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simulation runs+analysis

LATfield2 lead developer

relativistic N-body simulations

GR expert

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with additional work by Enea Di Dio and Chris Clarkson,
and some early work by Miki Obradovic

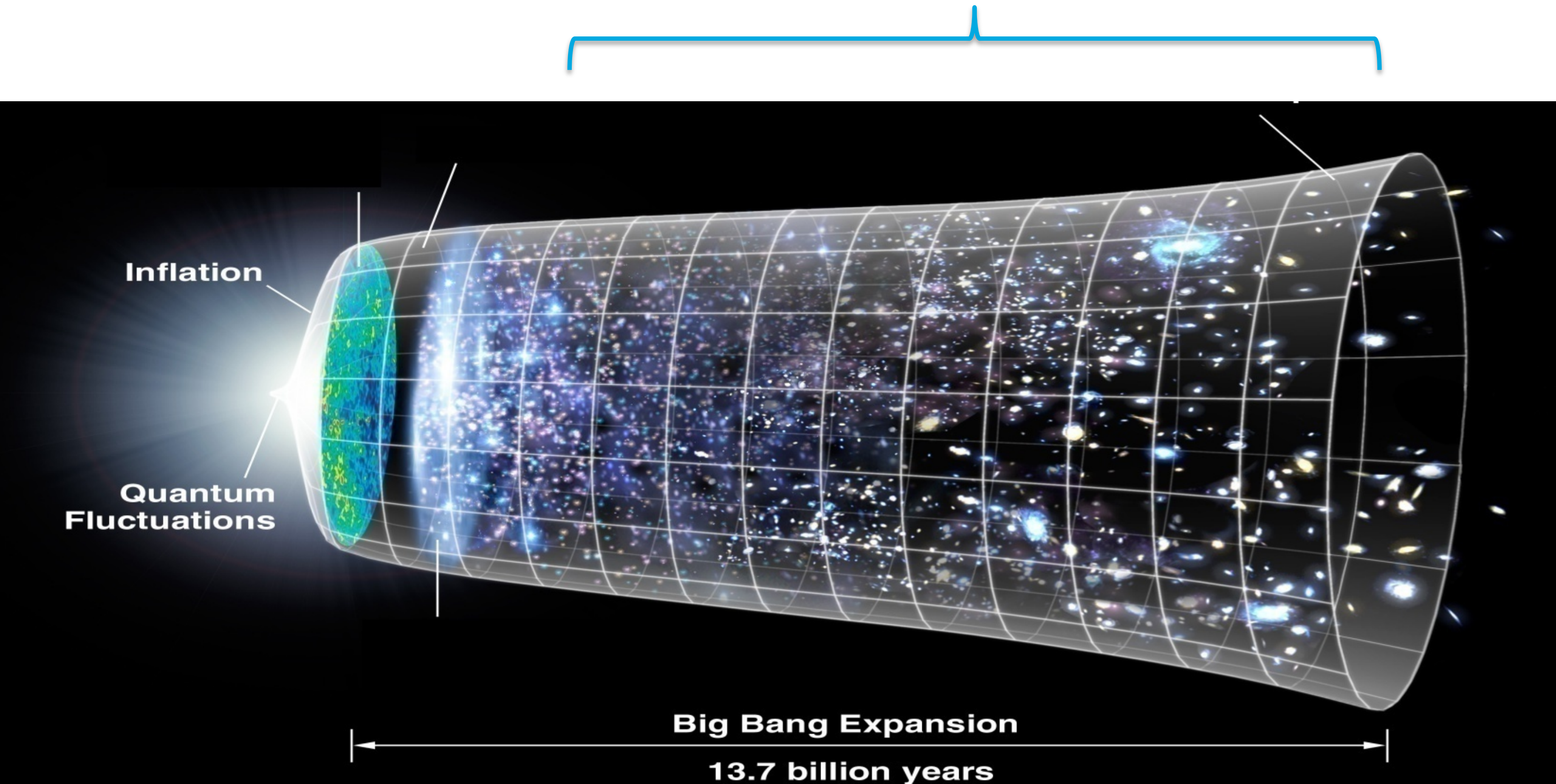
based on arXiv:1308.6524, arXiv:1401.3634,
arXiv:1408.2741, arXiv:1408.3352, arXiv:1509.01699
as well as ongoing work

