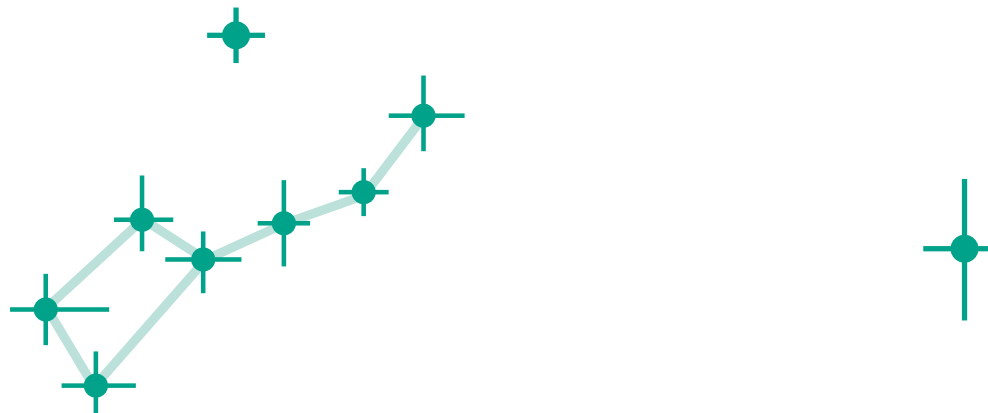




Bayesian Analysis of Weak Lensing on the Largest Scales

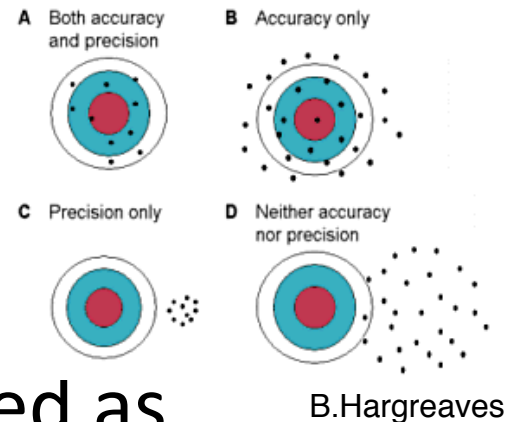
Alan Heavens

with Justin Alsing, Andrew Jaffe, Alina Kiessling, Ben Wandelt & Till Hoffmann



The opportunities and risks of surveys to test gravity

- Surveys such as Euclid and LSST will test the gravity law with exquisite *precision* (small error bars)
- Why should we be careful?
- *Inaccurate* results may be interpreted as new physics, when they should not be
- Incorrect analysis may lead to inaccuracies

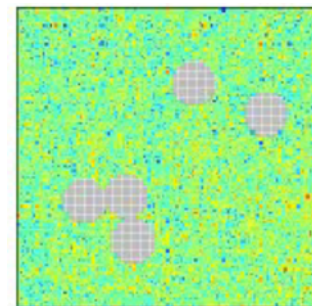
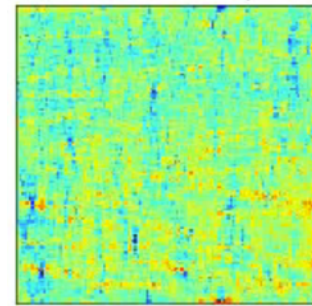
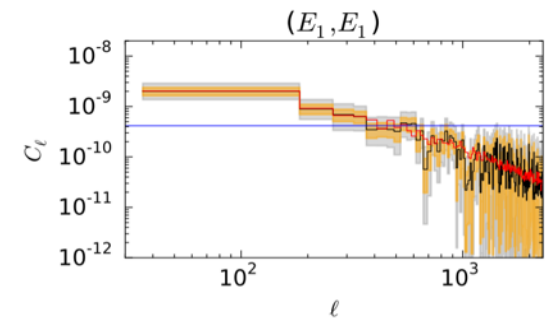


