HIGGS INFLATION & VACUUM STABILITY

Javier Rubio

based on Phys. Rev. D 92, 083512 F. Bezrukov, J.R., M.Shaposhnikov

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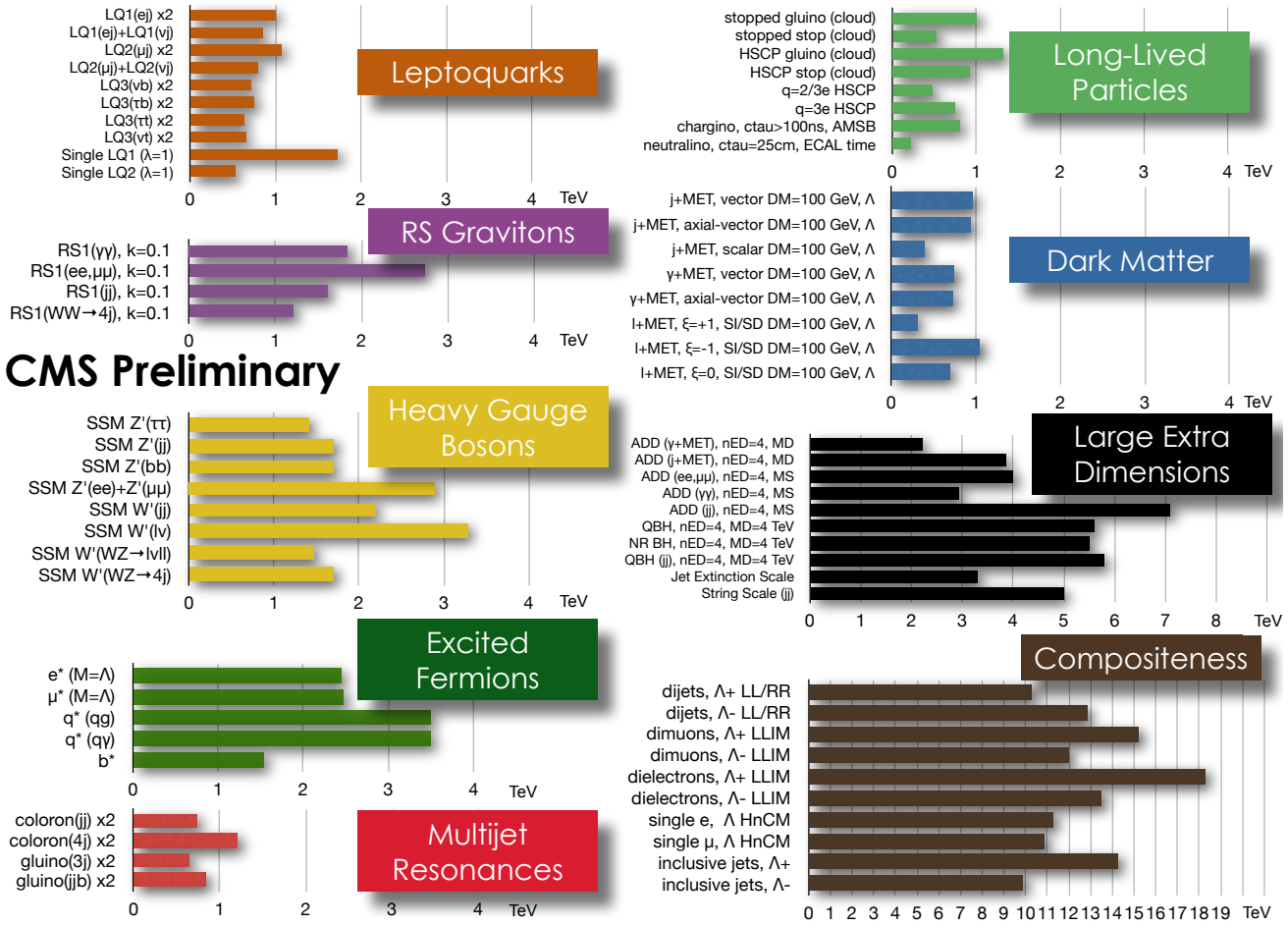
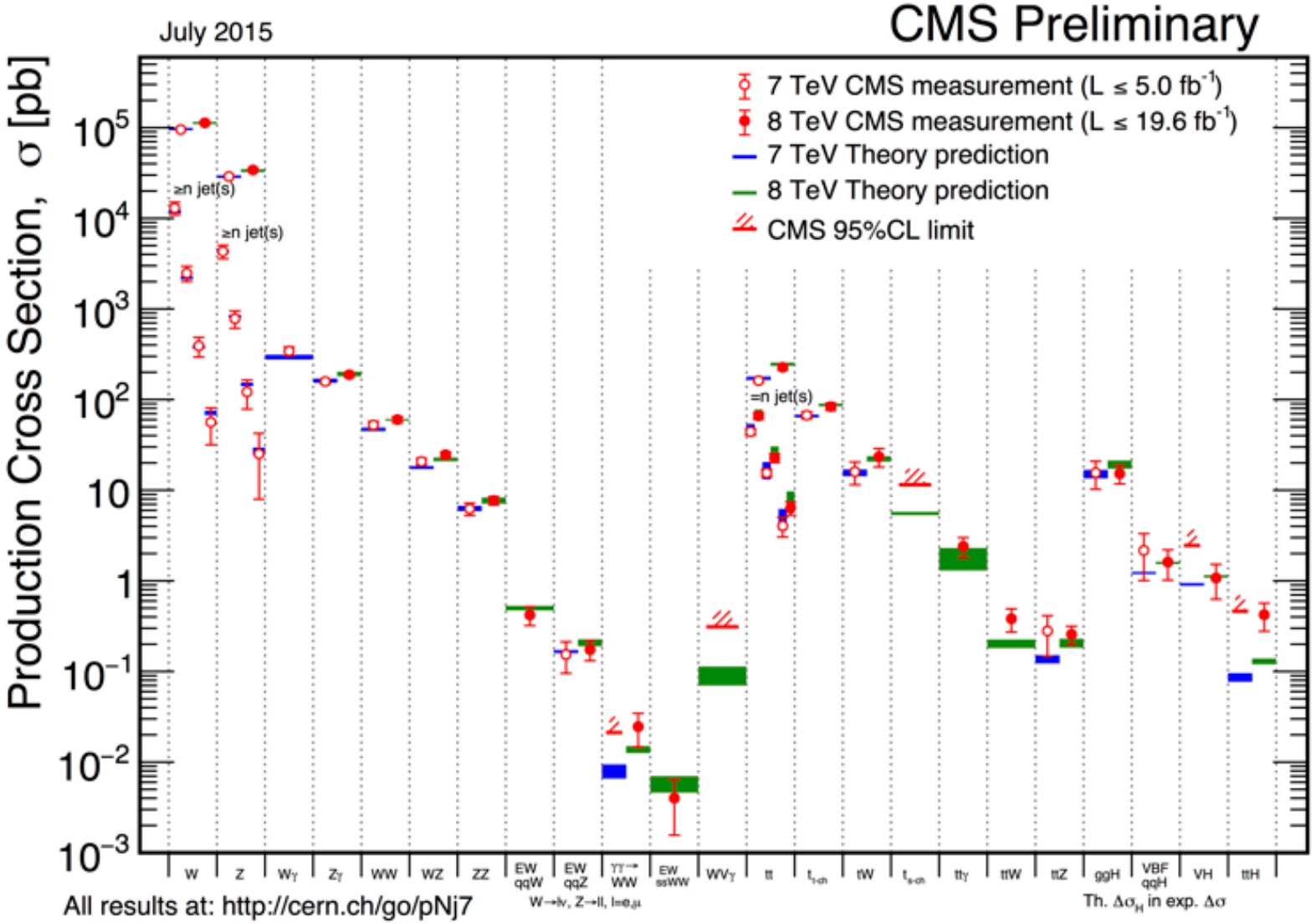
Physicists' Nightmare Scenario: The Higgs and Nothing Else

Many fear the LHC will cough up only the one particle they've sought for decades. Some would rather see nothing new at all



CMS Exotica Physics Group Summary – Moriond, 2015

The apotheosis of the SM



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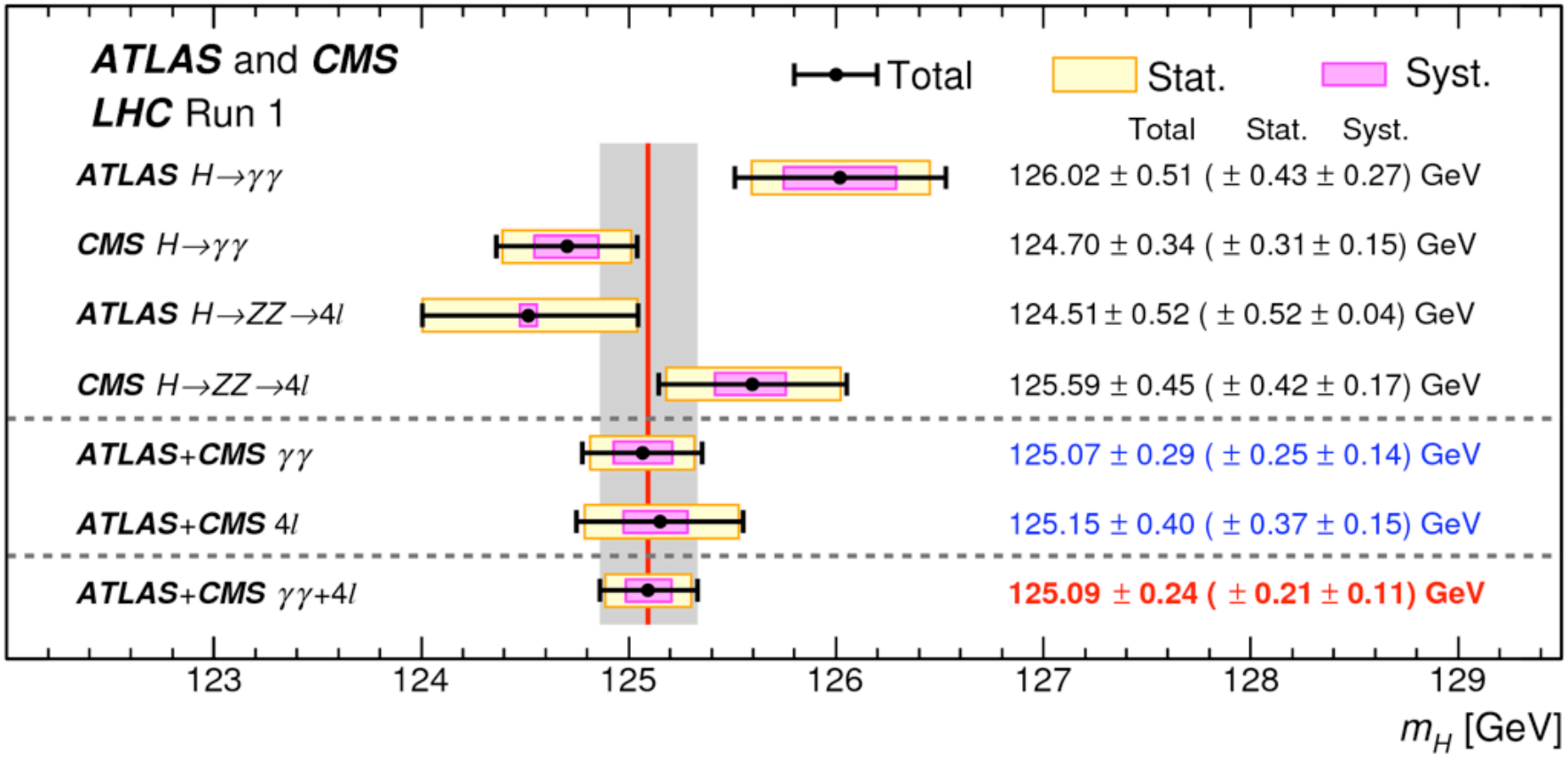


$$m_H = 125.09 \pm 0.21(\text{stat}) \pm 0.11\,(\text{syst})\ \text{GeV}$$

"low" or "high", depending on your taste...but certainly particular...

Higgs mass predictions

arXiv:0708.3344)

The oldest entry is: $m_H < 1020$ GeV by Dicus and Mathur (1973).

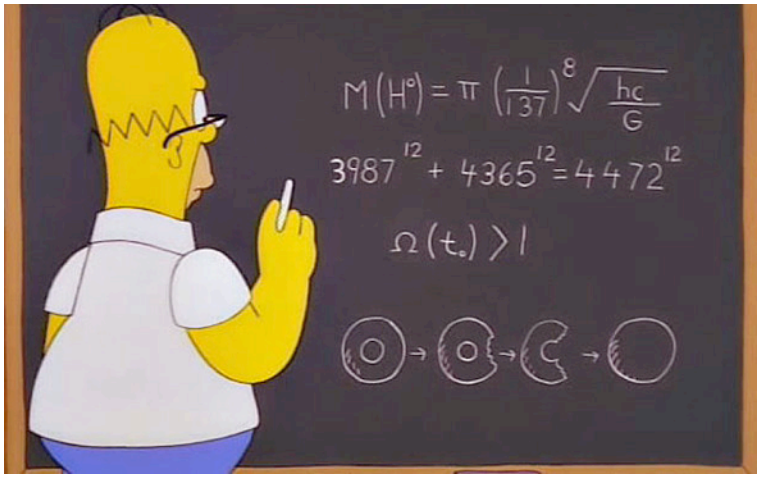
The most precise prediction is: $m_H = 161.8033989$ GeV by El Naschie (2005).

