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Advantages and unexpected shortcomings of extended theories of gravity

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MultiDark

Multimessenger Approach for Dark Matter Detection

GOBIERNO DE ESPAÑA

MINISTERIO DE CIENCIA E INNOVACIÓN

Outline

I. The degeneracy problem in extended theories

II. Averaging in Cosmology

- Acceleration, homogeneity and isotropy revisited
- Domains and averaged Hubble parameter

III. Backreaction mechanism in GR and extended theories

- Buchert formalism. Does backreaction work?
- E.g. 1: Quintessence
- E.g. 2: Brans-Dicke theories

IV. Limitations of Cosmographic approach in extended theories

- Biased results. Spotting ΛCDM
- Ruling out and reconstructing higher-order theories

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Motivation for extended theories of gravity

- General Relativity is consistent if treated in the frame of quantum effective field theories but it breaks down at Planck scale.

J.F. Donoghue and T. Torma, **gr-qc/9405057**

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✓ **Extended theories of gravity**

- must Emulate certain – gravitational – aspects of General Relativity
- must Explain the cosmological evolution in different eras
- are motivated by the cosmological constant (Λ) problem, dark energy, dark matter, singularities…

✓ **Proposals**

- Scalar/Vector-Tensor gravity: Brans-Dicke theories, ***f*(R) theories,** Horndeski
- Extra dimensions theories: **Brane-world theory**, String theory
- Massive gravity, Bi-metric gravity
- Born-Infeld inspired gravity
- Alternative geometries

Extended Theories of Gravity

✓ Main motivations (among others)

Scalar partners of the graviton naturally arise when quantizing or unifying gravity.

Coupling between scalar field(s) and matter: alleviation of the coincidence problem.

Explanations for Dark Matter: brane-world theories, axions, f(R),…

✓ Within FLRW (or other backgrounds) assumptions:

- provide identical background evolution as GR + dust + Λ
  AdlCD and A. Dobado, **Phys.Rev.D74:087501,2006**
- reconstruction methods, Elizalde, Odintsov, Sáez-Gomez *et al*.
- possible explanation for Dark Matter
  J. A. R. Cembranos, **Phys.Rev.Lett.102:141301,2009.**
- scalar perturbations may distinguish validity of theories/models
  AdlCD, A. Dobado and A. Maroto, **Phys.Rev.Lett.103:179001,2009.**

[See research at ITP Heidelberg and ACGC Cape Town]

The degeneracy problem

✓ Several extended gravity theories lead to identical results with either **General Relativity** or the **Concordance ΛCDM Model**

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✓ Consistency tests

- Evolution of geodesics and Raychaudhuri equation
- Importance of averaging and backreaction mechanism
- Evolution of scalar (growth rate) and tensor (Eg. CMB) perturbations
- Black holes properties and thermodynamics
- Stability issues, existence of ghosts, etc.
- Reliability of model-independent methods
- Dark matter: abundance, astrophysical fluxes and direct detection experiments

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  [The frames issue]

IV. Limitations of Cosmography in extended theories

Limitations in the Concordance Model

✓ Early universe (from BBN) is well described by the Concordance model:

- isotropic and homogeneous,
- with ordinary matter
- general relativity

✓ LSS and SNIa compatible with CMB data and discrepancy arises at $z < 1$

➢ At late times, distance and expansion rate are unpredicted in a **factor 2**.

✓ Standard explanation in FLRW models: "expansion has **accelerated**" (**!**)

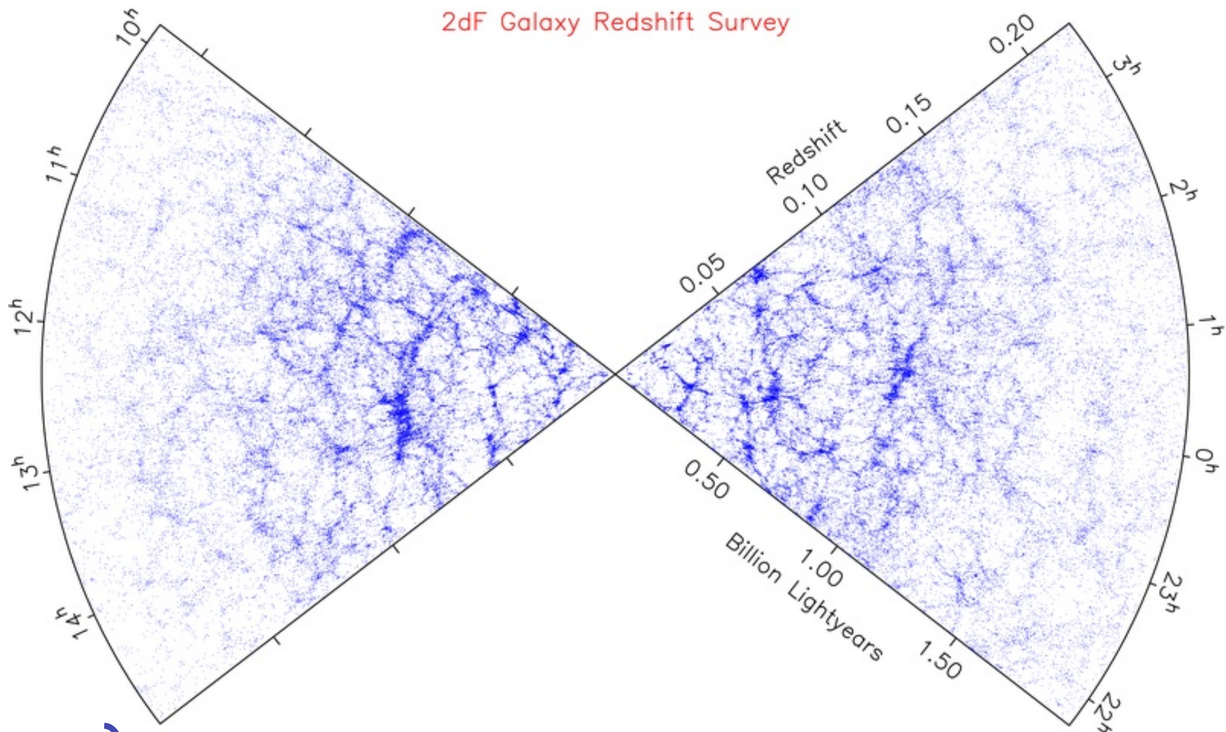
Alternative explanations?

❖ At least one of the Standard Cosmological Model assumptions might be wrong

- Exotic energy with negative pressure
  no evidence apart from accelerated expansion
- GR is not a complete theory
  modified gravity may alleviate this issue
- Homogeneity and Isotropy assumptions are not valid at late times
  both are violated due to formation of non-linear structures

Exact vs. Statistical Homogeneity & Isotropy

- Homogeneity scale
- Fundamental observers
- The distribution of non linear regions remains statistically HI on large scales (100 Mpc today)



- Box with non-linear regions
- Completely Smooth spacetime

evolve differently!

