Beta functions at large N_f

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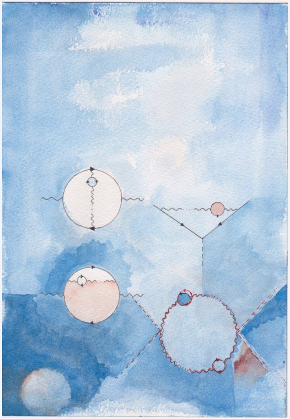
ITP, Heidelberg University, 17th July 2018

Outline

- Introduction and motivation
- Gauge-fermion theories
- Gauge-Yukawa theories
- Summary and outlook



Kaća Bradonjić

Based on:

Oleg Antipin, Nicola Andrea Dondi, Francesco Sannino, AET, and Zhi-Wei Wang [arXiv:1803.09770], to appear in PRD

Outline

Renormalization group flow

$$\mathcal{L} = -\tfrac{1}{4} F_{0,\mu\nu} F_0^{\mu\nu} + i\,\overline{\Psi}_0 \gamma^\mu (\partial_\mu - i g_0\, A_{0,\mu}) \Psi_0$$

Quantum corrections give infinite contributions

$$= -g_0^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\mathrm{Tr}\left[\gamma_\mu (\not{k} - \not{p}) \gamma_\nu \not{k}\right]}{k^2 (k-p)^2} = \infty$$

Calculability is recovered using dimensional regularization, $d = 4 - \epsilon$

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$$\simeq -i \frac{g_0^2}{12\pi^2} (p^2 g_{\mu\nu} - p_\mu p_\nu) \left[\frac{2}{\epsilon} + \frac{5}{3} - \gamma_E + \log\left(-\frac{4\pi}{p^2}\right)\right]$$

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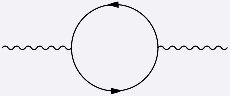

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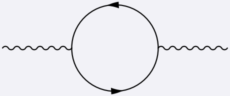

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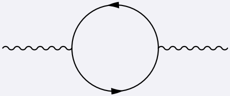

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$$g_0 = \mu^{\epsilon/2} Z_g g, \quad \text{where} \quad Z_g = 1 + \frac{1}{\epsilon} Z_g^{(1)} + \frac{1}{\epsilon^2} Z_g^{(2)} + \dots$$

As a result the renormalized coupling $g(\mu)$ gets a running

$$\beta_g = \frac{\mathrm{d}g}{\mathrm{d}\ln\mu} = \tfrac{1}{2}\left(-1 + g\frac{\partial}{\partial g}\right)(-\tfrac{1}{2}Z_g^{(1)}g) = \frac{g^3}{12\pi^2} + \mathcal{O}(g^5)$$

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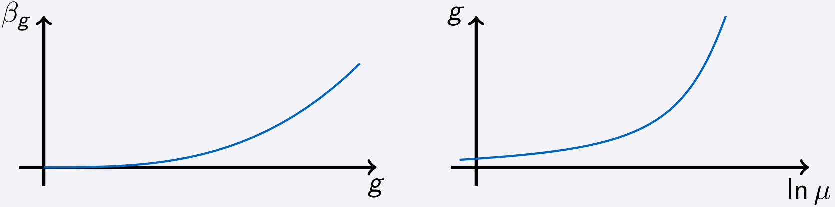
QED: Landau Pole



A fundamental theory must reach a FP in the UV

QCD: Asymptotic freedom

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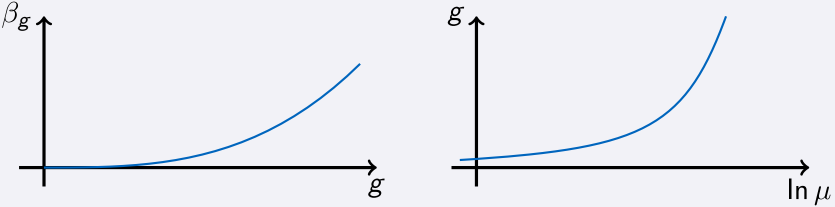

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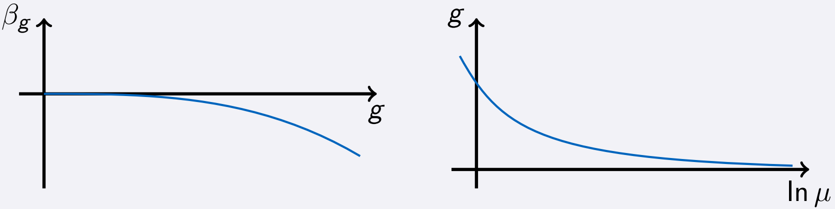