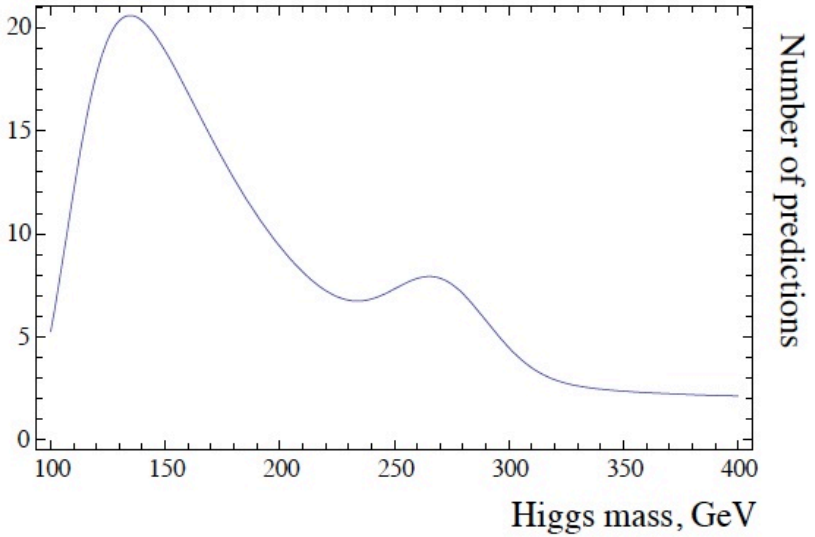
The highest number of predictions by a single co-author, Gogoladze, is 12.

Three intervals are still free of Higgs-mass predictions: 600–739 GeV, 781–1800 GeV and 2000–10^{18} GeV.

Homer Simpson's prediction 775 GeV (1998)

The highest prediction is: $m_H = 10^{18}$ GeV by Batakis and Kehagias (1991).





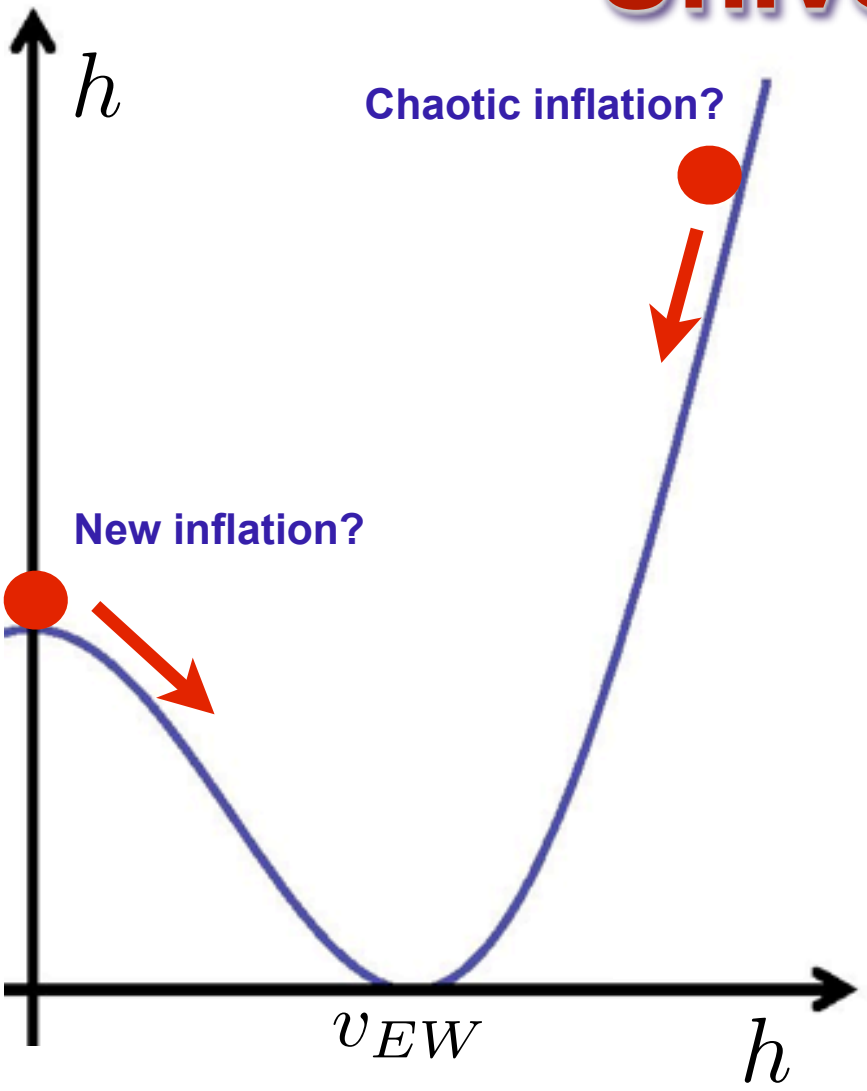
Higgs mass predictions

Hundreds of "very natural" models predict the "very natural" value

$$m_H = 125.09 \pm 0.21(\text{stat}) \pm 0.11\,(\text{syst})\ \text{GeV}$$

and all of them agree among themselves and with experiment

Can we "Higgsflate" the Universe?



New inflation? Linde '82

Too steep to inflate

Chaotic inflation? Linde '83

Too large primordial perturbations

$$\mathcal{P}_{\mathcal{R}} \sim 10^3 \lambda$$

Higgs inflation at tree level

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M_P^2 + \xi_h h^2}{2} R - \frac{1}{2}(\partial h)^2 - \frac{\lambda}{4}(h^2 - v_{EW}^2)^2$$

Scale invariance at $h \gg M_P/\sqrt{\xi_h}$

Moving to the Einstein frame

$\tilde{g}_{\mu\nu} = \Omega^2(h) g_{\mu\nu}$

$$\Omega^2 = 1 + \frac{\xi_h h^2}{M_P^2} \qquad \frac{\tilde{\mathcal{L}}}{\sqrt{-\tilde{g}}} = \frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - U(\phi)$$

All the non-linearities moved to the scalar sector

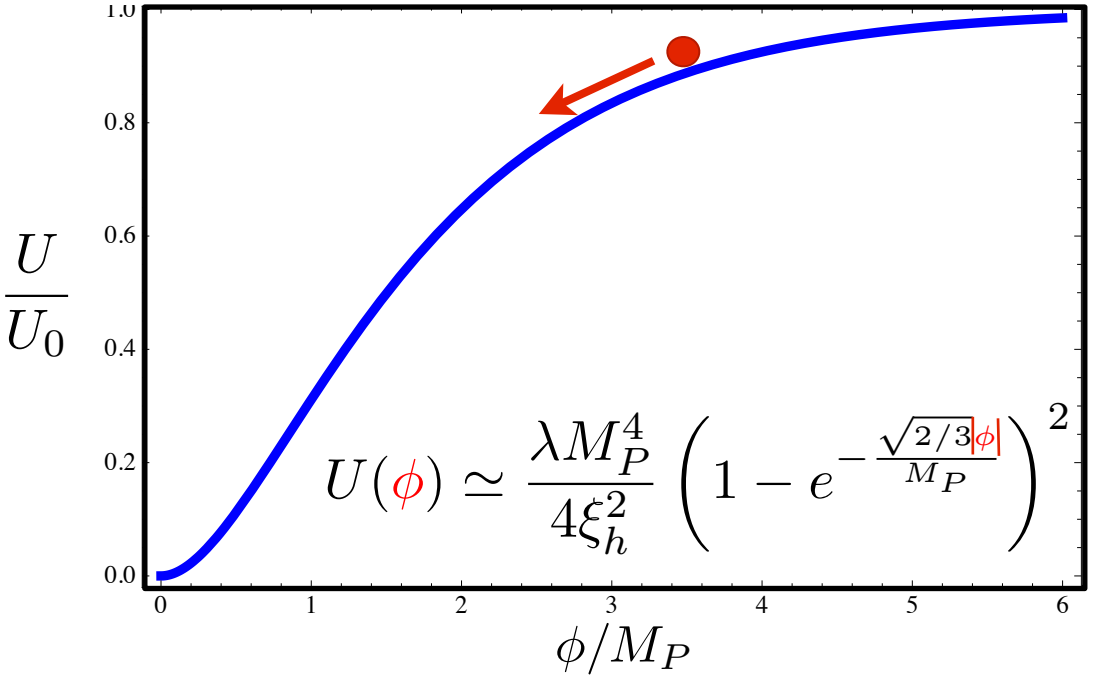
$$U(\phi) = \frac{\lambda}{4}(\phi^2 - v_{EW}^2)^2 \qquad \text{for} \quad h < M_P/\xi_h$$

$$U(\phi) = \frac{\lambda M_P^4}{4\xi_h^2}\left(1 - e^{-\frac{\sqrt{2/3}|\phi|}{M_P}}(1 + \frac{\xi_h v_{EW}^2}{M_P^2})\right)^2 \qquad \text{for} \quad h > M_P/\xi_h$$

F. L. Bezrukov and M. Shaposhnikov Phys. Lett. B659 (2008) 703–706

A sufficiently flat potential

Scale invariance JF --> shift symmetry EF



$$\frac{\delta T}{T} \sim 10^{-5}$$

$$\downarrow$$

$$\frac{\lambda}{\xi_h^2} \sim 10^{-11}$$

Only λ/ξ_h^2 is important

The primordial spectra

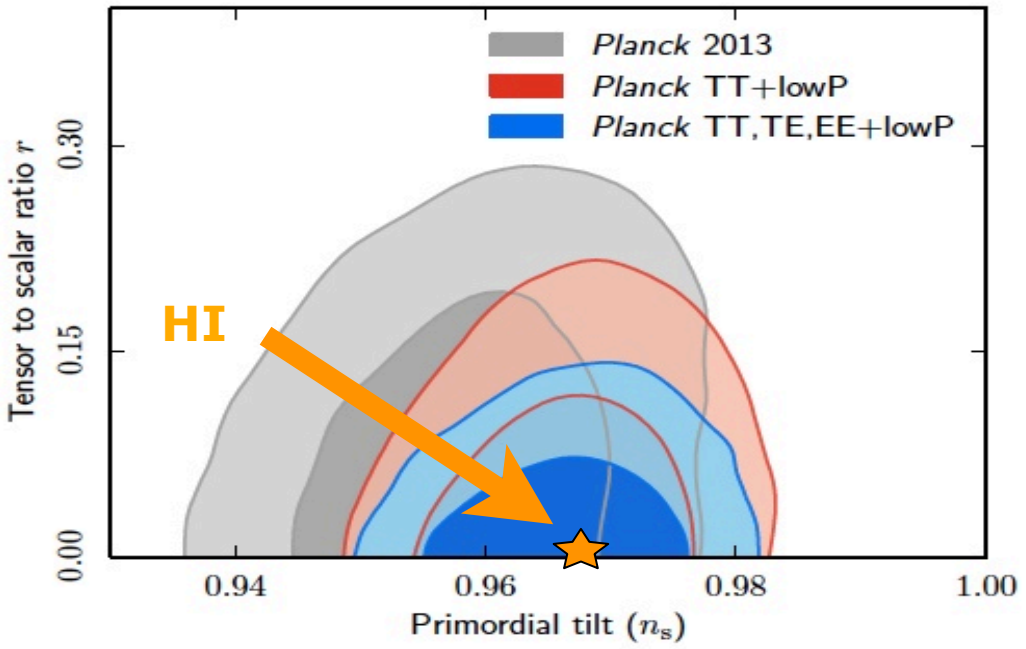
Scalar pert. $\mathcal{P}_S(k) = \mathcal{A}_s \left(\frac{k}{k^*}\right)^{n_s - 1 + \frac{1}{2}\alpha_s \ln(k/k^*) + \frac{1}{6}\beta_s (\ln(k/k^*))^2 + \ldots}$

