Recent Progress in Testing Alternate Theories of Gravity

Jeremy Sakstein
Institute of Cosmology and Gravitation, Portsmouth

Institute for Theoretical Physics
University of Heidelberg
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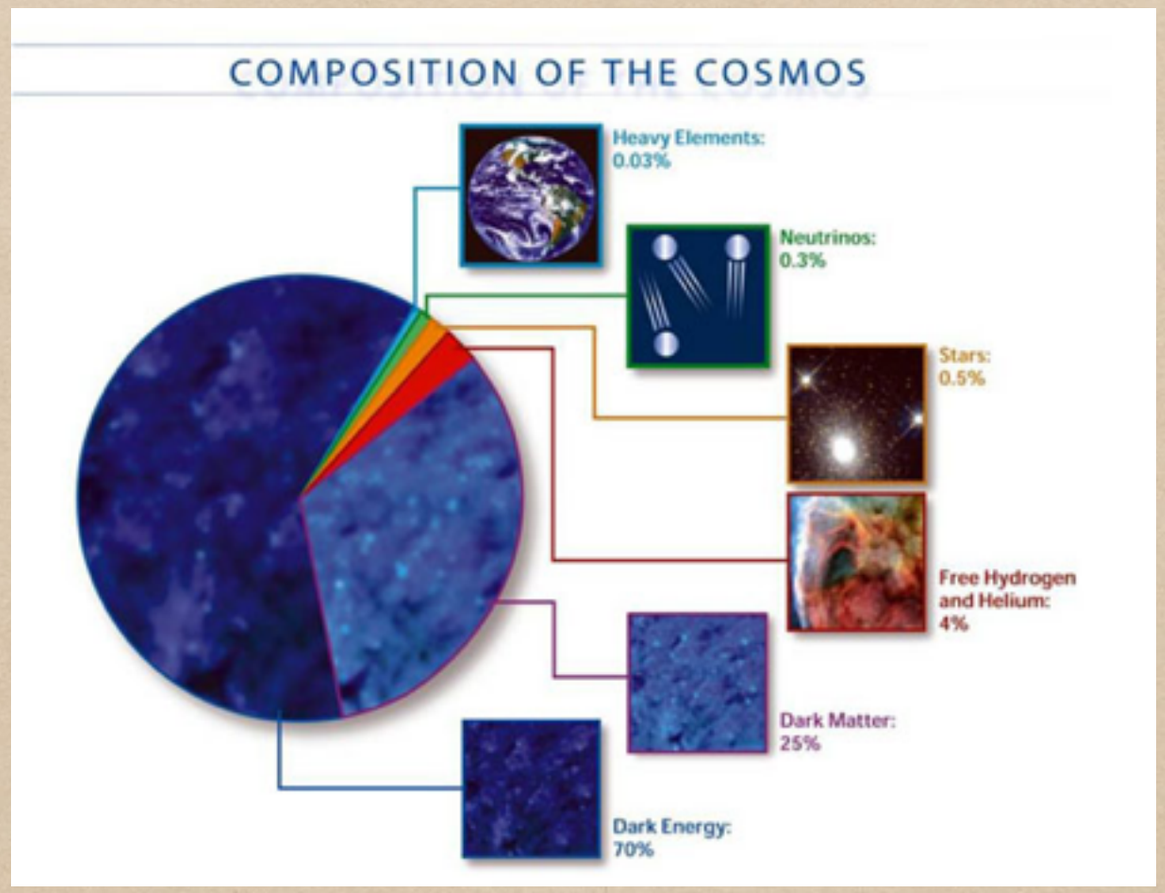
If it ain't broke, don't fix it

Why modify gravity?



The universe is accelerating, and we don't know why!

Can't GR accelerate?

GR can accelerate with a cosmological constant:

$$G_{\mu\nu} = 8\pi G \left(T_{\mu\nu} + \Lambda g_{\mu\nu} \right)$$

Get acceleration if $\Lambda \sim 10^{-12}$ eV^4

So, what's the problem?

Naturalness!

E.g. electron decoupling

$$\Lambda_{\text{eff}} = \underbrace{\Lambda}_{\text{bare}} + \underbrace{m_e^4}_{\text{quantum correction}}$$

$$\Lambda = -m_e^4 + 10^{-12}$$

Okay, so what happens next?

We have to do this for every particle:

$$\Lambda = -\underbrace{m_e^4}_{10^8\ \mathrm{eV}^4} - \underbrace{m_\mu^4}_{10^{28}\ \mathrm{eV}^4} - \underbrace{m_\tau^4}_{10^{36}\ \mathrm{eV}^4} - \cdots - \underbrace{m_H}_{10^{56}\ \mathrm{eV}^4} + 10^{-12}\ \mathrm{eV}^4$$

Have to fine-tune a classical number against a quantum correction to 56 decimal places!

This is not natural

Technical naturalness to the rescue

E.g. fermion masses are natural because of chiral symmetry

$$\Psi \to e^{i\alpha\gamma_5}\Psi$$

$$\mathcal{L} = \underbrace{\bar{\Psi} i\gamma^\mu \partial_\mu \Psi}_{\text{invariant}} - \underbrace{m\bar{\Psi}\Psi}_{\text{breaks symmetry}}$$

Symmetry restored when $m \to 0$

$$\Delta m^2 = m^2 + \mathcal{O}(1)m^2$$

A small fermion mass is natural.

How about gravity?

Can we find a theory of gravity with a technically natural vacuum?

- No naturalness problem
- No fine-tuning
- Self acceleration?

But wait? Ins't gravity already constrained?

Yes and no!

Gravity tested in:

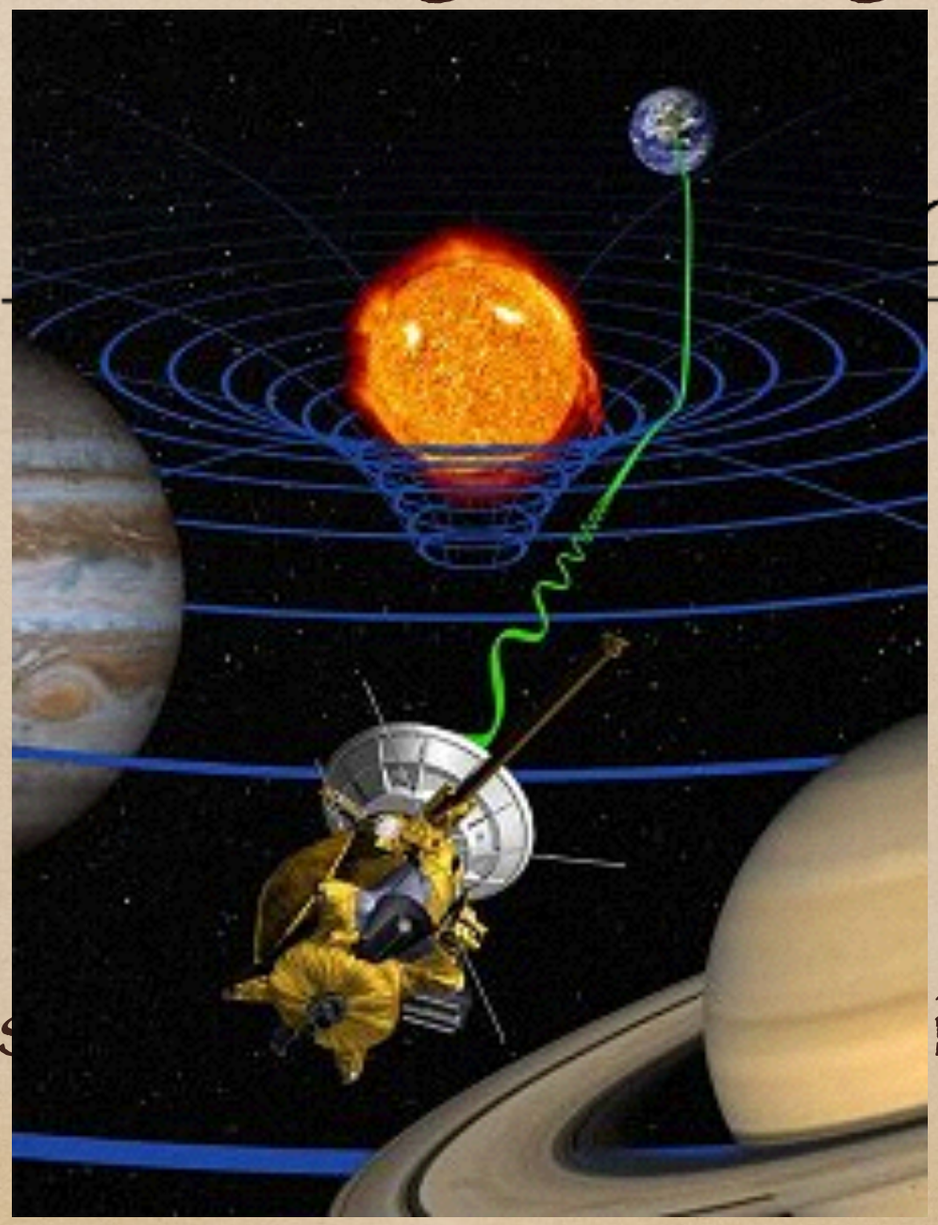
- Solar system - Newtonian and post-Newtonian
- Binary Pulsars - post-Newtonian

The fully relativistic structure has not been probed!

