SEARCHING FOR ASYMPTOTICALLY SAFE EXTENSIONS OF THE STANDARD MODEL.

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Running of the SM couplings

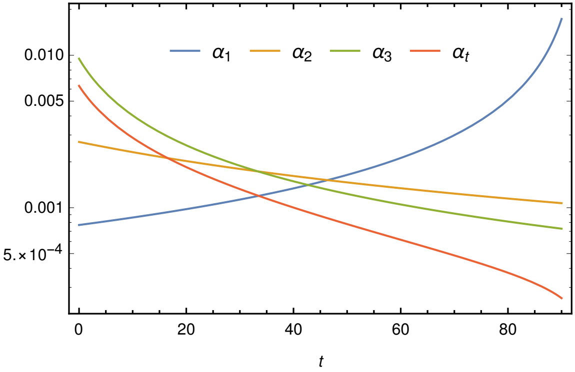


Figure : Running of the gauge couplings α_i and Yukawa α_t. On the horizontal axis $t = \ln(\mu/M_Z)$. Just above $t \simeq 40$ the three gauge couplings come close together. At larger values of t, α_1 begins its ascent towards the Landau pole.

Validity of the theory at high energies

- Exsitence of a Fixed Point (FP), $\beta_i = 0$.

- General form of the gauge-beta function

$$\beta_g = \frac{d\alpha_g}{dln\mu} = (-B + C\alpha_g - D\alpha_y)\alpha_g^2$$

$$\beta_y = \frac{d\alpha_y}{dln\mu} = (E\alpha_g - F\alpha_g)\alpha_y$$

- Cancellation among loop terms

Recap of FP analysis

- Definition

$$\beta_i(g_j) \equiv \mu \frac{dg_i}{d\mu} \to \beta_i(g_j^*) = 0\,.$$

- First condition for perturbativity

$$\alpha_i^* \equiv \left(\frac{g_i^*}{4\pi}\right)^2 < 1.$$

- Linearized flow

$$\frac{dy_i}{dt} = M_{ij} y_j,\; y_i \equiv g_i - g_i^*;\; (S^{-1})_{ij} M_{jl} S_{ln} = \delta_{in} \vartheta_n,\; z_i = (S^{-1})_{ij} y_j\,.$$

$$\frac{dz_i}{dt} = \vartheta_i z_i \quad \text{and} \quad z_i(t) = c_i\, e^{\vartheta_i t} = c_i \left(\frac{\mu}{\mu_0}\right)^{\vartheta_i}.$$

Theory space

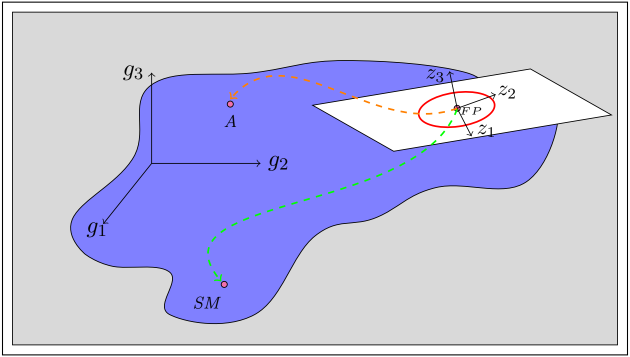


Figure : Theory space of couplings g_i where only 3 axes are shown for simplicity. For a given FP we show the UV safe surface (blue region), the approximated UV critical surface around the FP (white plane), the new set of coordinates z_i, a small region of possible initial points for the flow (red circle) and two UV safe trajectories ending at a given matching scale $\mathcal{M}$ (green and orange dashed lines, the former going to the SM, the latter going to a different IR physics A).

Analysis of the eigeinvalues

- For $\vartheta_i > 0$, as we increase μ we move away from the FP and z_i increases without control; the direction z_i is said to be *irrelevant*.
- If $\vartheta_i < 0$, as we increase μ we approach the fixed point; the direction z_i is called a *relevant* direction.
- If $\vartheta_i = 0$, we do not know the fate of z_i and we have to go beyond the linear order as explained below; the direction z_i is called *marginal* in this case.
- Perturbative eigenvalues (second condition for perturbativity)

$$\beta_i = -d_i g_i + \beta_i^q(g_j),$$

$$M_{ij} = -d_i \delta_{ij} + \frac{\partial \beta_i^q}{\partial g_j} \longrightarrow |\vartheta_i| < O(1)\,.$$

Marginal couplings

- Second order flow equation

$$\frac{dy_i}{dt} = M_{ij} y_j + P_{ijk} y_j y_k \,, \quad \text{where} \quad P_{ijk} = \frac{\partial^2 \beta_i}{\partial g_j \partial g_k}.$$

- Form of the Stability Matrix

$$\beta_i = (A^i + B^i_r \alpha_r + C^i_{rs} \alpha_r \alpha_s) \alpha_i^2,$$

$$M_{ij} = \left. \frac{\partial \beta_i}{\partial \alpha_j} \right|_{\alpha_i^*} = (B^i_j + 2 C^i_{jr} \alpha^*_r) \alpha_i^{*2} + 2(A^i + B^i_r \alpha^*_r + C^i_{rs} \alpha^*_r \alpha^*_s) \alpha^*_i \delta_{ij}.$$

$$M = \begin{bmatrix} m_{1,1} & m_{1,2} & \dots & m_{1,n} \\ m_{2,1} & m_{2,2} & \dots & m_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n-2,1} & m_{n-2,2} & \dots & m_{n-2,n} \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \end{bmatrix}$$