outline

- motivation and basic idea
- *gevolution* formalism and equations
- 1D simulations and illustrative results
    - clustering in real space, shell-crossing
    - comparison to exact GR solution
- the 3D code
    - *LATfield2* ‘computation engine’
    - accuracy in Schwarzschild geometry
    - initial 3D simulation simulation results
- some remarks on backreaction
- outlook

cosmological context

We are interested in how matter clumps together in a General-Relativistic context (i.e. going beyond Newtonian physics and linear perturbation theory)



motivation

- why use the ‘wrong’ theory (Newtonian gravity) if we can use GR (‘right’ or at least better approximation)?
- Future large surveys need predictions for relativistic effects. Some of them can be added ‘on top’ of N-body simulations, but it is impossible to assess the accuracy without doing it right once (as perturbation theory does not work on small scales). Do we believe ray-tracing results without vectors, tensors and gravitational slip?
- Some effects (like backreaction) need GR simulations as important terms are total derivatives in the Newtonian approximation.
- Including relativistic particles (neutrinos) & fields (DE/MG) appears also more natural with a relativistic simulation.

basic idea

- full numerical General Relativity is a killer (no global coordinate system, hard pde's, …)
- but in standard cosmology we are close to FLRW

$$ds^2 = a^2(\tau)\left[-(1+2\Psi)d\tau^2 + (1-2\Phi)d\mathbf{x}^2\right]$$

- and the potentials should remain small on *all* scales!

$$\Delta\Phi = 4\pi G a^2 \bar{\rho}\,\delta$$

($\Delta \sim k^2$ → small scales: k large, δ large, Φ stays small)

- use weak field approximation
  - metric perturbations stay small: all okay (?)
  - metric perturbations become large: uh oh (?)

approximation scheme

- beyond linear order vector and tensor perturbations couple to scalar perturbations, so need everything:

$$ds^2 = a^2(\tau)\left[-(1+2\Psi)d\tau^2 - 2B_i dx^i d\tau + (1-2\Phi)\delta_{ij}dx^i dx^j + h_{ij}dx^i dx^j\right].$$

- **metric perturbations** are supposed to remain small: keep them only to **linear order**
- **density perturbations** will become large: keep to **all orders**
- **velocities and gradients** of the metric pert's are intermediate: keep to **second order**
- the metric is a field on a grid, the matter phase-space is sampled by N-body particles → **particle-mesh**

formalism I : relativistic Poisson eq.

Now just 'crank the handle': compute Einstein and geodesic equations

example: 0-0 equation for LCDM (→ Poisson eq.):

$$(1+4\Phi)\,\Delta\Phi - 3\mathcal{H}\Phi' - 3\mathcal{H}^2\Psi + \frac{3}{2}\delta^{ij}\Phi_{,i}\Phi_{,j}$$
$$= 4\pi G a^2 \bar{\rho}\left[\delta + 3\Phi\,(1+\delta) + \frac{1}{2}\,(1+\delta)\,\langle v^2\rangle\right]$$

→ diffusion-type equation for Φ, estimate of diffusion to dynamical (free-fall) time scale for structure of size r:

$$\frac{t_{\rm diff}}{t_{\rm dyn}} \simeq \frac{r^2}{r_H^2}\sqrt{1+\delta}$$ <<1 for r << r_H

→ expect to be driven towards 'equilibrium' solution, which is given by solution of Poisson eq.

formalism II : ‘non-Newtonian’ quantities

traceless part of space-space Einstein equations:

$$\left(\delta^i_k\delta^j_l - \frac{1}{3}\delta^{ij}\delta_{kl}\right)\left[\frac{1}{2}h''_{ij} + \mathcal{H}h'_{ij} - \frac{1}{2}\Delta h_{ij} + B'_{(i,j)} + 2\mathcal{H}B_{(i,j)} + \chi_{,ij} - 2\chi\Phi_{,ij} + 2\Phi_{,i}\Phi_{,j} + 4\Phi\Phi_{,ij}\right]$$

$$= 8\pi G a^2\left(\delta_{ik}T^i_l - \frac{1}{3}\delta_{kl}T^i_i\right) \doteq 8\pi G a^2 \Pi_{kl}\,, \quad (2.9)$$