Local tests

E.g. Cassini measures light bending by the Sun

$$\mathrm{d}s^2 = (-1 - \qquad \frac{GM}{r})\,\mathrm{d}x^2$$



"How much s it rest mass"

What do local tests mean?

E.g. scalar-tensor theory - new scalar graviton

$$\nabla^2 \Phi_{\rm N} = 4\pi G \rho \qquad F_{\rm N} = -\nabla \Phi_{\rm N}$$

$$\nabla^2 \phi = 8\pi \alpha G \rho \qquad F_5 = -\alpha \nabla \phi$$

$$\phi = 2\alpha \Phi_{\rm N} \Rightarrow \frac{F_5}{F_{\rm N}} = 2\alpha^2$$

Cassini: $\alpha < 10^{-5} \Rightarrow$ Theory is GR on all scales

Screening mechanisms to the rescue

Non-linear effects decouple cosmological scales from the solar system

solar system

screened

astrophysics

partially screened

cosmology

unscreened

This talk

1. Vainshtein mechanism

- Astrophysics only partially screened
- Identify novel probes in stars and galaxies
- Place new constraints

This talk

2. Disformal Gravity

- Can constrain cosmology using solar system tests
- Interesting cosmological phenomenology
- Still many unsolved mysteries

The Vainshtein mechansim

Recall the problem:

$$\mathcal{L} = -\frac{1}{16\pi G}\partial_\mu\phi\partial^\mu\phi + \alpha\phi T$$

$$\nabla^2\phi = 8\alpha\pi G\rho$$

The Vainshtein mechanism

Try to fix this by adding new kinetic terms

E.g. cubic galileon:

$$\mathcal{L} = -\frac{1}{16\pi G}\partial_\mu\phi\partial^\mu\phi - \frac{1}{16\pi G\Lambda^3}\partial_\mu\phi\partial^\mu\Box\phi + \alpha\phi T$$

$$\nabla^2\phi + \frac{1}{\Lambda^3 r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\phi'^2\right) = 8\alpha\pi G\rho$$

Vainshtein Mechanism

We can integrate this once:

$$x = \frac{F_5}{F_{\mathrm{N}}}$$

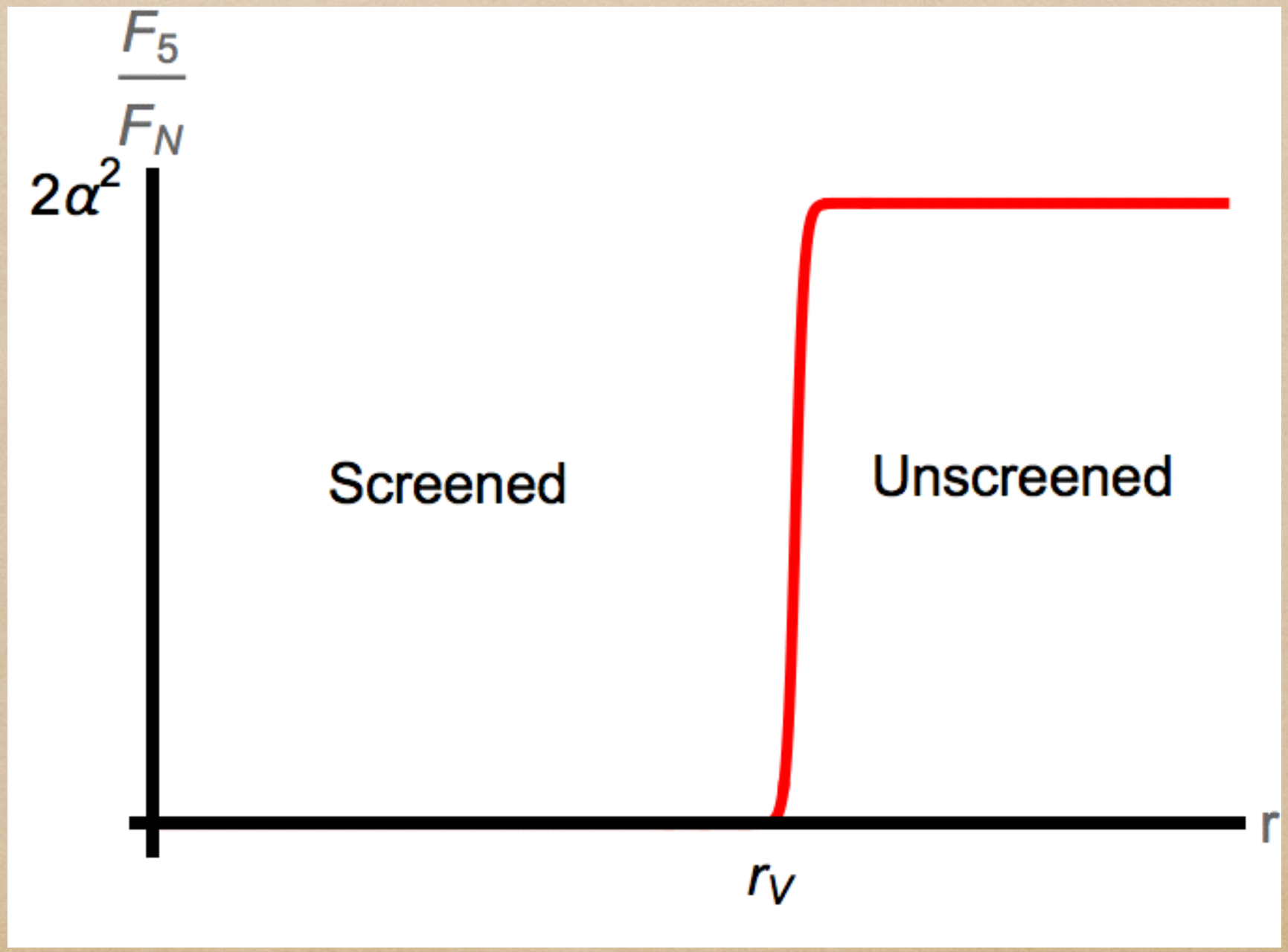
$$x + \left(\frac{r_{\mathrm{V}}}{r}\right)^3 x^2 = 2\alpha^2$$

$$r \ll r_{\mathrm{V}} \Rightarrow \frac{F_5}{F_{\mathrm{N}}} = 2\alpha^2 \left(\frac{r}{r_{\mathrm{V}}}\right)^{\frac{3}{2}} \ll 1$$

$$r \gg r_{\mathrm{V}} \Rightarrow \frac{F_5}{F_{\mathrm{N}}} = 2\alpha^2 \sim \mathcal{O}(1)$$

r_{V} - Vainshtein radius

Vainshtein Screening



Astrophysical Screening

$$r_{\mathrm{V}}^{\odot} \geq 10^2 \text{ pc}$$

Exhibited in:

- DGP - tension with data
- Covariant galileons - too much ISW
- Massive gravity - no FRW solutions
- Massive bigravity - unstable (or is it?)
- Beyond Horndeski - new and unexplored

Mechanism is partially broken in beyond Horndeski

Vainshtein Breaking

$$\mathrm{d}s^2 = -\left(1+2\Phi\right)\,\mathrm{d}t^2 + \left(1-2\Psi\right)\delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j$$

Motion of NR matter

Bending of Light

GR: $$\frac{\mathrm{d}\Phi}{\mathrm{d}r} = \frac{GM(r)}{r} \qquad \frac{\mathrm{d}\Psi}{\mathrm{d}r} = \frac{GM(r)}{r}$$