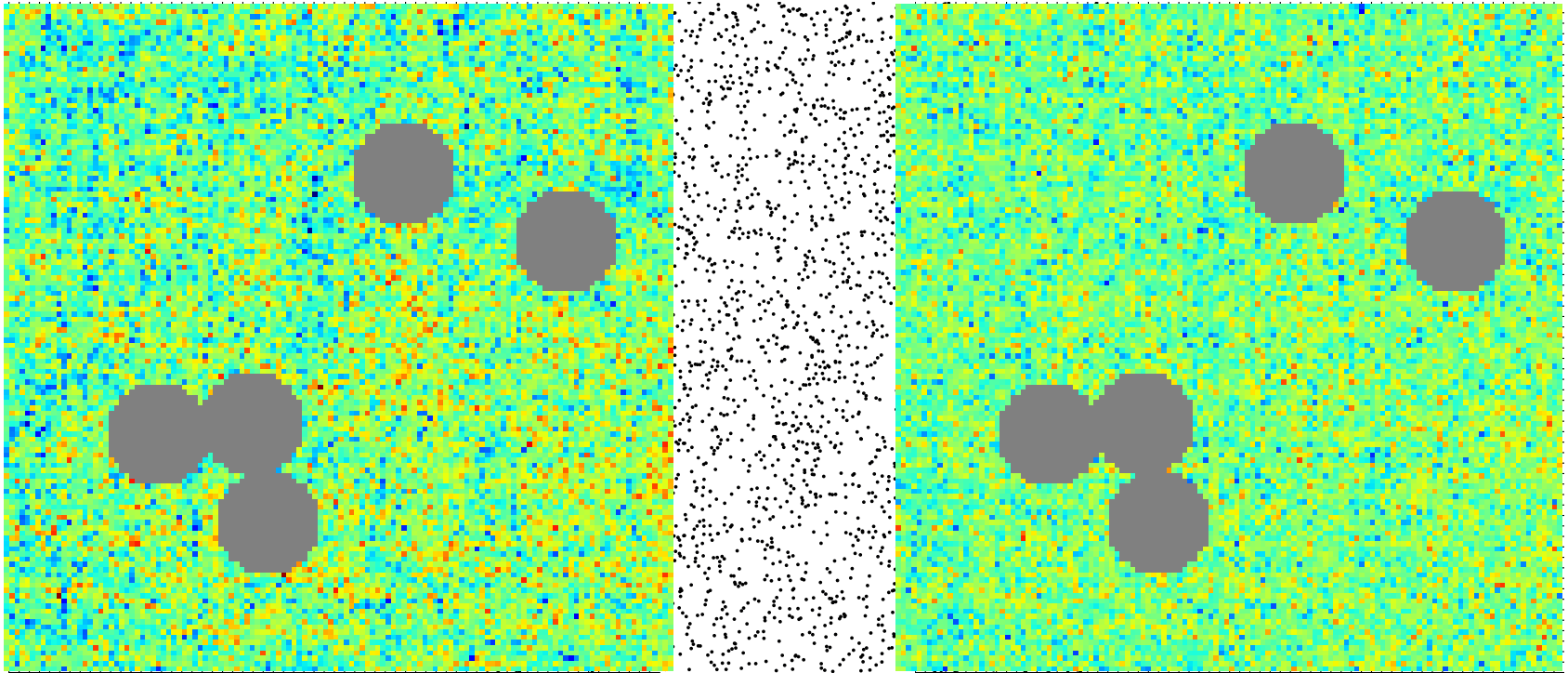
The goal of scientific inference

- Bayesian inference: given some new data, the posterior probability (of model parameters) encompasses all we know
- $P(\text{Parameters} \mid \text{Data, Model, Prior Information})^*$
- Complicated function - not analytic
- Alternative: *sample* from the posterior (cf MCMC)
- *If we can do it*, there is no reason not to do it this way
- * Also $P(\text{Model} \mid \text{Data})$

Bayesian Hierarchical Models

- Break the problem into steps:
- Parameters: Let $C =$ (various) power spectra
- \mathbf{s} = true shear map (many more parameters)
- Data: pixelised shear values $\mathbf{d} = \mathbf{s} + \mathbf{n}$ (noise)
- We typically want $p(C | \mathbf{d})$
- Conditional distributions, e.g. $p(\mathbf{s} | C)$, are often known





Credit: J. Alsing

Joint map-power spectrum inference

- Link between \mathbf{d} and C is the true map \mathbf{s}
- Natural to sample from C and \mathbf{s} jointly, conditioned on the data \mathbf{d} : $p(C, \mathbf{s} \mid \mathbf{d})$
- Marginalise over the map(s) \mathbf{s} to get $p(C \mid \mathbf{d})$
- Assume gaussian fields for large scales
- How to do this inverse problem?
- Consider the forward model:

TOMOGRAPHIC
SHEAR E/B-MODE
POWER SPECTRA

JEFFREYS
PRIOR

$P(\mathbf{C})$

\mathbf{C}

TOMOGRAPHIC
SHEAR FIELDS

$\mathbf{s}|\mathbf{C} \sim \mathcal{N}(\mathbf{0}, \mathbf{C})$
ELLIPTICITY NOISE