- $\chi=\Phi-\Psi$ is our second scalar variable
- we solve first the Φ equation and move Φ^2 terms to rhs
- we solve this equation in Fourier space where we can easily split it into spin components
    - one elliptic constraint for scalar χ
    - two parabolic evolution equations for B_i
    - two wave equations (hyperbol.) for h_{ij} (which atm we don’t solve)

formalism III : geodesic equation

- Finally, massive non-rel. particles follow geodesic eq:

$$\frac{\mathrm{d}^2 x^i_{(n)}}{\mathrm{d}\tau^2} + \mathcal{H}\frac{\mathrm{d}x^i_{(n)}}{\mathrm{d}\tau} + \delta^{ij}\left(\Psi_{,j} - \mathcal{H}B_j - B'_j - 2B_{[j,k]}\frac{\mathrm{d}x^k_{(n)}}{\mathrm{d}\tau}\right) = 0$$

- tensors do not contribute in non-relativistic limit, but vectors do (cf also Obradovic et al, arXiv:1106.5866)
- geodesic equation can be generalized to arbitrary momenta, in which case vectors, tensors and Φ contribute at same level as Ψ
- Newtonian gravity just retains first 3 terms
- We integrate the particle motion alternatingly with the field update using a staggered leapfrog

formalism IV : EM tensor, initial cond., etc

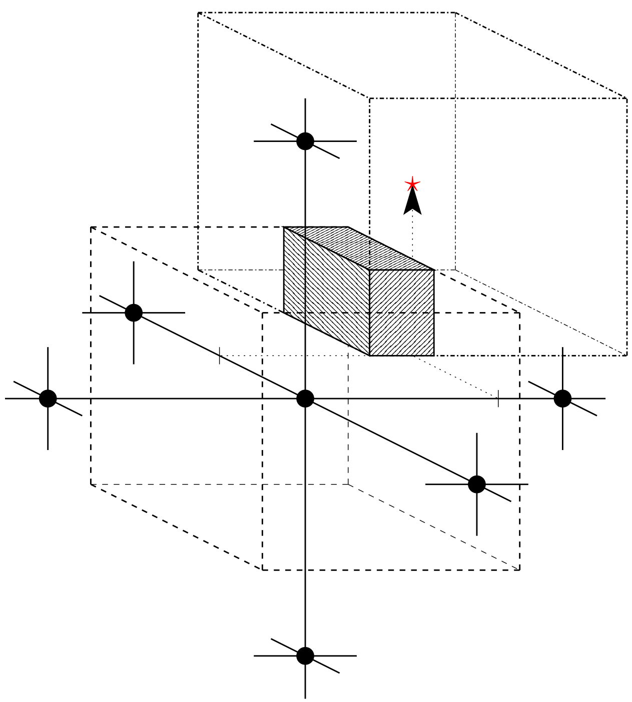
- Need to take metric pert's and velocities into account when interpreting N-body state, e.g. for usual (now 'bare') mass density:

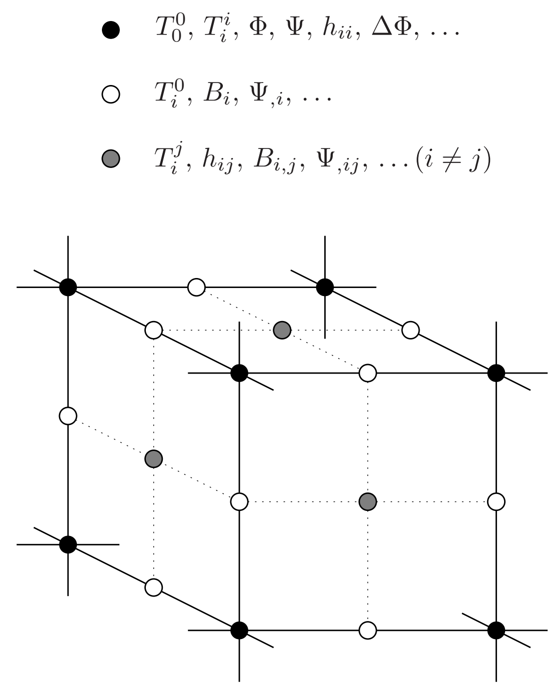
$$\rho_{\rm phys} \doteq -T_{\rm m}{}^0{}_0 = \left[1 + 3\Phi + \frac{1}{2}\langle v^2 \rangle\right] \rho$$

- Also the initial conditions are changed wrt Newtonian simulations, we use linear perturbation initial conditions in the longitudinal gauge.
- In the simplest case, we draw a realization of Φ=Ψ and derive a displacement field that is applied to an initial 'homogeneous' particle distribution (eg. a glass).
- Velocities are set with the Zel'dovich approximation.