Tensor pert. $\mathcal{P}_T(k) = \mathcal{A}_t \left(\frac{k}{k^*}\right)^{n_t + \frac{1}{2}\alpha_t \ln(k/k^*) + \ldots}$

$$n_{\rm s} \equiv 1 + \frac{{\rm d}\ln \mathcal{P}_{\rm s}}{{\rm d}\ln k} \qquad r(k) \equiv \frac{\mathcal{P}_{\rm T}(k)}{\mathcal{P}_{\rm s}(k)} \qquad \alpha_{\rm s} \equiv \frac{{\rm d}\, n_{\rm s}}{{\rm d}\ln k} \qquad \beta_s \equiv \frac{d^2 n_s}{d\ln k^2}$$

$$n_s \simeq 1 - \frac{2}{N} \simeq 0.97$$

$$r \simeq \frac{12}{N^2} \simeq 0.0033$$

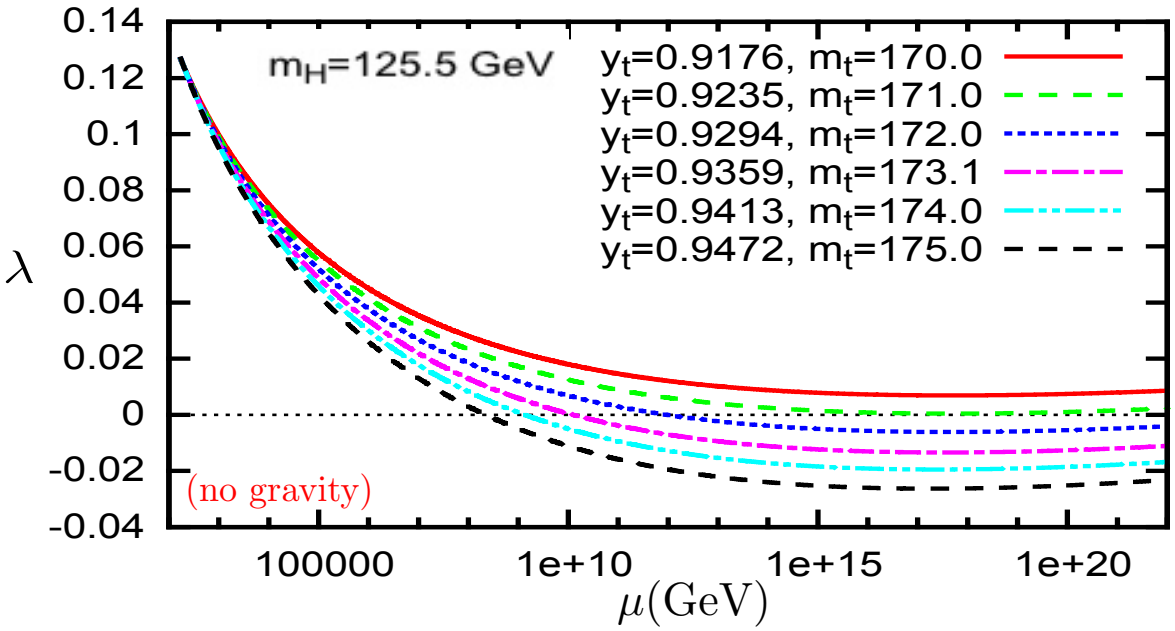


On the edge of stability

- ***SM remains perturbative all the way up till the inflat./Planck scale***
- ***Does the SM vacuum remains stable till those scales?***

$$\mu \frac{d\lambda(\mu)}{d\log(\mu)} = +\#\lambda^2 + \ldots - \#y_t^4$$

Non trivial interplay between Higgs self-coupling and top quark Yukawa coupling

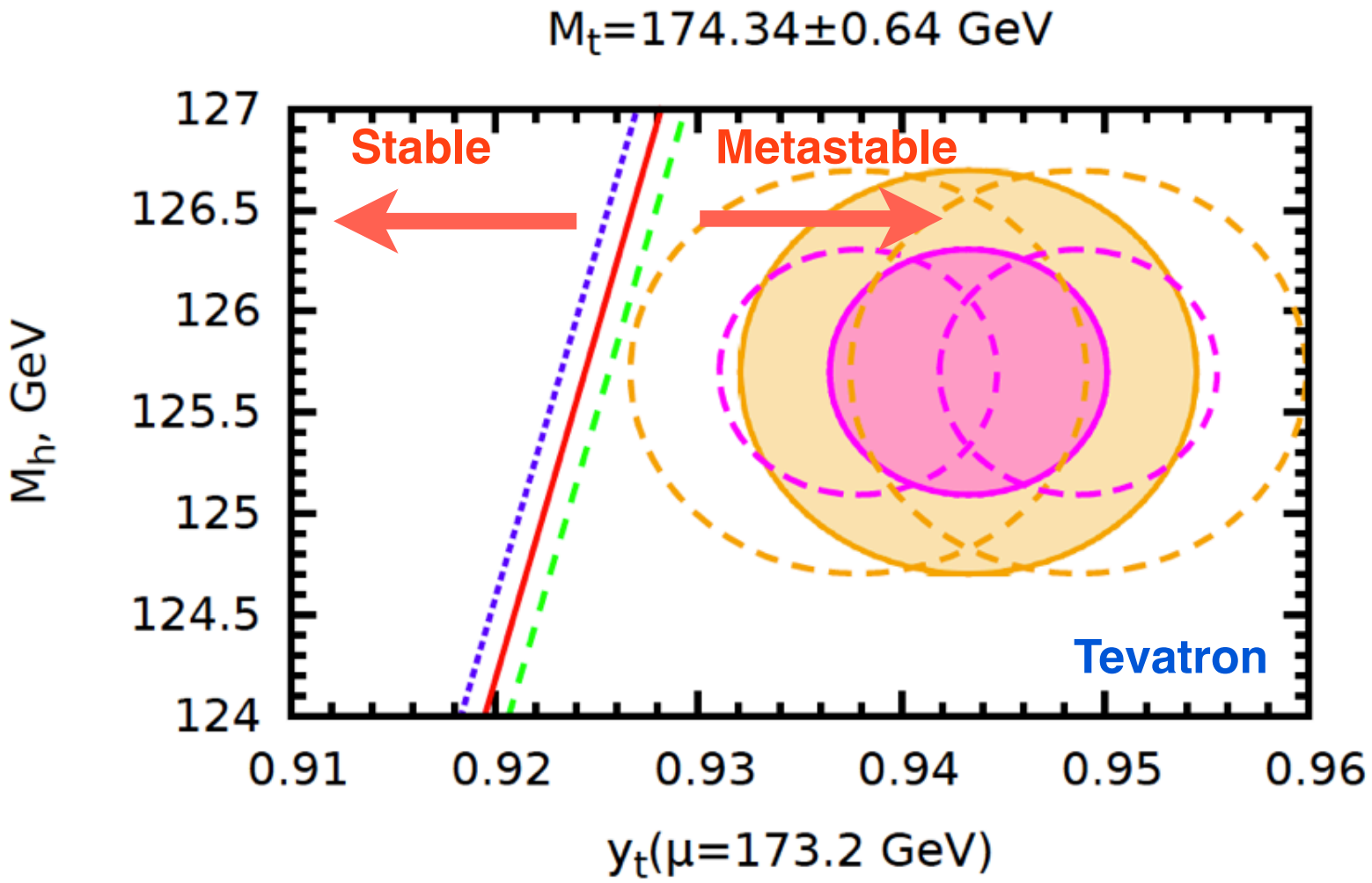


Is there a reason for that?

M. Lindner, M. Sher, and H. W. Zaglauer, Phys.Lett., B228, 139 (1989), C. Froggatt and H. B. Nielsen, Phys.Lett., B368,96 (1996), M. Shaposhnikov and C. Wetterich, Phys.Lett., B683, 196 (2010), M. Veltman, Acta Phys.Polon., B12, 437 (1981), B683, 196 (2010), M. Holthausen, K.S.Lim, M.Lindner and references therein....

Top quark & vac. instability

$$y_t^{\rm crit} = 0.9223 + 0.00118\left(\frac{\alpha_s - 0.1184}{0.0007}\right) + 0.00085\left(\frac{M_h - 125.03}{0.3}\right)$$



See F. Bezrukov, M. Shaposhnikov J.Exp.Theor.Phys. 120 (2015) 335-343 and references therein

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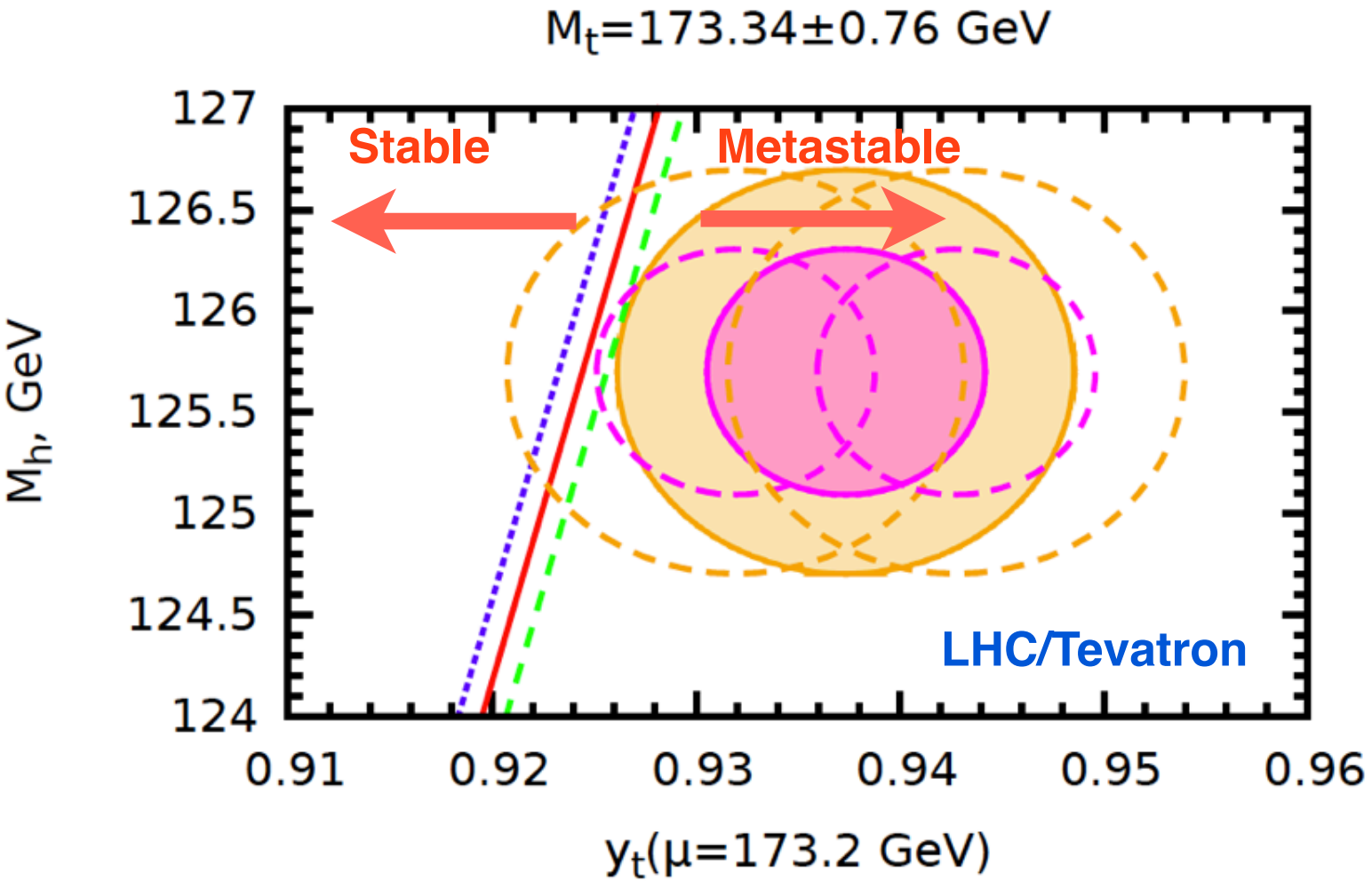

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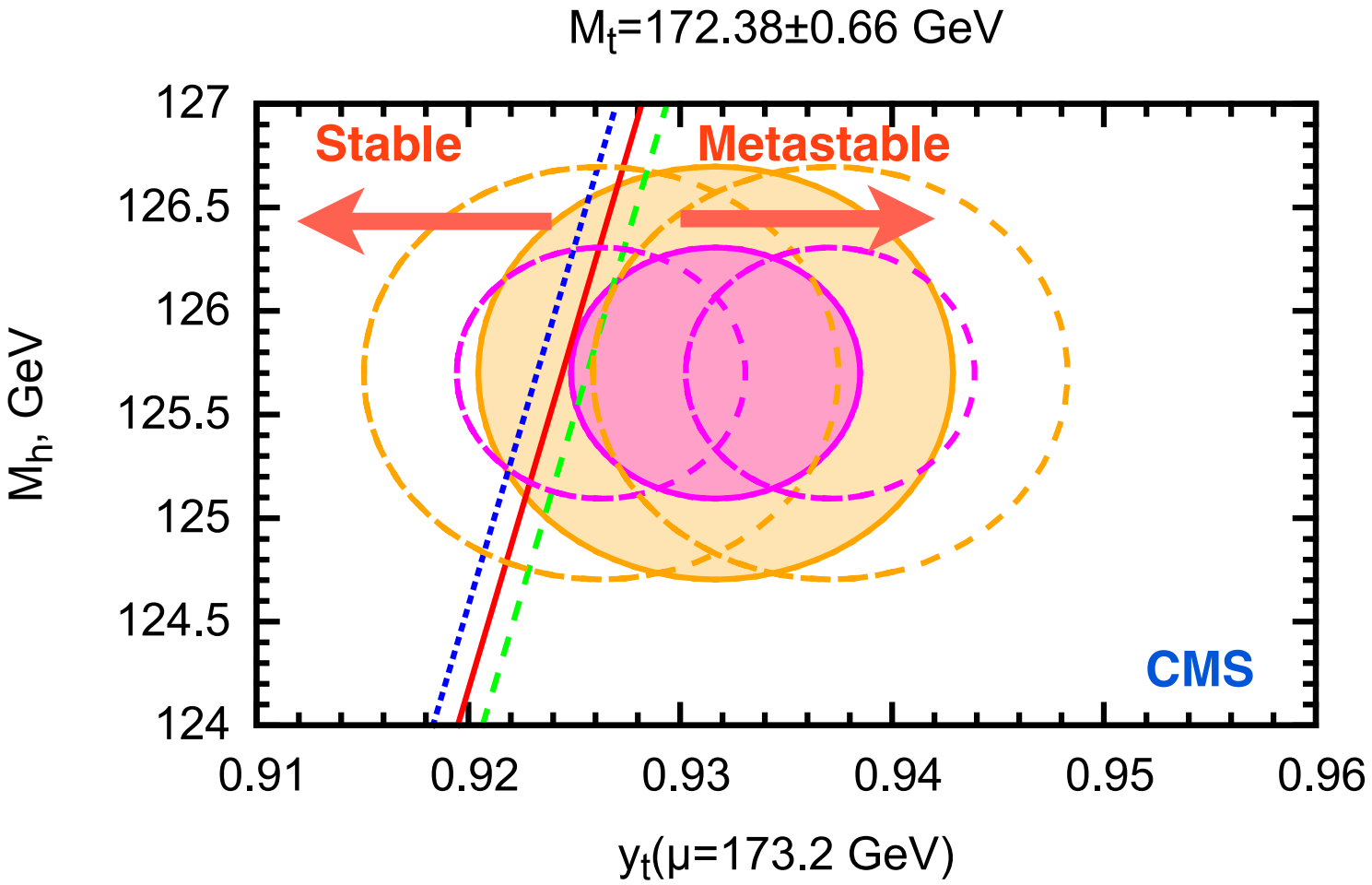

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Imagine that the top and Higgs masses are measured with a precision enough to conclude that our vacuum is not completely stable ...