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$$\beta_{\alpha_g} = \frac{4}{3}\left(\frac{N_f}{N_c} - \frac{11}{2} + f(\alpha_g, \alpha_y, \alpha_\lambda)\right)\alpha_g^2$$

A perturbative FP can be reached at $N_f, N_c \to \infty$

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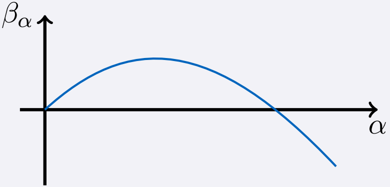

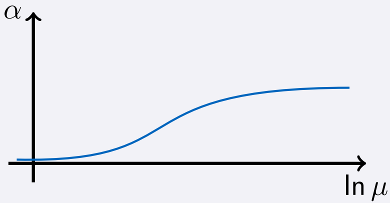

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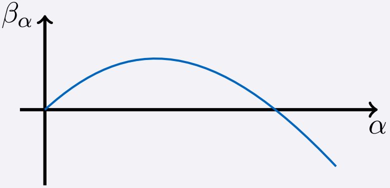

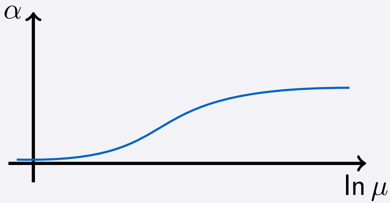

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Where the large N_f fits in

Idea: Organize the the computation as an expansion in $1/N_f$

- Computational control in a limit of QFT
- A new non-vanishing zero of the beta function

- A tool for model building

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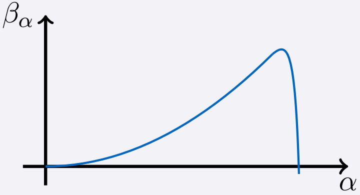

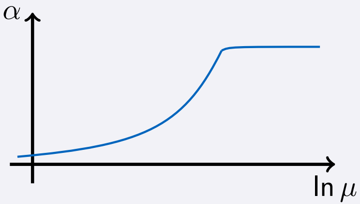

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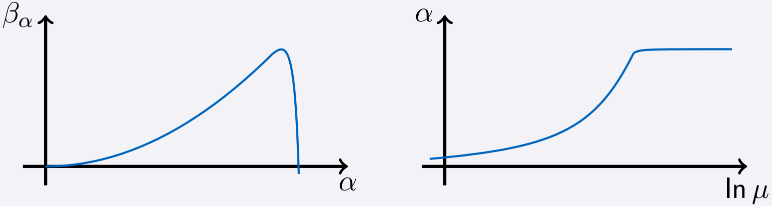

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$1/N_f$ counting

The goal is to expand the a gauge theory in $1/N_f$;

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{I=1}^{N_f} i\,\overline{\Psi}_I \gamma^\mu (\partial_\mu - ig\,A_\mu) \Psi^I$$

It is insufficient to take $N_f \to \infty$:

$$: \qquad \beta_g = \frac{\mathrm{d}g}{\mathrm{d}t} = \frac{1}{12\pi^2} g^3 N_f$$

Introduce a 't Hooft-like coupling

$$K = \frac{g^2 N_f}{4\pi^2}$$

$$1/N_f \ll 1 \text{ limit:} \qquad K = \text{cst.} \quad g \sim \sqrt{1/N_f}$$

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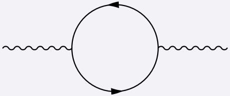
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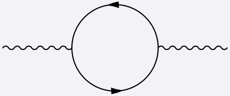 :

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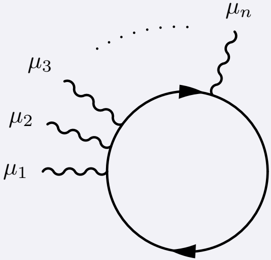

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- Loops with $n = 2$ are "free"; internal gauge lines must be dressed as

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- Loops with $n \geq 3$ decreases the order of a diagram.

