Relativistic effects in the galaxy number counts bispectrum

Giovanni Marozzi

University of Geneva
Department of Theoretical Physics and Center for Astroparticle Physics

Gravity at the Largest Scales
ITP, Heidelberg, 28 October 2015

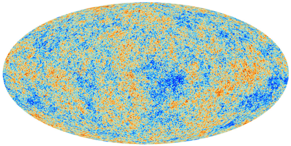
Mainly based on: E. Di Dio, R. Durrer, GM, F. Montanari, JCAP 12 (2014) 017;
E. Di Dio, R. Durrer, GM, F. Montanari, 1510.04202 [astro-ph.CO].

The Problem

Cosmology has entered a precision era.

The present and future main sources of data are:

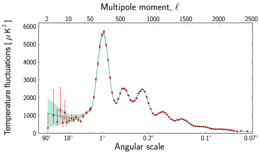
- CMB anisotropies
  (2 dimensional dataset).



CMB sky as seen by Planck

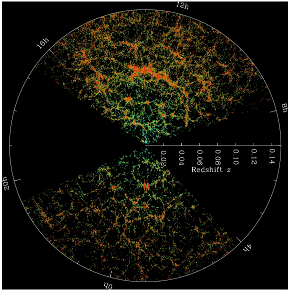
Link between data and models made mostly using linear perturbation theory.

$D_l = l(l+1)C_l/(2\pi)$
Planck Collaboration: Planck 2013 results XV
CMB power spectra and likelihood



The Problem

- Large scale structure observations (3 dimensional dataset).



Sloan Digital Sky Team

For LSS, on intermediate to small scales, non-linearities become important.

⇓

Much more information, but analysis more complicated.

We need not only accurate observations, but also an accurate model!

Observing the large scale structure of the Universe

- All observations are made over the past light-cone with redshift and incoming photons direction as observable coordinates.
- Both our observable coordinates and the observed volume are distorted by the presence of inhomogeneities.
- Standard Newtonian effects are usually described in k-space, where the result depends not only on the observations but also on the cosmological model assumed which relates redshifts and angles to distances.
- Relativistic lensing effects involve integral over the backward light-cone and their translation in k-space is not straightforward.
- We will report results from perturbation theory in ℓ-space and redshift space so that they can be directly compared with observations without any assumptions on the cosmology.

For 2-point correlation functions $\Rightarrow$ We have to go beyond Newtonian gravity!

For 3-point correlation functions $\Rightarrow$ We have to go beyond Newtonian gravity and beyond linear theory!

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Geodesic light-cone coordinates

An adapted light-cone coordinate system $x^\mu = (w, \tau, \tilde{\theta}^a)$, $a = 1, 2$ can be defined by the following metric (Gasperini, GM, Nugier, Veneziano (2011)):

$$ds^2 = \Upsilon^2 dw^2 - 2\Upsilon dw d\tau + \gamma_{ab}(d\tilde{\theta}^a - U^a dw)(d\tilde{\theta}^b - U^b dw) \; ; \qquad a, b = 1, 2 \; .$$

This metric depends on six arbitrary functions (Υ, the two-dimensional vector U^a and the symmetric tensor γ_{ab}) and is completely gauge fixed.

w is a null coordinate , $\partial_\mu \tau$ defines a geodesic flow

$k^\mu = g^{\mu\nu}\partial_\nu w = g^{\mu w} = -\delta^\mu_\tau \Upsilon^{-1}$ null geodesics connecting sources and observer

$$\Downarrow$$

Photons travel at constant w and $\tilde{\theta}^a$

The exact non-perturbative redshift is given by

$$1 + z_s = \frac{(k^\mu u_\mu)_s}{(k^\mu u_\mu)_o} = \frac{(\partial^\mu w \partial_\mu \tau)_s}{(\partial^\mu w \partial_\mu \tau)_o} = \frac{\Upsilon(w_o, \tau_o, \tilde{\theta}^a)}{\Upsilon(w_o, \tau_s, \tilde{\theta}^a)}$$

where the subscripts “o” and “s” denote, respectively, a quantity evaluated at the observer and source space-time position.