Transformation matrix

- Form of S

$$S = \begin{bmatrix} V_1^1 & V_1^2 & \dots & V_1^{n-2} & V_1^{n-1} & V_1^n \\ V_2^1 & V_2^2 & \dots & V_2^{n-2} & V_2^{n-1} & V_2^n \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ V_{n-2}^1 & V_{n-2}^2 & \dots & V_{n-2}^{n-2} & V_{n-2}^{n-1} & V_{n-2}^n \\ 0 & 0 & \dots & 0 & V_{n-1}^{n-1} & 0 \\ 0 & 0 & \dots & 0 & 0 & V_n^n \end{bmatrix}$$

- Form of S^{-1}

$$S^{-1} = \begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,n-2} & a_{1,n-1} & a_{1,n} \\ a_{2,1} & a_{2,2} & \dots & a_{2,n-2} & a_{2,n-1} & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ a_{n-2,1} & a_{n-2,2} & \dots & a_{n-2,n-2} & a_{n-2,n-1} & a_{n-2,n} \\ 0 & 0 & \dots & 0 & b & 0 \\ 0 & 0 & \dots & 0 & 0 & c \end{bmatrix}$$

Form of P_{ijk}

- Second order flow equation

$$P_{ijk} = \frac{\partial^2 \beta_i}{\partial \alpha_j \alpha_k}\bigg|_{\alpha_i^*} = 2C^i_{jk}\alpha_i^{*2} + 2(B^i_j + 2C^i_{jr}\alpha^*_r)\alpha^*_i\delta_{ik} + 2(B^i_k + 2C^i_{kr}\alpha^*_r)\alpha^*_i\delta_{ij} + 2(A^i + B^i_r\alpha^*_r + C^i_{rs}\alpha^*_r\alpha^*_s)\delta_{ij}\delta_{ik}$$

$$P_{ijk} = \begin{bmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \tilde{P}_i \end{bmatrix}$$

$$Q_{lmn} = \tilde{P}_i(S^{-1}_{li})S_{im}S_{in}, \; Q_{lmn} = \tilde{P}_i S_{im}\delta_{il}\delta_{im}\delta_{in}$$

$$Q_{lmn} = \begin{bmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \tilde{Q}_l \end{bmatrix}$$

Theory space

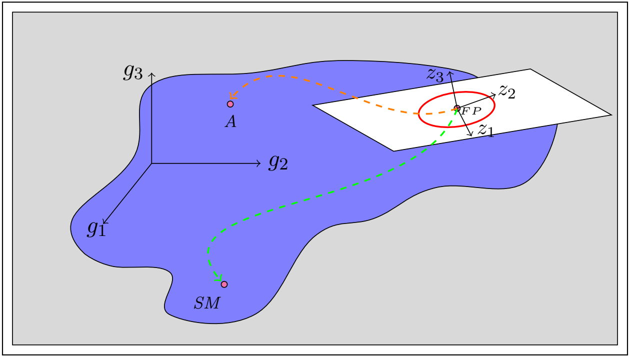


Figure : Theory space of couplings g_i where only 3 axes are shown for simplicity. For a given FP we show the UV safe surface (blue region), the approximated UV critical surface around the FP (white plane), the new set of coordinates z_i, a small region of possible initial points for the flow (red circle) and two UV safe trajectories ending at a given matching scale $\mathcal{M}$ (green and orange dashed lines, the former going to the SM, the latter going to a different IR physics A).

Standard Model matching

- For realistic models, the running to low energies has to match with the SM.
- We start the flow down to the at values near the FP.
- We define the initial condition for the couplings as

$$g_i^0 = g_i^* + S_{ij} z_j^0 \,.$$

- This equation puts strong constraints on the models.

Another test of perturbativity

- General structure of the gauge-beta functions at three-loops

$$\beta_i = \left(A^{(i)} + B_r^{(i)}\alpha_r + C_{rs}^{(i)}\alpha_r\alpha_s\right)\alpha_i^2,$$

- FP solutions means

$$0 = \beta_i = A_*^{(i)} + B_*^{(i)} + C_*^{(i)} \ ,$$

where $A_*^{(i)} = A^{(i)}\alpha_{i*}^2$, $B_*^{(i)} = B_r^{(i)}\alpha_{r*}\alpha_{i*}^2$, $C_*^{(i)} = B_{rs}^{(i)}\alpha_{r*}\alpha_{s*}\alpha_{i*}^2$.

- Perturbativity requires

$$\rho_i < \sigma_i < 1 \ , \quad \text{where} \quad \rho_i = |C_*^{(i)}/A_*^{(i)}| \quad \text{and} \quad \sigma_i = |B_*^{(i)}/A_*^{(i)}| \ .$$

Weyl consistency conditions

- General form of the action

$$\mathcal{L} = \mathcal{L}_{CFT} + g_i \mathcal{O}^i \,.$$

- Transformation under $\gamma_{\mu\nu} \to e^{2\sigma(x)}\gamma_{\mu\nu}$

$$W = Log\left[\int \mathcal{D}\Phi e^{i\int d^4x\mathcal{L}}\right] \quad ,$$

$$\Delta_\sigma \Delta_\tau W = \Delta_\tau \Delta_\sigma W \,.$$

- Important relation

$$\frac{\partial \beta^j}{\partial g_i} = \frac{\partial \beta^i}{\partial g_j} \,.$$

- $\exists$ a hierarchy in the β-functions. Gauge: N, Yukawa: $N-1$, Scalar quartic coupling: $N-2$.