Vainshtein Breaking

Stars and satellites behave differently

Cosmological field

BH:

$$\frac{\mathrm{d}\Phi}{\mathrm{d}r} = \frac{GM(r)}{r} + \frac{\Upsilon G}{4}\frac{\mathrm{d}^2 M(r)}{\mathrm{d}r^2}$$

$$\Upsilon = \left(\frac{\dot{\phi}_0}{\Lambda}\right)^4$$

$$\frac{\mathrm{d}\Psi}{\mathrm{d}r} = \frac{GM(r)}{r} - \frac{5\Upsilon G}{4r}\frac{\mathrm{d}M(r)}{\mathrm{d}r}$$

Light bent differently

Potential probes

- Stellar structure
- Galactic rotation curves
- Gravitational lensing

Stellar Structure Tests

Main idea:

- Stars burn fuel to stave off gravitational collapse
- Changing gravity changes the burning rate
- This alters the temperature, luminosity and life time

Vainshtein Stars

Gravity weaker

Slower burning rate

Dimmer and cooler stars that live longer

Polytropic stars

$$P = K\rho^{\frac{4}{3}}$$

Balls of gas that collapse under gravity - no physics

- No nuclear burning, convection etc.
- Can isolate new effects of MG
- Not realistic enough to compare with data

Mass-G-Luminosity relation

$$P_{\rm gas} = \frac{\rho k_{\rm B} T}{\mu m_{\rm H}}$$

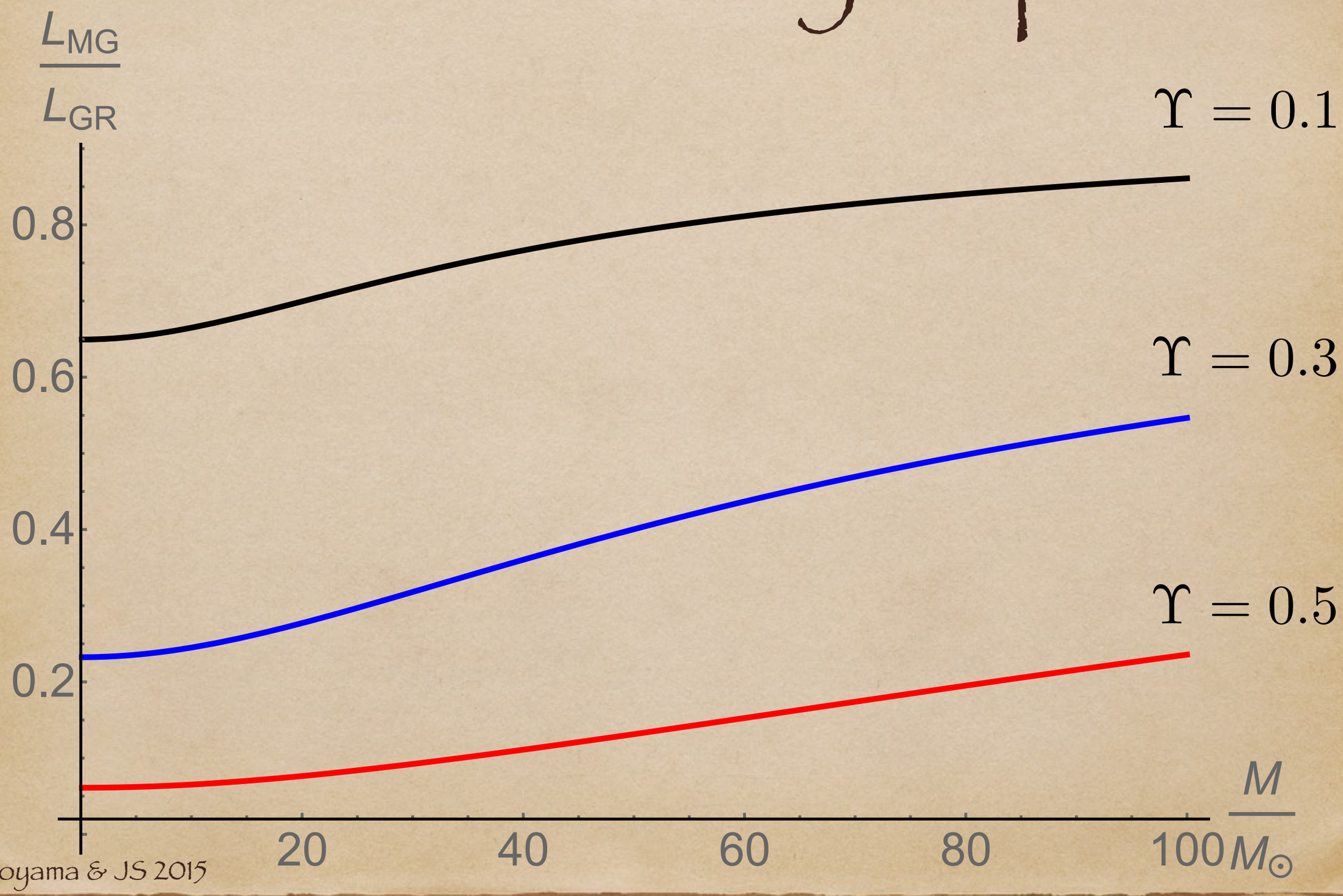
$$P_{\rm rad} = \frac{1}{3} a T^4$$

Gas pressure - $L \propto G^4 M^3$

Radiation pressure - $L \propto GM$

High-mass stars are more radiation pressure-supported

Vainshtein Polytropes

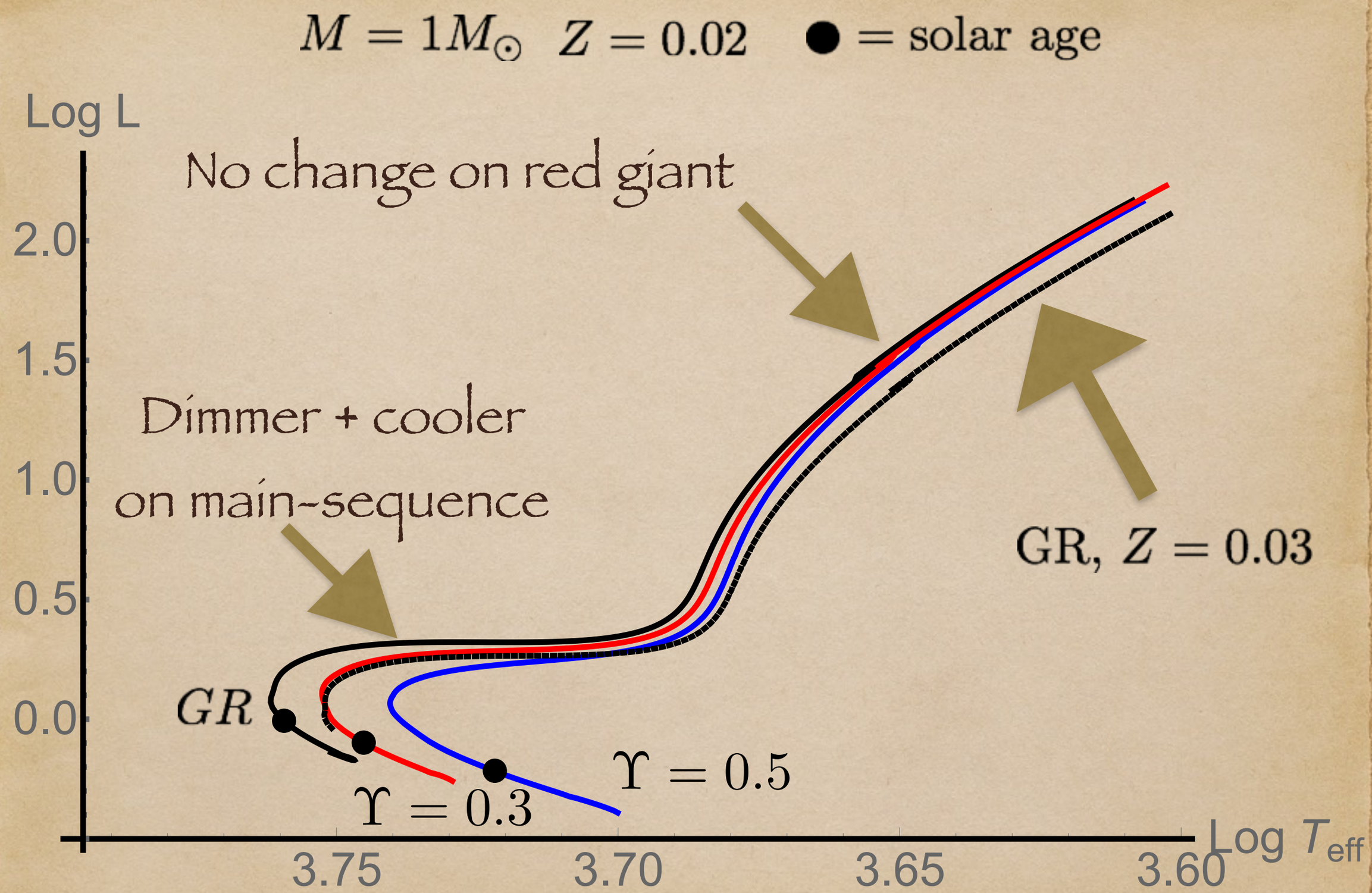


Koyama & JS 2015

Realistic stars

We have modified MESA to include MG:

- Fully consistent treatment of stellar structure
- No approximations
- Includes burning, convection, mass loss etc.
- Can compare with data



Koyama & JS 2015

Vainshtein vs. GR

- Main-sequence cooler and dimmer
- No change to red giant phase
- MS degenerate with GR + more metals

May be detectable by comparing MS and RG fits to globular clusters.