Galaxy Number Counts

Galaxy Number Counts= number N of sources (galaxies) per solid angle and redshift.
The fluctuation of the galaxy number counts in function of observed redshift and direction is given by

$$\Delta(\mathbf{n}, z) \equiv \frac{N(\mathbf{n}, z) - \langle N \rangle (z)}{\langle N \rangle (z)} ,$$

where

$$N(\mathbf{n}, z) = \rho(\mathbf{n}, z) V(\mathbf{n}, z) .$$

Considering the density and volume fluctuations per redshift bin dz and per solid angle $d\Omega$

$$V(\mathbf{n}, z) = \bar{V}(z)\left(1 + \frac{\delta V^{(1)}}{\bar{V}} + \frac{\delta V^{(2)}}{\bar{V}}\right)$$

$$\rho(\mathbf{n}, z) = \bar{\rho}(z)\left(1 + \delta^{(1)} + \delta^{(2)}\right) ,$$

we can give the directly observed number fluctuations

$$\Delta(\mathbf{n}, z) = \left[\delta^{(1)} + \frac{\delta V^{(1)}}{\bar{V}} + \delta^{(1)}\frac{\delta V^{(1)}}{\bar{V}} + \delta^{(2)} + \frac{\delta V^{(2)}}{\bar{V}} - \langle \delta^{(1)}\frac{\delta V^{(1)}}{\bar{V}}\rangle - \langle \delta^{(2)} \rangle - \langle \frac{\delta V^{(2)}}{\bar{V}} \rangle\right]$$

Volume Perturbation

The 3-dimensional volume element dV seen by a source with 4-velocity u^μ is

$$dV = \sqrt{-g}\epsilon_{\mu\nu\alpha\beta}u^\mu dx^\nu dx^\alpha dx^\beta \,.$$

In terms of the observed quantities (z, θ_o, ϕ_o)

$$dV = \sqrt{-g}\epsilon_{\mu\nu\alpha\beta}u^\mu \frac{\partial x^\nu}{\partial z}\frac{\partial x^\alpha}{\partial \theta_s}\frac{\partial x^\beta}{\partial \phi_s}\left|\frac{\partial(\theta_s, \phi_s)}{\partial(\theta_o \phi_o)}\right| dz d\theta_o d\phi_o \equiv v(z, \theta_o, \phi_o)\, dz d\theta_o d\phi_o \,.$$

Going to GLC we then have

$$dV = -\sqrt{-g}u^w \frac{\partial \tau}{\partial z} dz d\theta_o d\phi_o \,.$$

and

$$dV = \sqrt{|\gamma|}\left(-\frac{d\tau}{dz}\right) dz d\theta_o d\phi_o \,, \qquad \text{or} \qquad v = \sqrt{|\gamma|}\left(-\frac{d\tau}{dz}\right)$$

This is a non-perturbative expression for the volume element at the source in terms of the observed redshift and the observation angles in GLC gauge.

If we would know $\rho(\mathbf{n}, z)$ non-perturbatively we could write the number counts in an exact way in GLC.

Coordinates Trasformation

Let us consider a stochastic background of scalar perturbations on a conformally flat FLRW space-time to describe the inhomogeneities of our Universe at large scale.

Using spherical coordinates ($y^\mu = (\eta, r, \theta, \phi)$) in the Poisson gauge (PG) we have

$$g^{\mu\nu}_{NG} = a^{-2}(\eta)\,\mathrm{diag}\left(-1+2\Phi, 1+2\Psi, (1+2\Psi)\gamma_0^{ab}\right)$$

where $\gamma_0^{ab} = \mathrm{diag}\left(r^{-2}, r^{-2}\sin^{-2}\theta\right)$, $\Phi = \Psi^{(1)} + \Phi^{(2)} - 2(\Psi^{(1)})^2$ and $\Psi = \Psi^{(1)} + \Psi^{(2)} + 2(\Psi^{(1)})^2$.