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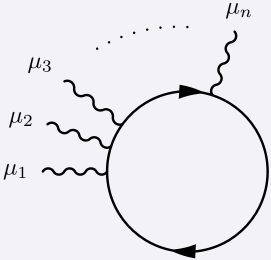

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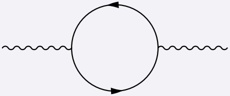
Large N_f beta function in QED

At LO in $1/N_f$ the beta function for the renormalized coupling, K, is

 :

$$\beta_K = \frac{\mathrm{d}K}{\mathrm{d}t} = \frac{2K^2}{3} + \dots$$

At NLO there is an infinite number of diagrams:

A. Planques-Mestre and P. Pascual '84

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F_1 ($\tilde{F}$) has a pole at $K = 15/2$ ($x = 5$)

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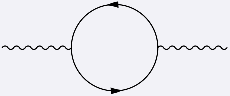

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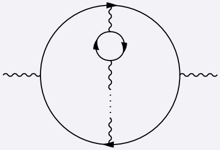

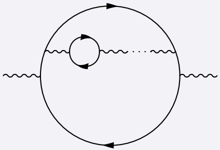

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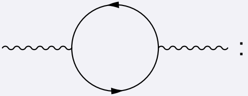

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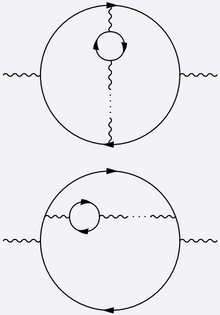

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Non-Abelian gauge theories

In simple non-Abelian theories a new renormalized coupling is introduced

$$K = \frac{g^2 N_f\, S_2(R_\Psi)}{4\pi^2}$$

Self-interacting gluons give new contributions at NLO:

J.A. Gracey [hep-ph/9602214] B. Holdom [1006.2119]

$$\beta_K = \frac{2K^2}{3}\left[1 + \frac{1}{N_f}\frac{d(G)}{d(R_\Psi)} H_1(K)\right] + \ldots$$

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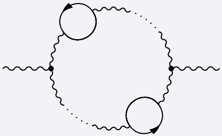

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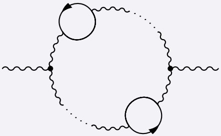

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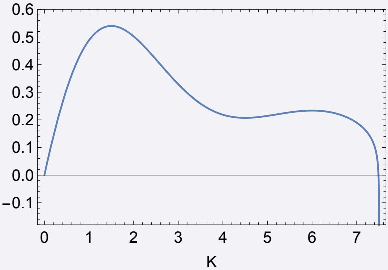
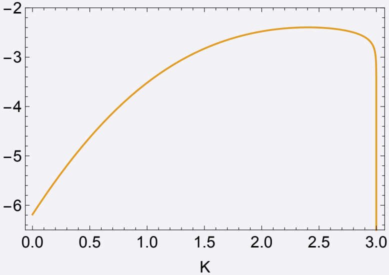
Zeros of the beta functions

Behavior of the NLO contributions ($N_c = 3$, $R_\Psi = \text{fund}$)



(a) Abelian, $F_1(K)$



(b) Non-Abelian, $H_1(K)$

$$\beta_K = \frac{2K^2}{3}\left[1 + \frac{1}{N_f} F_1(K)\right] \qquad \beta_K = \frac{2K^2}{3}\left[1 + \frac{1}{N_f}\frac{d(G)}{d(R_\Psi)} H_1(K)\right]$$

Semi-simple extension

Consider a gauge group $G = \times_\alpha G_\alpha$ and fermions $\Psi_I \in N_f \times (\otimes_\alpha R_\Psi^\alpha)$

New effective flavor number

$$\mathcal{N} \equiv N_f \prod_\alpha d(R_\Psi^\alpha) \quad \text{and} \quad K_\alpha = \frac{g_\alpha^2 \, \mathcal{N} \, S_2(R_\Psi^\alpha)}{4\pi^2 \, d(R_\Psi^\alpha)}$$

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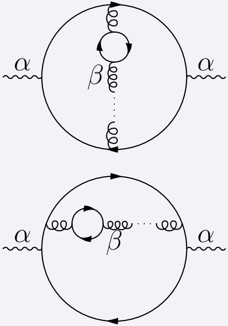

New mixed diagrams are like the Abelian NLO diagrams, but

$$g^4 N_f \longrightarrow g_\alpha^2 g_\beta^2 \mathrm{Tr}\left[T^A_{R_\Psi^\alpha} T^B_{R_\Psi^\alpha} T^C_{R_\Psi^\beta} T^C_{R_\Psi^\beta} \right]$$
$$= (4\pi^2)^2 \frac{d(G_\beta)}{\mathcal{N}} K_\alpha K_\beta \delta^{AB}$$