Galactic rotation curves

Circular velocity: $$v_{\rm circ}^2 = \frac{{\rm d}\Phi}{{\rm d}r}$$

New features in rotation curves?

$$\frac{{\rm d}\Phi}{{\rm d}r} = \frac{GM(r)}{r} + \frac{\Upsilon G}{4}\frac{{\rm d}^2 M(r)}{{\rm d}r^2}$$

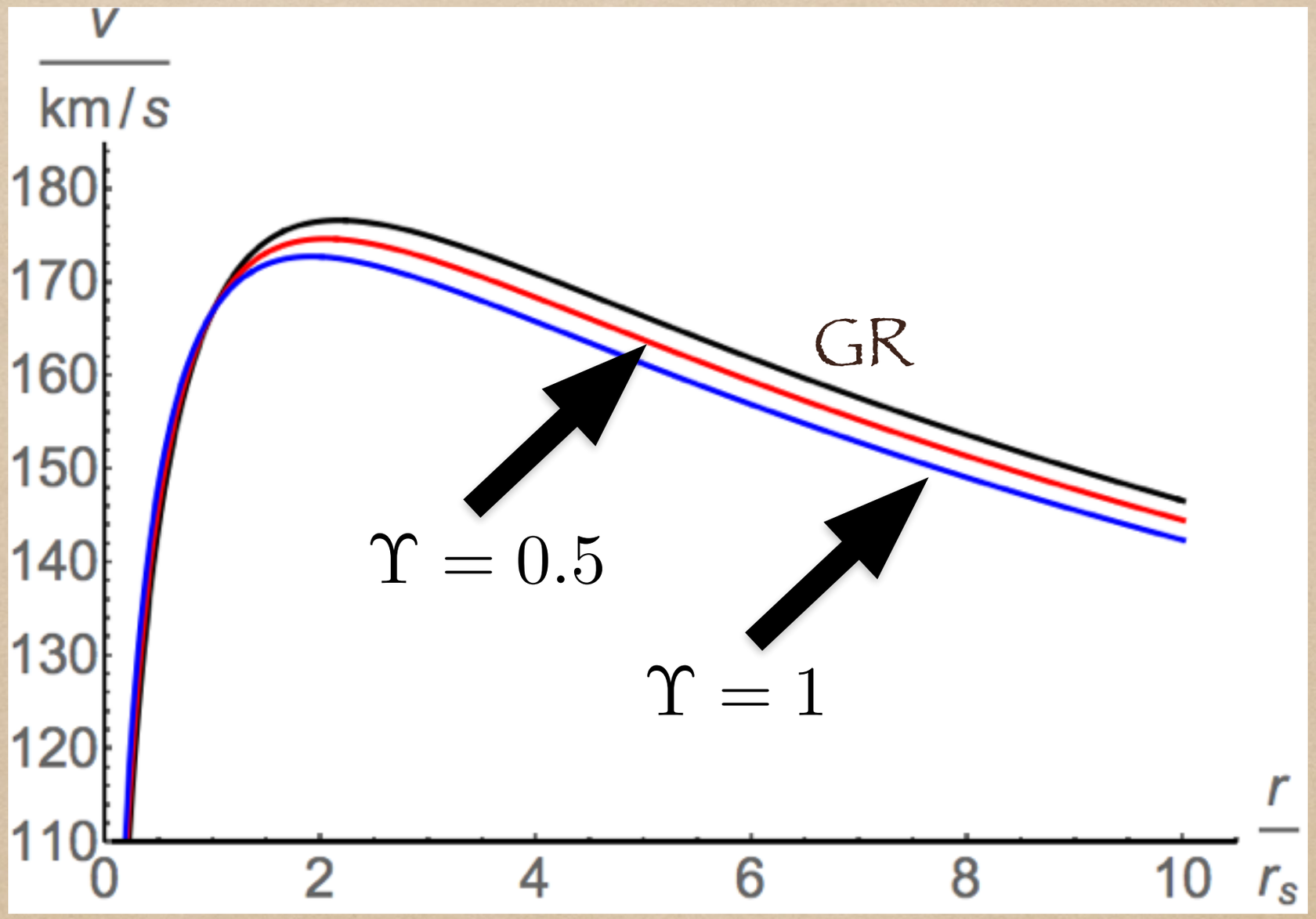
Galactic rotation curves

NFW density profile:

$$\rho(r) = \frac{\rho_{\rm s}}{\frac{r}{r_{\rm s}}\left(1+\frac{r}{r_{\rm s}}\right)^2}$$

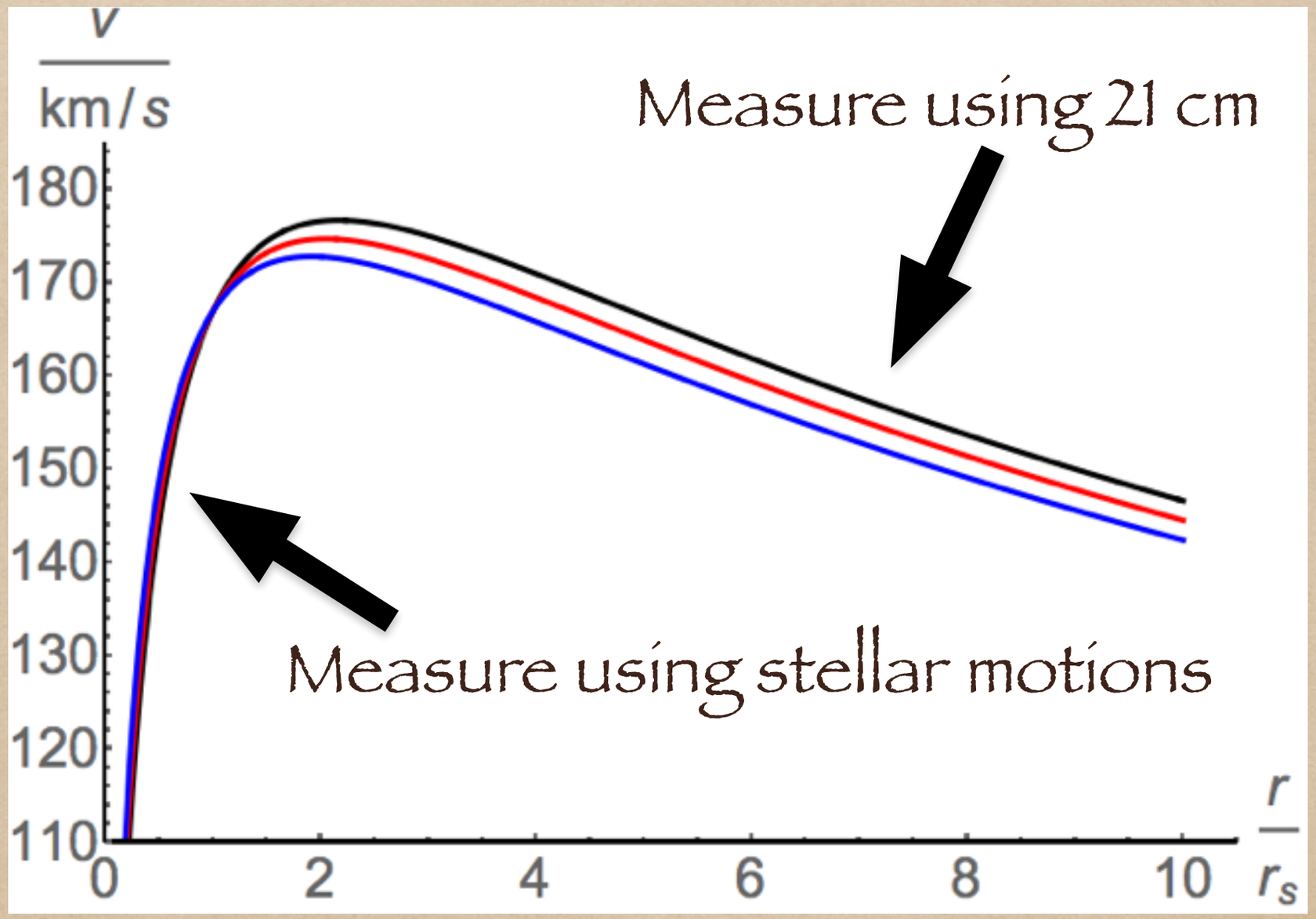
$$v_{\rm circ}^2 = \frac{4\pi G r_{\rm s}^3 \rho_{\rm s}}{r}\left[\ln\left(1+\frac{r}{r_{\rm s}}\right) - \left(1+\frac{r_{\rm s}}{r}\right)^{-1} + \frac{\Upsilon}{4}\frac{\left(\frac{r_{\rm s}}{r}-1\right)}{\left(1+\frac{r_{\rm s}}{r}\right)^3}\right]$$