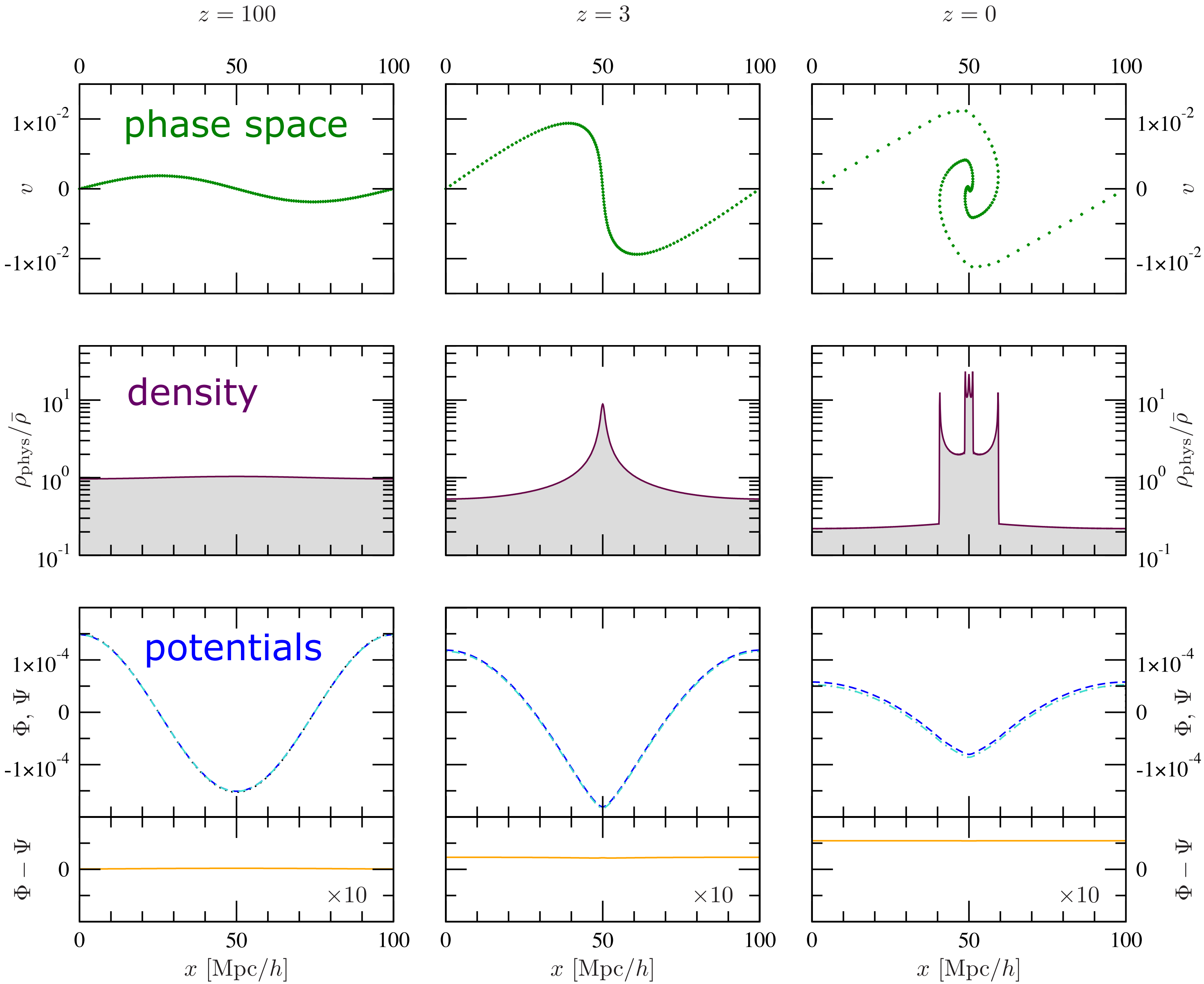
formalism V : particle-mesh implementation

- fields exist on a mesh, but not the particles
- we need to project particles on the mesh to get $T_{\mu\nu}$, and conversely interpolate the metric to the particle positions to compute the acceleration
- example of cloud-in-cell (CIC) projection:



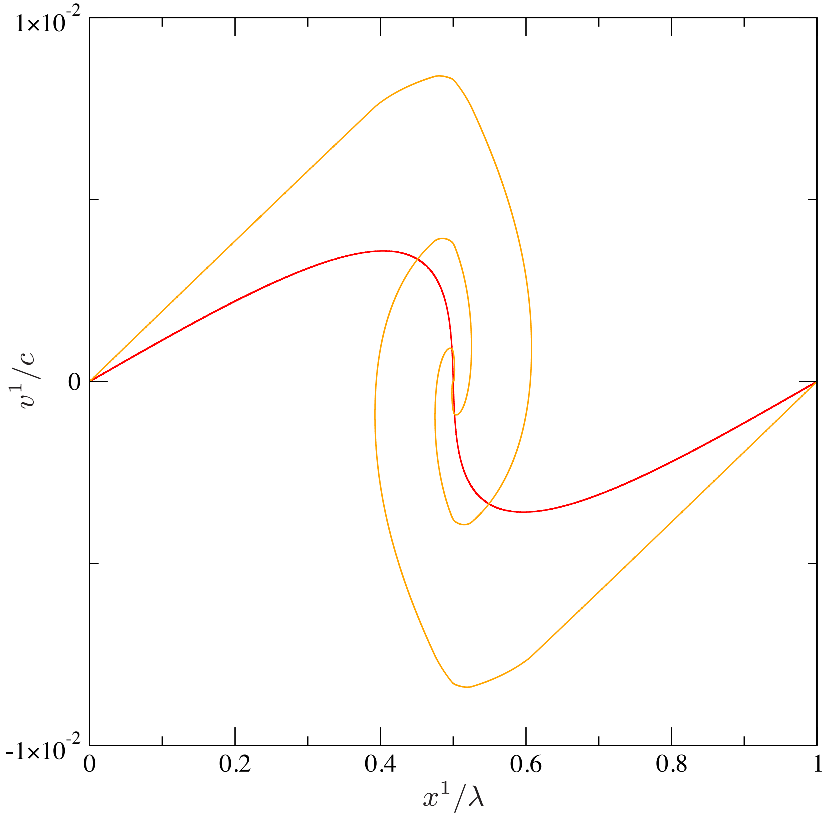


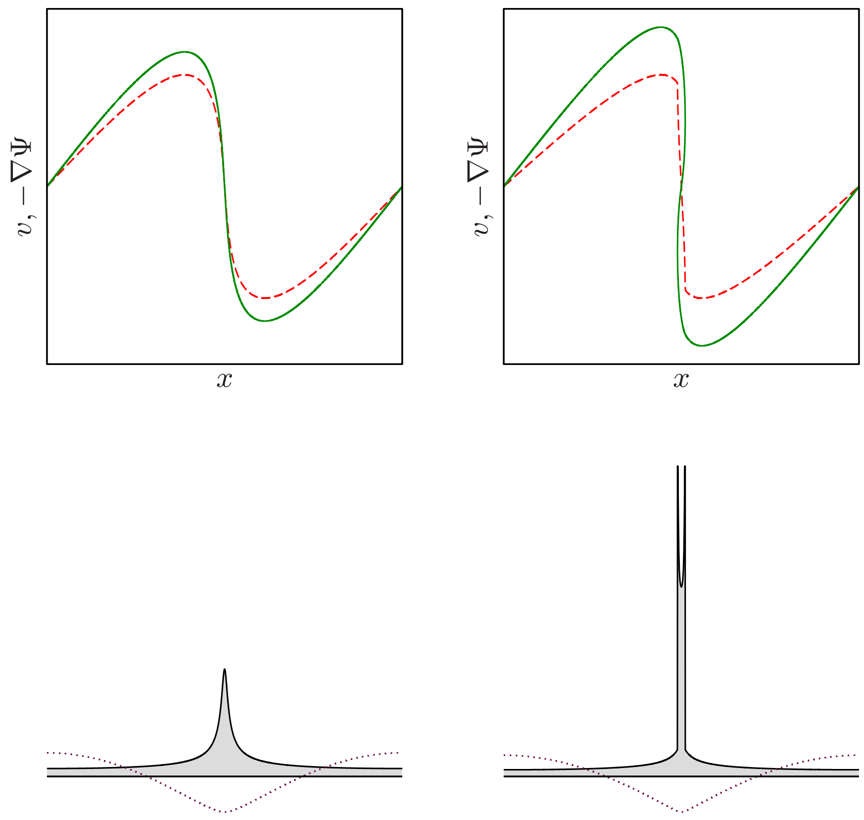
results for plane wave collapse