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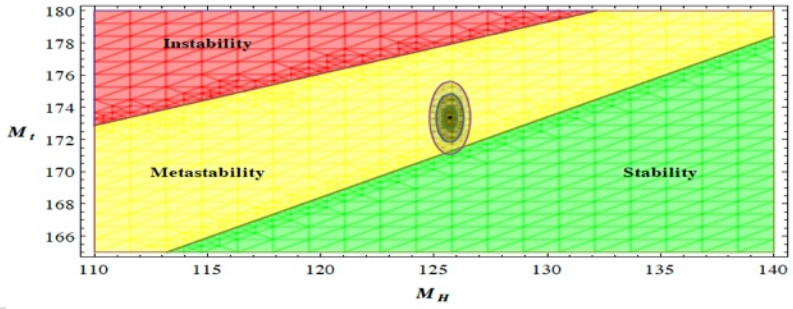
Is Higgs inflation dead?



Before the LHC Higgs discovery, it it was very tempting to identify the inflaton as the Higgs boson of the SM [...]. However, the recent LHC and Tevatron measurements [...] indicate that the Higgs potential. turns negative at a scale below the typical cosmic inflation scale. [...] new physics is indispensable to reach a stable electroweak vacuum after inflation "

by Anonymous Informer I

"If m_h and m_t are close to the measured central value, Higgs inflation is not possible and V_{eff} becomes negative much before M_P"

by Anonymous Informer II

HI is non-renormalizable

$$U(\phi) \simeq \frac{\lambda M_P^4}{4\xi_h^2}\left(1 - e^{-\frac{\sqrt{2/3}\phi}{M_P}}\right)^2$$

Non-polynomial

Quantum corrections should be introduced by interpreting the theory as an EFT in which a particular set of higher dim. operators are included but.....

Which set of operators?

The naive approach

In the absence of new physics the cutoff scale might be as large as the Planck scale, where gravitational interactions become important for sure

Consider all kind of Planck scale suppressed operators ...

...added in the Einstein frame

$$V \simeq \frac{\lambda M_P^4}{4\xi_h^2} \qquad \Delta V_n = c_n \frac{\chi^n}{M_P^{n-4}}$$

During inflation $\chi \sim M_P$

$$\frac{\Delta V}{V} \sim \frac{c_n \xi_h^2}{\lambda} \left(\frac{\chi}{M_P}\right)^n$$

Infamous cosmological hierarchy problem

...added in the Jordan frame

$$V \simeq \frac{\lambda}{4} h^4 \qquad \Delta V_n = c_n \frac{h^n}{M_P^{n-4}}$$

During inflation $h \sim M_P/\xi_h$

$$\frac{\Delta V}{V} \sim \frac{c_n}{\lambda} \left(\frac{h}{M_P}\right)^{n-4} \sim \frac{c_n}{\lambda \xi^{\frac{n-4}{2}}}$$

But is this self-consistent?

The self-consistent approach

A self-consistent approach is to define the cutoff from the theory itself by considering all possible reactions between the SM constituents....

.... and add all kind of operators suppressed by these cutoffs...

1. Compute the quadratic lagrangian $\phi = \bar{\phi} + \delta\phi \quad g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

$$\mathcal{L}^{(2)} = -\frac{M_P^2 + \xi\bar{\phi}^2}{8}\left(h^{\mu\nu}\Box h_{\mu\nu} + 2\partial_\nu h^{\mu\nu}\partial^\rho h_{\mu\rho} - 2\partial_\nu h^{\mu\nu}\partial_\mu h - h\Box h\right)$$
$$+\frac{1}{2}(\partial_\mu\delta\phi)^2 + \xi\bar{\phi}\left(\Box h - \partial_\lambda\partial_\rho h^{\lambda\rho}\right)\delta\phi\,,$$

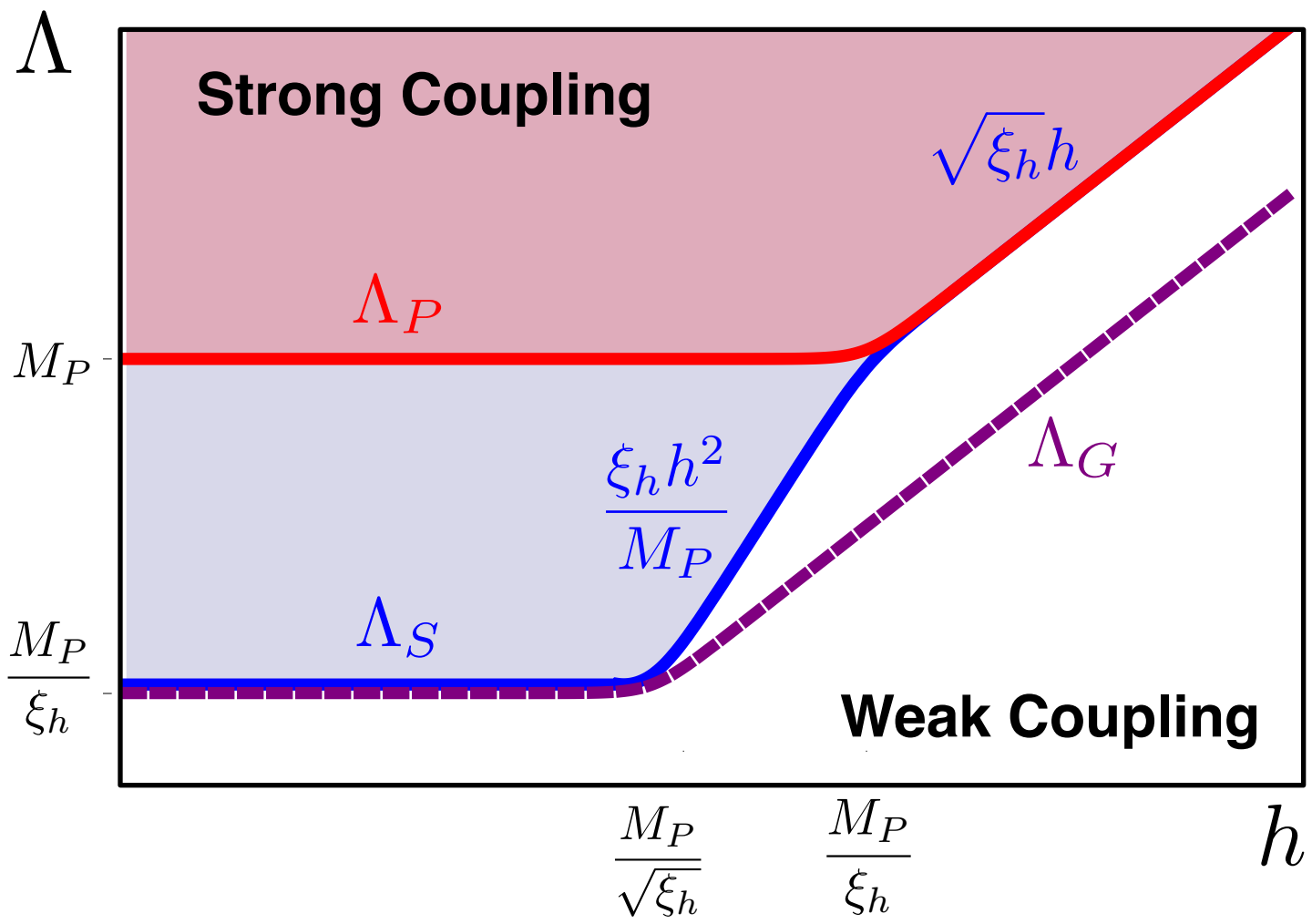
2. Get rid of the mixings in the quadratic action

$$\delta\phi = \sqrt{\frac{M_P^2 + \xi\bar{\phi}^2}{M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2}}\,\delta\hat{\phi}\,,$$

$$h_{\mu\nu} = \frac{1}{\sqrt{M_P^2 + \xi\bar{\phi}^2}}\,\hat{h}_{\mu\nu} - \frac{2\xi\bar{\phi}}{\sqrt{(M_P^2 + \xi\bar{\phi}^2)(M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2)}}\,\bar{g}_{\mu\nu}\,\delta\hat{\phi}$$

3. Read out the cutoff from higher order operators

$$\frac{\xi\sqrt{M_P^2 + \xi\bar{\phi}^2}}{M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2}(\delta\hat{\phi})^2\Box\hat{h}$$



A consistent EFT : Cutoffs are parametrically larger than all the energy scales involved in the history of the Universe

The minimal approach

Select a particular set of UV completions within the previous set.

Add only the higher dimensional operators generated by radiative corrections (i.e. those needed to make theory finite at every order in PT).

Fixed by the divergencies

Arbitrary

$$\delta\mathcal{L}_{ct} = \left(\frac{A_n}{\epsilon} + B_n\right)\mathcal{O}_n$$

(Partially) controllable link
between the low and high energy parameters of the model

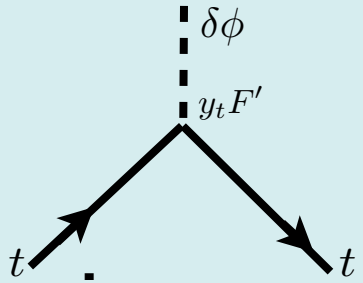
F. Bezrukov, A. Magnin, M. Shaposhnikov, and S. Sibiryakov JHEP 1101 (2011) 016,
See also C.P. Burguess, S.P. Patil, M.Trott JHEP 1406 (2014) 010

Top quark in Higgs background

$$\mathcal{L}_F = \frac{y_f}{\sqrt{2}} h\bar{\psi}\psi \quad \longrightarrow \quad \tilde{\mathcal{L}}_F = \frac{y_f}{\sqrt{2}}\frac{h}{\Omega}\bar{\psi}\psi \equiv \frac{y_f}{\sqrt{2}} F(\phi)\bar{\psi}\psi$$

$$\tilde{\mathcal{L}}_t(\phi+\delta\phi) = \frac{y_t}{\sqrt{2}} F(\phi+\delta\phi)\bar{\psi}_t\psi_t$$

$$= \frac{y_t}{\sqrt{2}} F(\phi)\bar{\psi}_t\psi_t + \frac{y_t}{\sqrt{2}}\frac{dF(\phi)}{d\phi}\delta\phi\bar{\psi}_t\psi_t + \dots$$



At low energies

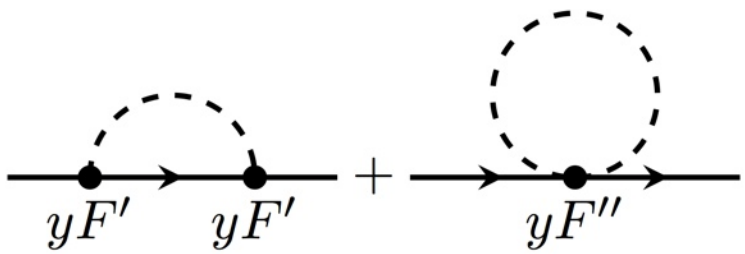
$$F = \phi \quad F'(0) = 1$$

At high energies

$$F = \frac{M_P}{\sqrt{\xi_h}}\left(1 - e^{-\alpha\kappa|\phi|}\right)^{1/2} \quad F'(\infty) = 0$$

Consider the propagation of the top quark

Add counterterms to cancel divergencies

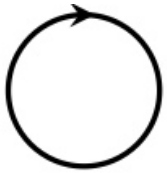


$$\delta\mathcal{L}_{\rm ct} \sim \left(\#\frac{y_t^3}{\bar{\epsilon}} + \delta y_{t1}\right) F'^2 F\bar{\psi}\psi + \left(\#\frac{y_t\lambda}{\bar{\epsilon}} + \delta y_{t2}\right) F''(F^4)''\bar{\psi}\psi$$

$\phi\psi\psi$ at small ϕ

One-loop effective potential

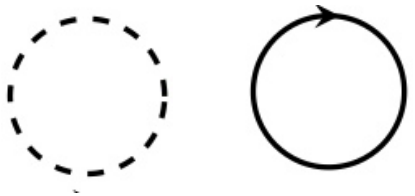
$$= \frac{1}{2}\,\mathrm{Tr}\ln\left[\Box - \left(\frac{\lambda}{4}(F^4)''\right)^2\right]$$



$$= -\,\mathrm{Tr}\ln\left[i\partial\!\!\!/ + y_t F\right]$$

At low energies	At high energies
$F = \phi \quad F'(0) = 1$	$F = const. \quad F'(\phi_0) = 0$

Add counterterms to cancel divergencies



$$\delta\mathcal{L}_{\rm ct} = \left(-\frac{2}{\bar{\epsilon}}\frac{9\lambda^2}{64\pi^2} + \delta\lambda_1\right)\left(F'^2 + \frac{1}{3}F''F^2\right)^2 F^4 + \left(\frac{2}{\bar{\epsilon}}\frac{y_t^4}{64\pi^2} - \delta\lambda_2\right)F^4$$

new

$\sim$ tree level

RG equations: truncation

The new finite parts should be promoted to new independent coupling constants with their own RG equations ...

$$\frac{d\lambda}{d\log\mu} = \beta_\lambda(\lambda, \lambda_1, \ldots) \qquad \frac{d\lambda_1}{d\log\mu} = \beta_{\lambda_1}(\lambda, \lambda_1, \ldots)$$

$$\frac{dy_t}{d\log\mu} = \beta_{y_t}(y_t, y_{t1}, \ldots) \qquad \frac{dy_{t1}}{d\log\mu} = \beta_{y_{t1}}(y_t, y_{t1}, \ldots)$$

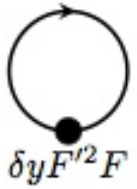
$$\vdots \qquad\qquad \vdots$$

but since we are dealing with a non-renormalizable theory, the set of RG equations is not closed...