The model

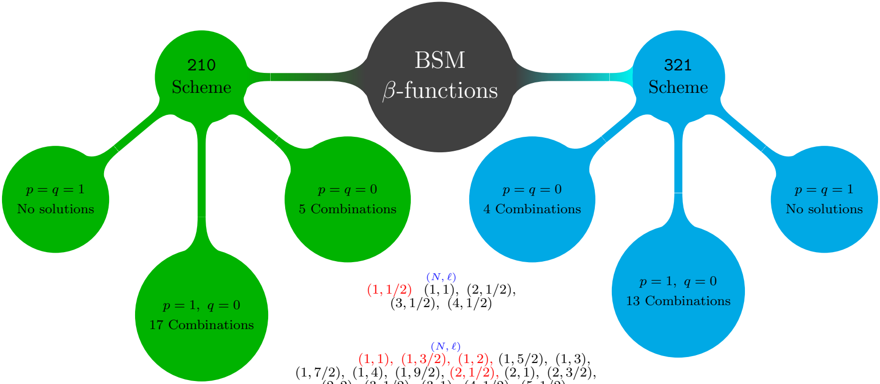
- Group: $SU_c(3) \times SU_L(2) \times U_Y(1)$.
- We take N_f families of vector-like fermions minimally coupled to the SM together whith a new type of Yukawa interactions.
- Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM} + \mathrm{Tr}(\bar{\psi} i \not{D} \psi) + \mathrm{Tr}(\partial_\mu S \partial_\mu S) - y\,\mathrm{Tr}(\bar{\psi}_L S \psi_R + \bar{\psi}_R S \psi_L).$$

- The model depends on the representations R_3 (p, q), R_2 (ℓ), Y and N_f.

The procedure

Parameter space
$N \in [1, 300, 1],\ \ell \in [1/2, 10, 1/2],\ Y \in [0, 10, 1/2]$



(N, ℓ)
$(1, 1/2)$ $(1, 1),\ (2, 1/2),$
$(3, 1/2),\ (4, 1/2)$

(N, ℓ)
$(1, 1),\ (1, 3/2),\ (1, 2),\ (1, 5/2),\ (1, 3),$
$(1, 7/2),\ (1, 4),\ (1, 9/2),\ (2, 1/2),\ (2, 1),\ (2, 3/2),$
$(2, 2),\ (3, 1/2),\ (3, 1),\ (4, 1/2),\ (5, 1/2).$

General FP in 210 for the singlet case

	α_1^*	α_2^*	α_3^*	α_t^*	α_y^*
P_1	0	0	0	0	0
P_2	0	$\alpha_2^*(p,q,\ell)$	$\alpha_3^*(p,q,\ell)$	0	$\alpha_y^*(p,q,\ell)$
P_3	0	$\alpha_2^*(p,q,\ell)$	$\alpha_3^*(p,q,\ell)$	$\alpha_t^*(p,q,\ell)$	$\alpha_y^*(p,q,\ell)$
P_4	0	$\alpha_2^*(p,q,\ell)$	$\alpha_3^*(p,q,\ell)$	0	0
P_5	0	$\alpha_2^*(p,q,\ell)$	$\alpha_3^*(p,q,\ell)$	$\alpha_t^*(p,q,\ell)$	0
P_6	$\alpha_1^*(p,q,\ell,Y)$	$\alpha_2^*(p,q,\ell,Y)$	$\alpha_3^*(p,q,\ell,Y)$	0	$\alpha_y^*(p,q,\ell,Y)$
P_7	$\alpha_1^*(p,q,\ell,Y)$	$\alpha_2^*(p,q,\ell,Y)$	$\alpha_3^*(p,q,\ell,Y)$	$\alpha_t^*(p,q,\ell,Y)$	$\alpha_y^*(p,q,\ell,Y)$
P_8	0	0	$\alpha_3^*(p,q,\ell)$	$\alpha_t^*(p,q,\ell)$	$\alpha_y^*(p,q,\ell)$
P_9	0	0	$\alpha_3^*(p,q,\ell)$	0	$\alpha_y^*(p,q,\ell)$
P_{10}	0	0	$\alpha_3^*(p,q,\ell)$	$\alpha_t^*(p,q,\ell)$	0
P_{11}	0	0	$\alpha_3^*(p,q,\ell)$	0	0

Table : List of all the FPs in the 210 approximation scheme. When non-zero, the dependence on the quantum numbers is indicated. Only the highlighted FPs appear in the following tables.