divergencies at shell crossing

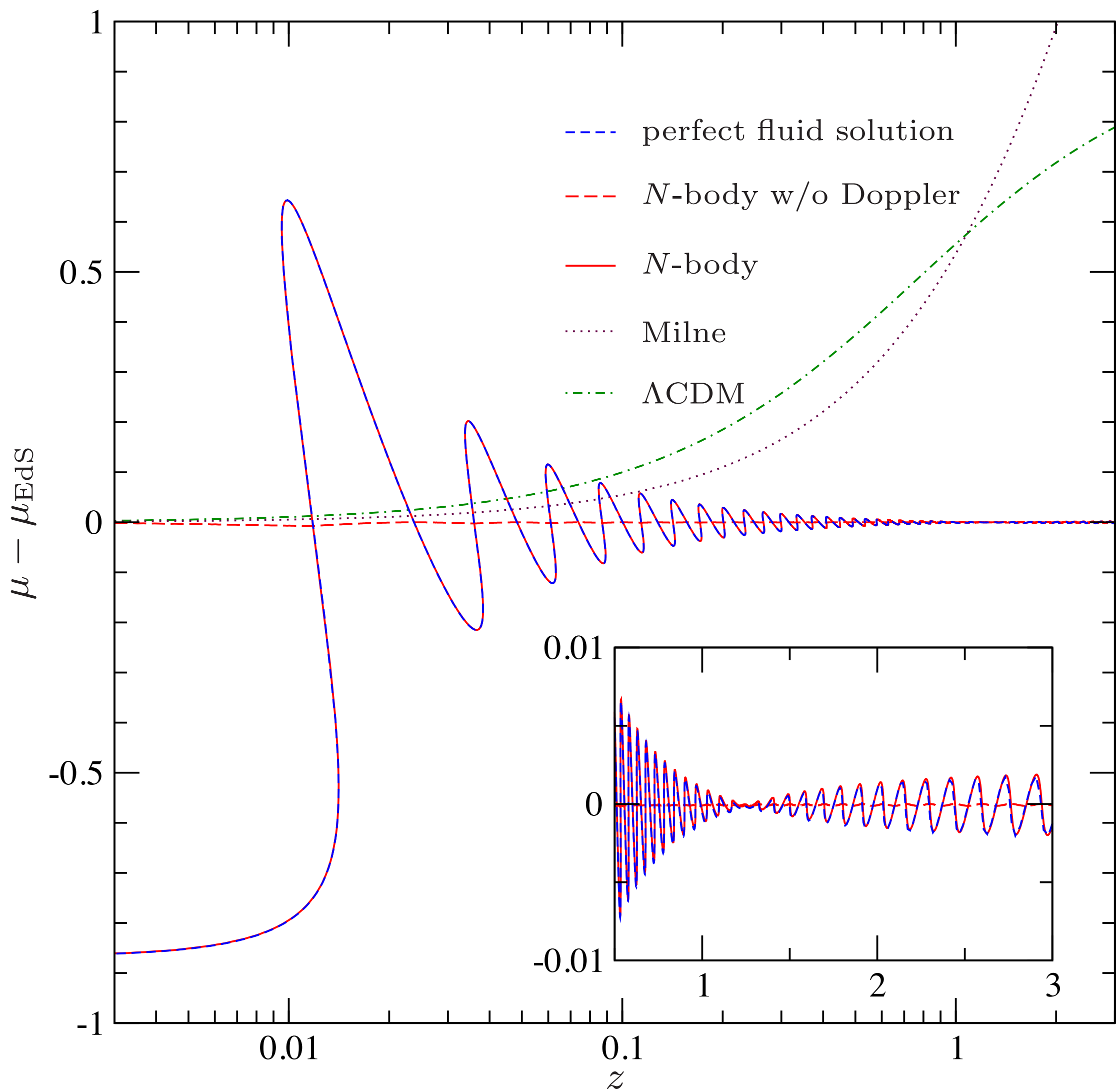
Eventually particle trajectories cross and δ diverges…





