GeV$
$1TeV$
quarks confined inside hadrons
deconfined quarks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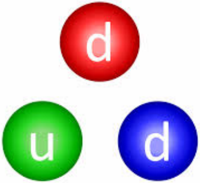
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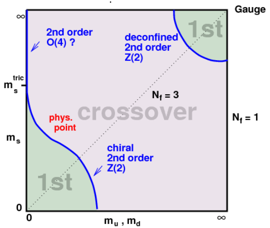
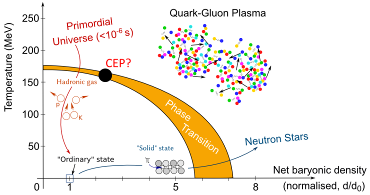
QCD Phase Diagram

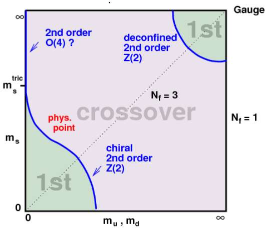




Several approaches on the market:

- Lattice QCD [de Forcrand, Philipsen, Rodriguez-Quintero, Mendes, ...]
- Dyson Schwinger Equations [Alkofer, Fischer, Huber, ...]
- Functional Renormalization Group [Pawlowski, Mitter, Schaefer...]
- Variational Approach [Reinhardt, Quandt, ...]
- Gribov-Zwanziger Action [Dudal, Oliveira, Zwanziger...]
- Matrix-, QM-, NJL-Model,... [Pisarski, Dumitru, Schaffner-B., Stiele, ...]
- Curci-Ferrari Model [Reinosa, Serreau, Tissier, Wschebor, ...]

Outline



- Lattice QCD
- Dyson Schwinger Equations
- Functional Renormalization Group
- Variational Approach
- Gribov-Zwanziger Action
- Matrix-, QM-, NJL-Model,...
- Curci-Ferrari Model

Part 1:

- generic aspects of the heavy quark region
- common to all approaches at one-loop order

Part 2:

- higher order corrections in one particular model
- Curci-Ferrari at two-loop order

Polyakov loops & effective potentials

At the YM point, a relevant order parameter for the deconfinement transition is the (anti-)Polyakov loop. It is related to the free energy F_q necessary to bring a quark into a "bath" of gluons.

$$\ell \equiv \frac{1}{3}\mathrm{tr}\left\langle P\exp\left(ig\int_0^\beta d\tau A_0^a t^a\right)\right\rangle \sim e^{-\beta F_q} \qquad \bar{\ell} \sim e^{-\beta F_{\bar{q}}}$$

Hence

$$\ell = 0 \leftrightarrow F_q = \infty \leftrightarrow \text{confinement} \qquad \ell \neq 0 \leftrightarrow F_q < \infty \leftrightarrow \text{deconfinement}$$

In all models, for each value of the temperature T, one then minimizes an effective potential

$$\boxed{V_{\text{glue}}(\ell,\bar{\ell})}$$

to find the physical position of the system. The particular form of this potential is model-dependent.

Polyakov loops & effective potentials

Introducing quarks, center symmetry is explicitly broken. For heavy quarks, this breaking is "soft", thus:

$$\ell \approx 0 \leftrightarrow F_q \approx \infty \leftrightarrow \text{confinement} \qquad \ell \not\approx 0 \leftrightarrow F_q < \infty \leftrightarrow \text{deconfinement}$$

Therefore $\ell, \bar{\ell}$ are still approximately good order parameters.