$P(\mathbf{s}|\mathbf{C})$

$\mathbf{d}|\mathbf{s} \sim \mathcal{N}(\mathbf{s}, \mathbf{N})$

\mathbf{N}

\mathbf{s}

NOISY SHEAR
MAPS

$P(\mathbf{d}|\mathbf{s}, \mathbf{N})$

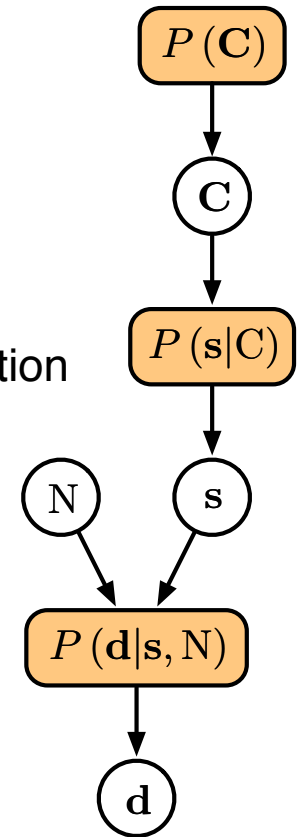
\mathbf{d}

Gibbs Sampling

$$\mathbf{C}^{i+1} \leftarrow P(\mathbf{C}|\mathbf{s}^i)$$

W^{-1} = Inverse Wishart distribution

$$\mathbf{s}^{i+1} \leftarrow P(\mathbf{s}|\mathbf{C}^{i+1}, \mathbf{N}, \mathbf{d})$$



WF = Wiener Filter:

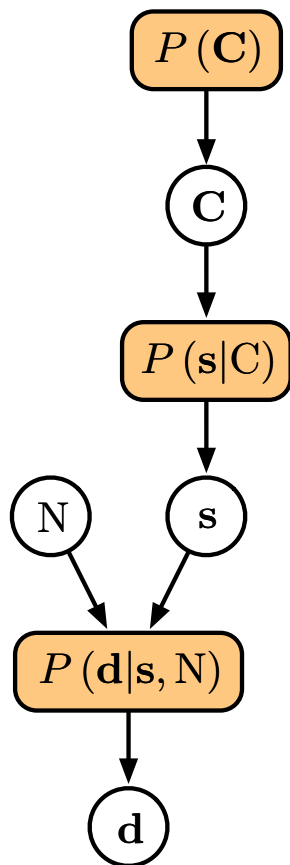
$$\mathbf{d}_{WF} = (\mathbf{C}^{-1} + \mathbf{N}^{-1})^{-1} \mathbf{N}^{-1} \mathbf{d}$$