Average evolution of a clumpy space is not the same as the evolution of a smooth space BACKREACTION (G. F. R. Ellis, 1983 *fitting problem*)

Leit-motiv

- Universe is only statistically homogeneous and isotropic
- Einstein's equations are not linear in the metric $g_{\mu\nu}$

$$G_{\mu\nu}(\langle g_{\mu\nu}\rangle) \neq \langle G_{\mu\nu}(g_{\mu\nu})\rangle$$

- Local inhomogeneity and anisotropies affect the background via the backreaction mechanism

G. F. R. Ellis and W. Stoeger, Class. Quant. Grav. **4** (1987) 1697.

T. Buchert and J. Ehlers, Astron. Astrophys. **320** (1997) 1.

T. Buchert, Gen. Rel. Grav. **32** (2000) 105.

- BASICS…

$$ds^2 = -dt^2 + g_{ij}(t,\vec{x})dx^i dx^j$$

$$\nabla_\beta u_\alpha = \frac{1}{3}h_{\alpha\beta}\theta + \sigma_{\alpha\beta} + \omega_{\alpha\beta}$$

$$\theta \equiv \nabla_\alpha u^\alpha$$ Local expansion rate

- Stress-energy tensor decomposition

$$T_{\mu\nu} = \rho u_\mu u_\nu + p h_{\mu\nu} + 2q_{(\mu}u_{\nu)} + \pi_{\mu\nu}$$

$$\rho \equiv T_{\mu\nu}u^\mu u^\nu \qquad q_\mu \equiv -T_{\alpha\beta}h^\alpha{}_\mu u^\beta$$

$$p \equiv \frac{1}{3}T_{\alpha\beta}h^{\alpha\beta} \qquad \pi_{\mu\nu} \equiv T_{\alpha\beta}h^\alpha_{\langle\mu}h^\beta_{\nu\rangle}\,.$$

- *Local Einstein Equations* $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu} = \sum_\alpha T^{(\alpha)}_{\mu\nu}$

Arnowitt-Deser-Misner decomposition

$$\left.\begin{cases} \frac{1}{2}\left(\mathcal{R} + \theta^2 - \theta^i_j\theta^j_i\right) = \rho\,, \\ \theta_{,i} - \theta^j_{\;i;\,j} = 8\pi G\, q_i\,, \end{cases}\right\} \text{Constraint eqns.}$$

$$\dot{\theta}^i{}_j = -\theta\theta^i{}_j - \mathcal{R}^i_j + \frac{1}{2}(\rho - 3p)\delta^i{}_j + T^i_j\,, \qquad \text{Evolution eqn.}$$

- *Local Einstein Equations* $$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu} = \sum_\alpha T^{(\alpha)}_{\mu\nu}$$

Arnowitt-Deser-Misner decomposition

$$\dot{\theta} = -2\sigma^2 - \frac{1}{3}\theta^2 - \frac{1}{2}(\rho + 3p)$$ Raychaudhuri eqn.

$$\partial_t \sigma^2 = -2\theta\sigma^2 - \sigma^i{}_j \mathcal{R}^i{}_j + \sigma^i{}_j \pi^j{}_i$$ Shear evolution eqn.

* No vorticity and strong energy condition implies $\dot{\theta} <0$

- *Conservation Equations* $$\nabla_\mu T^{\mu\nu} = 0$$

$$-u_\nu \nabla_\mu T^{\mu\nu} = \dot{\rho} + \theta(\rho + p) + \nabla_\mu q^\mu + \pi^{\mu\nu}\sigma_{\mu\nu} = 0\,,$$

$$h_{\alpha\nu}\nabla_\mu T^{\mu\nu} = \hat{\nabla}_\alpha p + \dot{q}_\alpha + \frac{4}{3}\theta q_\alpha + q^\mu \sigma_{\mu\alpha} + h_{\alpha\nu}\nabla_\mu \pi^{\mu\nu} = 0$$

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IV. Limitations of Cosmography in extended theories

DOMAINS and AVERAGED H_D

- *Key concept* Spatial average for an observable $O(t, \mathbf{x})$ at time t,

$$\langle O \rangle_D \equiv \frac{1}{V_D(t)} \int W_D(\mathbf{x}) O(t, \mathbf{x}) \sqrt{\det g_{ij}} \mathrm{d}\mathbf{x}$$

$$V_D(t) \equiv \int W_D(\mathbf{x}) \sqrt{\det g_{ij}} \mathrm{d}\mathbf{x}$$

$W_D(\mathbf{x})$ window function
[political decision]

- *Effective scale factor* a_D

$$\frac{a_D}{a_{D_0}} \equiv \left(\frac{V_D}{V_{D_0}} \right)^{1/3}$$

✓ doesn't describe a local behavior
✓ doesn't appear in the metric

- Effective Hubble rate H_D

$$H_D \equiv \frac{\dot{a}_D}{a_D} \equiv \langle \theta \rangle_{\cdot} / 3$$

DOMAINS and AVERAGED H_D

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$$V_D(t) \equiv \int W_D(\mathbf{x}) \sqrt{\det g_{ij}} d\mathbf{x}$$

$W_D(\mathrm{x})$ window function [political decision]