no problem: δ is 2nd derivative of metric, so Φ just has a kink

comparison: exact GR solution vs N-body



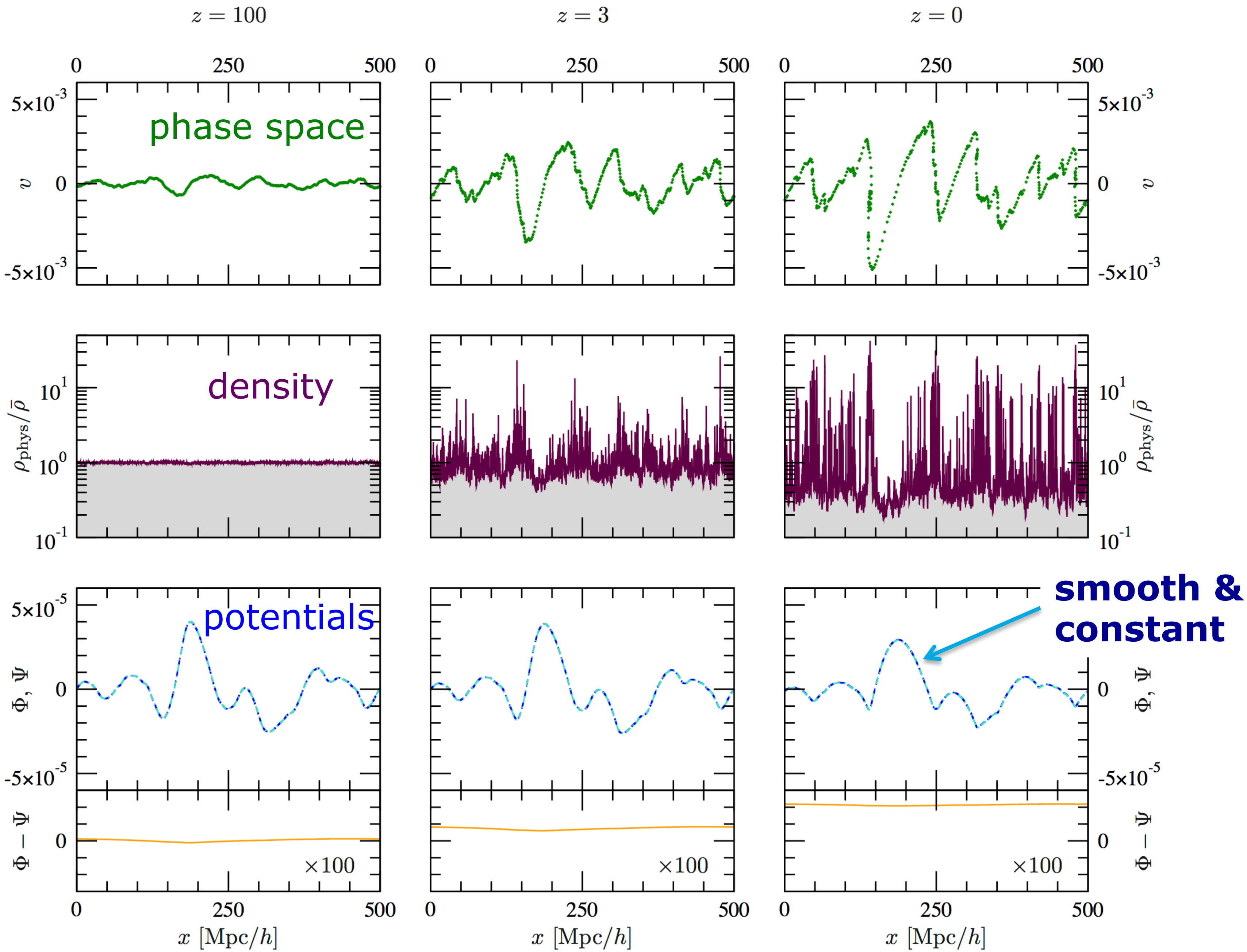
exact GR fluid solution and N-body agree extremely well!

(but can’t explain distance measurements if wavelength much smaller than horizon)

Main contribution to perturbations: Doppler (but gauge-dep. statement)