To use the previous results we have to re-express this metric in GLC form. We define the coordinates transformation using

$$g^{\rho\sigma}_{GLC}(x) = \frac{\partial x^\rho}{\partial y^\mu}\frac{\partial x^\sigma}{\partial y^\nu} g^{\mu\nu}_{NG}(y)$$

and imposing the following boundary conditions

- Non-singular transformation around the observer position at $r = 0$.
- The two-dimensional spatial section $r =$ const is locally parametrized at the observer position by standard spherical coordinates.

Cosmological Observables: redshift

The redshift up to second order in perturbation theory is

$$1+z=\frac{a(\eta_o)}{a(\eta_s)}\left[1+\delta^{(1)}z+\delta^{(2)}z\right]$$

with

$$\delta z^{(1)} = -\partial_r v_s^{(1)} - \Psi_s^{(1)} - 2\int_{\eta_s}^{\eta_o} d\eta'\,\partial_{\eta'}\Psi^{(1)}\left(\eta'\right)$$

$$\delta z^{(2)} = -\partial_r v_s^{(2)} - \Phi_s^{(2)} - \int_{\eta_s}^{\eta_0} d\eta'\,\partial_{\eta'}\left[\Phi^{(2)}+\Psi^{(2)}\right]\left(\eta'\right) + \frac{1}{2}\left(\partial_r v_s\right)^2 + \frac{1}{2}\left(\Psi_s\right)^2$$

$$+\left(-v_{||s}-\Psi_s\right)\left(-\Psi_s - 2\int_{\eta_s}^{\eta_o} d\eta'\,\partial_{\eta'}\Psi\left(\eta'\right)\right) + \frac{1}{2}\partial^a v_s\partial_a v_s + 2a\,\partial^a v_s\partial_a\int_{\eta_s}^{\eta_o} d\eta'\,\Psi\left(\eta'\right)$$

$$+4\int_{\eta_s}^{\eta_0} d\eta'\left[\Psi\left(\eta'\right)\partial_{\eta'}\Psi\left(\eta'\right) + \partial_{\eta'}\Psi\left(\eta'\right)\int_{\eta'}^{\eta_o} d\eta''\,\partial_{\eta''}\Psi\left(\eta''\right)\right.$$

$$\left.+\Psi\left(\eta'\right)\int_{\eta'}^{\eta_o} d\eta''\,\partial^2_{\eta''}\Psi\left(\eta''\right) - \gamma_0^{ab}\partial_a\left(\int_{\eta'}^{\eta_o} d\eta''\,\Psi\left(\eta''\right)\right)\partial_b\left(\int_{\eta'}^{\eta_o} d\eta''\,\partial_{\eta''}\Psi\left(\eta''\right)\right)\right]$$

$$+2\partial_a\left(\partial_r v_s + \Psi_s\right)\int_{\eta_s}^{\eta_o} d\eta'\,\gamma_0^{ab}\partial_b\int_{\eta'}^{\eta_o} d\eta''\,\Psi\left(\eta''\right)$$

$$+4\int_{\eta_s}^{\eta_o} d\eta'\,\partial_a\left(\partial_{\eta'}\Psi\left(\eta'\right)\right)\int_{\eta'}^{\eta_o} d\eta''\,\gamma_0^{ab}\partial_b\int_{\eta''}^{\eta_o} d\eta'''\,\Psi\left(\eta'''\right)$$

Ben-Dayan, GM, Nugier, Veneziano (2012), Fanizza, Gasperini, GM, Veneziano (2013) and GM (2015)
(see also Umeh, Clarkson, Maartens (2014))

Cosmological Observables

To obtain Δ in the PG, in function of the observed redshift and of the direction of observation (θ_o, φ_o), we have:

Step 1 $\rightarrow$ Expand the exact expression of Δ in function of the PG coordinate using the coordinate transformation.