General FP in 210 for the singlet case

	α_1^*	α_2^*	α_3^*	α_t^*	α_y^*
P_{12}	$\alpha_1^*(p,q,\ell,Y)$	0	$\alpha_3^*(p,q,\ell,Y)$	0	$\alpha_y^*(p,q,\ell,Y)$
P_{13}	$\alpha_1^*(p,q,\ell,Y)$	0	$\alpha_3^*(p,q,\ell,Y)$	$\alpha_t^*(p,q,\ell,Y)$	$\alpha_y^*(p,q,\ell,Y)$
P_{14}	$\alpha_1^*(p,q,\ell,Y)$	0	$\alpha_3^*(p,q,\ell,Y)$	0	0
P_{15}	$\alpha_1^*(p,q,\ell,Y)$	0	$\alpha_3^*(p,q,\ell,Y)$	$\alpha_t^*(p,q,\ell,Y)$	0
P_{16}	0	$\alpha_2^*(p,q,\ell)$	0	0	$\alpha_y^*(p,q,\ell)$
P_{17}	0	$\alpha_2^*(p,q,\ell)$	0	$\alpha_t^*(p,q,\ell)$	$\alpha_y^*(p,q,\ell)$
P_{18}	0	$\alpha_2^*(p,q,\ell)$	0	0	0
P_{19}	0	$\alpha_2^*(p,q,\ell)$	0	$\alpha_t^*(p,q,\ell)$	0
P_{21}	$\alpha_1^*(p,q,\ell,Y)$	0	0	0	0

Table : List of all the FPs in the 210 approximation scheme. When non-zero, the dependence on the quantum numbers is indicated. Only the highlighted FPs appear in the following tables.

General FP in 210 for the singlet case

	α_1^*	α_2^*	α_3^*	α_t^*	α_y^*
P_{22}	$\alpha_1^*(p,q,\ell,Y)$	0	0	$\alpha_t^*(p,q,\ell,Y)$	0
P_{23}	$\alpha_1^*(p,q,\ell,Y)$	0	0	$\alpha_t^*(p,q,\ell,Y)$	$\alpha_y^*(p,q,\ell,Y)$
P_{24}	$\alpha_1^*(p,q,\ell,Y)$	0	0	0	$\alpha_y^*(p,q,\ell,Y)$
P_{25}	$\alpha_1^*(p,q,\ell,Y)$	$\alpha_2^*(p,q,\ell,Y)$	0	0	$\alpha_y^*(p,q,\ell,Y)$
P_{26}	$\alpha_1^*(p,q,\ell,Y)$	$\alpha_2^*(p,q,\ell,Y)$	0	$\alpha_t^*(p,q,\ell,Y)$	$\alpha_y^*(p,q,\ell,Y)$
P_{28}	$\alpha_1^*(p,q,\ell,Y)$	$\alpha_2^*(p,q,\ell,Y)$	0	0	0
P_{29}	$\alpha_1^*(p,q,\ell,Y)$	$\alpha_2^*(p,q,\ell,Y)$	0	$\alpha_t^*(p,q,\ell,Y)$	0
P_{30}	$\alpha_1^*(p,q,\ell,Y)$	$\alpha_2^*(p,q,\ell,Y)$	$\alpha_3^*(p,q,\ell,Y)$	0	0
P_{31}	$\alpha_1^*(p,q,\ell,Y)$	$\alpha_2^*(p,q,\ell,Y)$	$\alpha_3^*(p,q,\ell,Y)$	$\alpha_t^*(p,q,\ell,Y)$	0

Table : List of all the FPs in the 210 approximation scheme. When non-zero, the dependence on the quantum numbers is indicated. Only the highlighted FPs appear in the following tables.

Singlet of $SU(3)$

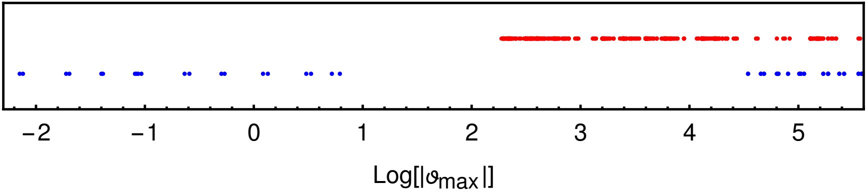


Figure : Distribution of the largest eigenvalues $\vartheta_{\max}$ of the stability matrix of the FPs of the colorless models. Blue dots: eigenvalues for the Y-independent solutions: there is a gap between 2.21 and 62.6. Red dots: eigenvalues for the Y-dependent solutions: there is no gap, the eigenvalues start around 10.

FP in 210 for the singlet case

(N_f, ℓ)	α_1^*	α_2^*	α_3^*	α_t^*	α_y^*	ϑ_1		ρ_2
$(1, \frac{1}{2})$	0	0.200	0	0	0.300	2.04	P_{16}	3.97
	0	0.213	0	0.106	0.319	2.21	P_{17}	4.33
	0	0.179	0	0	0	−1.61	P_{18}	3.28
	0	0.189	0	0.0943	0	−1.70	P_{19}	3.53
$(1, 1)$	0	0.0137	0	0	0.0411	0.333	P_{16}	0.194
	0	0.0140	0	0.0070	0.0420	0.341	P_{17}	0.198
	0	0.0103	0	0	0	−0.247	P_{18}	0.0963
	0	0.0105	0	0.0052	0	−0.251	P_{19}	0.0973
$(2, \frac{1}{2})$	0	0.104	0	0	0.117	1.0833	P_{16}	1.71
	0	0.108	0	0.0542	0.122	1.14	P_{17}	1.81
	0	0.0827	0	0	0	−0.744	P_{18}	1.19
	0	0.0856	0	0.0428	0	−0.770	P_{19}	1.23
$(3, \frac{1}{2})$	0	0.0525	0	0	0.0472	0.530	P_{16}	0.763
	0	0.0543	0	0.0272	0.0489	0.552	P_{17}	0.794
	0	0.0385	0	0	0	−0.346	P_{18}	0.471
	0	0.0394	0	0.0197	0	−0.355	P_{19}	0.483
$(4, \frac{1}{2})$	0	0.0189	0	0	0.0141	0.179	P_{16}	0.246
	0	0.0194	0	0.0097	0.0146	0.185	P_{17}	0.253
	0	0.0130	0	0	0	−0.117	P_{18}	0.141
	0	0.0132	0	0.0066	0	−0.119	P_{19}	0.143

Table : Set of FPs and eigenvalues for colorless vector-like fermions in the 210 approximation scheme. We highlight in green the that appear also in the 321 approximation. We show in the last column the ratio ρ_2 knowing that in 210 $A_*^{(2)} = B_*^{(2)}$.