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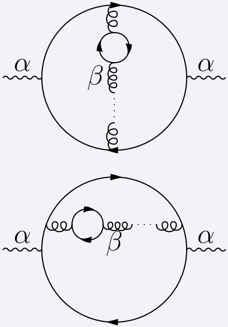
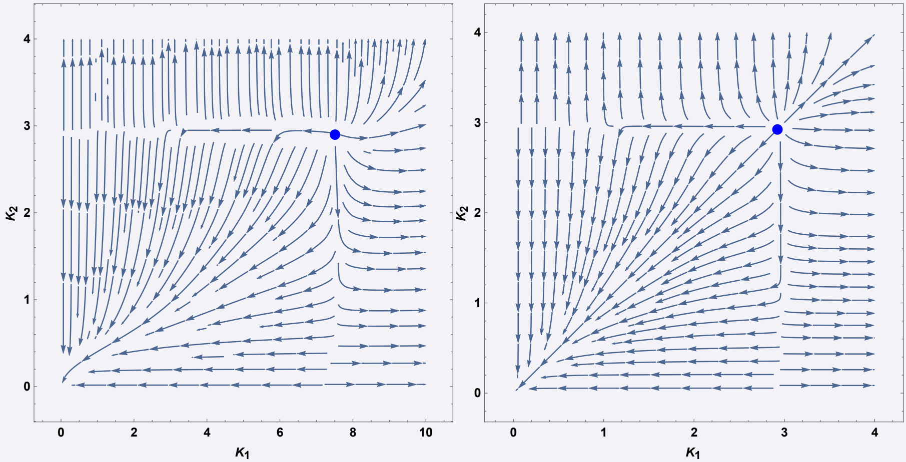


The full NLO beta function

$$\beta_{K_\alpha} = \frac{2K_\alpha^2}{3}\left[1 + \frac{d(G_\alpha)}{\mathcal{N}} H_1^{(\alpha)}(K_\alpha) + \sum_{\beta\neq\alpha} \frac{d(G_\beta)}{\mathcal{N}} F_1(K_\beta)\right]$$

Semi-simple phase diagram



(c) An Abelian and a non-Abelian group

(d) Two non-Abelian groups

Asymptotic safety?

F. Sannino and O. Antipin [1709.02354], B. Holdom [1006.2119]

- Large N_f expansion seems to be valid for $N_f \gtrsim 10N_c$ with $\Psi \in \boldsymbol{N_c}$
- Anomalous dimension as a check of the FP
  - Abelian: $\gamma_m \to \infty$ for $N_f \to \infty$
  - Non-Abelian: $\gamma_m \to 0$ for $N_f \to \infty$
- Series expansion of F_1 and H_1 seems to indicate worse convergence of the Abelian series

We should be skeptic of the Abelian fixed point

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Outline

Fields	$SO(1,3)^+$	$SU(N_f)$	$\times_\alpha G_\alpha$
Ψ	$\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)$	N_f	$\otimes_\alpha R^\alpha_\Psi$
χ	$\left(\frac{1}{2},0\right)$	1	$\otimes_\alpha R^\alpha_\chi$
ϕ	$(0,0)$	1	$\otimes_\alpha R^\alpha_\phi$

The most general Lagrangian with real scalars and Weyl fermions

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} + \sum_{I=1}^{N_f}\overline{\Psi}_I i\gamma^\mu D_\mu \Psi^I + i\bar{\chi}_i\bar{\sigma}^\mu(D_\mu\chi)^i + \frac{1}{2}(D_\mu\phi)^a(D_\mu\phi)^a$$
$$-\frac{1}{2}\left(y_{aij}\phi^a\chi^i\chi^j \; + \; \text{h.c.}\right) - \frac{1}{24}\lambda_{abcd}\phi^a\phi^b\phi^c\phi^d$$

Perturbative counting

$$\lambda \sim y^2 \sim g^2 \sim \frac{1}{N}$$

ensures that higher order contributions in regular loop counting can be consistently ignored.