$$\mathbf{C}_{WF} = (\mathbf{C}^{-1} + \mathbf{N}^{-1})^{-1}$$

Messenger fields

Elsner & Wandelt 2012, 2013

Jasche & Lavaux 2015



ISOTROPIC NOISE

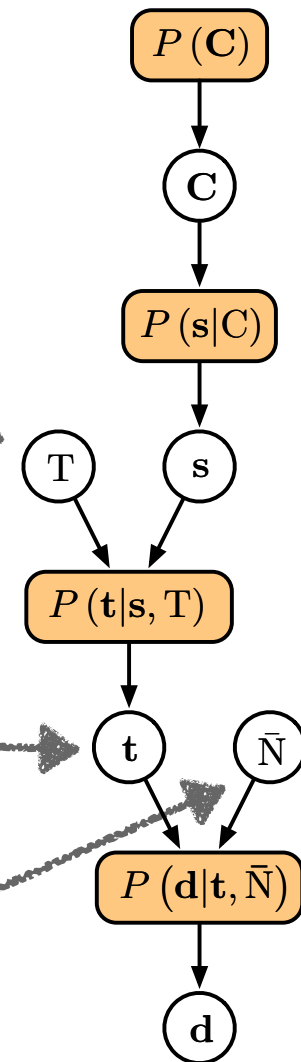