FP in 321 for the singlet case

(N_f, l)	α_1^*	α_2^*	α_3^*	α_t^*	α_y^*	α_λ^*	ϑ_1	σ_2	ρ_2
$(1, 1)$	0	0.0096	0	0.0048	0	0.0039	−0.244	0.918	0.0821
	0	0.0119	0	0.0060	0.0343	0.0048	0.301	0.8601	0.140
$(2, \frac{1}{2})$	0	0.0498	0	0.0259	0	0.0211	−0.592	0.581	0.418
	0	0.0567	0	0.0296	0.0734	0.0242	0.696	0.5012	0.499
$(3, \frac{1}{2})$	0	0.0291	0	0.0148	0	0.0120	−0.306	0.737	0.263
	0	0.0362	0	0.0184	0.0353	0.0150	0.403	0.645	0.354
$(4, \frac{1}{2})$	0	0.0117	0	0.0059	0	0.0048	−0.112	0.887	0.113
	0	0.0162	0	0.0081	0.0125	0.0066	0.161	0.823	0.177

Table : Fixed points and eigenvalues for colorless vector-like fermions, in the 321 approximation scheme. The last two columns give the values of the ratios σ_2 and ρ_2.

Behavior of eigenvalues

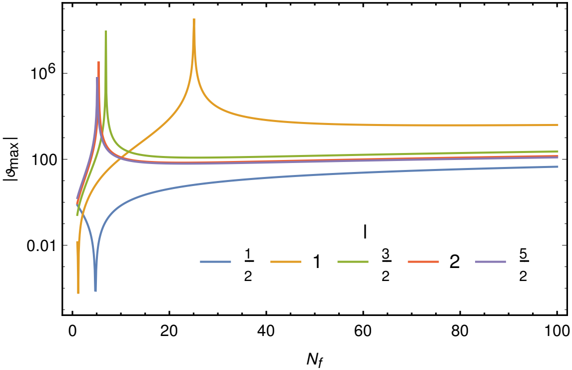


Figure : Behaviour of a given eigenvalue $|\vartheta|$ as a function of N_f for several values of ℓ in the colorless case. The scaling dimension increases very fast with N_f, and only small values of N_f, ℓ produce $|\vartheta| < O(1)$.

Matching to the SM

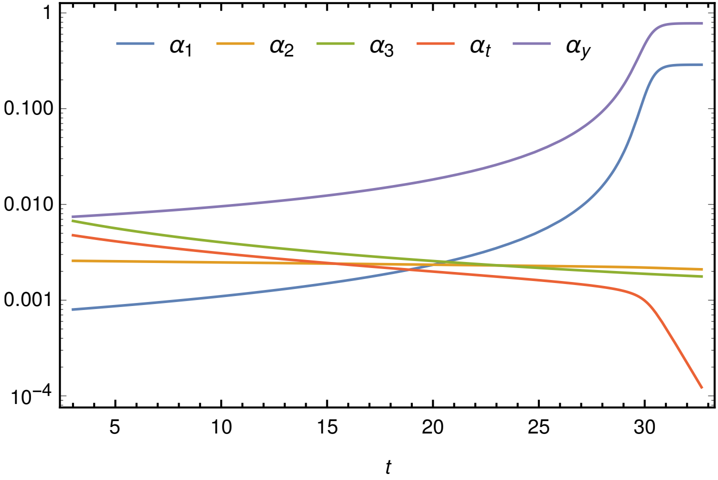


Figure : Evolution of the couplings with t in a logarithmic scale for the FP: $\alpha_1^* = 0.188,\ \alpha_2^* = 0,\ \alpha_3^* = 0,\ \alpha_t^* = 0,\ \alpha_y^* = 0.778,\ \vartheta_1 = 33.2,\ \vartheta_2 = -3.36,\ \vartheta_3 = -0.817,\ \vartheta_4 = 0,\ \vartheta_5 = 0,\ \rho_1 = 2.69$).

Fundamental representation of $SU(3)$

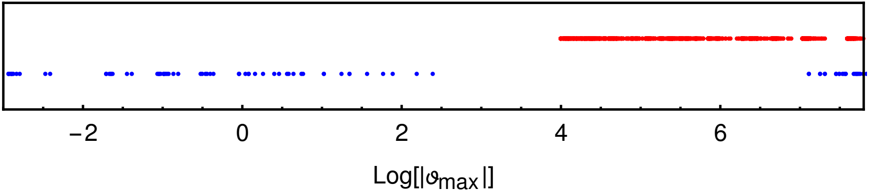


Figure : Distribution of the largest eigenvalues $\vartheta_{\max}$ of the stability matrix of the FPs of the $SU(3)$ fundamental representation. Blue dots: eigenvalues for the Y-independent solutions: there is a gap between 10.8 and 372. Red dots: eigenvalues for the Y-dependent solutions: there is no gap, the eigenvalues start at 52.1.

