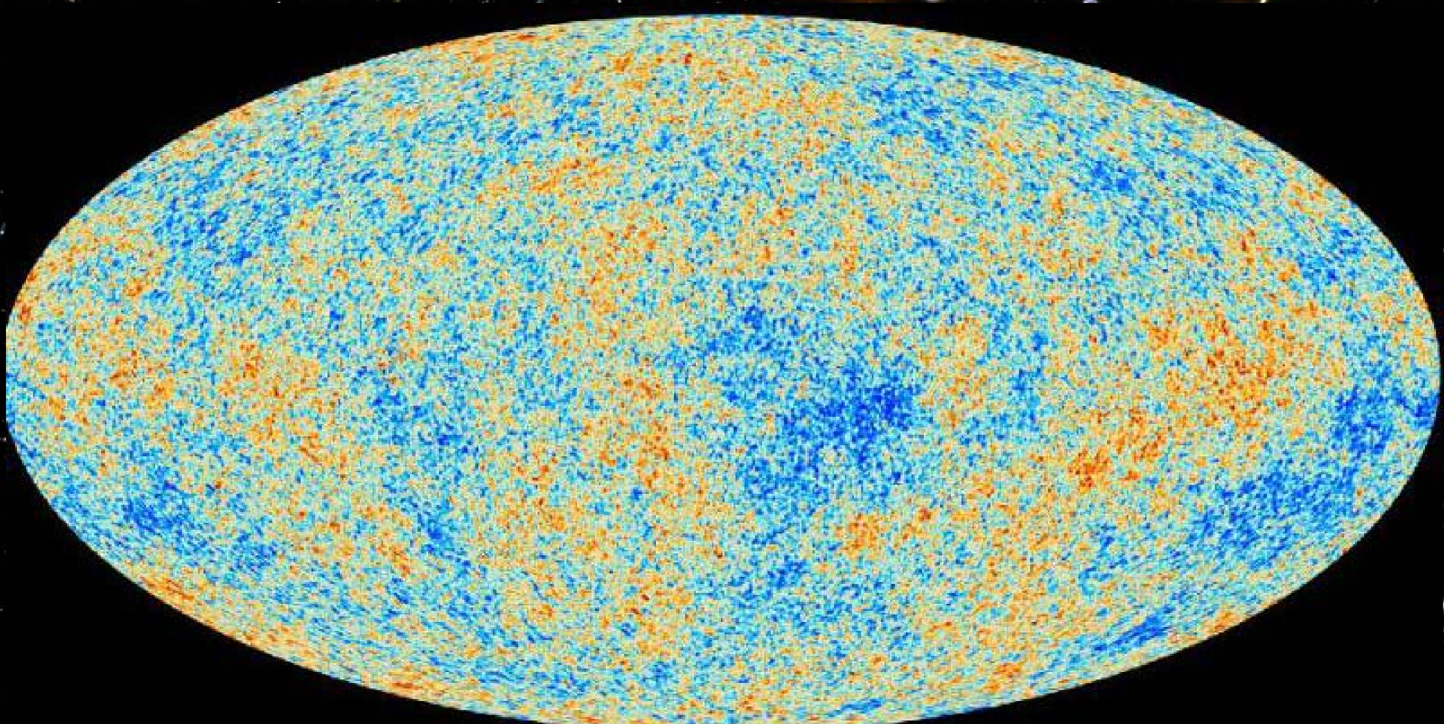
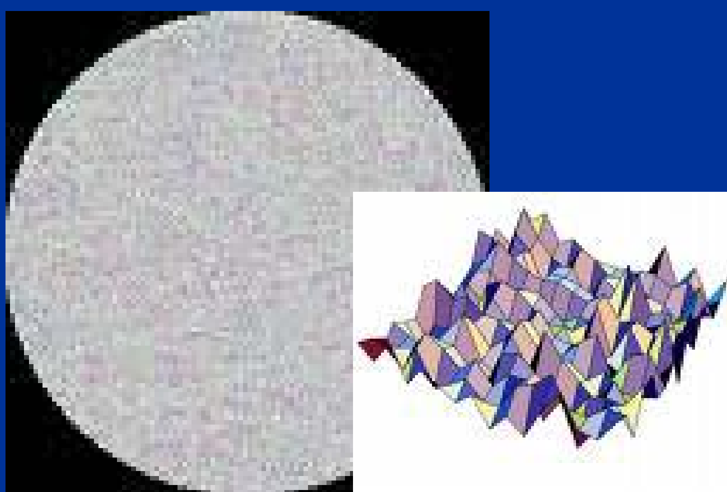
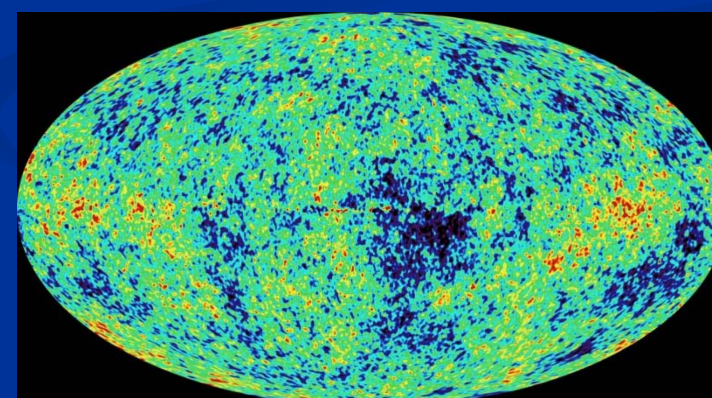
Can observations look back to the beginning of inflation ?