Generic gauge-Yukawa theory

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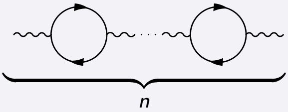
Large N_f machinery: the bubble chain

In the Landau gauge $\xi = 0$,

$D^{(n)}_{\mu\nu}(p) =$



$= \dfrac{-i}{p^2}\Delta_{\mu\nu}(p)\,\Pi_0^n(p^2)$

The single bubble contribution is ($d = 4 - \epsilon$ dimensions)

$$\Pi_0(p^2) = -2K_0\,\frac{\Gamma^2(2-\frac{\epsilon}{2})\Gamma(\frac{\epsilon}{2})}{\Gamma(4-\epsilon)}\left(-\frac{4\pi\mu^2}{p^2}\right)^{\epsilon/2}$$

The dimension-dependent contribution from the bubble shows up in every beta-function as $\Gamma_0^{-1}(\frac{2}{3}K) \longrightarrow$ pole at $K = 15/2$.

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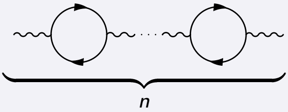

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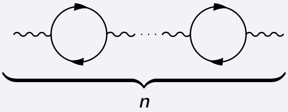
$D_{\mu\nu}^{(n)}(p) =$  $= \dfrac{-i}{p^2} \Delta_{\mu\nu}(p)\, \Pi_0^n(p^2)$

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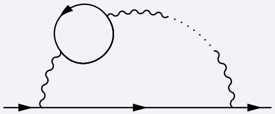
Fermion self-energy



n-bubble contribution to Weyl-spinor self-energy

$$-i\Sigma_{\chi}^{(n)}(p) = (i\tilde{g}_0)^2 C_2(R_\chi)\mu^\epsilon \int \frac{\mathrm{d}^d k}{(2\pi)^d} \bar{\sigma}^\mu \frac{i\sigma\cdot(p-k)}{(p-k)^2} \bar{\sigma}^\nu D_{\nu\mu}^{(n)}(k)$$

$Z_\chi^{(1)}$ is given by the simple pole of

$$\frac{\mathrm{d}\Sigma_\chi}{\mathrm{d}\bar{\sigma}\cdot p} = -\frac{9}{16\mathcal{N}} \frac{d(R_\Psi)C_2(R_\chi)}{S_2(R_\Psi)} \sum_{n=1}^{\infty} \left(-\frac{2K_0}{3}\right)^n \frac{1}{n\epsilon^n} H_\psi(n,\epsilon)$$

$H_\psi(n,\epsilon)$ is a regular function in $n\epsilon$ and ϵ. A. Planques-Mestre and P. Pascual '84

Resummation required to extract the $1/\epsilon$ pole

$$K_0 = Z_K^{-1} K = K\left[1 - \frac{2K}{3\epsilon} + O\left(\frac{1}{\mathcal{N}}\right)\right]$$

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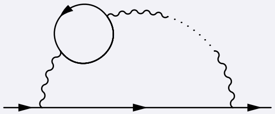

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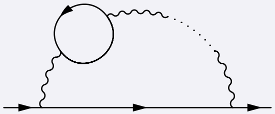

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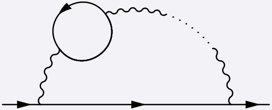
Self-energies



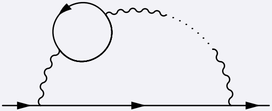
$$Z_\chi^{(1)} = \frac{d(R_\Psi)}{4\mathcal{N}\, S_2(R_\Psi)} C_2(R_\chi) \int_0^K \mathrm{d}x\, x H_0(\tfrac{2}{3}x)$$

The new function $H_0(x) = H_\psi(0,x)/x$ and is given by

$$H_0(x) = \frac{(1-\frac{x}{3})\Gamma(4-x)}{3\Gamma(3-\frac{x}{2})\Gamma^2(2-\frac{x}{2})\Gamma(1+\frac{x}{2})} = 1 - \frac{5}{12}x - \frac{35}{144}x^2 + \dots$$

The scalar self-energy is computed in a similar manner

$$Z_\phi^{(1)} = \frac{3\, d(R_\Psi)}{2\mathcal{N}\, S_2(R_\Psi)} C_2(R_\phi) \int_0^K \mathrm{d}x\, H_0(\tfrac{2}{3}x)$$



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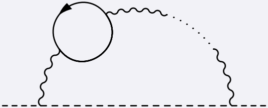

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The Yukawa beta function begins at order

$$\mathcal{O}(y^3) = \mathcal{O}(yg^2K^n) = \mathcal{O}\left(\frac{y}{\mathcal{N}}\right)$$

Only one kind of diagrams contribute to the vertex correction in the Landau gauge