Vainshtein rotation curves



Koyama & JS 2015

Vainshtein rotation curves



Deviations in 21 cm region compared with stellar prediction

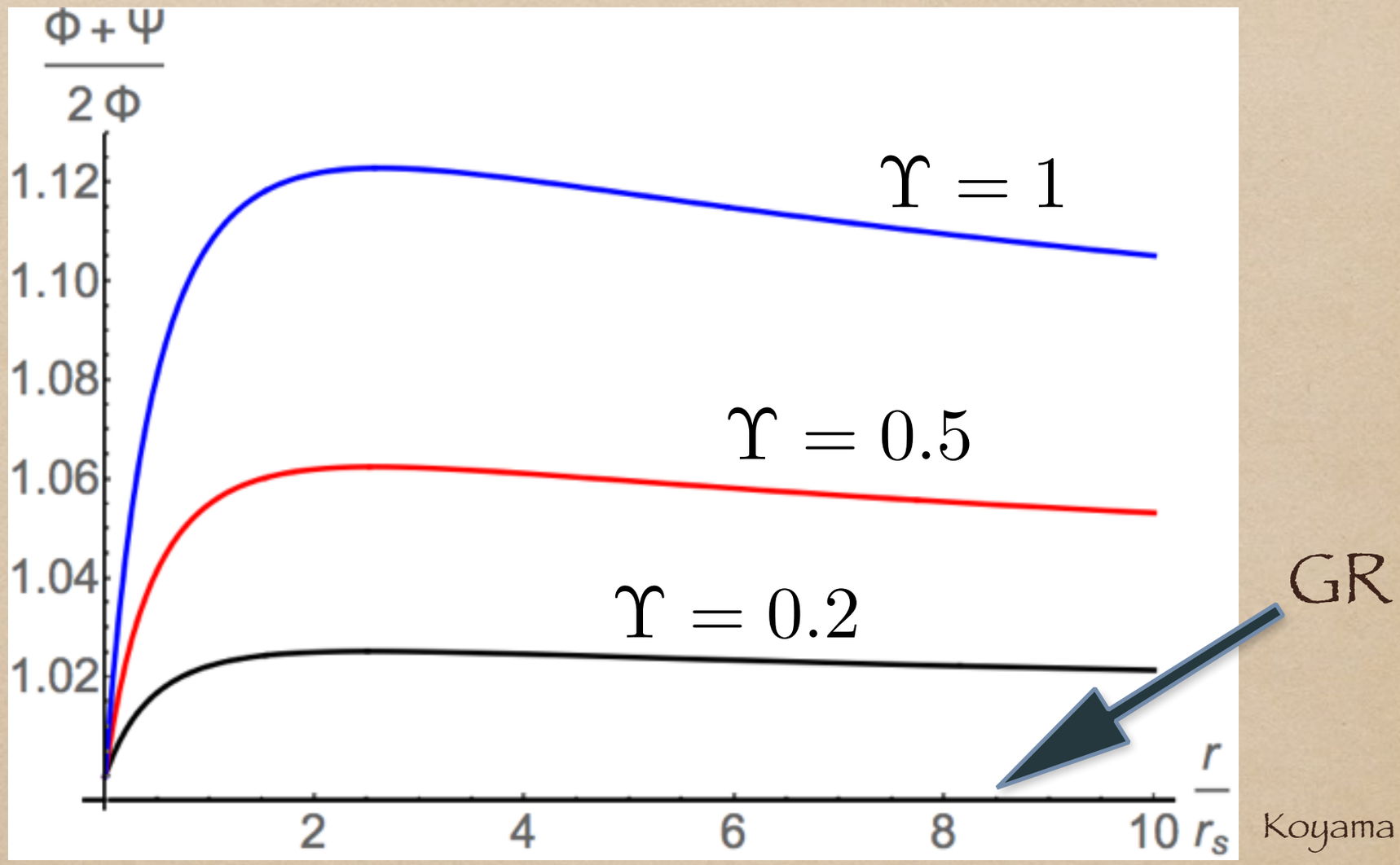
Gravitational lensing

Lensing of light (relative to NR force): $\dfrac{\Phi + \Psi}{2\Phi}$

GR: $\Phi + \Psi = 2\Phi$

BH: $$\Phi + \Psi = \frac{8\pi G r_{\rm s}^3 \rho_{\rm s}}{r}\left[\underbrace{\ln\left(1 + \frac{r}{r_{\rm s}}\right)}_{2\Phi_{\rm GR}} + \frac{\Upsilon}{8}\frac{\left(6 + \frac{r_{\rm s}}{r}\right)}{\left(1 + \frac{r_{\rm s}}{r}\right)^2}\right]$$

Vainshtein lensing



Koyama & JS 2015

Deviations from GR in the strong lensing regime

Disformal Gravity

$$S = \int \mathrm{d}^4x\sqrt{-g}\frac{M_{\rm pl}^2}{2}\left[\frac{R}{2} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right] + S_{\rm m}[\tilde{g}_{\mu\nu}]$$

$$\underbrace{\tilde{g}_{\mu\nu}}_{\text{Jordan frame}} = \underbrace{A^2(\phi)}_{\text{conformal}}\left[\underbrace{g_{\mu\nu}}_{\text{Einstein frame}} + \underbrace{\frac{B^2(\phi)}{\Lambda^2}}_{\text{disformal}}\partial_\mu\phi\partial_\nu\phi\right]$$

Matter moves on geodesics of $\tilde{g}_{\mu\nu}$ not $g_{\mu\nu}$

$\Rightarrow$ fifth-force

Beckenstein 1992

Potential Problem

Jordan frame metric can become singular:

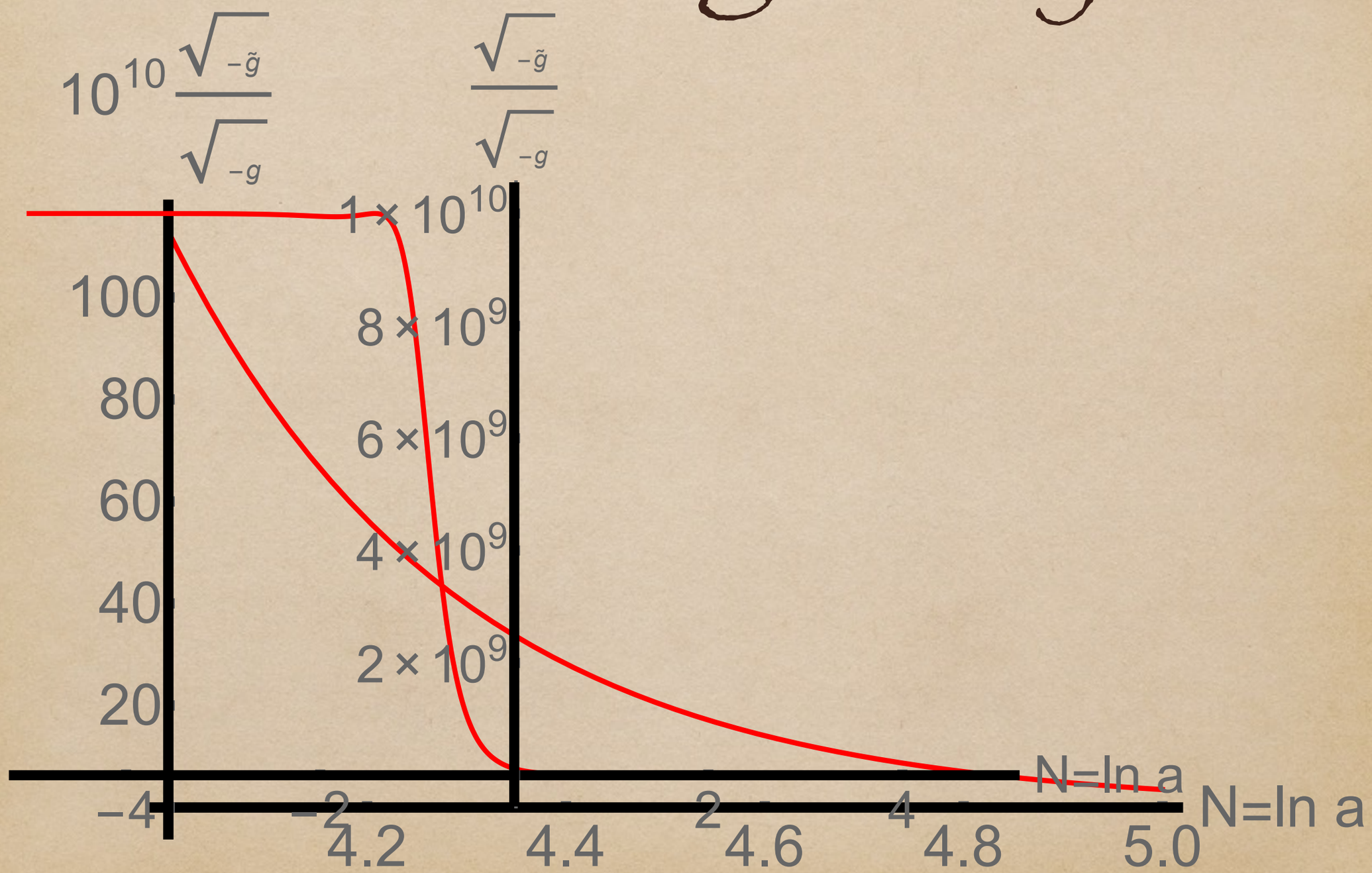
$$\frac{\sqrt{-\tilde{g}}}{\sqrt{-g}} = A^4\sqrt{1+\frac{B^2\left(\partial\phi\right)^2}{\Lambda^2}} = A^4\sqrt{1-\frac{\dot{\phi}_0}{\Lambda^2}}$$

Solutions slow down to avoid this, but no one knows why.

"Natural pathology resistance"

Koivisto, Mota & Zumalacarregui 2012

Metric Singularity?



Natural Pathology Resistance

Is this okay? No*, e.g. disformal only:

$$\mathrm{d}\tilde{s}^2 = -N^2\,\mathrm{d}t^2 + \cdots \qquad N^2 = 1 - \frac{\dot{\phi}^2}{\Lambda^2} \qquad \mathrm{d}T = N\mathrm{d}t$$

Proper time -> 0

Lapse -> 0 but so what? Two speeds:

$$\frac{c^2_{\text{tensors}}}{c^2_{\text{light}}} = N^2$$

Don't know what the true non-relativistic limit is!

Non-relativistic limit

EOM is horrible:

$$\chi\Box\phi - 8\pi G\frac{B^2}{\Lambda^2}T_{\mathrm{m}}^{\mu\nu}\nabla_\mu\nabla_\nu\phi =$$

$$-8\pi\alpha G T_{\mathrm{m}} - 8\pi G\frac{B^2}{\Lambda^2}\left(\alpha-\gamma\right)T_{\mathrm{m}}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \chi V(\phi)_{,\phi}$$

$$\chi = 1 + \frac{B^2(\partial\phi)^2}{\Lambda^2} \qquad \alpha = \frac{\mathrm{d}\ln A}{\mathrm{d}\phi} \qquad \gamma = \frac{\mathrm{d}\ln B}{\mathrm{d}\phi}$$

JS 2014

Non-relativistic limit

But we have calculated the PPN parameters:

Orbital precession

$$\tilde{g}_{00} = 1 + 2U - 2\beta U^2 + \cdots$$

Preferred frame

$$\tilde{g}_{0i} = -\frac{1}{2}(\cdots + \alpha_1 - \alpha_2 + \cdots)V_i - \frac{1}{2}(\cdots + \alpha_2 + \cdots)W_i$$

Light bending

$$\tilde{g}_{ij} = (1 + 2\gamma_{\mathrm{PPN}}U)\,\delta_{ij}$$

Non-relativistic limit

$$\tilde{g}_{00} = 1 + 2U - 2\beta U^2 + \cdots$$

$1 - \Upsilon$ (GR = 1)

$$\tilde{g}_{0i} = -\frac{1}{2}(\cdots + \alpha_1 - \alpha_2 + \cdots)V_i - \frac{1}{2}(\cdots + \alpha_2 + \cdots)W_i$$

$-\Upsilon$ (GR = 0)

-4Υ (GR = 0)

$$\tilde{g}_{ij} = (1 + 2\gamma_{\rm PPN} U)\,\delta_{ij}$$

$$\Upsilon = \left(\frac{B\dot{\phi}_0}{\Lambda}\right)^2$$

$1 - \Upsilon$ (GR = 1)

Constraints

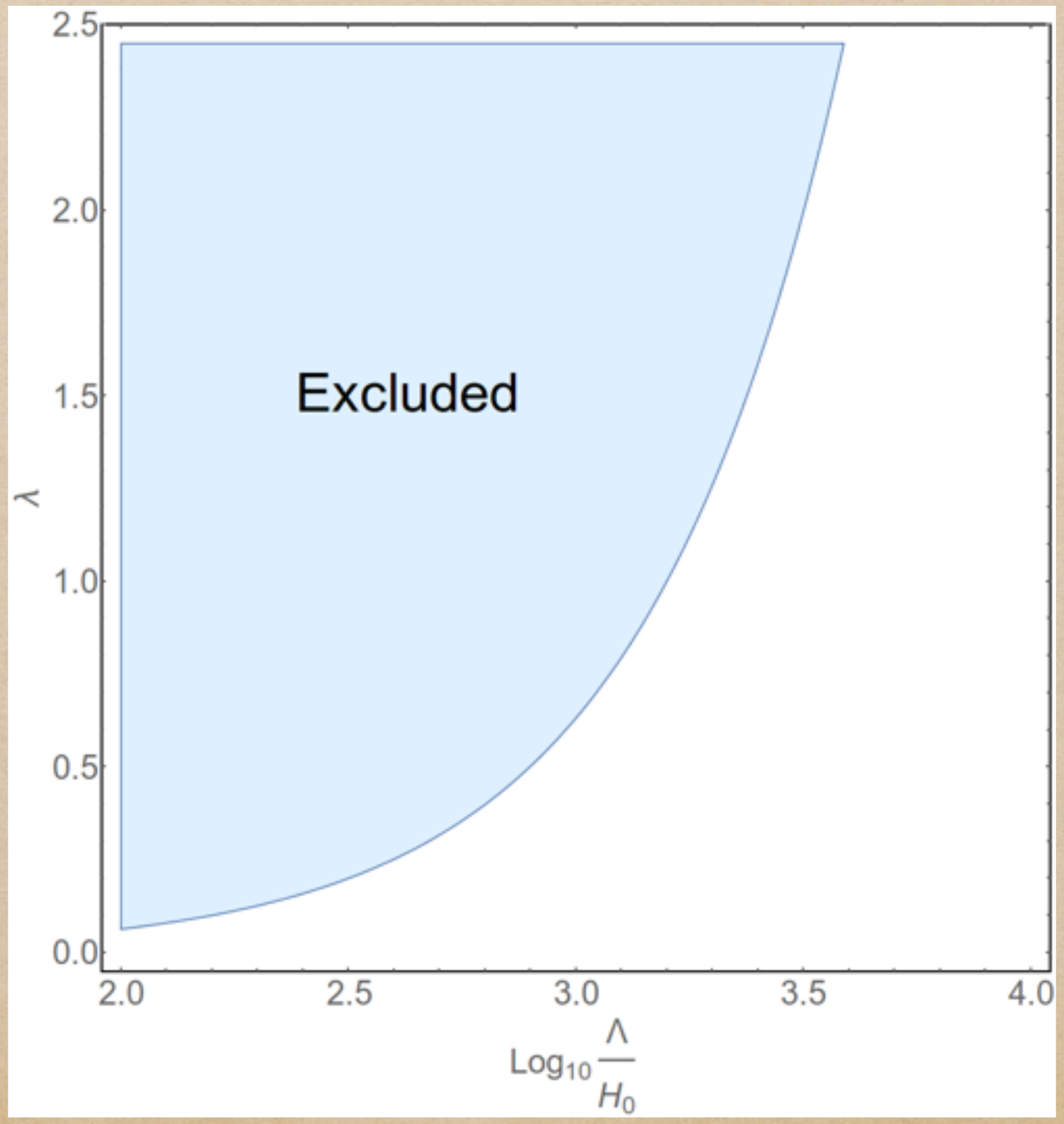
Strongest constraint comes from α_2

$$\alpha_2 < 10^{-7}$$

Simple model: $V(\phi) \propto e^{-\lambda\phi}$, $B = 1$

$$\lambda^2 \left(\frac{H_0}{\Lambda}\right)^2 < 4 \times 10^{-7}$$

Constraints



Ip, Schmidt & JS 2015

What does this mean?