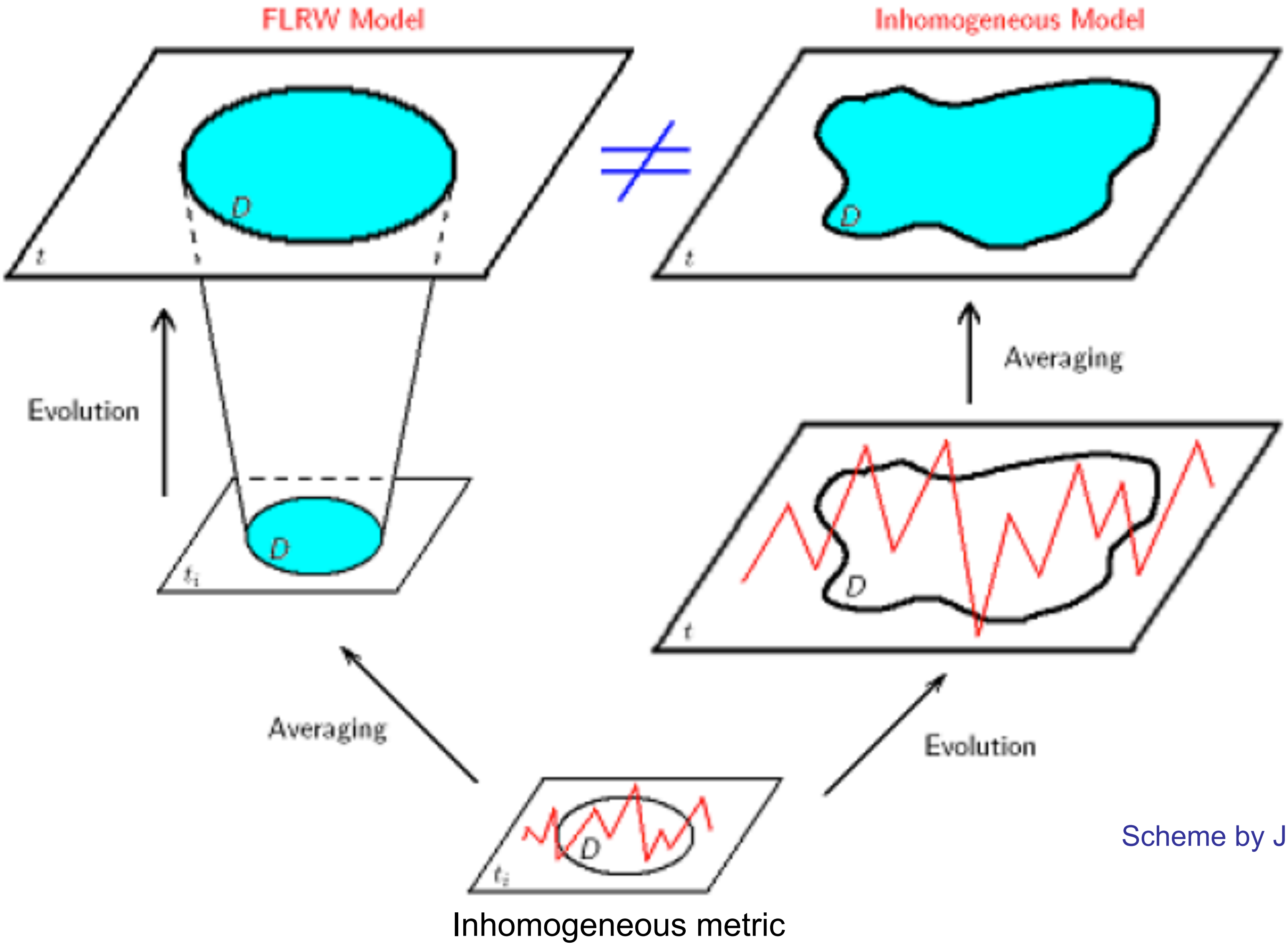
- *Commutation relations*

$$[\partial_t, \langle\rangle_{\mathcal{D}}]\, \mathcal{O} \equiv \partial_t \langle \mathcal{O} \rangle_{\mathcal{D}} - \langle \partial_t \mathcal{O} \rangle_{\mathcal{D}} = \langle \theta \mathcal{O} \rangle_{\mathcal{D}} - \langle \theta \rangle_{\mathcal{D}} \langle \mathcal{O} \rangle_{\mathcal{D}}$$

$$[\partial_t, \langle\rangle_{\mathcal{D}}]\, \mathcal{O} = \langle \delta\theta\, \delta\mathcal{O} \rangle_{\mathcal{D}} \quad \delta\mathcal{O} \equiv \mathcal{O} - \langle \mathcal{O} \rangle_{\mathcal{D}}.$$

↑ Second order effects

$$[\partial_t, \langle\rangle_D]\,\mathcal{O} \equiv \langle \mathcal{O}\theta\rangle_D - \langle\theta\rangle_D\langle\mathcal{O}\rangle_D$$



Scheme by Julien Larena

Buchert (1999) Formalism in GR

Buchert et al., 1002.3912, 0001056, 0707.2153

$$H_D^2 = \left(\frac{\dot{a}_D}{a_D}\right)^2 = \frac{8\pi G}{3}\rho_{\rm eff}$$

$$-\frac{\ddot{a}_D}{a_D} = \frac{4\pi G}{3}(\rho_{\rm eff} + 3p_{\rm eff})$$

$$\partial\langle\rho\rangle_D + 3\frac{\dot{a}_D}{a_D}\langle\rho\rangle_D = 0$$

Effective Friedmann equations in GR + dust universe

$$\rho_{\rm eff} \equiv \langle\rho\rangle_D - \frac{1}{16\pi G}\left(\langle Q\rangle_D + \langle\mathcal{R}\rangle_D\right)$$

$$p_{\rm eff} \equiv -\frac{1}{16\pi G}\left(\langle Q\rangle_D - \frac{1}{3}\langle\mathcal{R}\rangle_D\right)$$

$\rho_{\rm eff}$ effective density

$p_{\rm eff}$ effective pressure

$$\langle Q\rangle_D :\equiv \frac{2}{3}\left(\langle\theta^2\rangle - \langle\theta\rangle^2\right) - 2\langle\sigma^2\rangle$$ Kinematical backreaction

$\langle\mathcal{R}\rangle_D$: Averaged spatial curvature

Aren't the corrections just ~10^{-5}?

- No. Of course not. Λ is bs [Buchert, Kolb ...]
- Yes. Absolutely. Those guys are idiots. [Wald, Peebles ...]
- Well, maybe. I've no idea what's going on. [everyone else ... ?]
- Corrections from averaging enter Friedmann and Raychaudhuri equations

  - is this degenerate with 'dark energy'?
  - can we separate the effects [if there are any]?
  - or ... is it dark energy? neat solution to the coincidence problem

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- Well, maybe. I've no idea what's going on. [everyone else ... ?]

- Almost one year ago...
  Once structures virialise, the effect of backreaction **in GR** is negligible independently of initial conditions

  Can small scale structure ever affect cosmological dynamics?
  Julian Adamek, Chris Clarkson, Ruth Durrer, Martin Kunz
  Phys. Rev. Lett. 114, 051302 (2015) e-Print: **1408.2741 [astro-ph.CO]**

- Can average expansion rate speed up whereas the local one slows down ?

✓ Simple model: two regions [overdense vs. underdense]

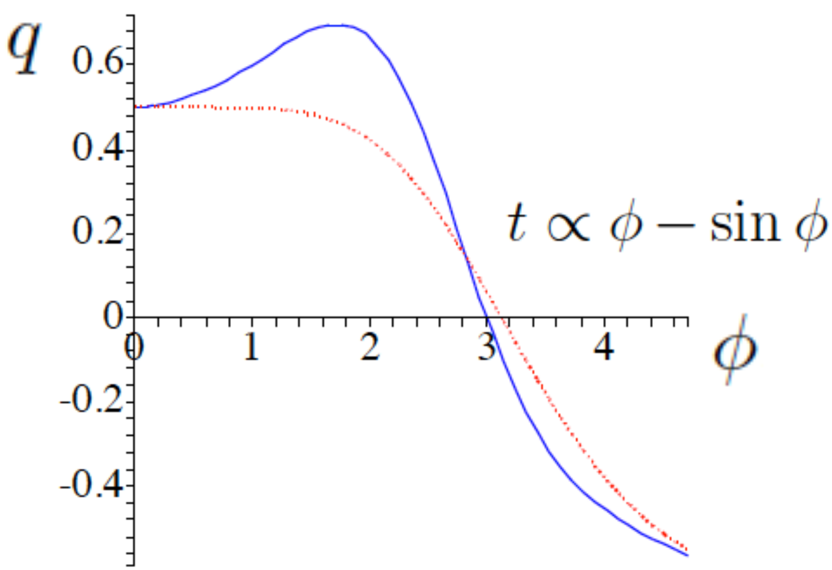
$$H = \frac{a_1^3}{a_1^3 + a_2^3} H_1 + \frac{a_2^3}{a_1^3 + a_2^3} H_2 \equiv v_1 H_1 + v_2 H_2$$

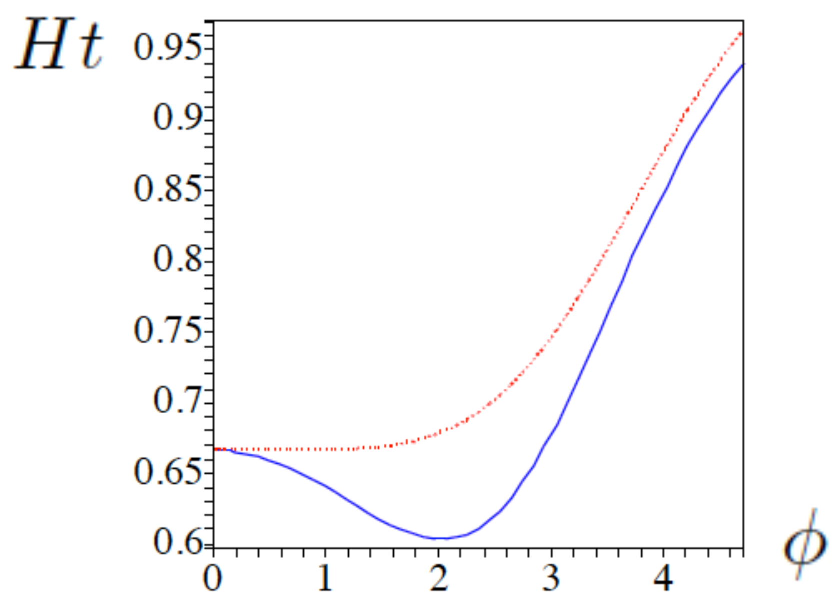
$$\frac{\ddot{a}}{a} = v_1 \frac{\ddot{a}_1}{a_1} + v_2 \frac{\ddot{a}_2}{a_2} + 2 v_1 v_2 (H_1 - H_2)^2$$