K. Kowalska and E.M. Sessolo [1712.06859]

$$\delta y^{(1)}_{aij} = -\frac{3d(R_\Psi)}{2\mathcal{N}} \frac{y_{akj} C_2(R_\chi)^k{}_i + y_{aik} C_2(R_\chi)^k{}_j - y_{bij} C_2(R_\phi)_{ab}}{2S_2(R_\Psi)} \times \int_0^K \mathrm{d}x \, (1 - \tfrac{1}{6}x) H_0(\tfrac{2}{3}x)$$

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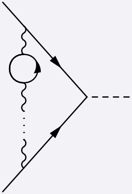

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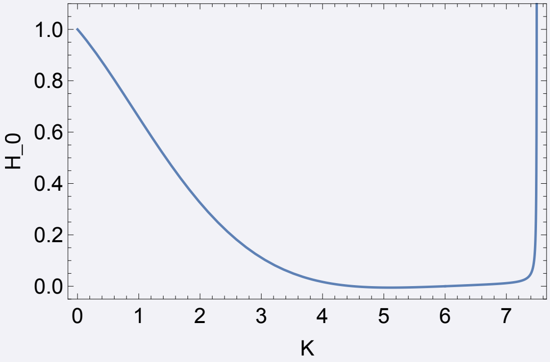
Yukawa beta function

The Yukawa beta function at LO in $1/\mathcal{N}$ including regular 1-loop contributions

$$\begin{aligned}\beta_{y,aij} =& \frac{1}{32\pi^2}\left(y_b y_b^\dagger y_a + y_a y_b^\dagger y_b\right)_{ij} + \frac{1}{32\pi^2}\mathrm{Tr}\left[y_a y_b^\dagger + y_a^\dagger y_b\right] y_{bij} \\ &+ \frac{1}{8\pi^2}(y_b y_a^\dagger y_b)_{ij} - \sum_\alpha \frac{d(R_\Psi^\alpha)}{8\mathcal{N}} \frac{y_{bij}\, C_2(R_\phi^\alpha)_{ab}}{S_2(R_\Psi^\alpha)} K_\alpha^2\, H_0(\tfrac{2}{3}K_\alpha) \\ &- \sum_\alpha \frac{3d(R_\Psi^\alpha)}{4\mathcal{N}} \frac{y_{akj}\, C_2(R_\chi^\alpha)^k{}_i + y_{aik}\, C_2(R_\chi^\alpha)^k{}_j}{S_2(R_\Psi^\alpha)} K_\alpha\, H_0(\tfrac{2}{3}K_\alpha)\end{aligned}$$

The H_0 function



$$H_0(\tfrac{2}{3}K) = \frac{2}{45\pi^2}\frac{1}{\frac{15}{2} - K} + \frac{2(1 + 60\ln 2)}{2025\pi^2} + \dots$$

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The Yukawa beta function at LO in $1/\mathcal{N}$ including regular 1-loop contributions

$$\beta_{y,aij} = \frac{1}{32\pi^2}\left(y_b y_b^\dagger y_a + y_a y_b^\dagger y_b\right)_{ij} + \frac{1}{32\pi^2}\mathrm{Tr}\left[y_a y_b^\dagger + y_a^\dagger y_b\right] y_{bij}$$
$$+\frac{1}{8\pi^2}(y_b y_a^\dagger y_b)_{ij} - \sum_\alpha \frac{d(R_\Psi^\alpha)}{8\mathcal{N}} \frac{y_{bij}\, C_2(R_\phi^\alpha)_{ab}}{S_2(R_\Psi^\alpha)} K_\alpha^2\, H_0(\tfrac{2}{3}K_\alpha)$$
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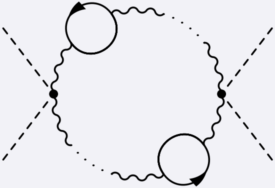
Quartic beta function

The quartic beta function begins at

$$\mathcal{O}(\lambda)^2 = \mathcal{O}(y)^4 = \mathcal{O}(\lambda y^2) = \mathcal{O}(\lambda g^2 K^n) = \mathcal{O}(g^4 K^n) = \mathcal{O}\left(\frac{1}{\mathcal{N}}\right)^2$$

Leading single gauge contribution to the Quartic beta function

G.M. Pelaggi *et. al.* [1708.00437]