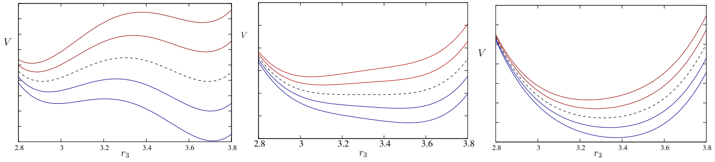
At leading order, the new effective potential is simply found by adding a quark part at one-loop level:

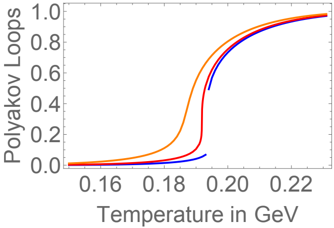
$$\boxed{V_{\text{glue}}(\ell, \bar{\ell}) + V_{\text{quark}}(\ell, \bar{\ell}, \mu)}$$

$\longrightarrow$ Let's look at some possible shapes of such a potential.

$\longrightarrow$ Let's look at some particular cases in more detail.

Order of the transition





⟶ Let's look at some particular cases in more detail.

Explicit Potentials in various Models

Gribov-Zwanziger: [JM, U.Reinosa, J.Serreau (2018)]

$$V_{GZ} = \underbrace{-\frac{d}{2}\frac{\sum_\kappa m_\kappa^4}{g^2 C_{\rm ad}} + \frac{d-1}{2}\sum_\kappa \int_Q^T \ln\frac{Q_\kappa^4 + m_\kappa^4}{Q_\kappa^2} - \frac{1}{2}\sum_\kappa \int_Q^T \ln Q_\kappa^2}_{V_{\rm glue}} \quad -{\rm Tr\,Ln}\,(\not\partial + M)$$

Curci-Ferrari: [U. Reinosa, J. Serreau, M. Tissier (2015)]

$$V_{CF} = \underbrace{\sum_\kappa \frac{T}{2\pi^2}\int_0^\infty dq\, q^2 \left\{3\ln\left[1 - e^{-\beta\varepsilon_q + ir_\kappa}\right] - \ln\left[1 - e^{-\beta q + ir_\kappa}\right]\right\}}_{V_{\rm glue}} \quad -{\rm Tr\,Ln}\left(\not\partial + M - ig\gamma_0 \bar{A}^k t^k\right)$$

Matrix-Models: [K.Kashiwa, R.D.Pisarski and V.V.Skokov (2012)]

$$V_M = -\frac{4\pi^2}{3}T^2 T_d^2\left(c_1 \sum_{i,j=1}^N V_1(q_i - q_j) + c_2 \sum_{i,j=1}^N V_2(q_i - q_j) + \frac{(N^2-1)}{60}c_3\right)$$

$$\underbrace{-\frac{(N^2-1)\pi^2}{45}T^4 + \frac{2\pi^2}{3}T^4 \sum_{i,j=1}^N V_2(q_i - q_j)}_{V_{\rm glue}} + \ln\det(\gamma^\mu\partial_\mu + q\delta^{\mu 4} + im)$$

Lattice: [M.Fromm, J.Langelage, S.Lottini and O.Philipsen (2012)]

$$Z_{\rm eff} = \int [dU_0] \underbrace{\left(\prod_{<ij>}\left[1 + 2\lambda_1 {\rm Re} L_i^* L_j\right]\right)}_{Z_{\rm glue}} \left(\prod_{\vec{x}} \det\left[\left(1 + h_1 W_{\vec{x}}\right)\left(1 + \bar{h}_1 W_{\vec{x}}^\dagger\right)\right]^{2N_f}\right)$$

Commonalities & Assumptions

- Potential v_{glue} is confining, with a minimum at $\ell = 0$ at zero temperature
- Quarks are added at one-loop level, in form of a Tr Ln

$$V = V_{\text{glue}} - \operatorname{Tr}\operatorname{Ln}\left(\partial\!\!\!/ + M\right)$$

Then in the heavy quark limit, the Tr Ln expands and one finds

$$\boxed{\beta^4 V(\ell, \beta, M) = v_{\text{glue}}(\ell, \beta) - 2N_f\, f(\beta M)\,\ell}$$

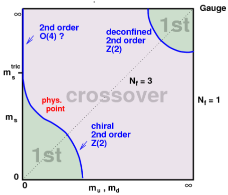
$$f(x) = (3x^2/\pi^2)K_2(x)$$

$K_2(x)$ is the modified Bessel function of the second kind

ℓ: Polyakov loop
β: inverse temp.
M: deg. quark mass

$\longrightarrow$ How do we find the 2nd order critical line?