Step 2 $\rightarrow$ Expand conformal time and radial PG coordinates around a fiducial model as $\eta_s = \eta_s^{(0)} + \eta_s^{(1)} + \eta_s^{(2)}$ and $r_s = r_s^{(0)} + r_s^{(1)} + r_s^{(2)}$ perturbatively solving

$$1+z_s = \frac{a(\eta_o)}{a(\eta_s^{(0)})} = \frac{a(\eta_o)}{a(\eta_s)}\left[1 + \delta^{(1)}z + \delta^{(2)}z\right] \quad , \quad w_o = \eta_s^{(0)} + r_s^{(0)} = w^{(0)} + w^{(1)} + w^{(2)}$$

Step 3 $\rightarrow$ Taylor expand the solution of Step 1 around the fiducial model using Step 2, and around the direction of observation using the fact that $\tilde{\theta}^a = \theta_o^a$ are constant along the line-of-sight and therefore

$$\theta^a = \theta^{a(0)} + \theta^{a(1)} = \theta_o^a - 2\int_{\eta_s^{(0)}}^{\eta_o} d\eta' \, \gamma_0^{ab} \partial_b \int_{\eta'}^{\eta_o} d\eta'' \, \Psi^{(1)}(\eta'', \eta_o - \eta'', \theta_o^a) \, .$$

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Galaxy Number Counts

The (second-order, non-homogeneous, non-averaged) expression of Δ in our perturbed background is so given (in a concise form) by

$$\Delta = \Delta^{(1)}(\mathbf{n}, z_s) + \Delta^{(2)}(\mathbf{n}, z_s)$$

To first order we have (Yoo, Fitzpatrick, Zaldarriaga (2009), Yoo (2010), Bonvin, Durrer (2011), Challinor, Lewis (2011))

$$\begin{aligned}\Delta^{(1)}(\mathbf{n}, z) &= \left(\frac{2}{\mathcal{H} r(z)} + \frac{\mathcal{H}'}{\mathcal{H}^2}\right)\left(\partial_r v^{(1)} + \Psi^{(1)} + 2\int_0^{r(z)} dr \partial_\eta \Psi^{(1)}\right) - \Psi^{(1)} \\ &\quad + 4\Psi_1 - 2\kappa + \frac{1}{\mathcal{H}}\left(\partial_\eta \Psi^{(1)} + \partial_r^2 v^{(1)}\right) + \delta^{(1)}\end{aligned}$$

with

$$\Psi_1(\mathbf{n}, z) = \frac{2}{r(z)}\int_0^{r(z)} dr \Psi^{(1)}(r) \quad , \quad 2\kappa = -\Delta_2 \psi = 2\int_0^{r(z)} dr \frac{r(z) - r}{r(z) r} \Delta_2 \Psi^{(1)}(r)$$

Galaxy Number Counts

Keeping only the leading (potentially observables) terms the number counts to second order turns to be

$$\Delta^{(2)} = \Sigma^{(2)} - \langle \Sigma^{(2)} \rangle$$

where

$$\begin{aligned}
\Sigma^{(2)}(\mathbf{n}, z) \quad = \quad & \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} - 2\kappa^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v \\
& + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\, \delta\right) - 2\delta\kappa + \nabla_a \delta \nabla^a \psi \\
& + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a \psi\right] + 2\kappa^2 - 2\nabla_b \kappa \nabla^b \psi \\
& - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z) - r}{r} \Delta_2 \left(\nabla^b \Psi_1 \nabla_b \Psi_1\right) - 2\int_0^{r(z)} \frac{dr}{r} \nabla^a \Psi_1 \nabla_a \kappa \,.
\end{aligned}$$

with

$$\kappa^{(2)} = \frac{1}{2}\int_0^{r(z)} dr \frac{r(z) - r}{r(z) r} \Delta_2 (\Psi + \Phi)^{(2)}(-r\mathbf{n}, \eta_0 - r)\,.$$

Magnification Bias

In practice, we cannot observe all galaxies, but only those with a flux which is larger than a certain limit $\bar{F}$.