Primordial fluctuations

- inflaton field : χ
- primordial fluctuations of inflaton become observable in cosmic microwave background





Does inflation allow us to observe properties of quantum vacuum ?

at which time ?

Which primordial epoch do we see ?

Does the information stored in primordial fluctuations concern

- beginning of inflation

or

- horizon crossing

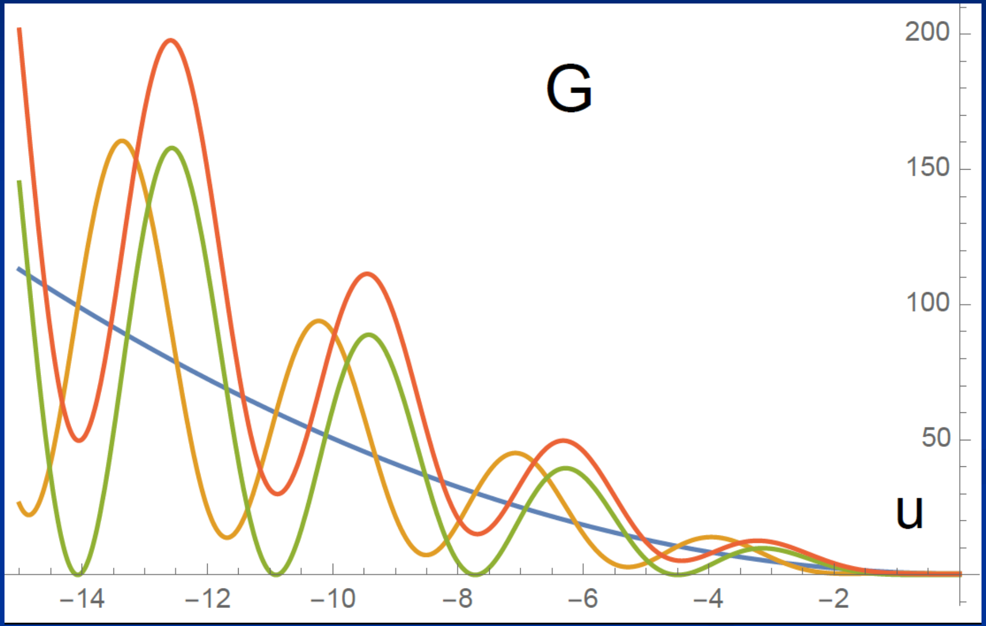
Initial conditions for correlation functions

set at beginning of inflation

- Is memory kept at the time of horizon crossing
- or do we find universal correlations , uniquely determined by properties of inflaton potential ?

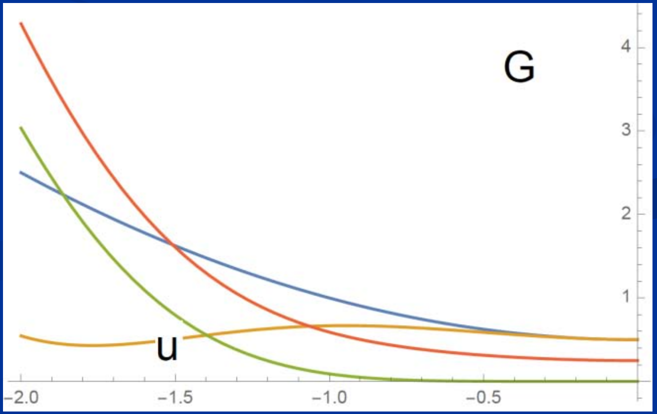
effective action for interacting inflaton coupled to gravity,
derivative expansion with up to two derivatives