Determination of the critical line



$$\beta^4 V(\ell,\beta,M) = v_{\rm glue}(\ell,\beta) - 2N_f\, f(\beta M)\,\ell$$

For a **fixed** N_f:

3 parameters: $\ell,\ \beta,\ \beta M$
3 equations: $\partial_\ell V = \partial_\ell^2 V = \partial_\ell^3 V = 0$

This yields:

$$\underbrace{\partial_\ell v_{\rm glue} = 2N_f\, f(\beta M)}_{\text{determines model-dep. } \beta M}\ , \qquad \underbrace{\partial_\ell^2 v_{\rm glue} = \partial_\ell^3 v_{\rm glue} = 0}_{\text{determines } \ell,\, \beta\text{, indep. of } \beta M,\, N_f}$$

$\longrightarrow$ $\boxed{N_f\, f(\beta M) = N_f'\, f(\beta M') = f(\beta M_s) + 2f(\beta M_{ud})}$ is const. on the critical line!

Determination at non-vanishing chemical potential

$$\beta^4 V = v_{\text{glue}}(\ell, \bar{\ell}, \beta) - N_f f(\beta M)(e^{-\beta\mu}\ell + e^{\beta\mu}\bar{\ell})$$

For a **fixed** N_f, μ:

4 parameters: ℓ, $\bar{\ell}$, β, βM
4 equations:

$$\partial_\ell V = \partial_{\bar{\ell}} V = 0\,, \tag{1}$$

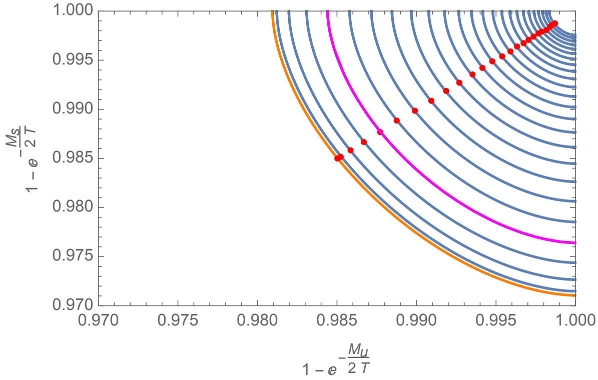
$$\underbrace{\partial_\ell^2 V \partial_{\bar{\ell}}^2 V - (\partial_\ell \partial_{\bar{\ell}} V)^2 = (a\partial_\ell - b\partial_{\bar{\ell}})^3 V = 0}_{\ell(\beta),\bar{\ell}(\beta)\,\text{indep. of}\,N_f,\mu} \tag{2}$$

with $a = \partial_{\bar{\ell}}^2 V|_c$ and $b = \partial_\ell \partial_{\bar{\ell}} V|_c$. The first two equations rewrite

$$N_f f(\beta M) = e^{\beta\mu}\,\partial_\ell v_{\text{glue}} = e^{-\beta\mu}\,\partial_{\bar{\ell}} v_{\text{glue}} \longrightarrow \underbrace{e^{-2\beta\mu} = \partial_\ell v_{\text{glue}}/\partial_{\bar{\ell}} v_{\text{glue}}}_{\ell(\beta(\mu)),\,\bar{\ell}(\beta(\mu))\,\text{indep. of}\,N_f} \tag{3}$$

$\longrightarrow$ $\boxed{N_f f(\beta M) = N_f' f(\beta M') = f(\beta M_s) + 2f(\beta M_{ud})}$ for each value of μ

$N_f f(\beta M) = N'_f f(\beta M')$ So what?