$$\mathbf{T} = \tau \mathbf{I}$$

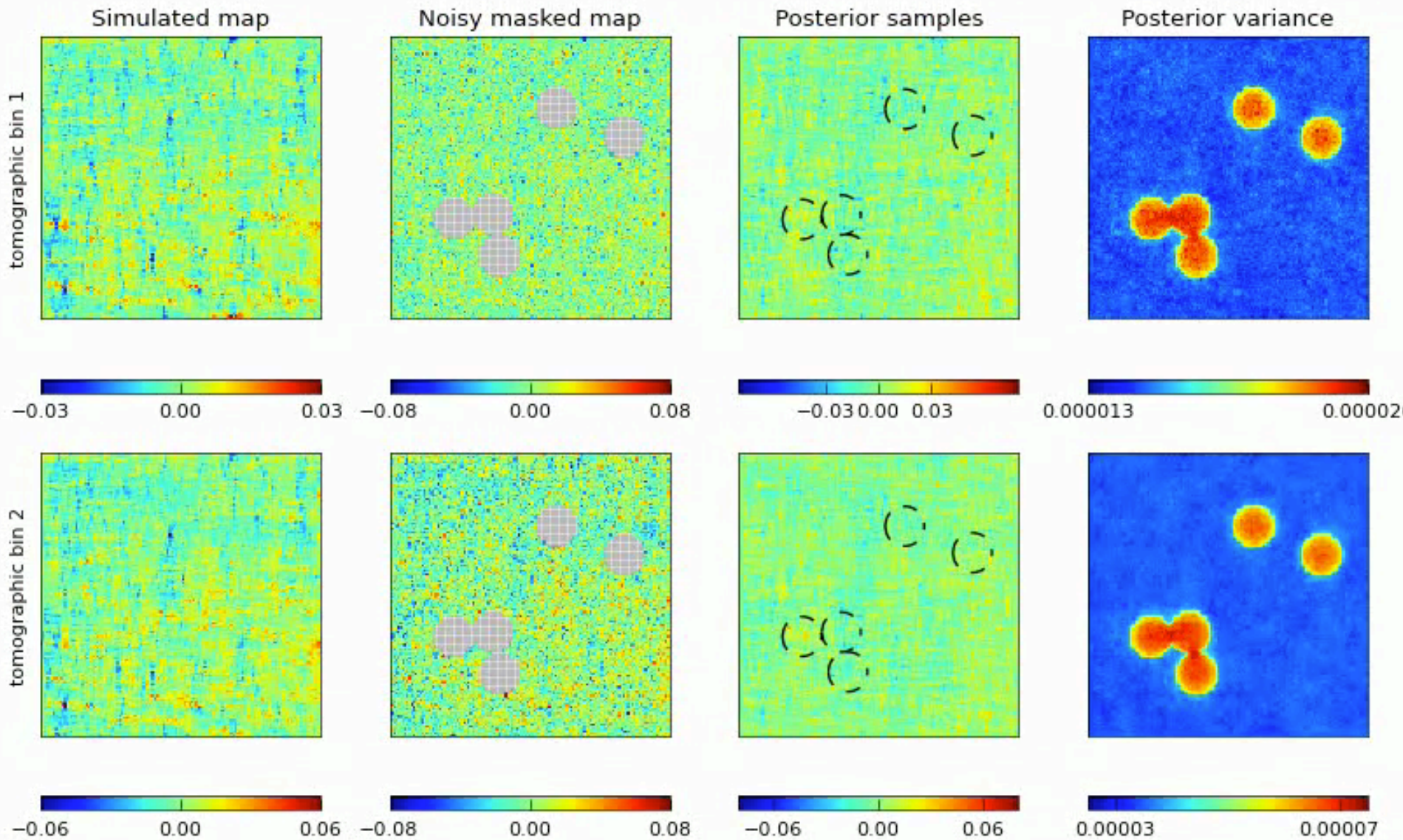


MESSENGER FIELD

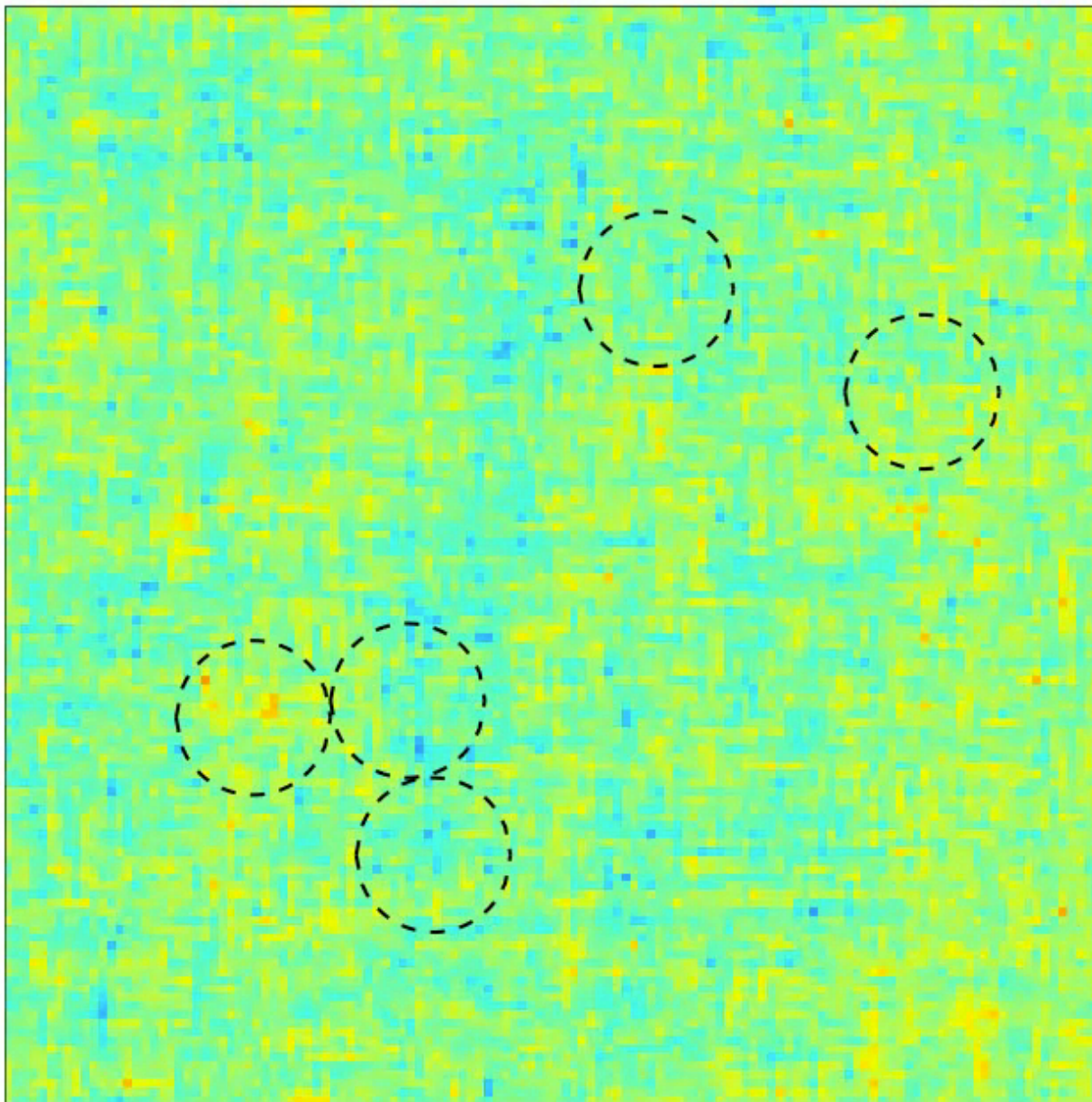
ANISOTROPIC NOISE