If the fluctuation of the source number density N depends on luminosity we have to further Taylor expand to obtain $\Delta(\mathbf{n}, \mathbf{z}, \mathbf{L})$.

This physical threshold impact only the part of the number counts that comes from the galaxy density ρ.

We then obtain (see also Bertacca (2014))

$$\begin{aligned}
N\left(\mathbf{n}, z, \bar{F}\right) &= N\left(\mathbf{n}, z\right) + \frac{\partial}{\partial L} N\left(\mathbf{n}, z, \bar{L}\right)\left(\delta L^{(1)} + \delta L^{(2)}\right) + \frac{1}{2}\frac{\partial^2}{\partial L^2} N\left(\mathbf{n}, z, \bar{L}\right)\left(\delta L^{(1)}\right)^2 \\
&= N(\bar{z}, \bar{L})\left[1 + \Delta^{(1)} + \Delta^{(2)} + \frac{\partial_L \bar{\rho}}{\bar{\rho}}\left(\delta L^{(1)} + \delta L^{(2)}\right) + \frac{1}{2}\frac{\partial_L^2 \bar{\rho}}{\bar{\rho}}\left(\delta L^{(1)}\right)^2 \right. \\
&\quad \left. + \frac{\left(\partial_L \rho - \partial_L \bar{\rho}\right)^{(1)}}{\bar{\rho}} \delta L^{(1)} + \frac{\partial_\eta\left(\partial_L \bar{\rho}\right)}{\bar{\rho}} \delta L^{(1)} \frac{\delta z^{(1)}}{\mathcal{H}}\right],
\end{aligned}$$

On the other hand, $F = L/(2\pi d_L^2)$ and at fixed flux $\delta L = \delta\left(d_L^2\right)$.

Magnification Bias

Defining

$$\left(\frac{\partial \ln \bar{\rho}}{\partial \ln L}\right)(z, \bar{L}) = -\frac{5}{2} s\,(z, \bar{L}) \quad , \quad \frac{\partial^2}{\partial (\ln L)^2}(\ln \bar{\rho})\,(z, \bar{L}) = -\frac{5}{2} t\,(z, \bar{L})$$

$$(1+\delta^{(1)})\frac{\partial \ln \rho}{\partial \ln L} - \frac{\partial \ln \bar{\rho}}{\partial \ln L} = -\frac{5}{2}(\delta s)^{(1)}(z, \bar{L})$$

We have, to first order (Challinor, Lewis (2011))

$$\begin{aligned}\Delta^{(1)}(\mathbf{n}, z) \quad = \quad & \left(\frac{2-5s}{\mathcal{H} r(z)} + 5s + \frac{\mathcal{H}'}{\mathcal{H}^2}\right)\left(\partial_r v^{(1)} + \Psi^{(1)} + 2\int_0^{r(z)} dr \partial_\eta \Psi^{(1)}\right) \\ & + (5s-1)\Psi^{(1)} + (2-5s)\left(2\Psi_1 - \kappa^{(1)}\right) \\ & + \frac{1}{\mathcal{H}}\left(\partial_\eta \Psi^{(1)} + \partial_r^2 v^{(1)}\right) + \delta^{(1)}\end{aligned}$$

Magnification Bias

While, the leading second order contribution becomes:

$$\Sigma^{(2)}(\mathbf{n}, z) = \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} - 2\left(1 - \frac{5}{2}s\right)\kappa^{(2)} + \mathcal{H}^{-2}\left[\left(\partial_r^2 v\right)^2 + \partial_r v \partial_r^3 v\right]$$
$$+\mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\,\delta\right) - 2\delta\kappa + \nabla_a\delta\nabla^a\psi$$
$$+\mathcal{H}^{-1}\left[-2\left(1 - \frac{5}{2}s\right)\partial_r^2 v\,\kappa + \nabla_a\partial_r^2 v\nabla^a\psi\right] + 2\left(1 - 5s + \frac{25}{4}s^2 - \frac{5}{2}t\right)\kappa^2$$
$$-2\left(1 - \frac{5}{2}s\right)\nabla_b\kappa\nabla^b\psi - \left(1 - \frac{5}{2}s\right)\frac{1}{2r(z)}\int_0^{r(z)} dr\frac{r(z) - r}{r}\Delta_2\left(\nabla^b\Psi_1\nabla_b\Psi_1\right)$$
$$-2\left(1 - \frac{5}{2}s\right)\int_0^{r(z)}\frac{dr}{r^2}\nabla^a\Psi_1\nabla_a\kappa + 5\left(\delta s\right)^{(1)}\kappa$$

If the number of galaxies depend on luminosity like a simple power law, $\rho \propto L^p$, we have $s = -2p/5$, $t = 0$ and $(\delta s)^{(1)} = s\delta^{(1)} = -2p\delta^{(1)}/5$.

For $p = -1$ we have $s = 2/5$, $t = 0$ and $(\delta s)^{(1)} = 2\delta^{(1)}/5$

$\Downarrow$

The pure lensing disappear at first and second order, while the terms $\nabla_a\delta\nabla^a\psi + \mathcal{H}^{-1}\nabla_a\partial_r^2 v\nabla^a\psi$ are not affected by magnification bias.

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Reduced bispectrum number counts

We define the bispectrum in real space as

$$B(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3, z_1, z_2, z_3) = \langle \Delta(\mathbf{n}_1, z_1)\, \Delta(\mathbf{n}_2, z_2)\, \Delta(\mathbf{n}_3, z_3) \rangle_c$$

Expanding the direction dependence of Δ in spherical harmonics

$$B(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3, z_1, z_2, z_3) = \sum_{\ell_i, m_i} B_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3}(z_1, z_2, z_3)\, Y_{\ell_1 m_1}(\mathbf{n}_1)\, Y_{\ell_2 m_2}(\mathbf{n}_2)\, Y_{\ell_3 m_3}(\mathbf{n}_3)$$

and

$$B_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3}(z_1, z_2, z_3) = \mathcal{G}_{\ell_1, \ell_2, \ell_3}^{m_1, m_2, m_3} \;\; b_{\ell_1, \ell_2, \ell_3}(z_1, z_2, z_3)$$

with

$\mathcal{G}_{\ell_1, \ell_2, \ell_3}^{m_1, m_2, m_3}$	Gaunt integral
$b_{\ell_1, \ell_2, \ell_3}(z_1, z_2, z_3)$	Reduced bispectrum

Reduced bispectrum number counts

Assuming Gaussian initial condition

$$\langle \Delta^{(1)}(\mathbf{n}_1, z_1)\Delta^{(1)}(\mathbf{n}_2, z_2)\Delta^{(1)}(\mathbf{n}_3, z_3)\rangle_c = 0$$

we compute the contribution coming from

$$\langle \Delta^{(2)}(\mathbf{n}_1, z_1)\,\Delta^{(1)}(\mathbf{n}_2, z_2)\,\Delta^{(1)}(\mathbf{n}_3, z_3)\rangle_c + \text{permutations}$$

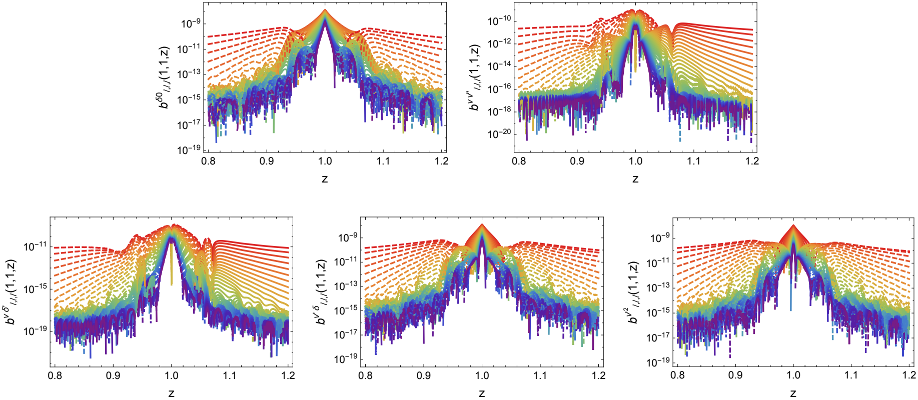
taking only the second order leading terms and $\Delta^{(1)} = \delta^{(1)}$.
We then divide our leading reduced bispectrum as follow