Backreaction $\langle Q \rangle_D$ variable

Simple model vs. ΛCDM

S. Rasanen arXiv:1012.0784 [astro-ph.CO]





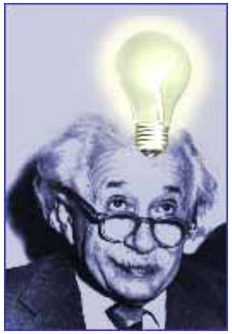
Shortcomings in – standard – Averaging

✓ Standard: general relativity assumed as the unique possible theory

✧ Why not other geometrical Lagrangians?

✓ Standard: only dust fluid in the matter side

✧ Why not other fluids (radiation, quasi-dust, multifluid…) ?



Any modified gravity, although not providing cosmological acceleration by itself, will present backreaction effects

Its relative importance might not be necessarily the same as in GR

Q1: Is GR backreaction distinguishable from modified backreaction?

Q2: Is the standard – Buchert-like – procedure to get averaged quantities valid?

a) Averaged Einstein equations

$$H_{\mathcal{D}}^2 = \frac{1}{3}\langle\rho\rangle_{\mathcal{D}} - \frac{1}{6}\langle\mathcal{R}\rangle_{\mathcal{D}} - \frac{1}{6}\langle Q\rangle_{\mathcal{D}} ,$$

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = -\frac{1}{6}\langle\rho + 3p\rangle_{\mathcal{D}} + \frac{1}{3}\langle Q\rangle_{\mathcal{D}} ,$$

$$\partial_t\langle\sigma^2\rangle_{\mathcal{D}} = -2\langle\theta\rangle_{\mathcal{D}}\langle\sigma^2\rangle_{\mathcal{D}} - \langle\theta\,\delta\sigma^2\rangle_{\mathcal{D}} + \langle\sigma^i{}_j\mathcal{C}^j{}_i\rangle_{\mathcal{D}},$$

$$\delta\sigma^2 \equiv \sigma^2 - \langle\sigma^2\rangle_{\mathcal{D}}$$
$$\mathcal{C}^j{}_i \equiv \pi^j_i - \mathcal{R}^j_{\perp\ i} ,$$

b) Averaged continuity equation

$$\partial_t\,\langle\rho\rangle_{\mathcal{D}} + \langle\theta\rangle_{\mathcal{D}}\,\langle\rho\rangle_{\mathcal{D}} + \langle\theta p\rangle_{\mathcal{D}} = \langle j\rangle_{\mathcal{D}}$$

$$j \equiv -\nabla_\mu q^\mu + \pi^{\mu\nu}\sigma_{\mu\nu}$$

a) Averaged Einstein equations

J. Beltrán, AdlCD, P. Dunsby, D. Sáez-Gómez
JCAP 1405 (2014) 031 **arXiv:1312.5680 [astro-ph.CO]**

combined

$$\frac{1}{2a_{\mathcal{D}}^{6}}\left[\partial_t\left(a_{\mathcal{D}}^{6}\left\langle Q\right\rangle_{\mathcal{D}}\right)+a_{\mathcal{D}}^{4}\partial_t\left(a_{D}^{2}\left\langle \mathcal{R}\right\rangle_{\mathcal{D}}\right)\right] = -\left\langle\delta\theta\,\delta p\right\rangle_{\mathcal{D}}+\left\langle j\right\rangle_{\mathcal{D}}$$

Integrability condition

b) Averaged continuity equation

J. Beltrán, AdlCD, P. Dunsby, D. Sáez-Gómez
JCAP 1405 (2014) 031 arXiv:1312.5680 [astro-ph.CO]

$$\frac{1}{2a_{\mathcal{D}}^6}\left[\partial_t\left(a_{\mathcal{D}}^6\langle Q\rangle_{\mathcal{D}}\right)+a_{\mathcal{D}}^4\partial_t\left(a_{\mathcal{D}}^2\langle\mathcal{R}\rangle_{\mathcal{D}}\right)\right]=-\langle\delta\theta\,\delta p\rangle_{\mathcal{D}}+\langle j\rangle_{\mathcal{D}}$$

Integrability condition

✧ Equation with NO analogy in Newtonian dynamics

✧ **Usual integrability condition in General Relativity is modified.**

✧ In extended theories, **the backreaction effects are not necessarily rapidly diluted**

$$\langle\mathcal{R}\rangle_{\mathcal{D}}\propto a_{\mathcal{D}}^{-2}\;\longrightarrow\;\langle Q\rangle_{\mathcal{D}}\propto a_{\mathcal{D}}^{-6}$$

✧ Second order perturbations with only first order (squared) terms.

✧ In other scenarios, pressure perturbations, mometum fluxes and anisotropic stress can source the kinematical backreaction.

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[The frames issue]

QUINTESSENCE (I) $S = \int d^4x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right)$

Single scalar field minimally coupled to gravity

$$\rho \equiv T_{\mu\nu}u^\mu u^\nu = \rho_m + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}h^{\alpha\beta}\hat{\nabla}_\alpha\phi\hat{\nabla}_\beta + V(\phi)$$

$$p \equiv \frac{1}{3}T_{\mu\nu}h^{\mu\nu} = p_m + \frac{1}{2}\dot{\phi}^2 - \frac{1}{6}h^{\alpha\beta}\hat{\nabla}_\alpha\phi\hat{\nabla}_\beta - V(\phi)$$

$$q_\mu \equiv -T_{\alpha\beta}h^\alpha_\mu u^\beta = -\dot{\phi}\hat{\nabla}_\mu\phi$$

$$\pi_{\mu\nu} \equiv T_{\alpha\beta}h^\alpha_{<\mu}h^\beta_{\nu>} = \hat{T}_{\mu\nu} - ph_{\mu\nu} \quad , \quad \hat{T}_{\mu\nu} \equiv h^\alpha_{(\mu}h^\beta_{\nu)}T_{\alpha\beta}$$

✓ Scalar field equation of motion

J. Beltrán, AdlCD, P. Dunsby, D. Sáez-Gómez
JCAP 1405 (2014) 031 arXiv:1312.5680 [astro-ph.CO]

$$\Box\phi = \frac{dV(\phi)}{d\phi} \qquad \ddot{\phi} + \theta\dot{\phi} - h^{\alpha\beta}\hat{\nabla}_\alpha\hat{\nabla}_\beta\phi + V_{,\phi} = 0$$

$$\partial_t\left(\partial_t\langle\phi\rangle_\mathcal{D} - \langle\delta\theta\,\delta\phi\rangle_\mathcal{D}\right) + \langle\theta\rangle_\mathcal{D}\left(\partial_t\langle\phi\rangle_\mathcal{D} - \langle\delta\theta\,\delta\phi\rangle_\mathcal{D}\right) + \underbrace{\langle V_{,\phi}\rangle_\mathcal{D}} = \underbrace{\left\langle h^{\alpha\beta}\hat{\nabla}_\alpha\hat{\nabla}_\beta\phi\right\rangle_\mathcal{D}}$$

$$\langle\phi\rangle_\mathcal{D} \simeq \phi_0 + \int \frac{\langle\delta\theta\,\delta\phi\rangle_\mathcal{D}}{a_\mathcal{D}H_\mathcal{D}}da_\mathcal{D}$$