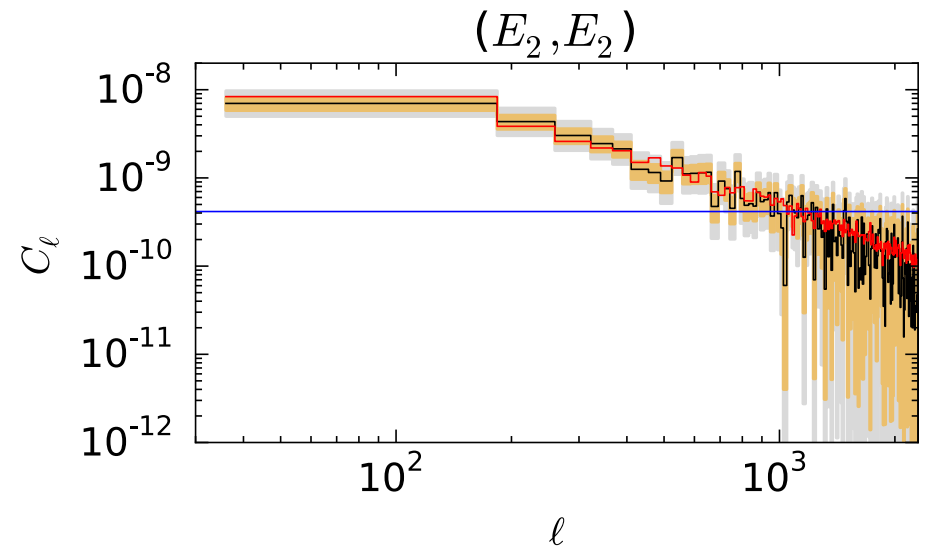
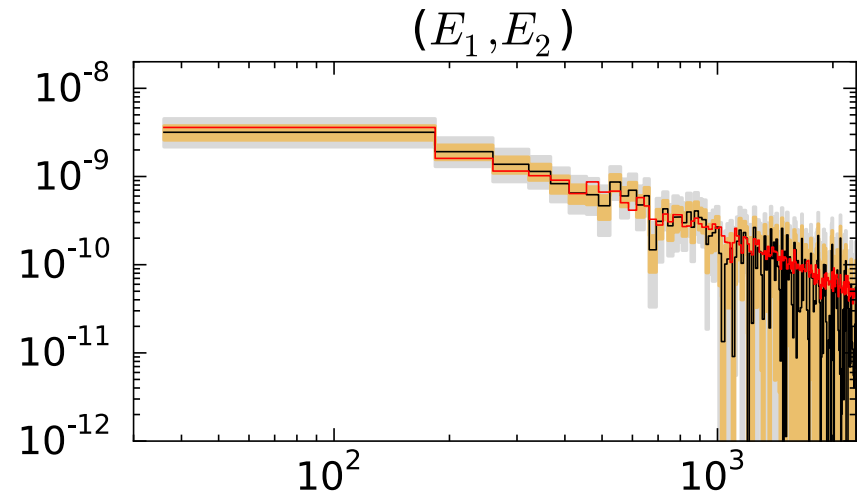
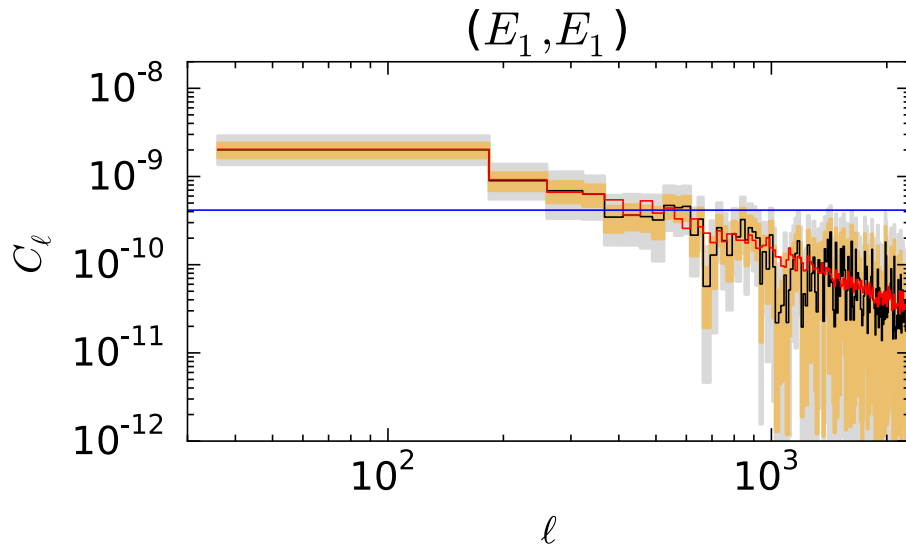
$$\bar{\mathbf{N}} = \mathbf{N} - \mathbf{T}$$