Red: Model-dep. $N_f = 3$ input determined from: $\partial_\ell V = \partial_\ell^2 V = \partial_\ell^3 V = 0$

Blue: Model-indep. line from: $N_f f(\beta M) = f(\beta M_s) + 2f(\beta M_{ud})$

$N_f f(\beta M) = N_f' f(\beta M')$ So what?

If expanded in large $R \equiv \beta M$, allows for the simple relation

$$R_{N_f'} - R_{N_f} \approx \ln \frac{N_f'}{N_f} \quad \longrightarrow \quad \boxed{Y_{N_f} \equiv \frac{R_{N_f} - R_1}{R_2 - R_1} \approx \frac{\ln N_f}{\ln 2}}$$

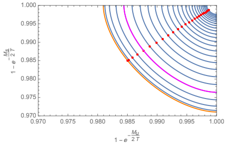
- satisfied both in continuum approaches as well as on the lattice
- robust against higher order corrections in the large βM expansion
- independent of chemical potential
- predict R_{N_f} for $N_f > 3$ or $\notin \mathbb{Z}$

Y_3	$\mu = 0$	$\mu = i\pi T/3$
Lattice	1.59	1.59
GZ1	1.58	1.57
GZ2	1.58	1.58
Matrix	1.59	1.56
CF	1.58	1.57

$$Y_3 \approx \frac{\ln 3}{\ln 2} \approx 1.58$$

Intermediate Summary for One-loop Models

- Heavy Quark region exhibits generic features among all one-loop models
- T_c constant along the critical line, whose shape is completely fixed, independently of μ
- Flavor dependence of the critical mass is independent of the gluon dynamics, as predicted by the universal quantity Y_{N_f}



$$Y_{N_f} \equiv \frac{R_{N_f} - R_1}{R_2 - R_1} \approx \frac{\ln N_f}{\ln 2}$$

Two assumptions were made:

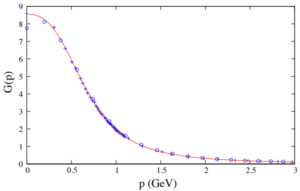
- large quark mass epansion
- quarks contribute at one-loop level

Curci-Ferrari and gluon mass term

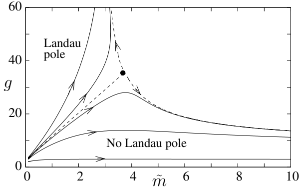
$$S = \int_x \left\{ \frac{1}{4}(F^a_{\mu\nu})^2 + \bar{\psi}(\not{D} + M + \mu\gamma_0)\psi \right\} + S_{FP} + \int_x \left\{ \frac{1}{2} m^2 (A^a_\mu)^2 \right\}$$

This gluon mass term can be motivated in several ways

- phenomenologically from lattice data of the Landau gauge gluon propagator saturating in the IR
- Residual ambiguity after non-complete gauge-fixing in Fadeev-Popov procedure due to presence of Gribov copies



one-loop gluon propagator against lattice data,
from [Tissier, Wschebor (2011)]
[Bogolubsky et al. (2009), Dudal, Oliveira, Vandersickel (2010)]



YM one-loop RG flow,
from [Serreau, Tissier (2012)]

Landau-DeWitt gauge [Braun, Pawlowski, Gies (2010)]

$$A^a_\mu = \bar{A}^a_\mu + a^a_\mu$$

In practice, at each temperature, the background field $\bar{A}^a_\mu$ is chosen such that the expectation value $\langle a^a_\mu \rangle$ vanishes in the limit of vanishing sources.

This corresponds to finding the absolute minimum of $\tilde{\Gamma}[\bar{A}] \equiv \Gamma[\bar{A}, \langle a \rangle = 0]$, where $\Gamma[\bar{A}, \langle a \rangle]$ is the effective action for $\langle a \rangle$ in the presence of $\bar{A}$.