Neglecting potential and spatial derivatives

QUINTESSENCE (and II)

Let´s assume a homogeneous field $\phi = \phi(t) = \langle \phi \rangle_{\mathcal{D}}$.

$$p = p_m + \frac{1}{2}(\partial_t \phi)^2 - V(\phi)$$

$$q_\mu = 0\,,$$

$$\pi^{\mu\nu} = g^{\mu\nu}\left[\frac{1}{2}(\partial_t \phi)^2 - V(\phi)\right]$$

and the integrability condition…

$$\frac{1}{2a_{\mathcal{D}}^6}\left[\partial_t\left(a_{\mathcal{D}}^6 \langle Q \rangle_{\mathcal{D}}\right) + a_{\mathcal{D}}^4 \partial_t\left(a_{\mathcal{D}}^2 \langle \mathcal{R} \rangle_{\mathcal{D}}\right)\right] = - \langle \delta\theta\, \delta p \rangle_{\mathcal{D}} = -\langle \theta\, \delta p_m \rangle_{\mathcal{D}}$$

J. Beltrán, AdlCD, P. Dunsby, D. Sáez-Gómez
JCAP 1405 (2014) 031 arXiv:1312.5680 [astro-ph.CO]

- Standard integrability condition **is recovered** in homogeneous quintessence scenarios provided that δp_m is negligible
- Homogeneous quintessence fields don’t contribute to averaged equations thanks to the **minimal coupling**

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[The frames issue]

IV. Limitations of Cosmography in extended theories

- Biased results depending on auxiliary variable choice
- Ruling out – reconstructing theories with higher orders

BRANS-DICKE THEORIES (I) $S = \frac{1}{2}\int d^4x\sqrt{-g}\left[\phi R - \frac{\omega_0}{\phi}\partial_\mu\phi\partial^\mu\phi\right] + S_M[g_{\mu\nu};\psi]$

- Non-minimal coupling
- Gravitational constant depends on the scalar field

$$\rho \equiv T_{\mu\nu}u^\mu u^\nu = \frac{\rho_m}{\phi} + \frac{\omega_0}{\phi^2}\left[\dot{\phi}^2 + \frac{1}{2}(\partial\phi)^2\right] + \frac{1}{\phi}\left(\ddot{\phi} + \Box\phi\right)$$

$$p \equiv \frac{1}{3}T_{\mu\nu}h^{\mu\nu} = \frac{p_m}{\phi} + \frac{\omega_0}{\phi^2}\left[h^{\mu\nu}\hat{\nabla}_\mu\phi\hat{\nabla}_\nu\phi - \frac{3}{2}(\partial\phi)^2\right] + \frac{1}{3\phi}\left(-2\Box\phi + \ddot{\phi}\right)$$

$$q_\mu \equiv -T_{\alpha\beta}h^\alpha_\mu u^\beta = -\frac{\omega_0}{\phi^2}\left[\dot{\phi}\hat{\nabla}_\mu\phi\right] - \frac{1}{\phi}h^\alpha{}_\mu u^\beta\nabla_\beta\left(h^\gamma_\alpha\nabla_\gamma\phi\right)$$

$$\pi_{\mu\nu} \equiv T_{\alpha\beta}h^\alpha{}_{<\mu}h^\beta{}_{\nu>} = \hat{T}_{\mu\nu} - ph_{\mu\nu} \quad , \quad \hat{T}_{\mu\nu} \equiv h^\alpha_{(\mu}h^\beta_{\nu)}T_{\alpha\beta}\,.$$

✓ Scalar field equation of motion

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JCAP 1405 (2014) 031 arXiv:1312.5680 [astro-ph.CO]

$$\Box\phi = -\frac{\rho_m - 3p_m}{2\omega_0 + 3} \qquad \ddot{\phi} + \theta\dot{\phi} - h^{\alpha\beta}\hat{\nabla}_\alpha\hat{\nabla}_\beta\phi - \frac{1}{2\omega_0 - 3}(\rho_m - 3p_m) = 0$$

$$\partial_{tt}\langle\phi\rangle_{\mathcal{D}} + \langle\theta\rangle_{\mathcal{D}}\partial_t\langle\phi\rangle_{\mathcal{D}} = \frac{1}{2\omega_0 - 3}\langle\rho_m - 3p_m\rangle_{\mathcal{D}} + \partial_t\langle\delta\theta\,\delta\phi\rangle_{\mathcal{D}} + \langle\theta\rangle_{\mathcal{D}}\langle\delta\theta\,\delta\phi\rangle_{\mathcal{D}} + \left\langle h^{\alpha\beta}\hat{\nabla}_\alpha\hat{\nabla}_\beta\phi\right\rangle_{\mathcal{D}}$$

Essentially the same as the one obtained in Quintessence

BRANS-DICKE THEORIES (and II)

Let´s again assume a homogeneous field $\phi = \phi(t) = \langle\phi\rangle_{\mathcal{D}}$.

$$
\begin{aligned}
p &= \frac{p_m}{\phi} + \frac{\omega_0}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{1}{3\phi}\left(\frac{2\rho_m}{2\omega_0+3} + \ddot{\phi}\right), \\
q_\mu &= 0, \\
\pi^{\mu\nu}\sigma_{\mu\nu} &= -\frac{1}{2\phi}\partial_t\phi\,\sigma^{ij}\partial_t g_{ij}
\end{aligned}
$$

and the integrability condition…

J. Beltrán, AdlCD, P. Dunsby, D. Sáez-Gómez
JCAP 1405 (2014) 031 arXiv:1312.5680 [astro-ph.CO]

$$
\begin{aligned}
\frac{1}{2a_{\mathcal{D}}^6}\left[\partial_t\left(a_{\mathcal{D}}^6\langle Q\rangle_{\mathcal{D}}\right) + a_{\mathcal{D}}^4\partial_t\left(a_{\mathcal{D}}^2\langle\mathcal{R}\rangle_{\mathcal{D}}\right)\right] &= \frac{-1}{3\phi(2\omega_0+3)}\Big[\left[\partial_t,\ \langle\rangle_{\mathcal{D}}\right]\left(2\rho_m + 3\left(2\omega_0+3\right)p_m\right)\Big] \\
&\quad - \frac{1}{2\phi}\partial_t\phi\left\langle\sigma^{ij}\partial_t g_{ij}\right\rangle_{\mathcal{D}}
\end{aligned}
$$

- Unlike Quintessence, the standard integrability condition is NOT recovered in homogeneous BD scenarios due to the inhomogeneous character of either density, pressure, the metric tensor or σ_{ij}

Conclusions and Prospects in Averaging

- Backreaction tries to account – at least partially - for the observed discrepancy between expected cosmological evolution and late accelerated era without Dark Energy

- If backreaction hypothesis is valid, its applicability to extended theories of gravity is a natural step to understand degenerate results. Precision Cosmology doesn't make sense prior to establishing the importance of backreaction effects

- For Quintessence and Brans-Dicke theories, the integrability condition is - isn't - recovered for homogeneous fields acting on inhomogeneous backgrounds