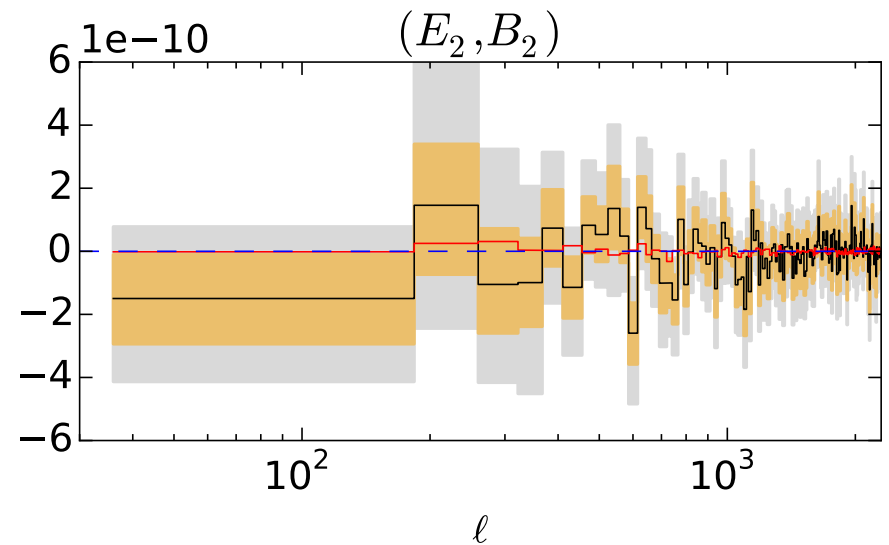
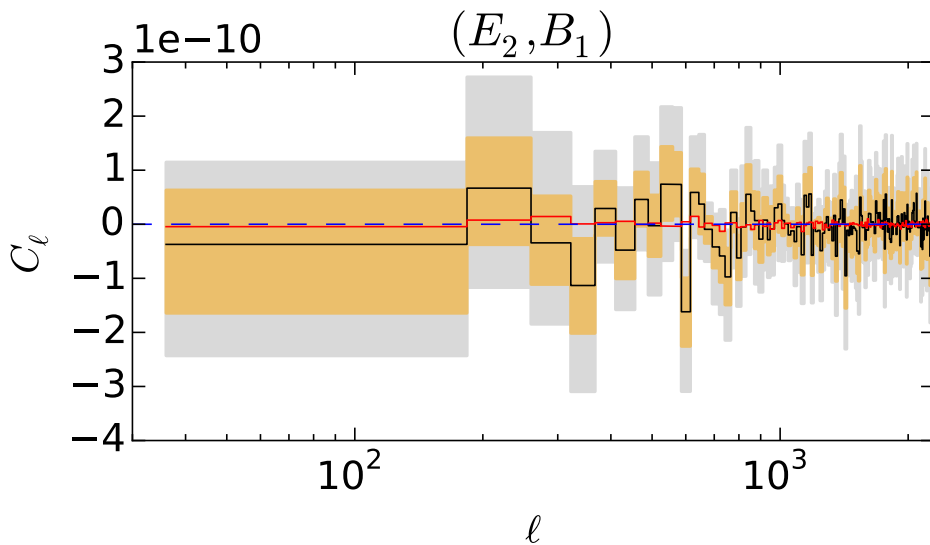
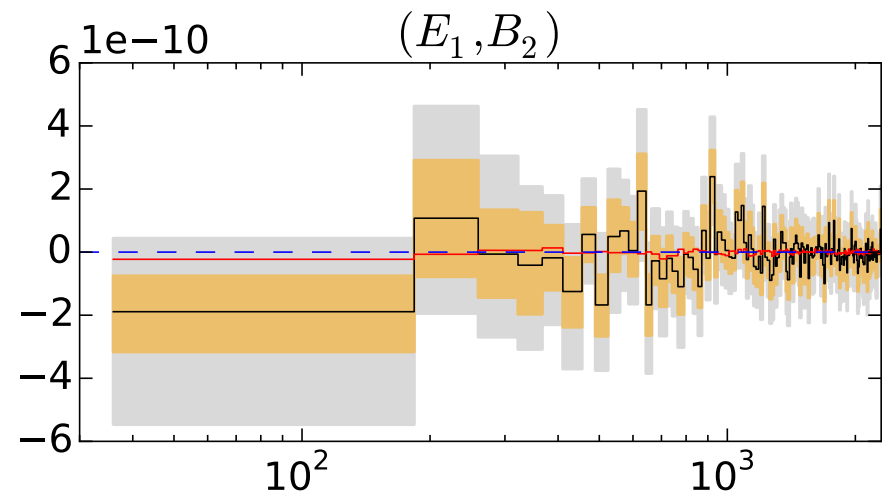
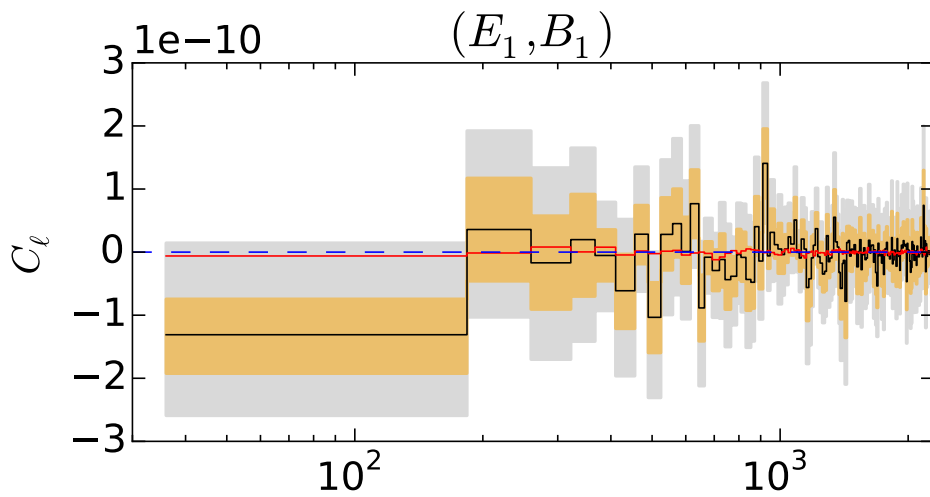


