Einstein-Cartan Theory as an Averaged Theory of Gravity

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$\Rightarrow$ Modifications can in principle act as a dark energy

$$G_{\mu\nu}(\langle g_{\mu\nu}\rangle) = 8\pi G \langle T_{\mu\nu}\rangle + 8\pi G T^{g}_{\mu\nu} + \Lambda \langle g_{\mu\nu}\rangle$$

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$\Rightarrow$ Such a manifold is called a **Riemann-Cartan spacetime** U_4.

- In a Riemann-Cartan spacetime the difference between the affine connection and the Levi-Civita connection defines the **contortion tensor**

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$\Rightarrow$
$$(L_4, g) \xrightarrow{Q=0} U_4 \xrightarrow{T=0} V_4 \xrightarrow{R=0} R_4$$

General Relativity

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Teleparallel Gravity

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$$\mathcal{L} = \frac{e}{16\pi G}\mathring{R}$$

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- In terms of Weitzenböck connection
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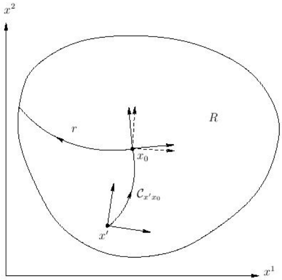
Averaging Process in General Relativity



Parallel transport along geodesics $\mathcal{C}_{xx'}$ realized by Wegner-Wilson line operator

$$V(x', x; \mathcal{C}_{xx'}) = \mathcal{P} \exp\left[- \int_{\mathcal{C}_{xx'}} dz^\mu \; \Gamma_\mu(z) \right]$$

where $\Gamma_\mu(x)$ are four matrices with components $(\Gamma_\mu(x))^\lambda_\nu = \Gamma^\lambda_{\mu\nu}(x)$

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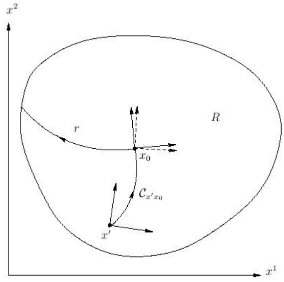

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$$\langle T^\mu{}_\nu \rangle(x) =$$
$$\frac{1}{V_\Sigma} \int_\Sigma V^\mu{}_{\mu'}(x, x'; \mathcal{C}_{xx'}) \widehat{V}_\nu{}^{\nu'}(x, x'; \mathcal{C}_{xx'}) T^{\mu'}{}_{\nu'}(x') \sqrt{-g(x')} d^4x'$$

Averaging Process in Teleparallel Gravity

$$\langle T^{\mu}{}_{\nu}\rangle(x) = \frac{1}{V_\Sigma}\int_\Sigma P^{\mu}{}_{\mu'}(x,x')P_{\nu}{}^{\nu'}(x,x')T^{\mu'}{}_{\nu'}(x')e(x')d^4x'$$

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- Path independent parallel transporters
  $P^{\mu}{}_{\mu'}(x,x') = e_a{}^{\mu}(x)e^{a}{}_{\mu'}(x')$ and $P_{\nu}{}^{\nu'}(x,x') = e^{a}{}_{\nu}(x)e_a{}^{\nu'}(x')$

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- where $e(x') = \det\left(e^{a}{}_{\mu}(x')\right)$
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- Problem: The metric is invariant under both averaging processes!

Einstein-Cartan Theory

Hehl, von der Heyde, Kerlick, and Nester:

- Lagrangian invariant under Poincaré gauge transformations

$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi G} R + \mathcal{L}_m(\Psi, \partial\Psi, g, \partial g, T)$$

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- They are related according to

$$\mu^{\nu\lambda\mu} = \frac{1}{2} \left(\tau^{\nu\lambda\mu} - \tau^{\lambda\mu\nu} + \tau^{\mu\nu\lambda} \right)$$

and

$$\Sigma^{\mu\nu} = \sigma^{\mu\nu} + (\nabla_{\lambda} - T^{\rho}{}_{\lambda\rho}) \mu^{\mu\nu\lambda}$$

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⇒ Combined field equation

$$\mathring{R}^{\mu\nu} - \frac{1}{2} \mathring{R} g^{\mu\nu}$$

$$= 8\pi G \sigma^{\mu\nu} + \frac{(8\pi G)^2}{4} (-2\tau_{\lambda}{}^{\mu\rho} \tau_{\rho}{}^{\nu\lambda} + 2\tau^{\lambda\mu}{}_{\lambda} \tau^{\rho\nu}{}_{\rho} - 2\tau^{\lambda\mu\rho} \tau_{\lambda}{}^{\nu}{}_{\rho}$$

$$+ \tau^{\mu\rho\lambda} \tau^{\nu}{}_{\rho\lambda} + \frac{1}{2} g^{\mu\nu} (2\tau_{\lambda\rho}{}^{\sigma} \tau_{\sigma}{}^{\rho\lambda} - 2\tau^{\lambda}{}_{\rho\lambda} \tau^{\sigma\rho}{}_{\sigma} + \tau^{\lambda\sigma\rho} \tau_{\lambda\sigma\rho}))$$

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$$H^{\mu\nu}{}_{,\nu} = \frac{4\pi}{c} J^\mu \text{ with } H^{\mu\nu} = \langle F^{\mu\nu} \rangle + 4\pi P^{\mu\nu}$$

- Einstein equations in Jordan's form

$$C_{\mu\nu\rho\sigma}{}^{;\sigma} = \kappa J_{\mu\nu\rho}$$

with $C_{\mu\nu\rho\sigma}$: Weyl tensor and $J_{\mu\nu\rho} = -\left(T_{\rho[\mu;\nu]} - \frac{1}{3}g_{\rho[\mu}T_{,\nu]}\right)$

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with the polarization tensor

$$P_{\mu\nu\rho\sigma} = \frac{1}{2}\left(-Q'{}_{\rho\sigma\epsilon[\mu}{}^{,\epsilon} - \frac{1}{3}\eta_{\rho[\mu} Q'{}^{\gamma}{}_{\sigma\gamma\epsilon}{}^{,\epsilon}\right)_{,\nu]}$$

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- Generalize formalism to more general metrics