A Ghost Story II

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in collaboration with Y. Akrami, L. Amendola, and H. Nersisyan

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The Cosmological Constant ...

- is technically unnatural
- might not be quantum mechanically consistent
- is theoretically allowed because of ...?
- does not challenge GR

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Let K be a tensor that naturally depends on a pseudo-Riemannian metric and

- has rank 2,
- is divergence-free,
- is of 2nd order in the derivatives of the coefficients of $g_{\mu\nu}$,
- is linear in these 2nd derivatives,
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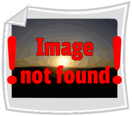

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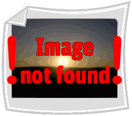

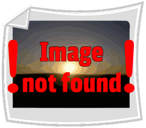

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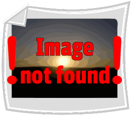

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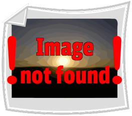

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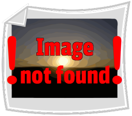

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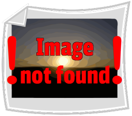

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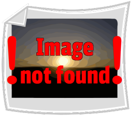

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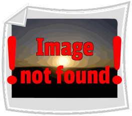

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Theorem of Navarro-Sancho

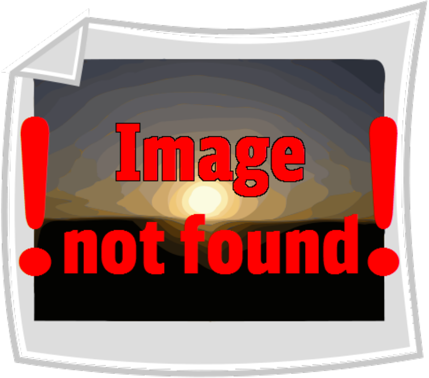
Let K be a tensor that naturally depends on a pseudo-Riemannian metric and

- has rank 2,
- is divergence-free,
- has weight $w > -2$.

Then K is, up to constant factors, either the Einstein tensor (with weight $w = 0$) or the metric (with weight $w = 2$).

Navarro & Sancho (2008)

Why are we so scared about ghosts?



(credit: Luisa Jaime)

Ostrogradsky's theorem

nth derivatives in the Lagrangian will usually introduce $2n$th order der. in the EOM:

$$\frac{\mathrm{d}^n}{\mathrm{d}t^n}\frac{\partial L}{\partial x^{(n)}} - \dots + \frac{\mathrm{d}^2}{\mathrm{d}t^2}\frac{\partial L}{\partial \ddot{x}} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{x}} + \frac{\partial L}{\partial x} = 0$$

example: $n = 2$

$$H = P_1\dot{Q}_1 + P_2\dot{Q}_2 - L$$

with

$$Q_1 = x, \quad Q_2 = \dot{x}, \quad P_1 = \frac{\partial L}{\partial \dot{x}} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \ddot{x}}, \quad P_2 = \frac{\partial L}{\partial \ddot{x}}$$

$\Rightarrow$ **Hamiltonian is unbounded from below!**

unbounded Hamiltonian from below

- could introduce classical instabilities
- classical perturbative solution might hide negative energy solutions
- no stable vacuum!
- $\Rightarrow$ ghosts will **immediately rule out** a theory

Vacuum decay

simple example:

$$S_{int} = \lambda \int \mathrm{d}^D x e^{-x_0^2/T^2} \phi^2(x) \psi^2(x)$$

where $\Delta t \sim 2T$ denotes time since the creation of vacuum

$$\Rightarrow \qquad \mathcal{P} = \int \mathrm{d}^4 p_1 \int \mathrm{d}^4 p_2 \int \mathrm{d}^4 k_1 \int \mathrm{d}^4 k_2 |\mathcal{M}|^2$$

and

$$\mathcal{M} = (2\pi)^D \lambda \delta^{D-1} (P + K) \frac{T}{2\sqrt{\pi}} e^{-\left(P^0 + K^0\right)^2 T^2/4}$$

Vacuum decay

simple example:

$$S_{int} = \lambda \int \mathsf{d}^D x e^{-x_0^2/T^2} \phi^2(x)\psi^2(x)$$

$$\mathcal{P} \propto \int \mathsf{d}^4 P ... \propto \int \mathsf{d}^3 \vec{v} \sqrt{1+\vec{v}^2} \int \mathsf{d}s s ...$$

with $v_\mu \equiv -P_\mu/\sqrt{s}$

Avoiding an **instantaneous** decay of the vacuum requires a modification of the momentum integral!

One possibility:

Breaking Lorentz invariance

Kaplan & Sundrum (2005)

Garriga & Vilenkin (2013)

Vacuum decay

simple example:

$$S_{int} = \lambda \int \mathrm{d}^D x e^{-x_0^2/T^2} \phi^2(x)\psi^2(x)$$

$$\Rightarrow \mathcal{P} \sim \lambda^2 \mathcal{E}^2 s_{\mathsf{max}}$$

Kaplan & Sundrum (2005)

Massive Gravity with (friendly) Ghosts

$$S = M_P^2 \int \mathsf{d}^4 x \sqrt{-g} \left[R + 2m^2 \left(\left[\sqrt{g^{-1}\eta} \right] - \frac{1}{2} \left(\alpha(t) - 1 \right) \left(\left[\sqrt{g^{-1}\eta} \right]^2 - \left[\sqrt{g^{-1}\eta} \right] \right) \right) \right]$$

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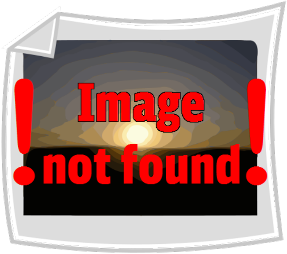
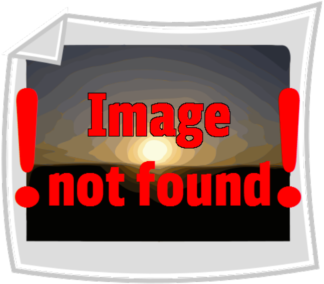
- lowest order interactions that introduce BD ghost
- has ghost-free limit
- allows for dynamical FLRW solutions
- additional features, e.g. particle production

Conclusions





- Ghosts rule out all Lorentz invariant theories!
- broken Lorentz invariance: vacuum might not decay instantaneously
- enlarges the class of viable theories of modified gravity