SUNGLASS simulations (Kiessling et al 2011)





E -modes are recovered,
well below the shot noise
at high- l



EB cross-power
 consistent with zero (as
 expected from parity)

Cosmology

- Sampling the power spectrum first has advantages:
- Gibbs sampling is efficient (gaussian fields)
- Further cosmological parameter inference is straightforward:

$$\boldsymbol{\theta} \sim P(\boldsymbol{\theta}|\mathbf{d}) = P(\mathbf{C}(\boldsymbol{\theta})|\mathbf{d}) \frac{P(\boldsymbol{\theta})}{P(\mathbf{C}(\boldsymbol{\theta}))}$$

- Different theoretical models can be investigated (notably intrinsic alignment contribution)

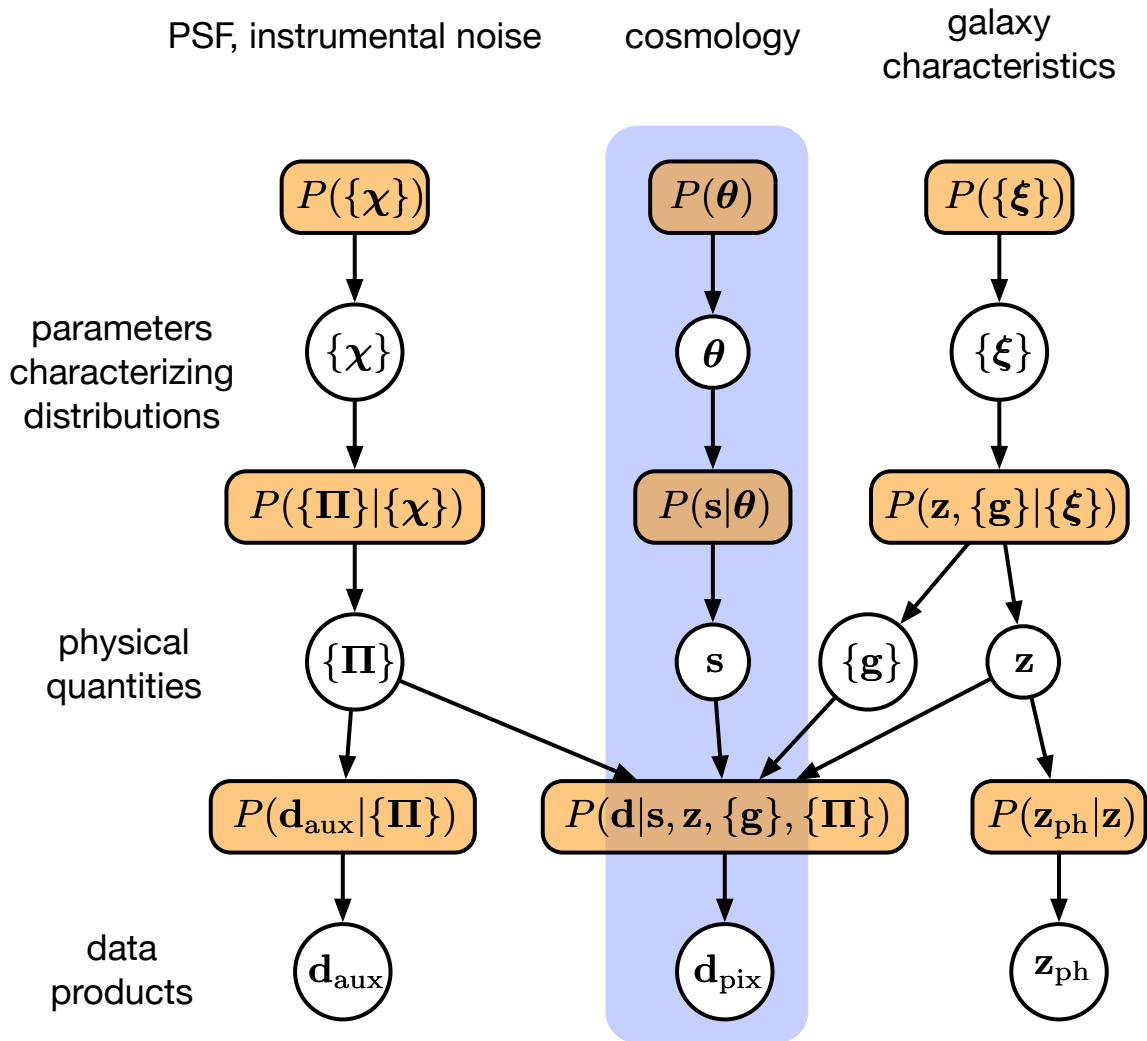
Feasibility

- 2 bin (128^2 pixels) tomography runs on quad-core desktop, generating ~ 4 million samples in a few days

$$\text{FFT, SPH} \sim n_{\text{pix}} \log(n_{\text{pix}}), \quad n_{\text{pix}}^{3/2}$$

- 10-bin tomography (Euclid-like survey) is probably feasible now with supercomputers

Global BHM:



Alsing, J., AFH, et al., 2015
Schneider M., et al., 2014

Conclusions

- Bayesian hierarchical models are the natural way to do principled Bayesian statistical inference from weak lensing
- Messenger fields now make it possible
- Joint map and power spectrum inference with $\sim 10^5$ parameters (or more) is feasible
- Masks and intrinsic alignments are easily included
- In progress: CFHTLenS analysis; non-gaussian likelihoods; possible Euclid pipeline
- Alsing et al. [arXiv:1505.07840](https://arxiv.org/abs/1505.07840)