correlation function for different initial conditions



u = k η

no loss of memory
at horizon crossing
(u = -1)



separate initial condition for every k-mode

amplitude not fixed

memory of initial spectrum

$$\Delta^2 = \frac{(A_p + 1)V}{24\pi^2 \epsilon M^4}$$

$$n_s = 1 + n_p - 6\epsilon + 2\eta$$

scalar correlation function

$$G(x,y) = \langle\tilde{\phi}(x)\tilde{\phi}(y)\rangle - \langle\tilde{\phi}(x)\rangle\langle\tilde{\phi}(y)\rangle$$

$$G(y,x) = G(x,y)$$

$$G(\eta,\vec{x};\eta',\vec{y}) = G(\vec{r},\eta,\eta')$$

$$G(\vec{r},\eta,\eta') = \int_k G(\vec{k},\eta,\eta')e^{i\vec{k}\vec{r}}$$

$$G(k,\eta) = G(k,\eta,\eta)$$

$$\Delta^2(k) \approx \frac{k^3H^2}{4\pi^2\dot{\varphi}^2}G(k,\eta)_{|hc} = A_s\left(\frac{k}{k_s}\right)^{n_s-1}$$

correlation function from quantum effective action

- no operators needed
- no explicit construction of vacuum state
- no distinction between quantum and classical fluctuations
- one relevant quantity : correlation function

quantum theory as functional integral

$$Z[j] = \int \mathcal{D}\tilde{\phi} \exp\left(- S + \int_x J\tilde{\phi} \right)$$

$$S = \int_x e L[\tilde{\phi}, e_\mu^m].$$

background geometry given by vierbein or metric

$$e = \det(e_\mu^m)$$

$$\bar{g}_{\mu\nu} = e_\mu^m e_\nu^n \delta_{mn}$$

homogeneous and isotropic cosmology

$$e_k^m = a(\eta)\delta_k^m \ , \ e_0^m = ia(\eta)\delta_0^m$$

$$e = \bar{e}a^4$$

M: $\bar{e} = i$

E: $\bar{e} = 1$

quantum effective action

$$W[J] = \ln Z[J]$$

$$\frac{\partial W}{\partial J(x)} = \langle \phi(x) \rangle = \phi(x)$$

$$\Gamma[\phi] = -W[J] + \int_x J\phi$$

exact field equation

$$\frac{\partial \Gamma}{\partial \phi(x)} = J(x)$$

exact propagator equation

$$\Gamma^{(2)} W^{(2)} = 1$$

$$\Gamma^{(2)} = \partial^2 \Gamma / \partial \phi(x) \partial \phi(y)$$

$$W^{(2)} = \partial^2 W / \partial J(x) \partial J(y)$$

correlation function and quantum effective action

$$G(x,y) = \langle\tilde{\phi}(x)\tilde{\phi}(y)\rangle - \langle\tilde{\phi}(x)\rangle\langle\tilde{\phi}(y)\rangle = \frac{\partial^2 W}{\partial J(x)\partial J(y)}$$

$$G(x,y) = \left(\frac{\partial^2 \Gamma}{\partial\phi(x)\partial\phi(y)}\right)^{-1}$$

propagator equation for interacting scalar field in homogeneous and isotropic cosmology

$$\tilde{D}_\eta G(k,\eta,\eta') = -\frac{i}{a^2}\delta(\eta-\eta'),$$

$$\tilde{D}_\eta = \partial_\eta^2 + 2\mathcal{H}\partial_\eta + k^2 + m^2 a^2$$

$$\mathcal{H}(\eta) = \frac{\partial \ln a(\eta)}{\partial\eta}\;,\;\; m^2(\eta) = \frac{\partial^2\mathcal{U}}{\partial\varphi^2}_{|\bar{\varphi}(\eta)}$$

general solution and mode functions

$$G_>(k,\eta,\eta') = \frac{\alpha(k)+1}{2} w_k^-(\eta) w_k^+(\eta') + \frac{\alpha(k)-1}{2} w_k^+(\eta) w_k^-(\eta') + \zeta(k) w_k^+(\eta) w_k^+(\eta') + \zeta^*(k) w_k^-(\eta) w_k^-(\eta')$$

$$\tilde{D}_\eta \psi_k(\eta) = 0 \ , \ \ \psi_k(\eta) = c_+ w_k^+(\eta) + c_- w_k^-(\eta)$$

$$w_k^-(\eta) = \left(w_k^+(\eta)\right)^*$$

$$\partial_\eta \left[w_k^-(\eta) w_k^+(\eta') - w_k^+(\eta) w_k^-(\eta') \right]_{|\eta=\eta'} = -\frac{i}{a^2(\eta)}$$