FP in 210 for the fundamental representation

(N_f, l)	α_1^*	α_2^*	α_3^*	α_t^*	α_y^*	ϑ_1		ρ
$(1, \frac{1}{2})$	0	0.0411	0	0	0.0264	0.378	P_{16}	0.522
	0	0.0422	0	0.0211	0.0271	0.389	P_{17}	0.537
	0	0.0385	0	0	0	-0.346	P_{18}	0.471
	0	0.0394	0	0.0197	0	-0.355	P_{19}	0.483
$(1, 1)$	0	0	0.417	0	0	-6.67	P_{11}	20.9
	0	0	0.521	0	0.417	10.8	P_9	31.8
$(1, \frac{3}{2})$	0	0	0.176	0	0	-2.81	P_{11}	5.45
	0	0	0.205	0.365	0	3.84	P_{10}	7.21
	0	0	0.195	0	0.120	3.49	P_9	6.60
	0	0	0.232	0.413	0.143	4.83	P_8	9.06
$(1, 2)$	0	0	0.0982	0	0	-1.57	P_{11}	2.42
	0	0	0.108	0.193	0	1.88	P_{10}	2.88
	0	0	0.105	0	0.0526	1.78	P_9	2.73
	0	0	0.117	0.208	0.0586	2.15	P_8	3.30
$(1, \frac{5}{2})$	0	0	0.0600	0	0	-0.960	P_{11}	1.27
	0	0	0.0646	0.115	0	1.08	P_{10}	1.44
	0	0	0.0632	0	0.0266	1.04	P_9	1.39
	0	0	0.0683	0.121	0.0288	1.18	P_8	1.59
$(1, 3)$	0	0	0.0412	0.0733	0.0150	0.689	P_8	0.839
	0	0	0.0388	0	0.0141	0.632	P_9	0.758
	0	0	0.0395	0.0702	0	0.647	P_{10}	0.778
	0	0	0.0372	0	0	-0.596	P_{11}	0.707

Table : Fixed points and eigenvalues for vector-like fermions in the fundamental representation of $SU_c(3)$, in the 210 approximation scheme, with $N_f = 1$. We highlight in green the FPs that appear also in the 321 approximation scheme. The last column gives the values of the ratio ρ for α_2 or α_3 depending on the case.

FP in 210 for the fundamental representation

(N_f, l)	α_1^*	α_2^*	α_3^*	α_t^*	α_y^*	ϑ_1		ρ
$(1, \frac{7}{2})$	0	0	0.0221	0	0	−0.354	P_{11}	0.384
	0	0	0.0232	0.0413	0	0.376	P_{10}	0.415
	0	0	0.0229	0	0.0073	0.370	P_9	0.406
	0	0	0.0241	0.0428	0.0077	0.394	P_8	0.441
$(1, 4)$	0	0	0.0114	0	0	−0.182	P_{11}	0.182
	0	0	0.0118	0.0210	0	0.191	P_{10}	0.195
	0	0	0.0117	0	0.0033	0.188	P_9	0.191
	0	0	0.0122	0.0217	0.0035	0.197	P_8	0.205
$(1, \frac{9}{2})$	0	0	0.0033	0	0	−0.0530	P_{11}	0.0495
	0	0	0.0034	0.0061	0	0.0550	P_{10}	0.0523
	0	0	0.0034	0	0.0009	0.0544	P_9	0.0516
	0	0	0.0035	0.0063	0.0009	0.0566	P_8	0.0547
$(2, \frac{1}{2})$	0	0	0.176	0	0	−2.81	P_{11}	5.45
	0	0	0.205	0.365	0	3.84	P_{10}	7.21
	0	0	0.260	0	0.260	5.91	P_9	11.1
	0	0	0.330	0.588	0.330	8.99	P_8	17.4
$(2, 1)$	0	0	0.0600	0	0	−0.960	P_{11}	1.27
	0	0	0.0646	0.115	0	1.08	P_{10}	1.44
	0	0	0.0727	0	0.0529	1.30	P_9	1.77
	0	0	0.0795	0.141	0.0578	1.50	P_8	2.07

Table : Fixed points and eigenvalues for vector-like fermions in the fundamental representation of $SU_c(3)$, in the 210 approximation scheme. We highlight in green the FPs that appear also in the 321 approximation scheme. The last column gives the values of the ratio ρ_3.