$$\begin{aligned} b_{\ell_1\ell_2\ell_3} = {} & b^{\delta^{(2)}}_{\ell_1\ell_2\ell_3} + b^{v^{(2)'}}_{\ell_1\ell_2\ell_3} + b^{v'^2}_{\ell_1\ell_2\ell_3} + b^{vv''}_{\ell_1\ell_2\ell_3} + b^{v\delta'}_{\ell_1\ell_2\ell_3} + b^{v'\delta}_{\ell_1\ell_2\ell_3} \\ & + b^{\kappa^{(2)}}_{\ell_1\ell_2\ell_3} + b^{\kappa\delta}_{\ell_1\ell_2\ell_3} + b^{\nabla\delta\nabla\psi}_{\ell_1\ell_2\ell_3} + b^{v'\kappa}_{\ell_1\ell_2\ell_3} + b^{\nabla v'\nabla\psi}_{\ell_1\ell_2\ell_3} \\ & + b^{\kappa^2}_{\ell_1\ell_2\ell_3} + b^{\nabla\kappa\nabla\psi}_{\ell_1\ell_2\ell_3} + b^{\int\nabla\kappa\nabla\Psi_1}_{\ell_1\ell_2\ell_3} + b^{\int\Delta_2(\nabla\Psi_1\nabla\Psi_1)}_{\ell_1\ell_2\ell_3} \end{aligned}$$

where the color coding indicates

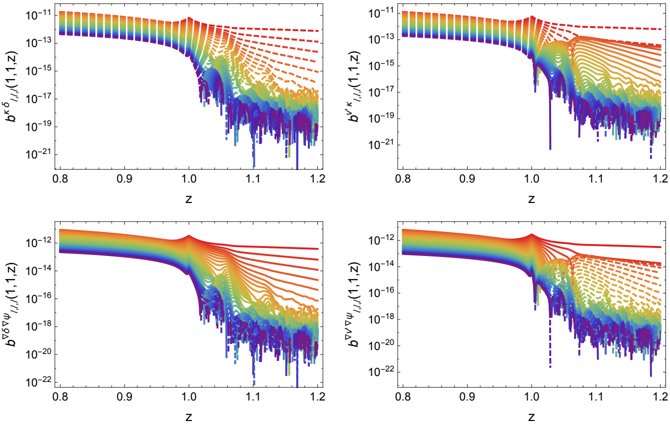
Newtonian terms | Newtonian x lensing terms | Lensing terms

Newtonian terms



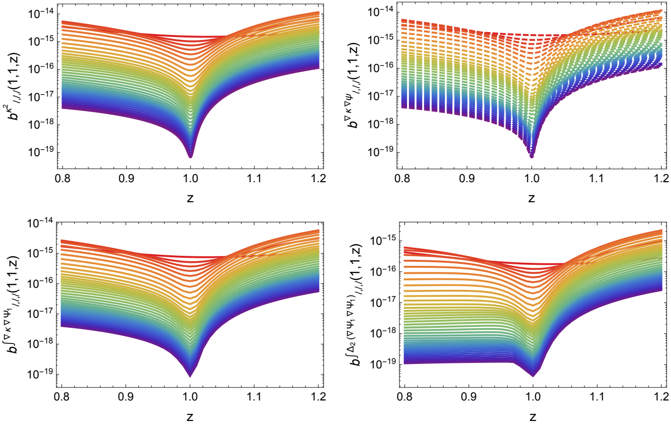
We plot the contributions from the Newtonian terms to the bispectrum for different values of $\ell = \ell_1 = \ell_2 = \ell_3$, from $\ell = 4$ (red) to $\ell = 400$ (purple), as a function of the third redshift $z_3 = z$ for $z_1 = z_2 = 1$.