- Prospects

1. Determination of H_D in extended theories and comparison with - for instance - Supernovae catalogues.

2. Perturbative backreaction: for scalar perturbations, study their effects in the averaged evolution.

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II. Averaging in Cosmology

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III. Backreaction mechanism in GR and extended theories

- (Buchert) formalism. Does backreaction work?
- E.g. 1: Quintessence
- E.g. 2: Brans-Dicke theories

  [The frames issue]

IV. Limitations of Cosmographic approach in extended theories

- Biased results depending on auxiliary variable choice
- Ruling out – reconstructing theories with higher orders

✓ Cosmography rudiments

- In order to test GR and the Copernican Principle, a useful tool is to use frameworks able to encompass a large class of models/theories

- Such model independent methods - instead of a case-by-case approach – have been used to infer the Dark Energy EoS and reconstruct classes of DE theories

- Cosmography approach just relies on the **Copernican principle** and the expression of the **scale factor in terms of an auxiliary variable** (redshift, time, etc.)

$$H = \frac{\dot{a}}{a}\,,\; q = -\frac{\ddot{a}}{aH^2}\,,\; j = \frac{a^{(3)}}{aH^3}\,,\; s = \frac{a^{(4)}}{aH^4}\,,\; l = \frac{a^{(5)}}{aH^5}\,,\ldots$$

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$$\boxed{z=0} \quad H = H_0 + H_{z0} z + \frac{H_{zz0}}{2} z^2 + \ldots \quad |z| < 1$$

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$z = 0$ $\quad H = H_0 + H_{z0} z + \frac{H_{zz0}}{2} z^2 + \ldots \quad |z| < 1$

or $y = \dfrac{z}{1+z}$ as alternative independent variable

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✓ Differences between auxiliary variables: y vs. z

- Mock data generated from a fiducial flat ΛCDM model with redshift distribution Union2.1 catalogue and $\sigma_\mu = 0.15$

$$\Omega_m = 0.3$$
$$H_0 = 73.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

- Two sets of parameters and 100 simulations

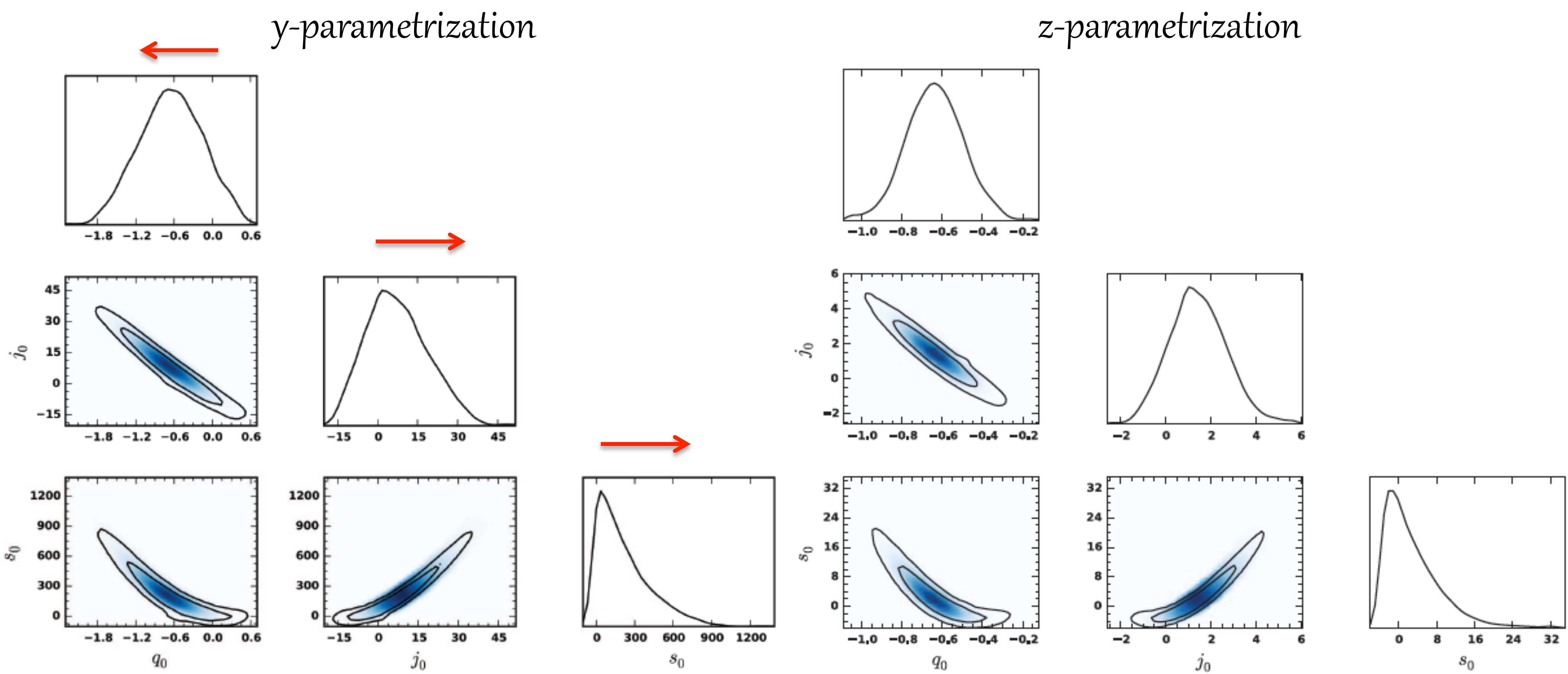
$$\boldsymbol{\theta}_1 = \{H_0, q_0, j_0, s_0\} \qquad \boldsymbol{\theta}_2 = \{H_0, q_0, j_0, s_0, l_0\}$$

- How frequent the true cosmographic values fall in 1, 2, 3σ confidence regions

	$\boldsymbol{\theta}_1$						$\boldsymbol{\theta}_2$					
	y			z			y			z		
	1σ	2σ	3σ	1σ	2σ	3σ	1σ	2σ	3σ	1σ	2σ	3σ
q_0	26	32	42	67	27	6	82	12	6	82	18	0
j_0	10	45	45	64	29	7	93	5	2	88	12	0
s_0	10	67	23	83	15	2	92	7	1	93	6	1
l_0	-	-	-	-	-	-	100	0	0	100	0	0