FP in 210 for the fundamental representation

(N_f, l)	α_1^*	α_2^*	α_3^*	α_t^*	α_y^*	ϑ_1		ρ
$(2, \frac{3}{2})$	0	0	0.0221	0	0	−0.354	P_{11}	0.384
	0	0	0.0232	0.0413	0	0.376	P_{10}	0.415
	0	0	0.0252	0	0.0144	0.417	P_9	0.475
	0	0	0.0266	0.0473	0.0152	0.448	P_8	0.520
$(2, 2)$	0	0	0.0033	0	0	−0.0530	P_{11}	0.0495
	0	0	0.0034	0.0061	0	0.0550	P_{10}	0.0523
	0	0	0.0036	0	0.0017	0.0587	P_9	0.0579
	0	0	0.0038	0.0068	0.0018	0.0612	P_8	0.0616
$(3, \frac{1}{2})$	0	0	0.0600	0	0	−0.960	P_{11}	1.27
	0	0	0.0646	0.115	0	1.08	P_{10}	1.44
	0	0	0.0882	0	0.0784	1.77	P_9	2.47
	0	0	0.0985	0.175	0.0876	2.10	P_8	3.01
$(3, 1)$	0	0	0.0114	0	0	−0.182	P_{11}	0.182
	0	0	0.0118	0.0210	0	0.191	P_{10}	0.195
	0	0	0.0143	0	0.0095	0.237	P_9	0.264
	0	0	0.0150	0.0267	0.0100	0.252	P_8	0.288
$(4, \frac{1}{2})$	0	0	0.0221	0	0	−0.354	P_{11}	0.384
	0	0	0.0232	0.0413	0	0.376	P_{10}	0.415
	0	0	0.0335	0	0.0268	0.607	P_9	0.763
	0	0	0.0361	0.0642	0.0289	0.670	P_8	0.866
$(5, \frac{1}{2})$	0	0	0.0033	0	0	−0.0530	P_{11}	0.0495
	0	0	0.0343	0.0061	0	0.0550	P_{10}	0.0523
	0	0	0.0052	0	0.0038	0.0850	P_9	0.1010
	0	0	0.0055	0.0097	0.0040	0.0903	P_8	0.111

Table : Fixed points and eigenvalues for vector-like fermions in the fundamental representation of $SU_c(3)$, in the 210 approximation scheme. We highlight in green the FPs that appear also in the 321 approximation scheme. The last column gives the values of the ratio ρ_3.

FP in 321 for the fundamental representation

(N_f, l)	α_1^*	α_2^*	α_3^*	α_t^*	α_y^*	α_λ^*	ϑ_1	σ	ρ
$(1, \frac{1}{2})$	0	0.0291	0	0.0148	0	0.0120	−0.306	0.737	0.263
	0	0.0305	0	0.0155	0.0209	0.0126	0.322	0.719	0.281
$(1, \frac{5}{2})$	0	0	0.0346	0	0	0	−0.748	0.577	0.423
	0	0	0.0355	0	0.0167	0	−0.774	0.559	0.441
$(1, 3)$	0	0	0.0252	0	0	0	−0.501	0.676	0.323
	0	0	0.0258	0	0.0101	0	−0.516	0.664	0.336
$(1, \frac{7}{2})$	0	0	0.0171	0	0	0	−0.315	0.771	0.228
	0	0	0.0177	0.0358	0	0.0221	0.969	0.758	0.242
	0	0	0.0175	0	0.0058	0	−0.324	0.763	0.237
	0	0	0.0182	0.0368	0.0061	0.0227	0.998	0.748	0.252
$(1, 4)$	0	0	0.098	0	0	0	−0.170	0.864	0.136
	0	0	0.0102	0.0193	0	0.0119	0.521	0.856	0.144
	0	0	0.0101	0	0.0029	0	−0.175	0.859	0.141
	0	0	0.0104	0.0198	0.0030	0.0123	0.536	0.8505	0.149
$(1, \frac{9}{2})$	0	0	0.0032	0	0	0	−0.0519	0.955	0.0451
	0	0	0.0033	0.0059	0	0.0037	0.159	0.952	0.0476
	0	0	0.0032	0	0.0008	0	−0.0532	0.953	0.0469
	0	0	0.0033	0.0061	0.0009	0.00038	0.1635	0.9505	0.0495

Table : Fixed points and eigenvalues for vector-like fermions in the fundamental representation of $SU_c(3)$, in the 321 approximation scheme. The last two columns give the values of the ratio σ and ρ for α_2 or α_3 depending on the case.

FP in 321 for the fundamental representation

(N_f, l)	α_1^*	α_2^*	α_3^*	α_t^*	α_y^*	α_λ^*	ϑ_1	σ	ρ
$(2, 1)$	0	0	0.346	0	0	0	−0.748	0.577	0.423
	0	0	0.0381	0	0.0319	0	−0.846	0.5077	0.492
$(2, \frac{3}{2})$	0	0	0.0171	0	0	0	−0.315	0.771	0.228
	0	0	0.0177	0.0358	0	0.0221	0.969	0.758	0.242
	0	0	0.0187	0	0.0113	0	−0.350	0.737	0.263
$(2, 2)$	0	0	0.0032	0	0	0	−0.0519	0.955	0.0451
	0	0	0.0033	0.0059	0	0.0037	0.159	0.952	0.0476
	0	0	0.0035	0	0.0016	0	−0.0570	0.948	0.0521
	0	0	0.0036	0.0065	0.0017	0.0040	0.1756	0.945	0.552
$(3, \frac{1}{2})$	0	0	0.0346	0	0	0	−0.748	0.577	0.423
	0	0	0.0417	0	0.0440	0	−0.950	0.431	0.569
$(3, 1)$	0	0	0.0098	0	0	0	−0.170	0.864	0.136
	0	0	0.0102	0.0193	0	0.119	0.521	0.856	0.144
	0	0	0.0118	0	0.0081	0	0.208	0.819	0.181
	0	0	0.0123	0.0237	0.0085	0.0147	0.641	0.8062	0.194
$(4, \frac{1}{2})$	0	0	0.0171	0	0	0	−0.315	0.771	0.228
	0	0	0.0177	0.0358	0	0.0221	0.969	0.758	0.242
	0	0	0.0226	0	0.0196	0	0.439	0.647	0.353
$(5, \frac{1}{2})$	0	0	0.0033	0	0	0	−0.0519	0.955	0.0451
	0	0	0.0033	0.0059	0	0.0037	0.159	0.952	0.0476
	0	0	0.0048	0	0.0035	0	0.0798	0.914	0.0859
	0	0	0.0050	0.0092	0.0037	0.0057	0.248	0.9066	0.0934

Table : Fixed points and eigenvalues for vector-like fermions in the fundamental representation of $SU_c(3)$, in the 321 approximation scheme. The last two columns give the values of the ratio σ_3 and ρ_3.