Newtonian x lensing terms



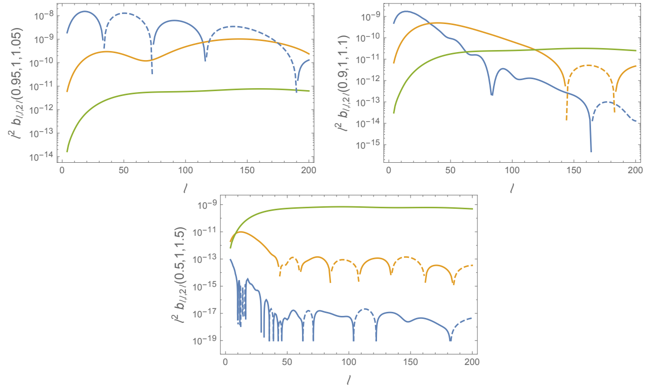
We plot the contributions from the Newtonian $\times$ lensing terms to the bispectrum for different values of $\ell = \ell_1 = \ell_2 = \ell_3$, from $\ell = 4$ (red) to $\ell = 400$ (purple), as a function of the third redshift $z_3 = z$ for $z_1 = z_2 = 1$.

Lensing terms



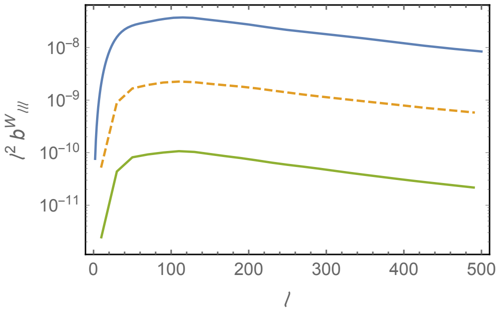
We plot the contributions from the pure lensing terms to the bispectrum for different values of $\ell = \ell_1 = \ell_2 = \ell_3$, from $\ell = 4$ (red) to $\ell = 400$ (purple), as a function of the third redshift $z_3 = z$ for $z_1 = z_2 = 1$.

Reduced bispectrum number counts: redshift separation



We plot the contributions from the Newtonian terms (blue), the Newtonian $\times$ lensing terms (yellow) and the pure lensing terms (green) for $z_1 = 0.95$, $z_2 = 1$ and $z_3 = 1.05$ (top left), for $z_1 = 0.9$, $z_2 = 1$ and $z_3 = 1.1$ (top right) and for $z_1 = 0.5$, $z_2 = 1$ and $z_3 = 1.5$ (bottom) as function of $\ell = \ell_1 = \ell_2 = \ell_3/2$. Dashed lines correspond to negative values.

Reduced bispectrum number counts: redshift bin



We plot the contributions to the bispectrum with window function of width $\Delta z = 1$ and mean redshift $z = 1$, for Newtonian (blue), Lensing $\times$ Newtonian (yellow) and Lensing (green). Dashed lines correspond to negative values.

Conclusions

- We have presented the geodesic light-cone coordinates, a coordinate system adapted to an observer and his past light-cone.
- In the framework of the GLC we can write LSS observables in an exact, non-perturbative way.
- We have show the leading perturbative expressions for the number counts at second order as a function of the observed redshift and the direction of the observation.
- We have defined the number counts reduced bispectrum in the directly observable spherical-harmonics-redshift space.
- In particular configurations the integrated relativistic terms can dominate the signal/be not negligible
  - Well separated redshifts.
  - Broad window functions.

Outlook: Evaluation of the signal-to-noise to investigate whether planed surveys can detect the lensing signal when it dominates.

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THANKS FOR THE ATTENTION!