The one-loop diagrams associated to the new counterterms are given by

$$\sim \delta\lambda[(F'^2 + \frac{1}{3}F''F)^2F^4]''\lambda(F^4)''$$

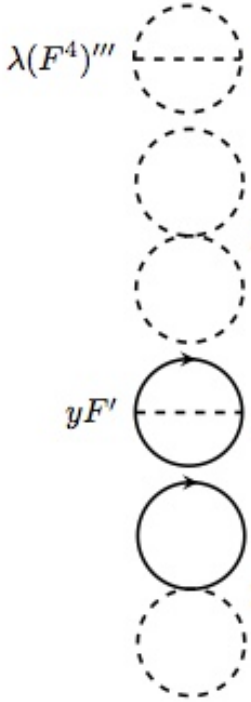
$$\delta\lambda[(F'^2 + \tfrac{1}{3}F''F)^2F^4]''$$



$$\sim \delta y F'^2 F y^3 F^3$$

$$\delta y F'^2 F$$

The two-loop contributions generated by the original Lagrangian



$$\lambda(F^4)''' \quad \lambda(F^4)''' \sim (\lambda(F^4)''')^2\lambda(F^4)'',$$

$$\lambda(F^4)'''' \sim \lambda(F^4)''''(\lambda(F^4)'')^2$$

$$yF' \quad yF' \sim (yF')^2(yF)^4,$$

$$yF'' \sim yF''(yF)^3\lambda(F^4)'',$$

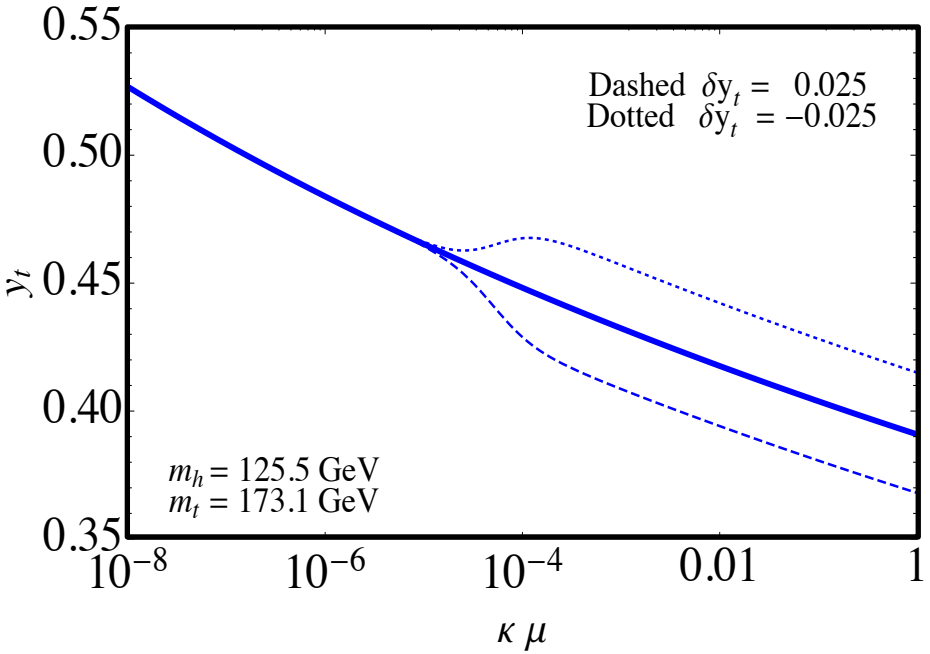
In order to have a good perturbative expansion, the finite parts must be of the same order (in power counting) than the loops producing them.

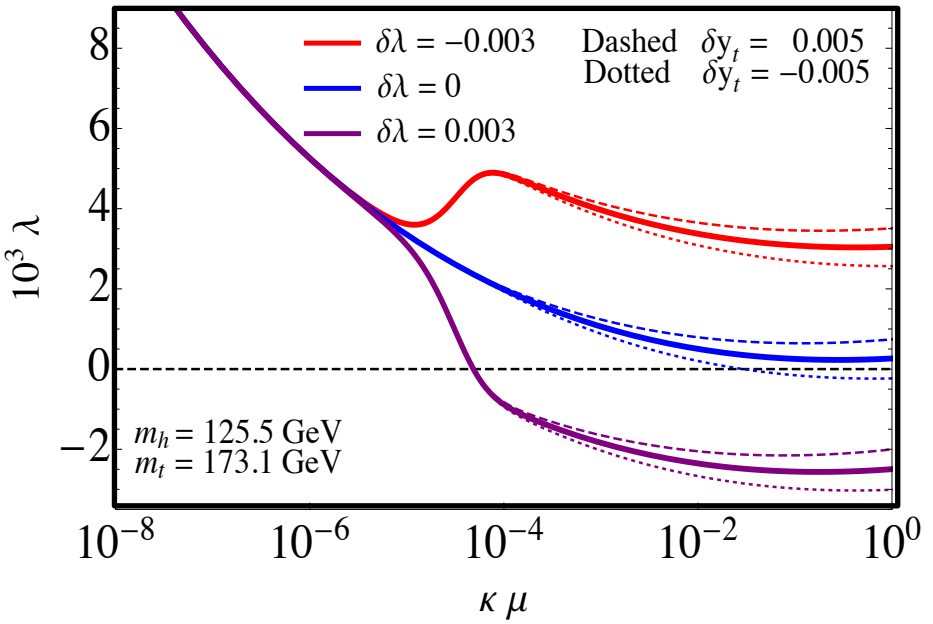
$$\delta\lambda \sim \mathcal{O}(\lambda^2, y^4) \qquad \delta y \sim \mathcal{O}(y^3, y\lambda) \qquad \lambda \sim \mathcal{O}(y^2)$$

Eff. behaviour of coupling constants

Neglecting the running of $\delta\lambda_1$ and $\delta_{y_{t1}}$
between $\mu \sim M_P/\xi_h$ and $M_P/\sqrt{\xi_h}$

$$y_t(\mu) \longrightarrow y_t(\mu) + \delta y_t \left[F'^2 - 1\right] + \ldots \qquad \lambda(\mu) \longrightarrow \lambda(\mu) + \delta\lambda \left[\left(F'^2 + \frac{1}{3}F''F\right)^2 - 1\right]$$





The shape of the asymptotics is maintained

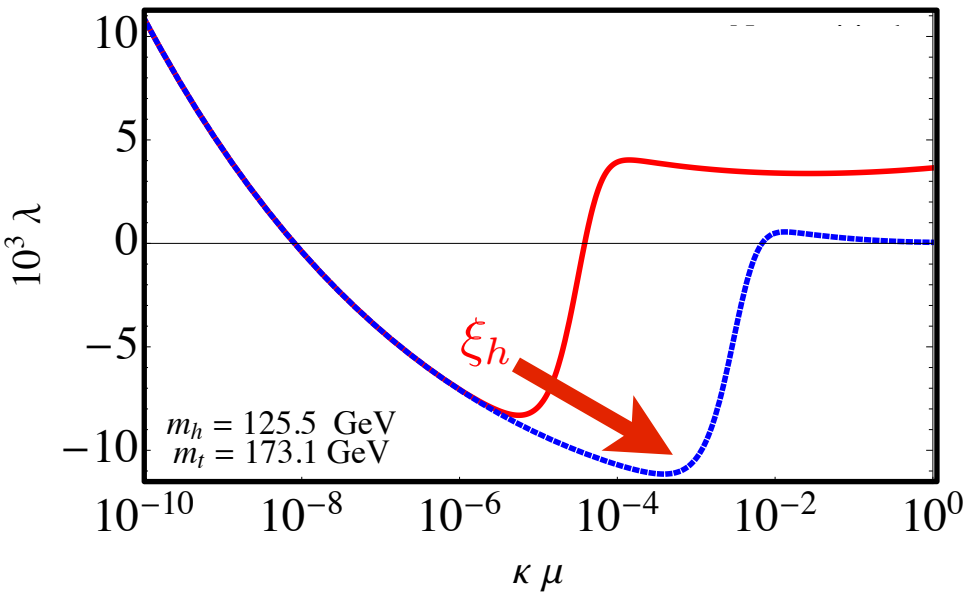
Restoring Higgs inflation

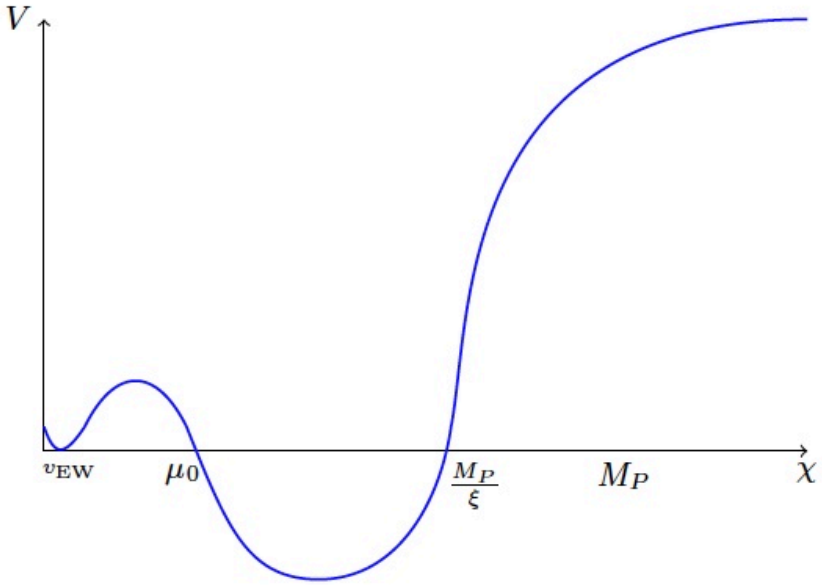
$$\delta\lambda \ll \lambda(M_P/\xi)$$
$$\delta y_t \ll y_t(M_P/\xi)$$

- **Higgs inflation requires absolute stability of the SM vacuum**

~~$\delta\lambda \ll \lambda(M_P/\xi)$~~
~~$\delta y_t \ll y_t(M_P/\xi)$~~

- **Higgs inflation can be possible even in the case of a metastable vacuum.**

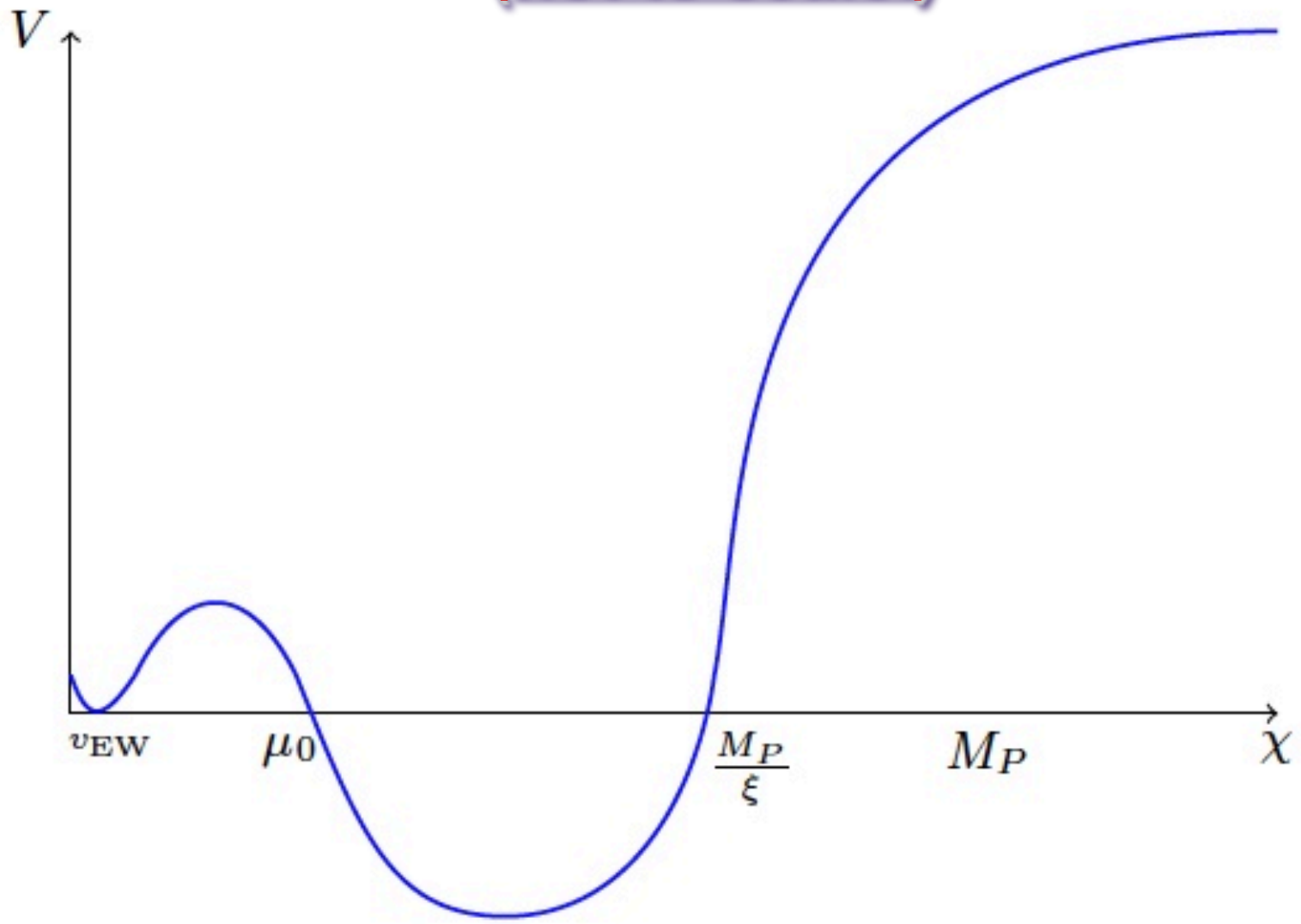




But how to avoid finishing in the wrong vacuum ???