Adjoint representation of $SU(3)$

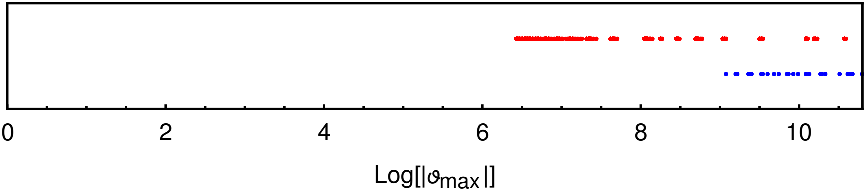


Figure : Distribution of the largest eigenvalue $\vartheta_{\max}$ of the stability matrix of the FPs of the $SU(3)$ adjoint representation. Blue: eigenvalues for the Y-independent solutions. Red: eigenvalues for the Y-dependent solutions. In both cases, there is no gap and the eigenvalues start at very large values.

Benchmark scenarios in Bond et al.

	(R_3, R_2, N_f)	α_2^*	α_3^*	α_y^*	ϑ_1	ϑ_2	ϑ_3	ρ
A	$(1,4,12)$	0.241	0	0.338	210	−1.90	0	45.3
B	$(10,1,30)$	0	0.129	0.116	338	−2.06	0	107
		0.277	0.129	0.116	341	−2.08	0.897	107
C	$(10,4,80)$	0	0.332	0.0995	23258	−2.18	0	9138
		0.0753	0.0503	0.0292	1499	328	−2.77	630
		0.800	0	0.150	145193	−2.12	0	57378
D	$(3,4,290)$	0.0615	0.0416	0.0057	943	45.3	−2.29	371
		0.0896	0	0.0067	1984	−2.11	0	781
E	$(3,3,72)$	0.218	0.150	0.0471	896	112	−1.78	326

Table : 210 Approximation scheme

	(R_3, R_2, N_f)	α_2^*	α_3^*	α_y^*	ϑ_1	ϑ_2	ϑ_3	ρ_3
A	$(1,4,12)$	0	0	0.1509	−4.83	0	0	–
B	$(10,1,30)$	0	0.0138	0	−20.02	2.24	0	3.14
		0	0	0.0594	−4.75	0	0	–
C	$(10,4,80)$	0	0	0.0187	−4.501	0	0	–
		0	0.0036	0	−49.4	2.28	0	9.29
D	$(3,4,290)$	0	0	0.0115	−6.95	0	0	–
		0	0.0108	0	−36.7	1.015	0	5.81
E	$(3,3,72)$	0	0	0.0357	−5.79	0	0	–
		0	0.0305	0	−21.8	1.098	0	2.66

Table : 321 Approximation scheme

Two-families model

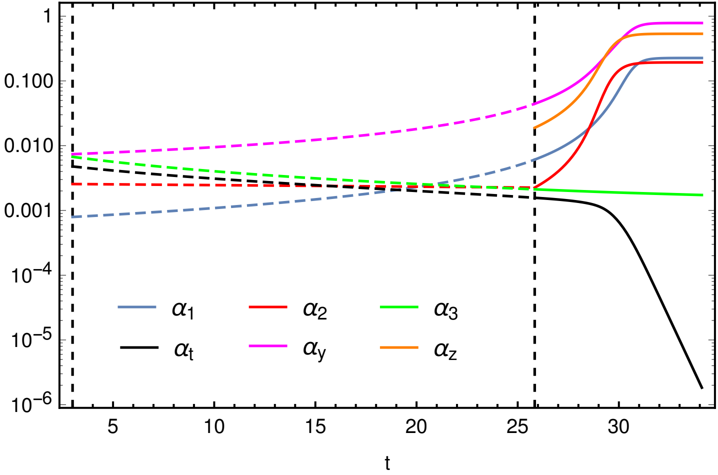


Figure : Evolution of the couplings with t for the FP: within the 210 approximation with 3 fields in $(1, 2, 3/2)$ and 8 fields in $(1, 5, 0)$. This running provides a trajectory in the theory space connecting the FP to a matching scale around 2 TeV passing through another matching (for the quintuplets) at about 10^{13} TeV.

Conclusions

- A systematic scan of possible extensions of the SM based on vector-like fermions shows that there are no FPs that satisfy the minimal criteria to make them perturbatively stable and therefore physical.
- Most of those that appear in the `210` approximation scheme are difficult to identify when probed in the `321` approximation scheme. Those that seem to be present in both schemes always contain a trivial solution.
- The same applies for other modesl appearing in the litterature.
- Using non-perturbative tools seems necessary.
- The inclusion of gravity is also wanted.