$$\lambda^2 \left(\frac{H_0}{\Lambda}\right)^2 < 4 \times 10^{-7}$$

Need $\left(\frac{H_0}{\Lambda}\right)^2 \sim \mathcal{O}(1)$ for novel cosmology

Van de Bruck & Morrice 2015

Cosmology is identical to ΛCDM

Cosmology

Friedmann equations same as GR

Field equations are coupled:

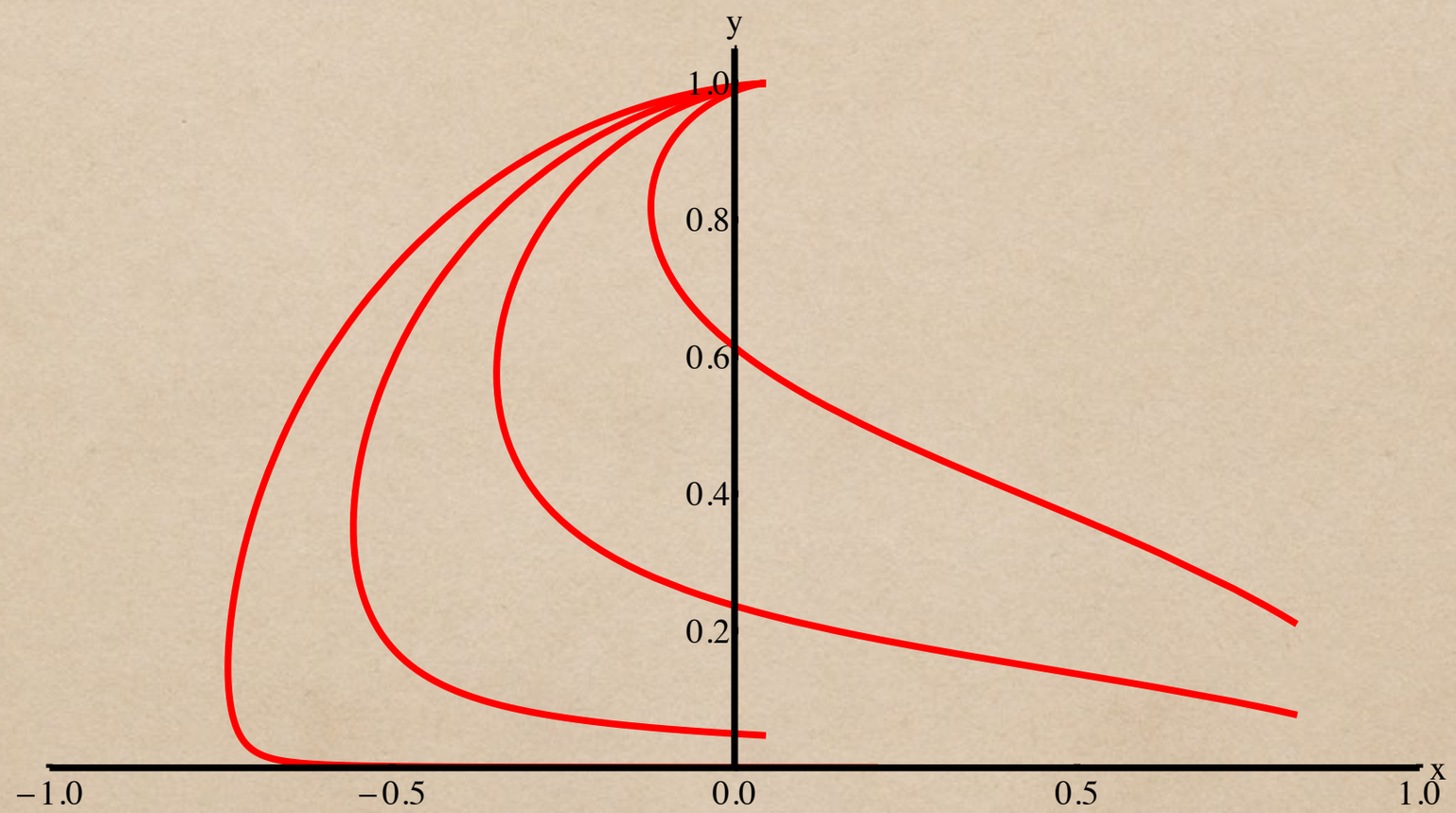
$$\dot{\rho} + 3H\rho = Q_0 \dot{\phi}_\infty$$

$$\ddot{\phi}_\infty + 3H\dot{\phi}_\infty + V'(\phi_\infty) = -Q_0$$

$$Q_0 = 8\pi G\rho \frac{\alpha + \frac{B^2}{\Lambda^2}\left([\gamma - \alpha]\,\dot{\phi}_\infty^2 - 3H\dot{\phi}_\infty - V_{,\phi}\right)}{1 + \frac{B^2}{\Lambda^2}\left(8\pi G\rho - \dot{\phi}_\infty^2\right)}$$

Dynamical Systems

Different initial conditions, common late-time behaviour



$$x = \frac{\phi'}{\sqrt{6}} \qquad y = \frac{\sqrt{V}}{\sqrt{3}H}$$

Why is this useful?

- Classify the entire solution space
- Know cosmological properties at the fixed points
- Identify models that have the properties we want
- Tells us which models to focus future efforts on

What we want

Fixed points with

- Dark energy domination - match observations
- Finite metric determinant - well-defined NR limit

Recap: Conformal Case

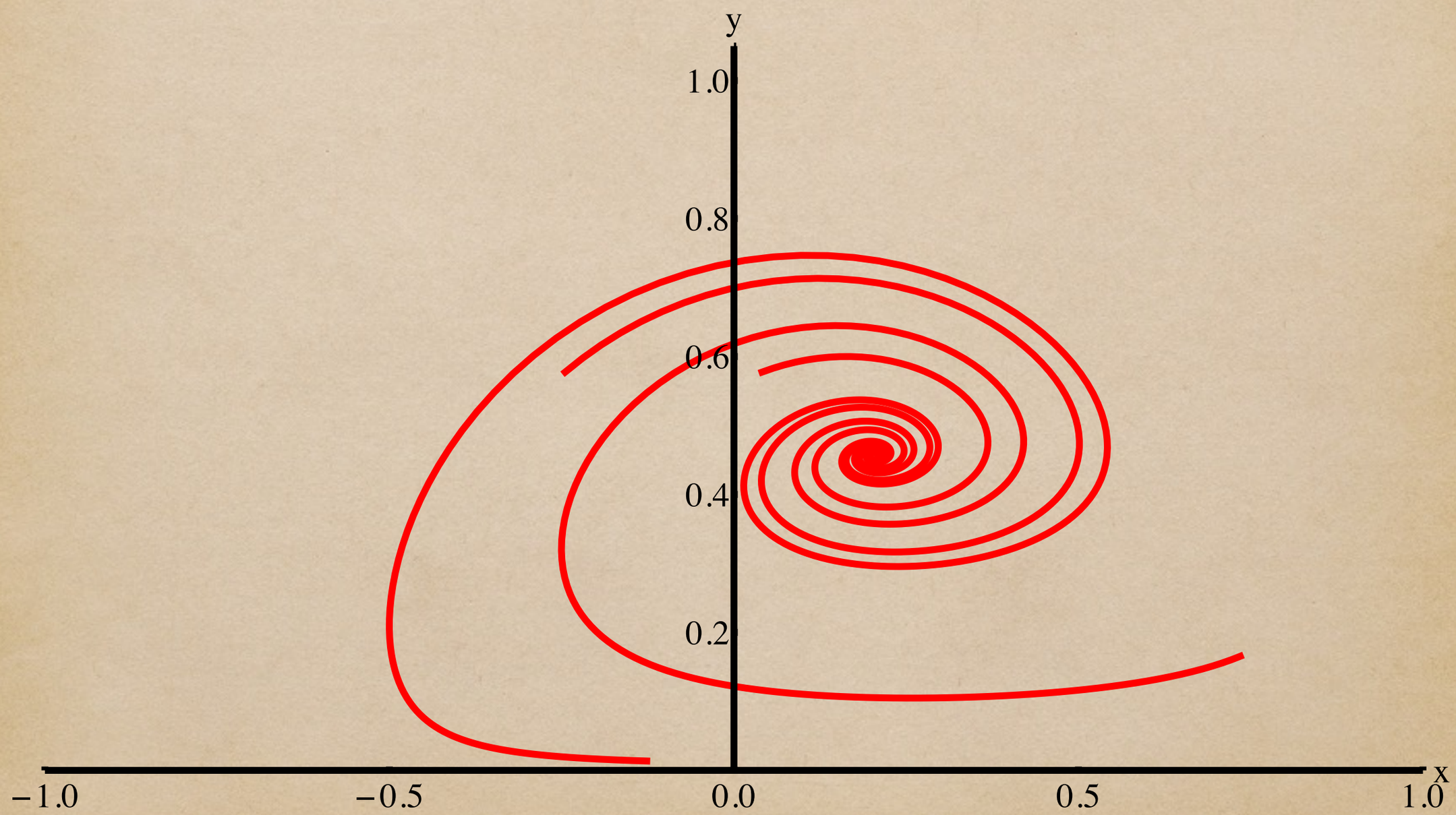
$$V(\phi) = m_0^2 e^{-\lambda\phi} \qquad A(\phi) = e^{\alpha\phi} \qquad N = \ln a$$