Bunch – Davies vacuum

α = 1 , ζ = 0 for all k

$$
\begin{aligned}
G_>(k,\eta,\eta') &= \frac{\alpha(k)+1}{2} w_k^-(\eta) w_k^+(\eta') \\
&+ \frac{\alpha(k)-1}{2} w_k^+(\eta) w_k^-(\eta') \\
&+ \zeta(k) w_k^+(\eta) w_k^+(\eta') + \zeta^*(k) w_k^-(\eta) w_k^-(\eta')
\end{aligned}
$$

initial conditions

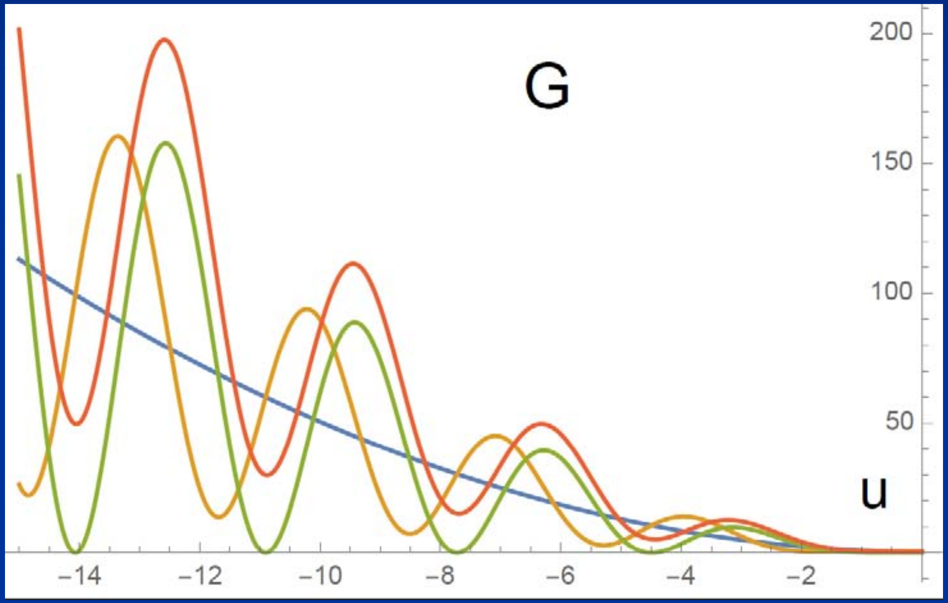
$$G_>(k,\eta,\eta') = \sum_i p_i \psi_k^{(i)}(\eta)\left(\psi_k^{(i)}(\eta')\right)^*$$

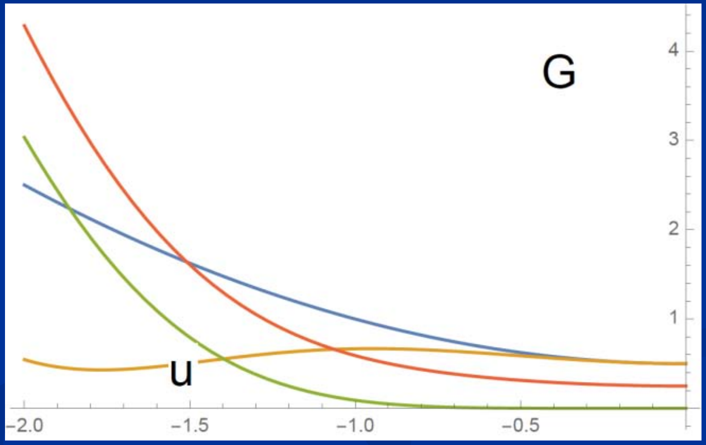
mixed state : positive probabilities p_i

$$\alpha(k) \geq 1\ ,\ \ \beta^2(k) + \gamma^2(k) \leq \alpha^2(k) - 1$$

$\beta^2 + \gamma^2 = 4\,\zeta\,\zeta^*$

absence of loss of memory of initial correlations





amplitude for each k-mode is free integration constant !