Seek the minima in the subspace of configurations $\bar{A}$ that respect the symmetries of the system at finite temperature.
$\longrightarrow$ One restricts to temporal and homogenous backgrounds:

$$\bar{A}_\mu(\tau, \mathbf{x}) = \bar{A}_0 \delta_{\mu 0}$$

$\longrightarrow$ functional $\tilde{\Gamma}[\bar{A}]$ reduces to an effective potential $V(\bar{A}_0)$ for the constant matrix field $\bar{A}_0$.

One can always rotate this matrix $\bar{A}_0$ into the Cartan subalgebra:

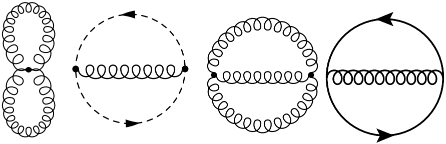
$$\beta g \bar{A}_0 = r_3 \frac{\lambda_3}{2} + r_8 \frac{\lambda_8}{2}$$

Then $V(\bar{A}_0)$ reduces to a function of 2 components $V(r_3, r_8)$.

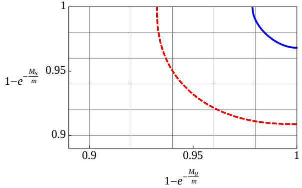
	r_3	r_8
$\mu = 0$	$\mathbb{R}$	0
$\mu \in i\,\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$
$\mu \in \mathbb{R}$	$\mathbb{R}$	$i\,\mathbb{R}$

Two-loop Expansion

$$
\begin{aligned}
V(r_3, r_8) \quad = \quad & -\mathrm{Tr}\,\mathrm{Ln}\left(\partial\!\!\!/ + M + \mu\gamma_0 - ig\gamma_0 \bar{A}^k t^k\right) \\
& + \frac{3}{2}\mathrm{Tr}\,\mathrm{Ln}\left(\bar{D}^2 + m^2\right) - \frac{1}{2}\mathrm{Tr}\,\mathrm{Ln}\left(\bar{D}^2\right) \\
& +
\end{aligned}
$$



Vanishing chemical potential



$$R_{N_f} \equiv \frac{M_c(N_f)}{T_c(N_f)}$$

$\mathcal{O}(1)$: $\quad M_{\text{bare}} = M_{\text{ren.}}$
$\mathcal{O}(g^2)$: $\; M_{\text{bare}} = Z_M \, M_{\text{ren.}} + C_M$

$\longrightarrow$ hard to compare between different approaches!

However, Z_M, C_M are independent of N_f at $\mathcal{O}(g^2)$, and observing

$$\frac{T_c(N_f=3) - T_c(N_f=1)}{T_c(N_f=1)} \approx 0.2\%$$

allows for:

$$\overbrace{R_{N_f'}/R_{N_f} \approx M_c(N_f')/M_c(N_f)}^{\text{if } C_M=0} \qquad \overbrace{Y_{N_f} \equiv \frac{R_{N_f} - R_1}{R_2 - R_1}}^{\text{if } C_M \neq 0}$$

is scheme indep. & comparable to other approaches up to higher order corrections.

Vanishing chemical potential

$\mu = 0$	R_1	R_2	R_3	R_2/R_1	R_3/R_1	Y_3
Matrix [1]	8.04	8.85	9.33	1.10	1.16	1.59
GZ1 [2]	7.09	7.92	8.40	1.12	1.19	1.58
GZ2 [2]	9.45	10.25	10.72	1.08	1.13	1.58
CF 1-loop [3]	6.74	7.59	8.07	1.13	1.20	1.58
CF 2-loop [2]	7.53	8.40	8.90	1.12	1.18	1.57
Lattice [4]	7.23	7.92	8.33	1.10	1.15	1.59
DSE [5]	1.42	1.83	2.04	1.29	1.43	1.51