The divergence occurs at $p = 0$, where the loop integral is insensitive to the location of the bubbles;

$$\operatorname{div}\Lambda^{(n)} = \tfrac{1}{2} n K_{\alpha,0}^{n+2}\, L(n, \epsilon)$$

$$\delta\lambda^{(1)}_{abcd} = \frac{24\pi^2 d^2(R^\alpha_\Psi)}{\mathcal{N}^2 S_2^2(R^\alpha_\Psi)} A^\alpha_{abcd}\, K^2_\alpha (1 - \tfrac{1}{6} K_\alpha) H_0(\tfrac{2}{3} K_\alpha)$$

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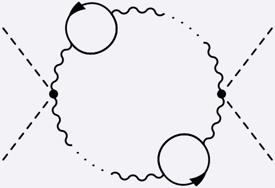

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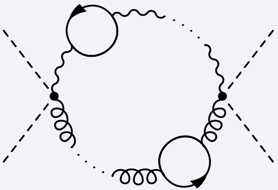
The leading mixed gauge contribution to the quartic beta function differs from the simple gauge case



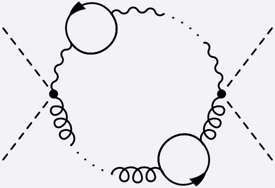
The divergence takes the form

$$\operatorname{div}\Lambda^{(n)} = K_{\alpha,0}K_{\beta,0}\sum_{m=0}^{n} K_{\alpha,0}^{n-m}K_{\beta,0}^{m}\,L(n,\epsilon)$$
$$= \frac{K_{\alpha,0}K_{\beta,0}}{K_{\alpha,0}-K_{\beta,0}}\left(K_{\alpha,0}^{n+1}-K_{\beta,0}^{n+1}\right)L(n,\epsilon).$$

$$\delta\lambda^{(1)}_{abcd} = B^{\alpha,\beta}_{abcd}\frac{48\pi^2}{\mathcal{N}^2}\frac{d(R^{\alpha}_{\Psi})\,d(R^{\beta}_{\Psi})}{S_2(R^{\alpha}_{\Psi})\,S_2(R^{\beta}_{\Psi})}\frac{K_\alpha K_\beta}{K_\alpha - K_\beta}\int_{K_\beta}^{K_\alpha}\mathrm{d}x\,(1-\tfrac{1}{6}x)H_0(\tfrac{2}{3}x)$$

Quartic beta function

The leading mixed gauge contribution to the quartic beta function differs from the simple gauge case



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Quartic beta function

The quartic beta function at LO in $1/\mathcal{N}$

$$
\begin{aligned}
\beta_{\lambda,abcd} = {} & \frac{1}{16\pi^2} \underbrace{L_{abcd}}_{3\times\lambda^2} - \frac{1}{4\pi^2} \underbrace{H_{abcd}}_{6\times y^4} + \frac{1}{32\pi^2} \left[\lambda_{abce} \mathrm{Tr}(y_e y_d^\dagger + y_e^\dagger y_d) \; + \; 3 \text{ perm.}\right] \\
& - \sum_\alpha \frac{3\, d(R_\psi^\alpha)}{4\mathcal{N}\, S_2(R_\psi^\alpha)} \left[\lambda_{abce} C_2(R_\phi^\alpha)_{ed} \; + \; 3 \text{ perm.}\right] K_\alpha \, H_0(\tfrac{2}{3} K_\alpha) \\
& + \frac{24\pi^2}{\mathcal{N}^2} \sum_\alpha A_{abcd}^\alpha \frac{d^2(R_\psi^\alpha)}{S_2^2(R_\psi^\alpha)} \left[(K_\alpha^2 - \tfrac{1}{3} K_\alpha^3)\, H_0(\tfrac{2}{3} K_\alpha) + \tfrac{2}{3} (K_\alpha^3 - \tfrac{1}{6} K_\alpha^4)\, H_0'(\tfrac{2}{3} K_\alpha)\right] \\
& + \frac{48\pi^2}{\mathcal{N}^2} \sum_{\alpha<\beta} B_{abcd}^{\alpha,\beta} \frac{d(R_\psi^\alpha)\, d(R_\psi^\beta)}{S_2(R_\psi^\alpha)\, S_2(R_\psi^\beta)} \frac{K_\alpha K_\beta}{K_\alpha - K_\beta} \\
& \qquad \times \left[(K_\alpha - \tfrac{1}{6} K_\alpha^2)\, H_0(\tfrac{2}{3} K_\alpha) - (K_\beta - \tfrac{1}{6} K_\beta^2)\, H_0(\tfrac{2}{3} K_\beta)\right]
\end{aligned}
$$