memory of initial spectrum

α = 1 + A_p

$$\Delta^2 = \frac{(A_p+1)V}{24\pi^2\epsilon M^4}$$

example :

$$A_p(k) = \frac{A}{2}\left(1-\frac{2}{\pi}arctg\ x\right)\ ,\ x=\Delta^{-1}\ln\left(\frac{k}{k_0}\right)$$

$$\tilde{p}M\frac{a_{in}}{a_{hc}} = H_0, \quad \tilde{p}=\frac{H_0 a_{hc}}{M a_{in}} = e^{N_{in}}\frac{H_0}{M}$$

$$x = \Delta^{-1}\left(N_{in}-\ln\left(\frac{M}{H_0}\right)\right)$$

spectral index :

$$n_s = 1 + n_p - 6\epsilon + 2\eta$$

$$\begin{aligned} n_p &= \frac{\partial \ln\left(1+A_p(k)\right)}{\partial \ln k} \\ &= -\left[\Delta(1+x^2)\left(\frac{\pi}{2}-arctg\ x+\frac{\pi}{A}\right)\right]^{-1} \end{aligned}$$

$$n_p(x=0(1)) = -\frac{2}{\pi\Delta}\left(1+\frac{2(4)}{A}\right)^{-1}$$

predictivity of inflation ?

- initial spectrum at beginning of inflation : gets only processed by inflaton potential
- small tilt in initial spectrum is not distinguishable from small scale violation due to inflaton potential
- long duration of inflation before horizon crossing of observable modes : one sees UV-tail of initial spectrum . If flat , predictivity retained !

loss of memory beyond approximation of derivative expansion of effective action ?

- seems likely for long enough duration of inflation before horizon crossing of observable modes
- sufficient that modes inside horizon reach Lorentz invariant correlation for flat space
- not yet shown
- equilibration time unknown

vacuum in cosmology

- simply the (averaged) state of the Universe
- result of time evolution
- minimal “particle number” ?

  depends on definition of particle number

  as observable

end

propagator equation in homogeneous and isotropic cosmology

$$\tilde{D}_\eta G(k,\eta,\eta') = -\frac{i}{a^2}\delta(\eta-\eta'),$$
$$\tilde{D}_\eta = \partial_\eta^2 + 2\mathcal{H}\partial_\eta + k^2 + m^2 a^2$$

$\eta > \eta'$: $G_> = G_s + G_a$

$$G_s(\eta',\vec{y};\eta,\vec{x}) = G_s(\eta,\vec{x};\eta',\vec{y}),$$
$$G_a(\eta',\vec{y};\eta,\vec{x}) = -G_a(\eta,\vec{x};\eta',\vec{y})$$

$$\tilde{D}_\eta G_s = 0 \ , \ \tilde{D}_\eta G_a = 0 \ , \ \partial_\eta G_{a_{|\eta=\eta'}} = -\frac{i}{2a^2}$$

evolution equation for equal time correlation function

$$
\begin{aligned}
\langle \varphi(\eta,\vec{k})\varphi^*(\eta,\vec{k}')\rangle_c &= G_{\varphi\varphi}(k,\eta)\delta(k-k') \\
Re\big(\langle \partial_\eta\varphi(\eta,\vec{k})\varphi^*(\eta,\vec{k}')\rangle_c\big) &= G_{\pi\varphi}(k,\eta)\delta(k-k') \\
\langle \partial_\eta\varphi(\eta,\vec{k})\partial_\eta\varphi^*(\eta,\vec{k}')\rangle_c &= G_{\pi\pi}(k,\eta)\delta(k-k')
\end{aligned}
$$

$$
\tilde{G}_{\varphi\varphi} = 2a^2 k G_{\varphi\varphi}\ ,\ \tilde{G}_{\pi\varphi} = 2a^2 G_{\pi\varphi}\ ,\ \tilde{G}_{\pi\pi} = \frac{2a^2}{k} G_{\pi\pi}
$$

$$
\begin{aligned}
\partial_u \tilde{G}_{\varphi\varphi} &= -\frac{2\tilde{h}}{u}\tilde{G}_{\varphi\varphi} + 2\tilde{G}_{\pi\varphi}, \\
\partial_u \tilde{G}_{\pi\varphi} &= \tilde{G}_{\pi\pi} - \left(1+\frac{\hat{m}^2}{u^2}\right)\tilde{G}_{\varphi\varphi}, \\
\partial_u \tilde{G}_{\pi\pi} &= \frac{2\tilde{h}}{u}\tilde{G}_{\pi\pi} - 2\left(1+\frac{\hat{m}^2}{u^2}\right)\tilde{G}_{\pi\varphi}
\end{aligned}
$$

u = k η

massless scalar in de Sitter space :

$$
\tilde{G}_{\varphi\varphi} = \alpha(k)\left(1+\frac{1}{u^2}\right) + \beta(k)\left[\left(1-\frac{1}{u^2}\right)\cos(2u) + \frac{2}{u}\sin(2u)\right] + \gamma(k)\left[\frac{2}{u}\cos(2u) - \left(1-\frac{1}{u^2}\right)\sin(2u)\right]
$$