Phase space is 2-dimensional

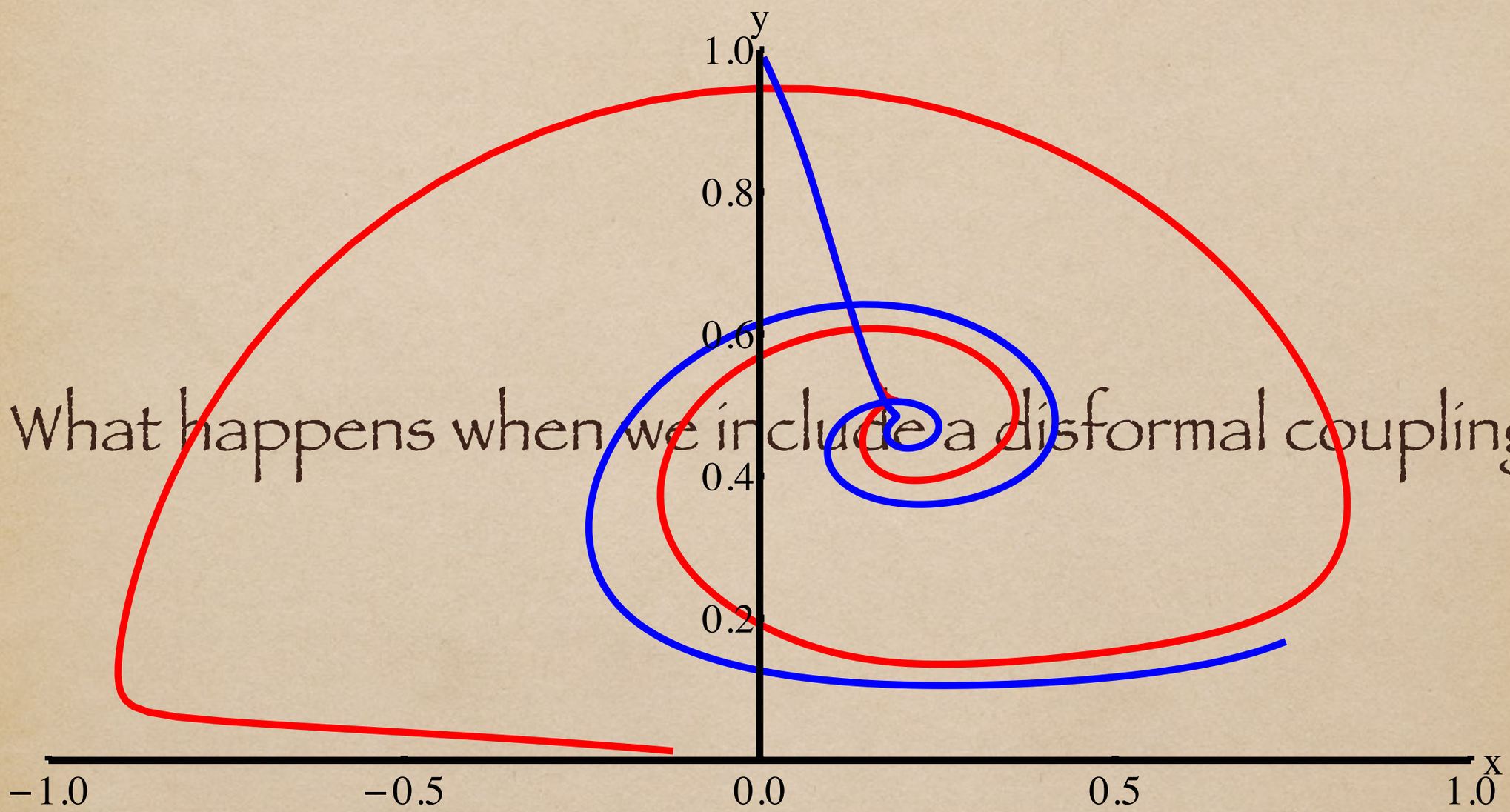
$$x = \frac{\phi'}{\sqrt{6}} \qquad y = \frac{\sqrt{V}}{\sqrt{3}H} \qquad \lambda = -\frac{V'}{V} = \text{constant}$$

$$\ddot{\phi}_\infty + 3H\dot{\phi}_\infty + V'(\phi) = 8\pi\alpha G\rho$$

Conformal phase space



Disformal System



What happens when we include a disformal coupling?

Old attractor is a saddle point - what's going on?

Disformal System

$$x = \frac{\phi'}{\sqrt{6}} \qquad y = \frac{\sqrt{V}}{\sqrt{3}H} \qquad \lambda = -\frac{V'}{V} = \text{constant}$$

Can't eliminate terms of x and y

Phase space is 3D

$$Q_0 = 8\pi G\rho \frac{\alpha + \frac{B^2}{\Lambda^2}\left([\gamma - \alpha]\,\dot{\phi}_\infty^2 - 3H\dot{\phi}_\infty - V_{,\phi}\right)}{1 + \frac{B^2}{\Lambda^2}\left(8\pi G\rho - \dot{\phi}_\infty^2\right)}$$

$$z = \frac{BH}{\Lambda} = \frac{Z}{Z+1}$$

$$0 \leq Z \leq 1 \qquad \gamma \text{ constant}$$

Stability is altered

Perturbations in the z - direction are unstable

(3 eigenvalues instead of 2, one can be positive)

New dark energy dominated fixed point at

$x = 0, y = 1, Z = 1$

But: $\sqrt{-\tilde{g}} = 0$

Metric singularity at late times

Special Case

$$\gamma = \lambda/2 \Rightarrow zy = \text{const}$$

New fixed point with interesting properties

$$\Omega_{\rm DE} = \Omega_{\rm DE}\left(\lambda, \frac{\Lambda}{m_0}\right) \quad \omega_{\rm DE} = \omega_{\rm DE}\left(\frac{\Lambda}{m_0}\right)$$

Can always match any measurement without fine-tuning

BUT: $\sqrt{-\tilde{g}} = 0$

What does this mean?

Only viable models are identical to conformal theories!*

* Possible loopholes are models whose dynamics are not described by this analysis

Summary

Vainshtein

- Vainshtein broken in beyond Horndeski
- Main-sequence stars are dimmer and cooler
- Circular velocity is lower
- Deviations from GR in strong lensing regime

Summary

Disformal

- Metric singularity not well-understood
- Solar system constraints give ΛCDM
- Cosmological solutions evolve towards singularity
- Only viable models have no disformal properties
- Still a lot to understand!

Thank you!
(and to my collaborators)

<u>Vainshtein</u>

Kazuya Koyama (ICG)

<u>Papers</u>

1502.06872

<u>Disformal</u>

Fabian Schmidt (MPA)

Hiu Yan Ip (MPA)

<u>Papers</u>

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