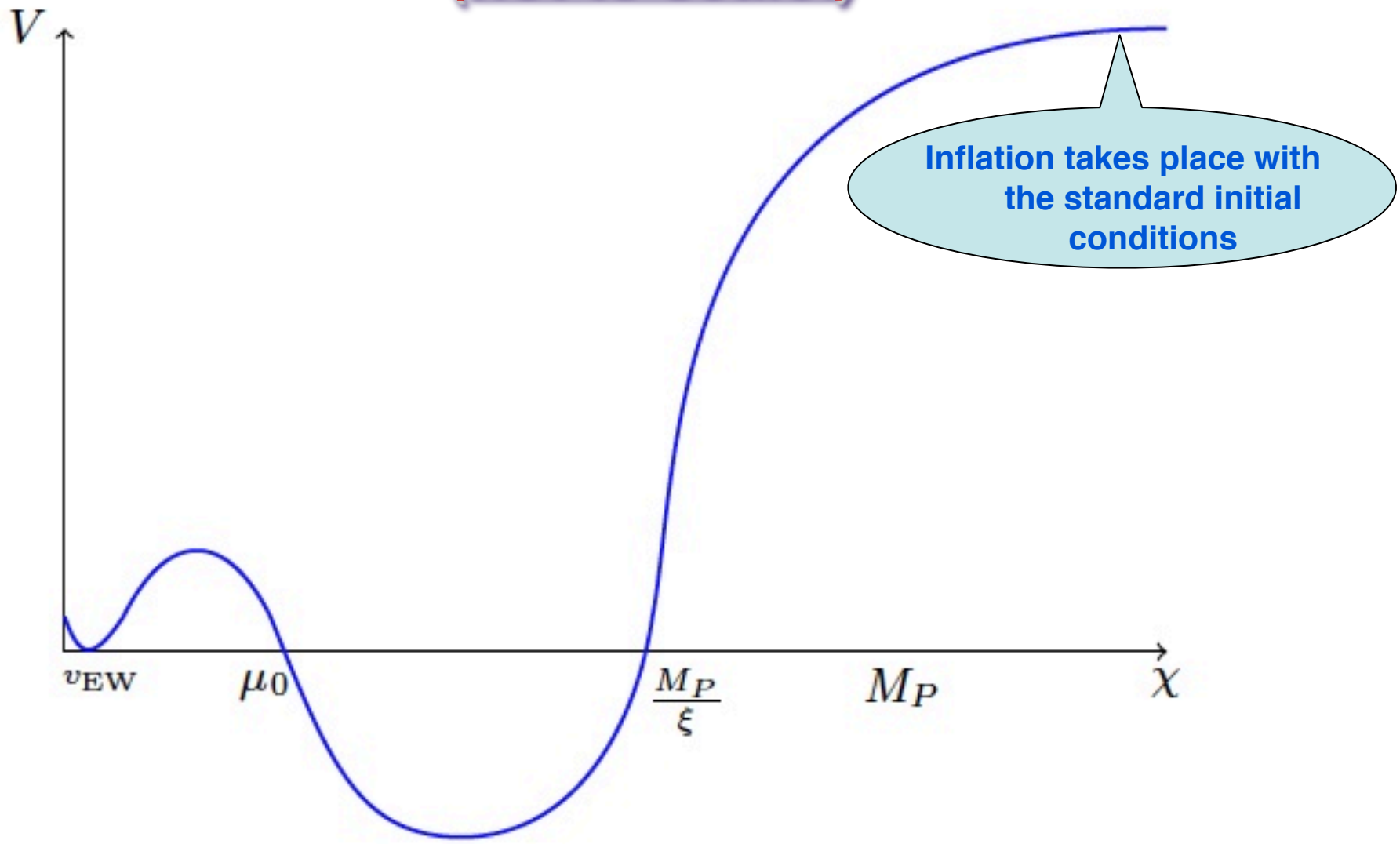
Sketch of effective potential

(not to scale!)



Sketch of effective potential

(not to scale!)



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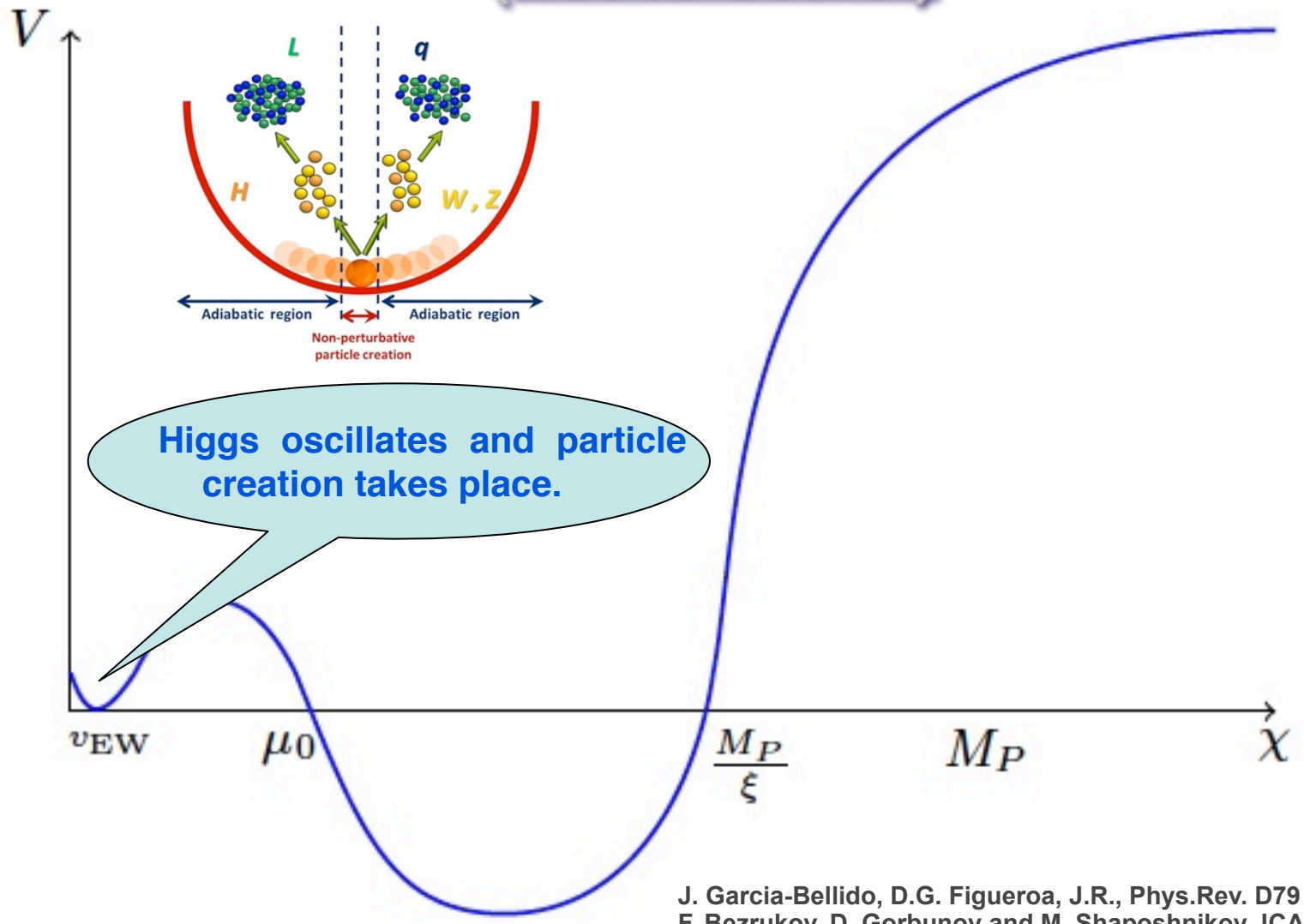
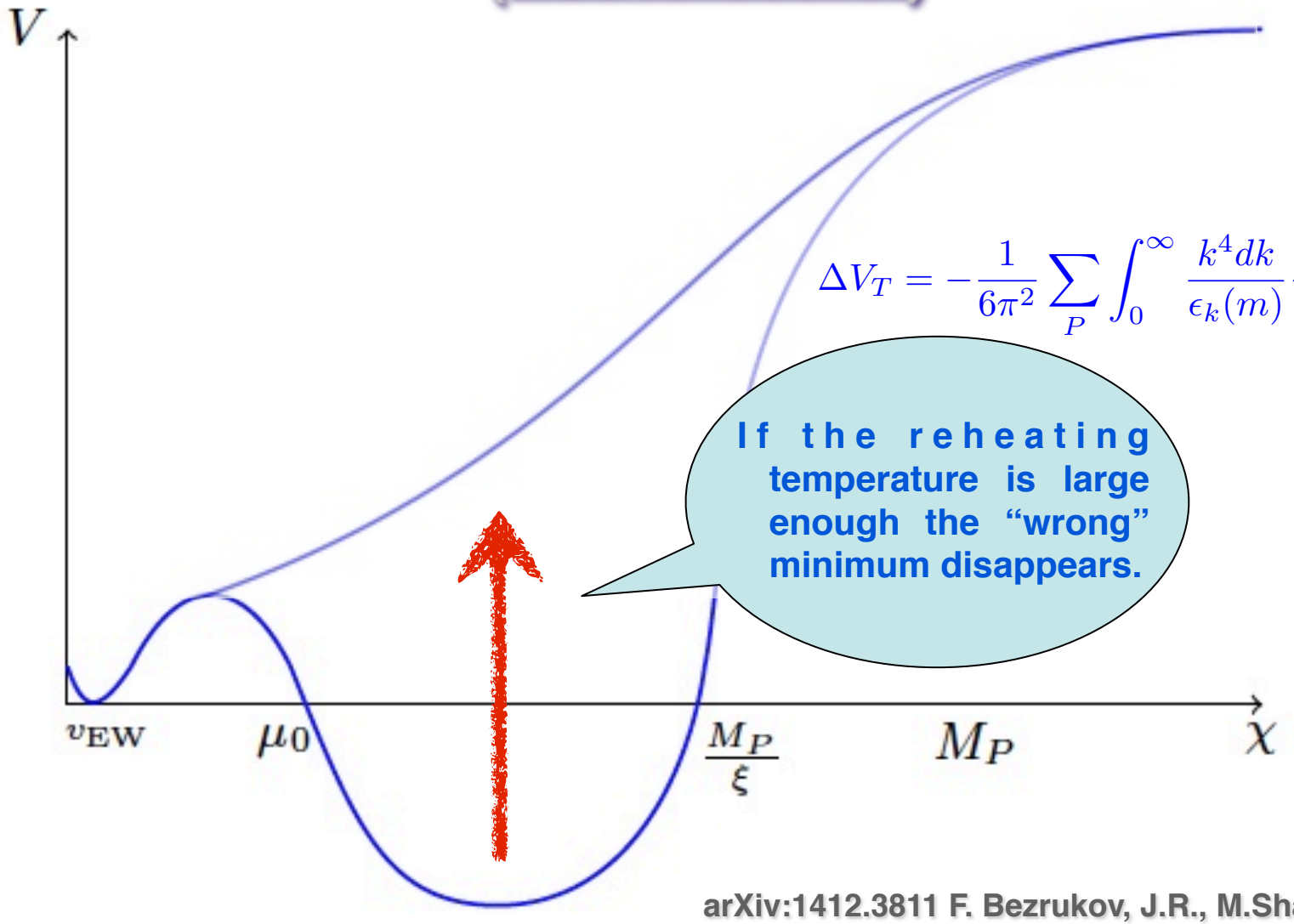


J. Garcia-Bellido, D.G. Figueroa, J.R., Phys.Rev. D79 (2009) 063531
F. Bezrukov, D. Gorbunov and M. Shaposhnikov JCAP 0906 (2009) 029
J. Repond, J. R, M. Shaposhnkov, in preparation

Sketch of effective potential

(not to scale!)