V. Busti, AdlCD, P. Dunsby, D. Sáez-Gómez
arXiv:1505.5503 [astro-ph.CO]

✓ Differences between auxiliary variables: y vs. z



One simulation, $\theta_1 = \{H_0, q_0, j_0, s_0\}$

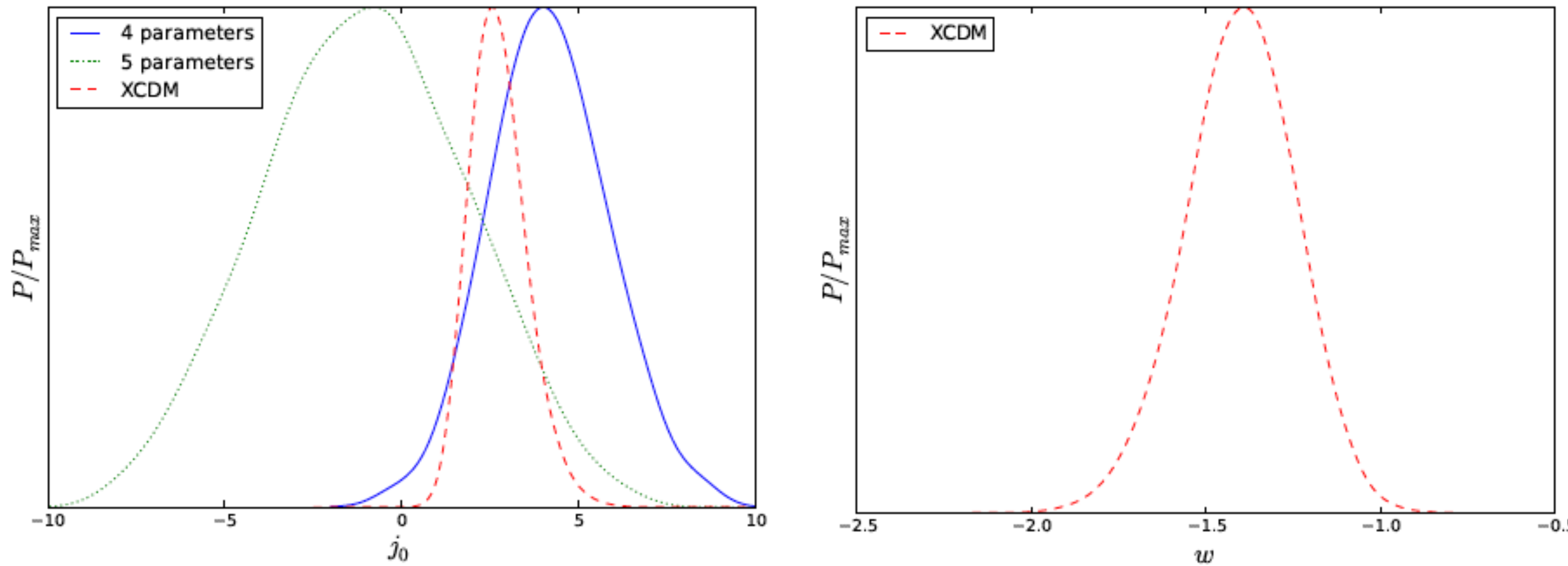
V. Busti, AdlCD, P. Dunsby, D. Sáez-Gómez
arXiv:1505.5503 [astro-ph.CO]

✓ Is Cosmography able to spot the correct *X*CDM model?

- Mock realizations of data for a flat *X*CDM $\Omega_m = 0.3 \quad w = -1.3 \quad j_0 = 1.945$

- Constraints for $\boldsymbol{\theta}_1$ (fourth order) $\boldsymbol{\theta}_2$ (fifth order) and direct constraint of parameters

V. Busti, AdlCD, P. Dunsby, D. Sáez-Gómez
arXiv:1505.5503 [astro-ph.CO]



Fittng to the model spots deviations from ΛCDM with less effort

Some evidence of $j_0 \neq 1$ when considering $\boldsymbol{\theta}_1$, but dissapears assuming $\boldsymbol{\theta}_2$

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- Cosmography as a tool to reconstruct DE models

Capozziello et al., Bamba et al. Astrophys. Space Sci. 342, 155 (2012)

- Nonetheless in theories with higher derivatives, the appearance of extra parameters apart from the cosmographic ones, imposes some limitations in the method

E.g. 1: K-essence

$$\mathcal{S} = \int dx^4 \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} \omega(\phi) \partial_\mu \phi \partial^\mu \phi - V(\phi) + \mathcal{L}_m \right]$$

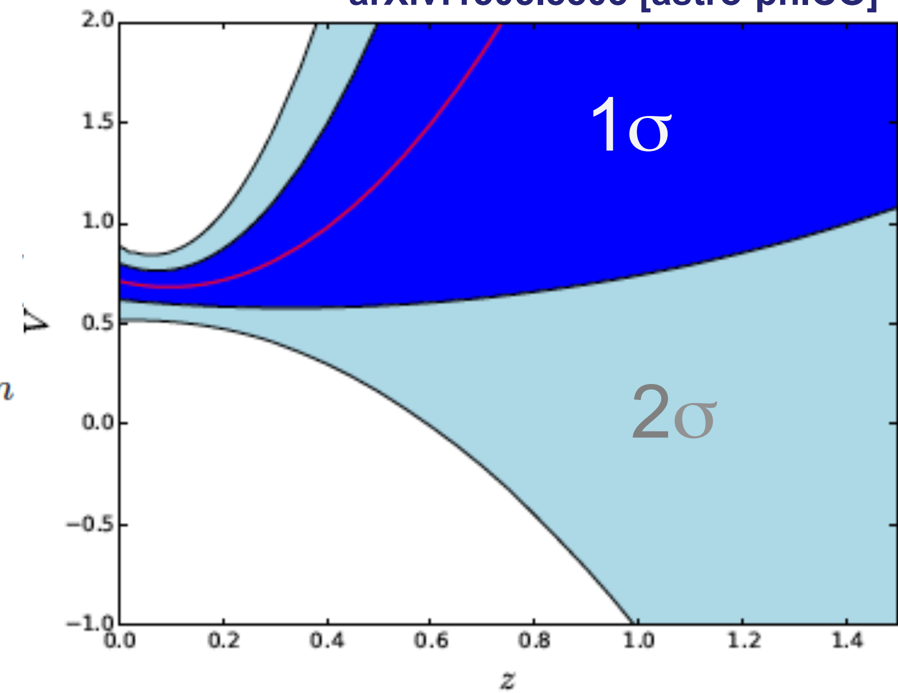
V. Busti, AdlCD, P. Dunsby, D. Sáez-Gómez
arXiv:1505.5503 [astro-ph.CO]

$$\frac{V_0}{H_0^2} = 2 - q_0 - \frac{3\Omega_m}{2} \, ,$$

$$\frac{V_{z0}}{H_0^2} = 4 + 3q_0 - j_0 - \frac{9\Omega_m}{2} \, ,$$

$$\frac{V_{2z0}}{H_0^2} = 4 + 8q_0 + j_0(4 + q_0) + s_0 - 9\Omega_m \, ,$$

$$\frac{V_{3z0}}{H_0^2} = j_0^2 - l_0 - q_0 j_0 (7 + 3q_0) - s_0(7 + 3q_0) - 9\Omega_m$$



- Generic realization of ΛCDM, fourth-order expansion
- It requires assumption on the model today

$$\Omega_m \approx 2/3(1 + q_0)$$

E.g. 2: *f*(R) theories

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}f(R) + \mathcal{L}_m\right]$$

$$\frac{f_0}{6H_0^2} = -\alpha q_0 + \Omega_m + 6\beta(2 + q_0 - j_0) \ ,$$

$$\frac{f_{z0}}{6H_0^2} = \alpha(2 + q_0 - j_0) \ ,$$

$$\frac{f_{2z0}}{6H_0^2} = 6\beta(2 + q_0 - j_0)^2 + \alpha[2 + 4q_0 + (2 + q_0)j_0 + s_0]$$

$$\left.\frac{\mathrm{d}f}{\mathrm{d}R}\right|_{R=R_0} = \alpha \quad ; \quad \left.\frac{\mathrm{d}^2 f}{\mathrm{d}R^2}\right|_{R=R_0} = \frac{\beta}{H_0^2}$$