Abelian: $K_\alpha \to 15/2$ in the UV forces $\lambda \sim -\exp(\mathcal{N})$

Non-Abelian: $K_\alpha \to 3$ in the UV opens an interacting FP

take e.g. $y \to 0$

Quartic beta function

The quartic beta function at LO in $1/\mathcal{N}$

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$$-\sum_\alpha \frac{3\,d(R_\psi^\alpha)}{4\mathcal{N}\,S_2(R_\psi^\alpha)}\left[\lambda_{abce}C_2(R_\phi^\alpha)_{ed} \;+\; 3\text{ perm.}\right] K_\alpha\, H_0(\tfrac{2}{3}K_\alpha)$$

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Outline

Summary

- The zeros of the gauge beta functions persists when generalized to semi-simple gauge theories
- The leading $1/N_f$ gauge contributions to β_y and β_λ share the pole of $H_0(\frac{2}{3}K_\alpha)$
- If the non-Abelian gauge couplings reach FPs, then the Yukawa and quartic coupling can become safe

Future prospects

- Other limits to explore T. Alanne and S.Blasi [1806.06954]
- Check that the 0 of the beta function corresponds to a FP

Summary and outlook

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Resummation

Here $H(n,\epsilon) = \sum_{j=0}^{\infty}(n\epsilon)^j H^{(j)}(\epsilon)$ and $K_0 = K(1 - \frac{2K}{3\epsilon} + \ldots)^{-1}$. All functions are taken to be regular in all arguments.

$$\sum_{n=1}^{\infty}\left(-\frac{2K_0}{3}\right)^n \frac{1}{n\epsilon^n} H(n,\epsilon)\bigg|_{1/\epsilon}$$

$$= \sum_{m=1}^{\infty}\left(-\frac{2K_0}{3}\right)^m \sum_{j=0}^{m-1}\frac{H^{(j)}(\epsilon)}{\epsilon^{m-j}} \sum_{K=0}^{m-1}\binom{m-1}{k}(-1)^k (m-k)^{j-1}\Bigg|_{1/\epsilon}$$

$$= \sum_{m=1}^{\infty}\left(\frac{2K_0}{3}\right)^m \frac{1}{m}\frac{H^{(0)}(\epsilon)}{\epsilon^m}\bigg|_{1/\epsilon} = \frac{1}{\epsilon}\sum_{m=1}^{\infty}\left(\frac{2K_0}{3}\right)^m \frac{H_m^{(0)}}{m}$$

$$= -\frac{2}{3\epsilon}\int_0^K \mathrm{d}x\, H^{(0)}(\tfrac{2}{3}x).$$

A-dimensional bare couplings are

$$y_{0,aij}\,\mu^{-\epsilon/2} = y_{aij} - \frac{1}{2\epsilon}\left(Z_1^{(1)} + Z_2^{(1)} + Z_\phi^{(1)}\right) y_{aij} + \frac{1}{\epsilon}\delta y_{aij}^{(1)} + \sum_{k=2}^{\infty}\frac{1}{\epsilon^k} y_{aij}^{(k)},$$

$$\lambda_{0,abcd}\,\mu^{-\epsilon} = \lambda_{abcd} - \frac{2}{\epsilon}\left((Z_\phi^{(1)})_{ae}\lambda_{ebcd} + 3\text{perm.}\right) + \frac{1}{\epsilon}\delta\lambda_{abcd}^{(1)} + \sum_{k=2}^{\infty}\frac{1}{\epsilon^k}\lambda_{abcd}^{(k)},$$

$$K_0\,\mu^{\epsilon} = K - \frac{1}{\epsilon}Z_A^{(1)}K + \sum_{k=2}^{\infty}\frac{1}{\epsilon^k}K^{(k)}.$$

The beta functions depend on the simple poles of the bare couplings

$$\beta_{K_\alpha} = \left(-1 + K_\beta\frac{\partial}{\partial K_\beta}\right) K_\alpha^{(1)},$$

$$\beta_{y,aij} = \left(-\frac{1}{2} + K_\beta\frac{\partial}{\partial K_\beta} + \frac{y_{ekl}}{2}\frac{\partial}{\partial y_{ekl}}\right) y_{aij}^{(1)},$$

$$\beta_{\lambda}{}^{ab}{}_{cd} = \left(-1 + K_\beta\frac{\partial}{\partial K_\beta} + \frac{y_{ekl}}{2}\frac{\partial}{\partial y_{ekl}} + \lambda_{efgh}\frac{\partial}{\partial\lambda_{efgh}}\right)\lambda_{abcd}^{(1)}.$$