arXiv:1412.3811 F. Bezrukov, J.R., M.Shaposhnikov

Sketch of effective potential

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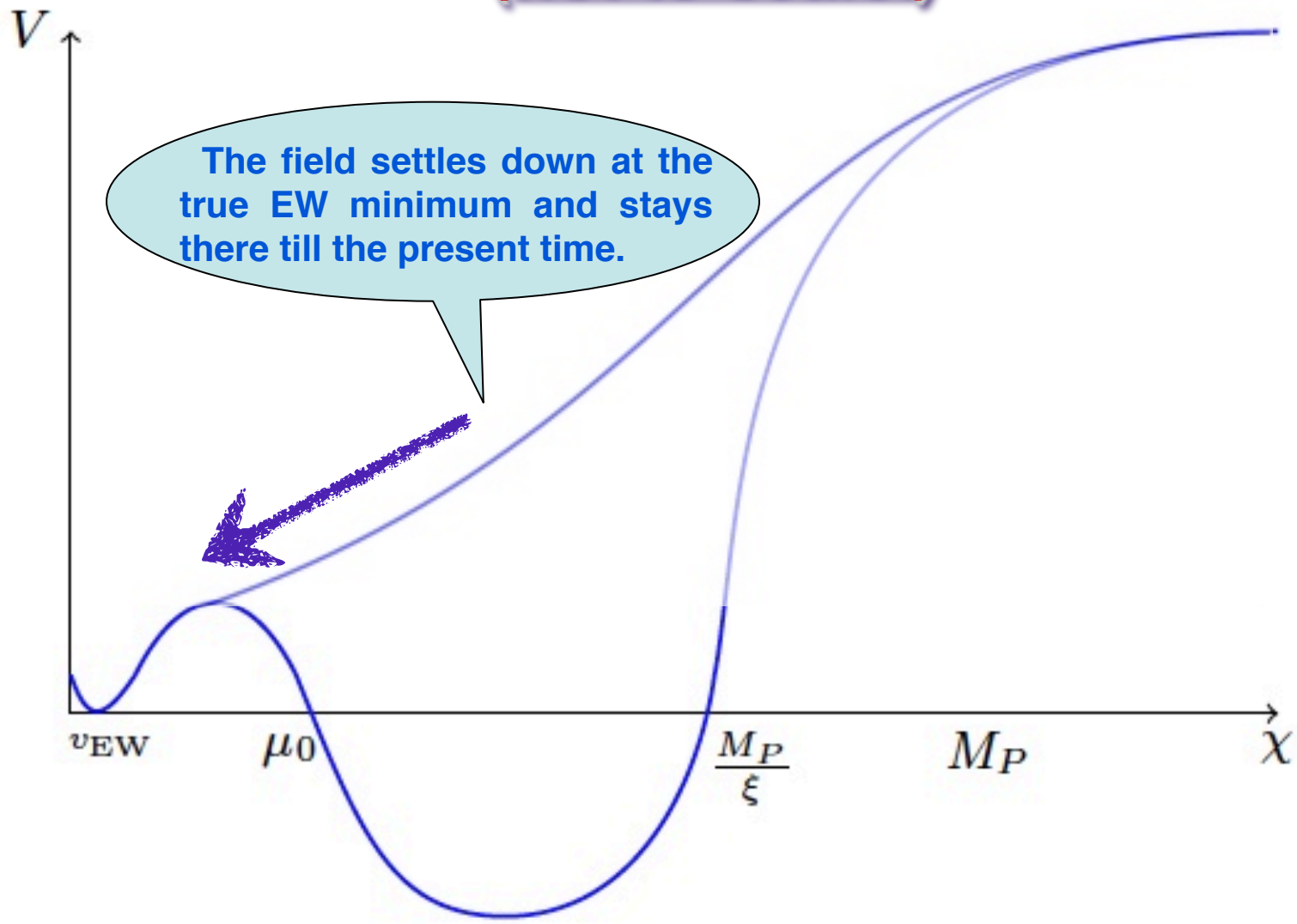
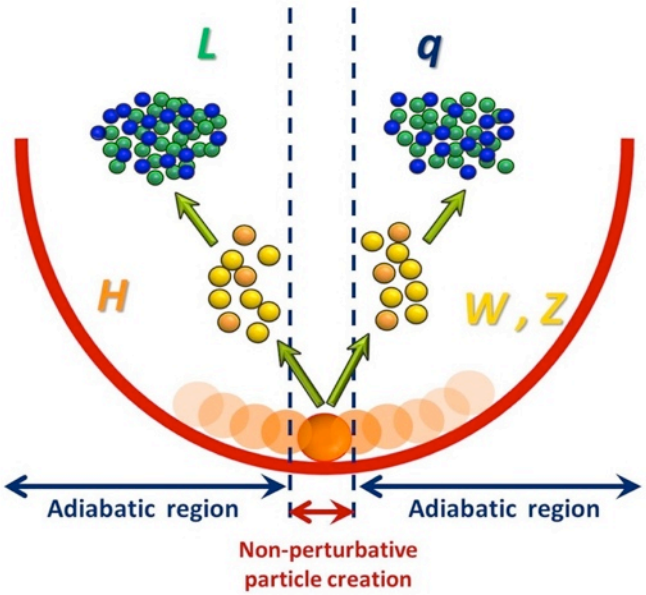
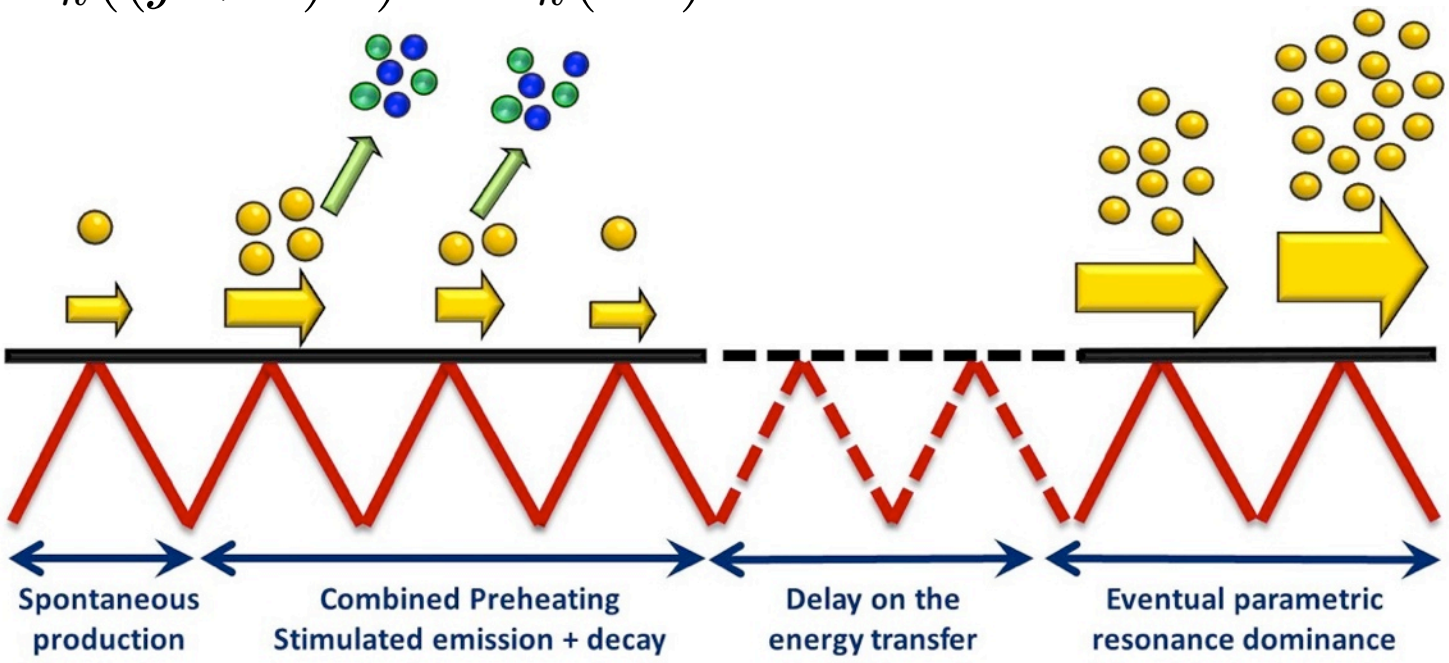


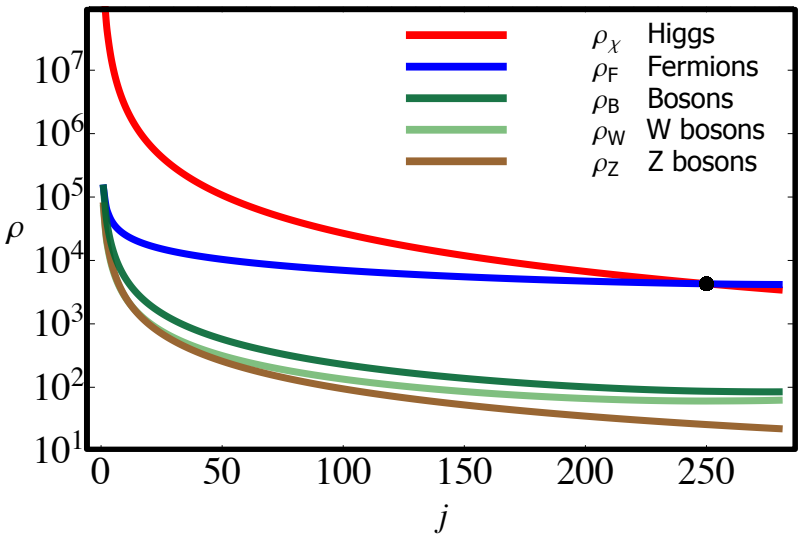
Combined Preheating

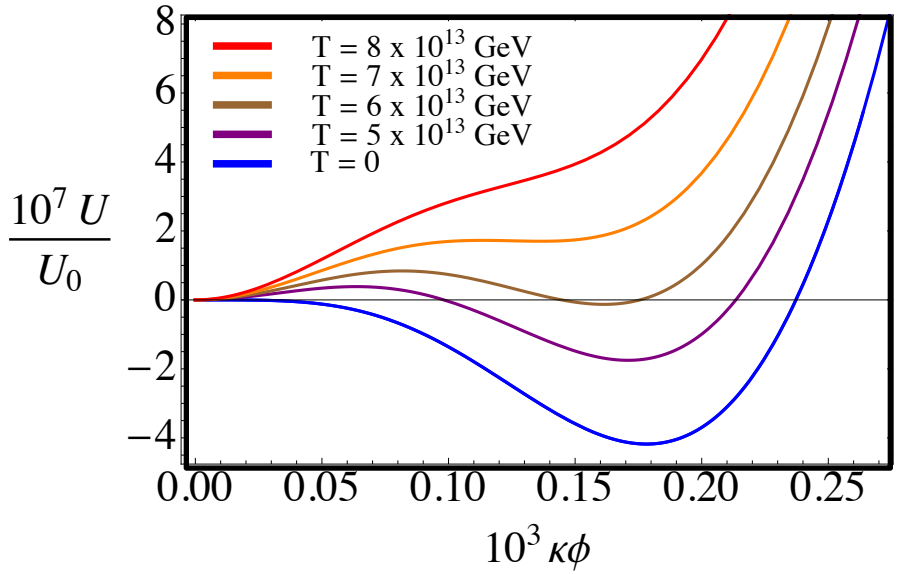


$$n_k((j+1)^+) = n_k(1^+)\, e^{-\gamma F_\Sigma(j)}\, e^{2\pi \sum_{i=1}^{j} \mu_k(i+1)}$$



Symmetry restoration





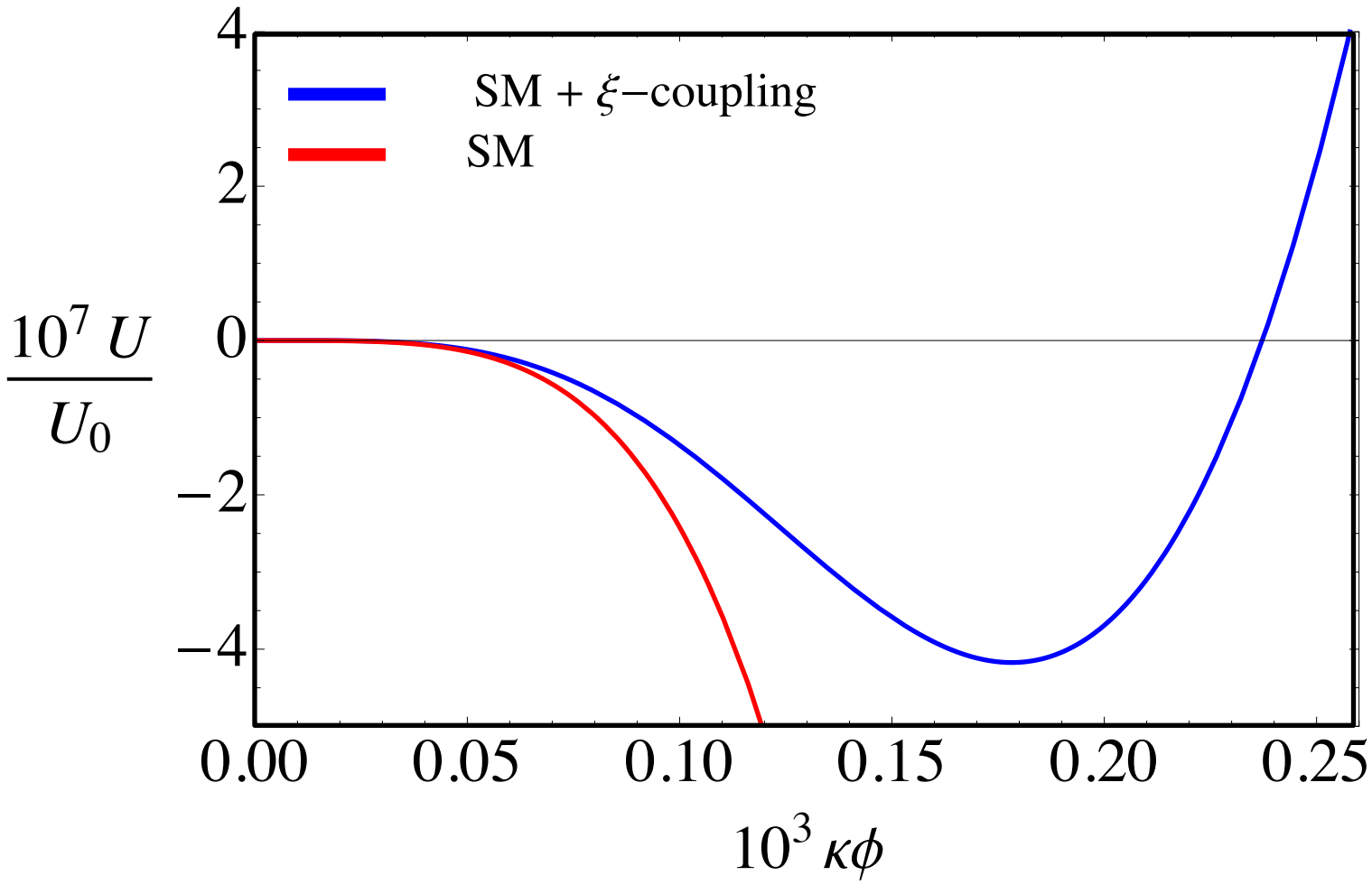
$T_R \simeq 1.8 \times 10^{14}\,\mathrm{GeV}$

$T_+ \simeq 7 \times 10^{13}\,\mathrm{GeV}$

$T_R > T_+$

Higgs inflation can be possible
***even if* our vacuum is not completely stable**

What about lifetime?