Two extra parameters

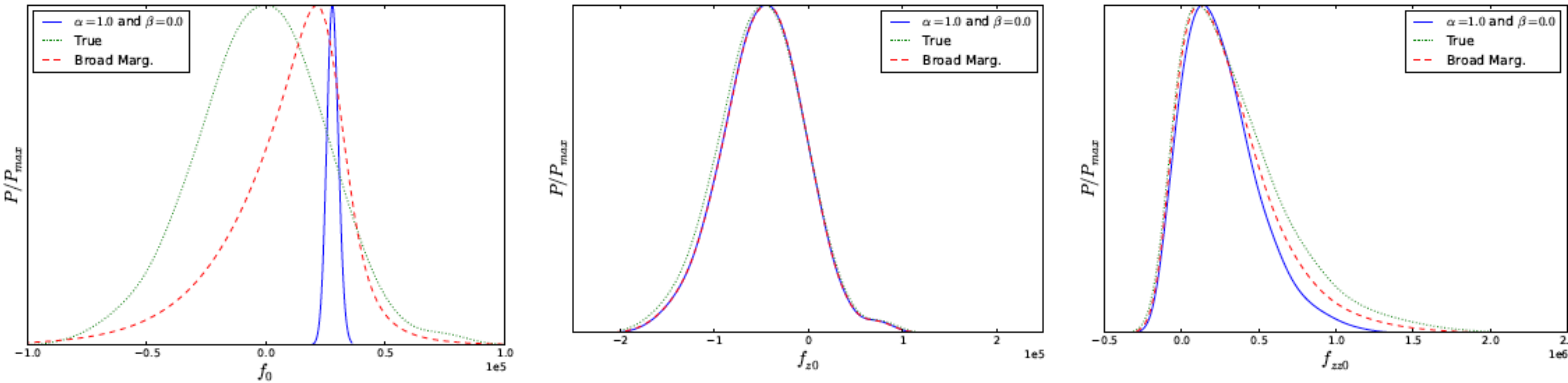
- ✓ Cosmological values $\alpha \neq 1$ and $\beta \neq 0$ may still produce viable cosmological models
- ✓ One-to-one correspondence between *f*(R)-derivatives and cosmographic parameters must be abandoned. Sensible priors for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are required

E.g. 2: *f*(R) theories

$$f(R) = R + aR^2 + bR^3$$

$$\alpha = 2.81$$
$$\beta = 0.06$$

- ✓ Mock data generated from the given *f*(R) model
- ✓ Simulations: true values, $\{\alpha = 1, \beta = 0\}$, and broad marginalization $\alpha \sim N(1, 0.05)$, $\beta \sim N(0.07, 0.05)$



$$\{f_0, f_{z0}, f_{zz0}\}$$

V. Busti, AdlCD, P. Dunsby, D. Sáez-Gómez
arXiv:1505.5503 [astro-ph.CO]

- Values for the broad marginalization do not cover the true values of f_0
- N.B.: Wide range of free parameters in more viable models (e.g. Hu-Sawicki) lie within 1σ region in the figures

Conclusions on Cosmographic Approach

- Cosmography results do depend upon both the chosen auxiliary variable (redshifts z or y) and the expansion order

- Reliability of cosmography to spot ΛCDM around close-enough XCDM competitors, remains limited with results again depending upon the expansion order

- For extended theories, the method provides a sort of clear picture for theories with no higher-order derivatives, although not competitive (larger errors) with other mehods

- For theories with higher derivatives in either geometrical or matter sector, there are extra free parameters requiring marginalization and large errors emerge

- Other neglected limitations: spatial cuvature (Clarkson 2011), lensings effects (Wald 1998, Bacon 2014) and local gravitational redshift (Wojtak 2015) may lead to extra scatter in Hubble diagrams

- Any hope?
    1. Clear definition of auxiliary variables and extensive testing against mock data
    2. Establish a trade-off between number of data points, number of cosmological parameters and Bayesian evidence, so criteria can be provided
    3. Motivated priors over extra parameters

Astrophysics, Cosmology and Gravitation Research Centre

- 19+ Academic Staff
- 16+ Postdoctoral Fellows
- 36+ Postgraduate Students

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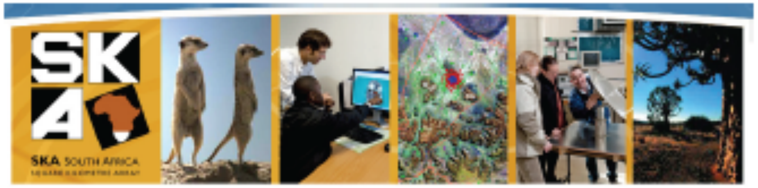

http://www.acgc.uct.ac.za

Extended Outline

I. A kind of motivation for extended theories: the degeneracy problem

II. Averaging in Cosmology

- Acceleration, homogeneity and isotropy revisited
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III. Backreaction mechanism in GR and extended theories

- Buchert formalism. Does backreaction work?
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[The frames issue]

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DIFFERENT FRAMES IN MODIFIED GRAVITY THEORIES

$$S_{BD} = \int d^4x\sqrt{-g}\left[\phi R - \frac{\omega_0}{\phi}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi) + 2\mathcal{L}_m\right]$$

Jordan frame

- Non-minimal coupling
- Particles follow geodesics

✓ Conformal transformation $g_{E\mu\nu} = \Omega^2 g_{\mu\nu} \quad \Omega^2 = \phi$

$$S_E = \int d^4x\sqrt{-g_E}\left[R_E - \frac{2\omega_0+3}{2\phi^2}\partial_\mu\phi\partial^\mu\phi - \frac{V(\phi)}{\phi^2} + \frac{2}{\phi^2}\mathcal{L}_{Em}\right]$$

Einstein frame

$$\mathcal{L}_{Em} = \mathcal{L}_m\left(\phi, g_{E\mu\nu}\right). \qquad \nabla_{E\mu}T_E^{\mu\nu(m)} \neq 0.$$

- Minimal coupling Einstein-like
- Particles don't follow geodesics

✓ In principle, averaging is possible in both frames (but not that trivial…)

Einstein frame

$$\dot{\rho} + \theta(\rho+p) + \nabla_\mu q^\mu + \pi^{\mu\nu}\sigma_{\mu\nu} + a^\mu q_\mu = 0$$

$$\partial_t\langle\rho\rangle_{\mathcal{D}} + \langle\rho\rangle_{\mathcal{D}}\langle\theta\rangle_{\mathcal{D}} = -\langle p\theta\rangle_{\mathcal{D}} - \langle\pi_{\mu\nu}\sigma^{\mu\nu}\rangle_{\mathcal{D}} - \langle a^\mu q_\mu\rangle_{\mathcal{D}}$$

✓ Frame-choice shortcomings

1. Averaging vs. frame changing don't commute

2. Spatial averages are performed in a volume

$$\mathrm{d}\Sigma = \sqrt{h}\,\mathrm{d}^3x = \Omega^3(t,\mathrm{x})\sqrt{g_E^{(3)}}\,\mathrm{d}^3x = \Omega^3(t,\mathrm{x})\,\mathrm{d}\Sigma_E$$

   - Riemannian volumes of the domains do not exhibit direct correspondence.
   - Even for conformally invariant quantities, $\mathcal{O}\sqrt{h} = \mathcal{O}_E\sqrt{g_E^{(3)}}$ averages would not be invariant.

3. Volume integral: evaluated along a different path (conformal transformation depends upon the spacetime points).

4. Are the domains in Einstein frame well-defined?
   - Usually domains are defined w.r.t. rest frame of matter content (comoving with matter)
   - In Einstein frame there is a fifth force (congruences accelerate).

Hint: Calculations seem more straightforward and technically less challenging in the Jordan frame where matter and geometry are minimally coupled.