$\longrightarrow$ The Y_3 values are still satisfied to very good approximation which underlines its importance as a universal quantitiy

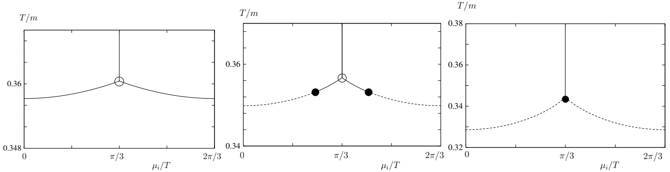
$\longrightarrow$ The overall good agreement seems to suggest that the underlying dynamics is well-described within (Curci-Ferrari) perturbation theory.

[1] Kashiwa, Pisarski, Skokov (2012) [2] JM, Reinosa, Serreau (2017+18)

[3] Reinosa, Serreau, Tissier (2015) [4] Fromm, Langelage, Lottini, Philipsen (2012)

[5] Fischer, Luecker, Pawlowski (2015)

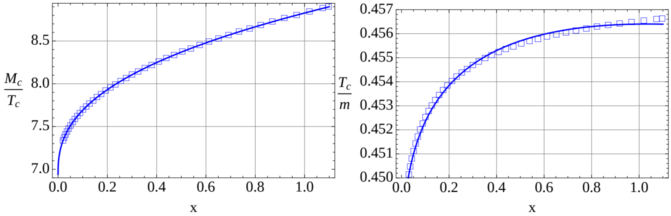
Imaginary chemical potential $\mu = i\mu_i$



The vicinity of the tricritical point is approximately described by the mean field scaling behavior

$$\frac{M_c(\mu_i)}{T_c(\mu_i)} = \frac{M_{\text{tric.}}}{T_{\text{tric.}}} + K\left[\left(\frac{\pi}{3}\right)^2 - \left(\frac{\mu_i}{T_c}\right)^2\right]^{\frac{2}{5}}$$

[de Forcrand, Philipsen (2010); Fischer, Luecker, Pawlowski (2015)]



$$x \equiv (\pi/3)^2 + (\mu/T_c)^2 = (\pi/3)^2 - (\mu_i/T_c)^2$$

Imaginary chemical potential $\mu = i\mu_i$

$\mu = i\pi T/3$	R_1	R_2	R_3	R_2/R_1	R_3/R_1	Y_3
Matrix [1]	5.00	5.90	6.40	1.18	1.28	1.56
GZ1 [2]	5.02	5.92	6.43	1.18	1.28	1.57
GZ2 [2]	7.51	8.34	8.82	1.11	1.17	1.58
CF 1-loop [3]	4.74	5.63	6.15	1.19	1.30	1.57
CF 2-loop [2]	5.47	6.41	6.94	1.17	1.27	1.57
Lattice [4]	5.56	6.25	6.66	1.12	1.20	1.59
DSE [5]	0.41	0.85	1.11	2.07	2.70	1.59

$\longrightarrow$ The Y_3 values are in overall very good agreement between all cases, one loop models and higher order ones.

[1] Kashiwa, Pisarski, Skokov (2012) [2] JM, Reinosa, Serreau (2017+18)

[3] Reinosa, Serreau, Tissier (2015) [4] Fromm, Langelage, Lottini, Philipsen (2012)

[5] Fischer, Luecker, Pawlowski (2015)

Conclusion

One-loop:

- Heavy Quark region exhibits generic features among all one-loop models
- T_c constant along the critical line, whose shape is completely fixed, independently of μ
- Flavor dependence of the critical mass is independent of the gluon dynamics, as predicted by the universal quantity Y_{N_f}

Higher order:

- updated Y_3 values still agree with one-loop predictions
- suggests that the perturbative description of the phase diagram within the CF model is robust

Outlook:

- Can we describe the chiral transition in the lower left part of the Columbia plot?