Z. Lalak,M. Lewicki, P. Olszewki JHEP 1405 (2014) 119 ,
V. Branchina, E. Messina Phys.Rev.Lett. 111 (2013) 241801 etc...

For SM computation see : J. R. Espinosa and M. Quiros, Phys. Lett. B 353 (1995) 257
J. R. Espinosa, G. F. Giudice and A. Riotto, JCAP 0805 (2008) 002

CONCLUSIONS

✓ **The Higgs field can inflate the Universe**

✓ **HI provides universal predictions if the UV completion respects SI**

$$n_s \simeq 1 - \frac{2}{N} \simeq 0.97 \qquad r \simeq \frac{12}{N^2} \simeq 0.0033$$

✓ **The relation of these predictions to LE observables contains an irreducible theoretical uncertainty.**

UV completion?

✓ **Higgs inflation can be possible *even if our vacuum is metastable***

✓ **The HI scenario is just a particular realization of a general idea. Vacuum instability is not necessarily a problem if:**

- **The potential is modified below the scale of inflation**
- **The reheating process is efficient enough as to make the wrong minimum disappear temporally.**

BACKUP SLIDES

Higgs effective inflation

Can repeat the analysis done by others allowing for the theoretical uncertainty represented by these unknown Wilson coefficients, and compute the effects on CMB observables. C.P.Burgess, S.P.Patil, M.Trott, arXiv:1402.1476

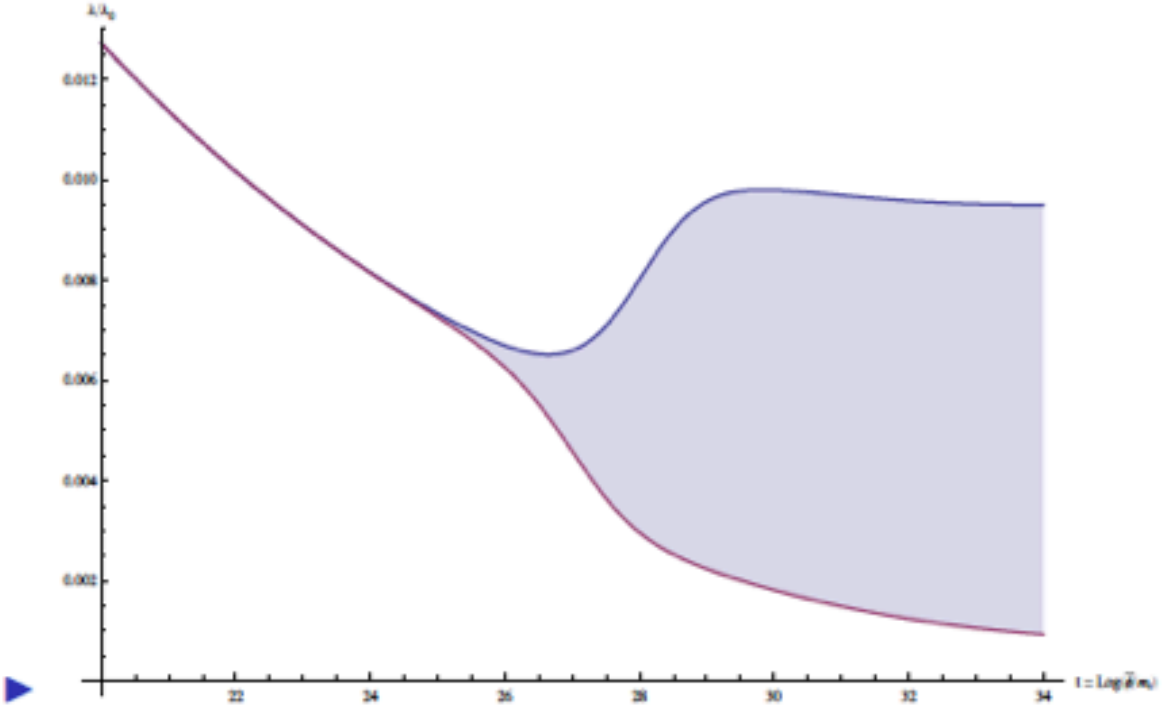


Figure: The effect of the unknown UV completion on the running of the quartic coupling in the Higgs inflation scenario.

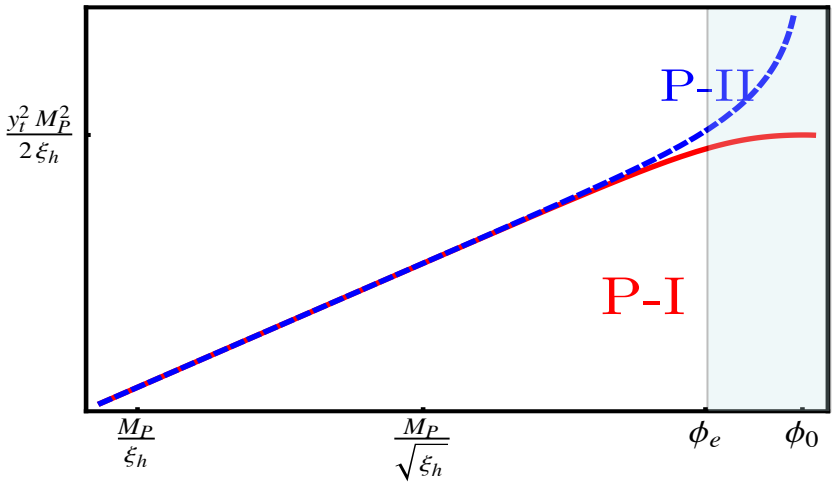
Respect scale invariance -> Dimensional regularization

The choice of μ

In renormalizable theories, it is arbitrary and field-independent

We modify this prescription in our non-renormalizable theory

$$\lambda = \mu^{2\epsilon}\left(\lambda_R + \sum_n \frac{a_n}{\epsilon_n}\right)$$



Jordan frame def.

P-I	$M_P^2 + \xi_h h^2$
P-II	M_P^2

Einstein frame equiv.

P-I	M_P^2
P-II	$M_P^4/(M_P^2 + \xi_h h^2)$

RG equations Chiral Phase

$$16\pi^2\mu\frac{\partial}{\partial\mu}g' = \left(\frac{1}{6} - \frac{1}{12} + \frac{20n_f}{3}\right)g'^3 \qquad 16\pi^2\mu\frac{\partial}{\partial\mu}g = -\left(\frac{43}{16} + \frac{1}{12} - \frac{4n_f}{3}\right)g^3$$

$$16\pi^2\mu\frac{\partial}{\partial\mu}g_3 = -7g_3^2 \qquad 16\pi^2\mu\frac{\partial}{\partial\mu}y_t = \left(-\frac{17}{12}g'^2 - \frac{3}{2}g^2 - 8g_3^2 + 3y_t^2\right)y_t$$

$$16\pi^2\mu\frac{\partial}{\partial\mu}\xi_h = -\left(\frac{3}{2}g'^2 + 3g^2 - 6y_t^2\right)\xi_h$$

From the eq. for the vev

$$16\pi^2\mu\frac{\partial}{\partial\mu}\lambda = -6y_t^4 + \frac{3}{8}\left(2g^2 + (g'^2 + g^2)^2\right) + \left(-3g'^2 - 6g^2 + 12y_t^2\right)\lambda$$

No λ^2. No Landau-pole behaviour. It enters in strong coupling but remains finite

$$16\pi^2\mu\frac{\partial}{\partial\mu}\left(\frac{\lambda}{\xi_h^2}\right) = \frac{1}{\xi_h^2}\left(-6y_t^4 + \frac{3}{8}\left(2g^2 + (g'^2 + g^2)^2\right)\right)$$

No terms prop. to λ

Smaller running for heavier Higgs mass

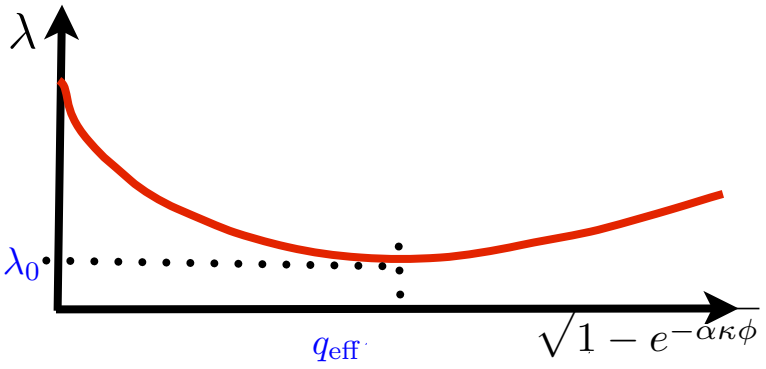
Let's look at the asymptotics

$$\lambda(\mu(\phi)) = \lambda_0 + b\log^2\left(\frac{\sqrt{1-e^{-\alpha\kappa\phi}}}{q_{\rm eff}}\right)$$

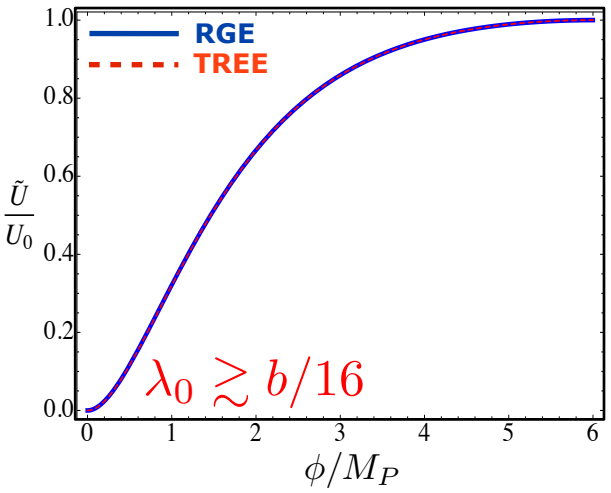
$$b \simeq 2.3\times 10^{-5}$$

$$\lambda_0 = \lambda_0(m_h^*, m_t^*) \ll 1$$

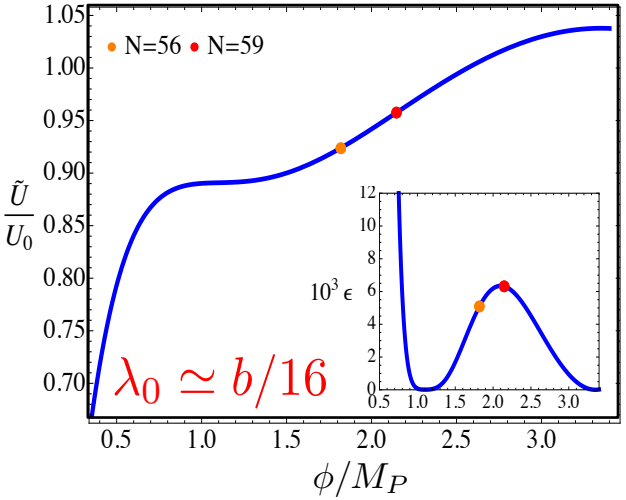
$$q_{\rm eff} = q_{\rm eff}(m_h^*, m_t^*) \sim \mathcal{O}(1)$$



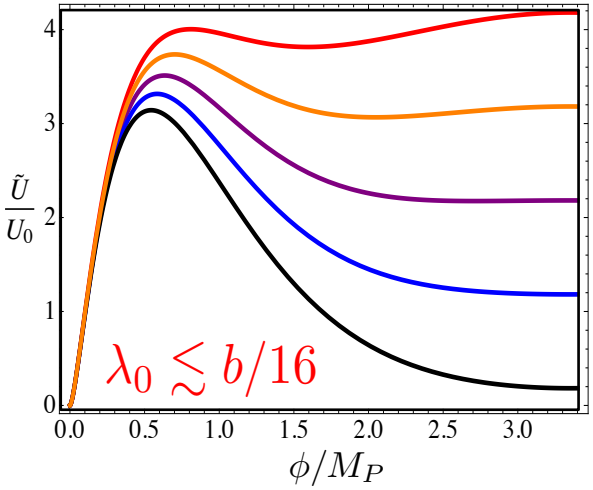
UNIVERSALITY



CRITICALITY



NO INFLATION



Outline

Could the Higgs field itself be responsible for inflation?

1. Reminder of HI

What happens if our vacuum becomes unstable below the scale of inflation?

2. Potential problems with vacuum inst.

Which is the relation between the SM parameters and the inflationary observables?
Which are the requirements for this scenario to be self-consistent?

3. Self-consistent approach to HI

4. HI in the metastability scenario

5. Conclusions