notice: distances are not single valued ☺

the '1D' universe



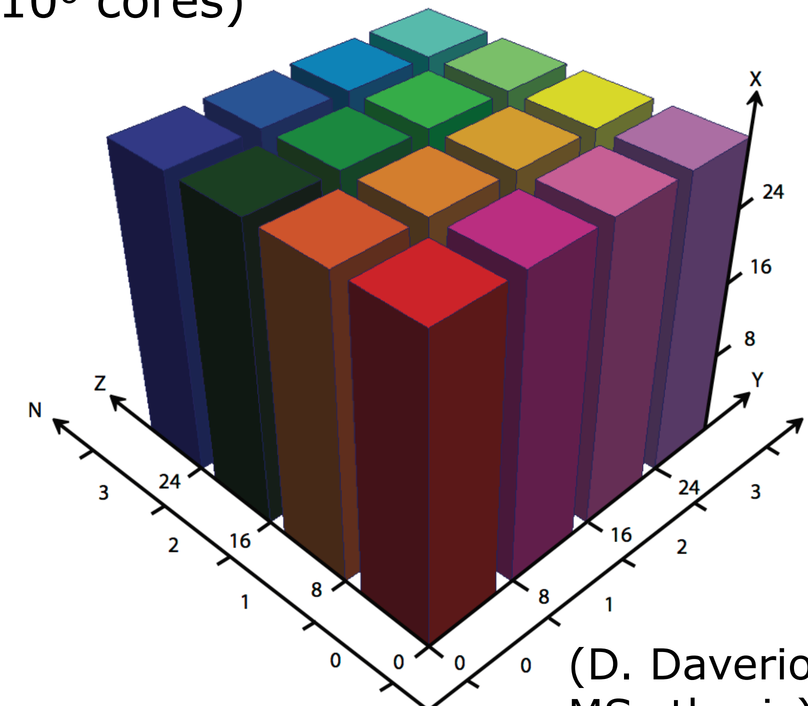
The 3D code

- 3D is computationally much harder than 1D
- Luckily we had just improved our field theory / cosmic string simulation framework LATfield2:
    - 2D (rod) parallelization w/ MPI
    - transparent handling of fields
    - I/O server (providing Tb/s bandwidth to I/O cores)
    - fully distributed FFT with excellent speed-up
- LATfield2 is available at latfield.org
- LATfield2 is also handling our particle ensemble and projection/interpolation (not yet part of public release)
- gevolution will also be soon available at https://github.com/gevolution-code/gevolution-1.0.git
- current runs take ~5h on 16k cores for $(4096)^3$ grids

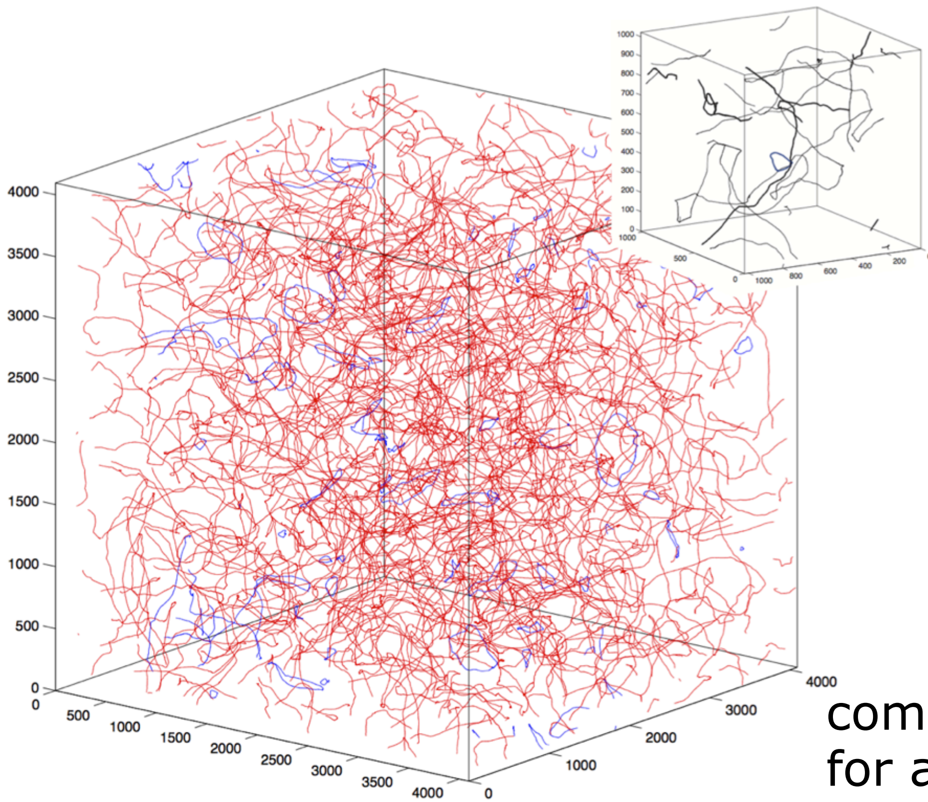
3D simulation framework: LATfield2

A C++ framework for parallel field simulations. Hides all the parallelization. No need to think about it from 4 cores to …. (tested up to 72,000, designed to scale to > 10^6 cores)

focus: easy to use & efficient



(D. Daverio MSc thesis)



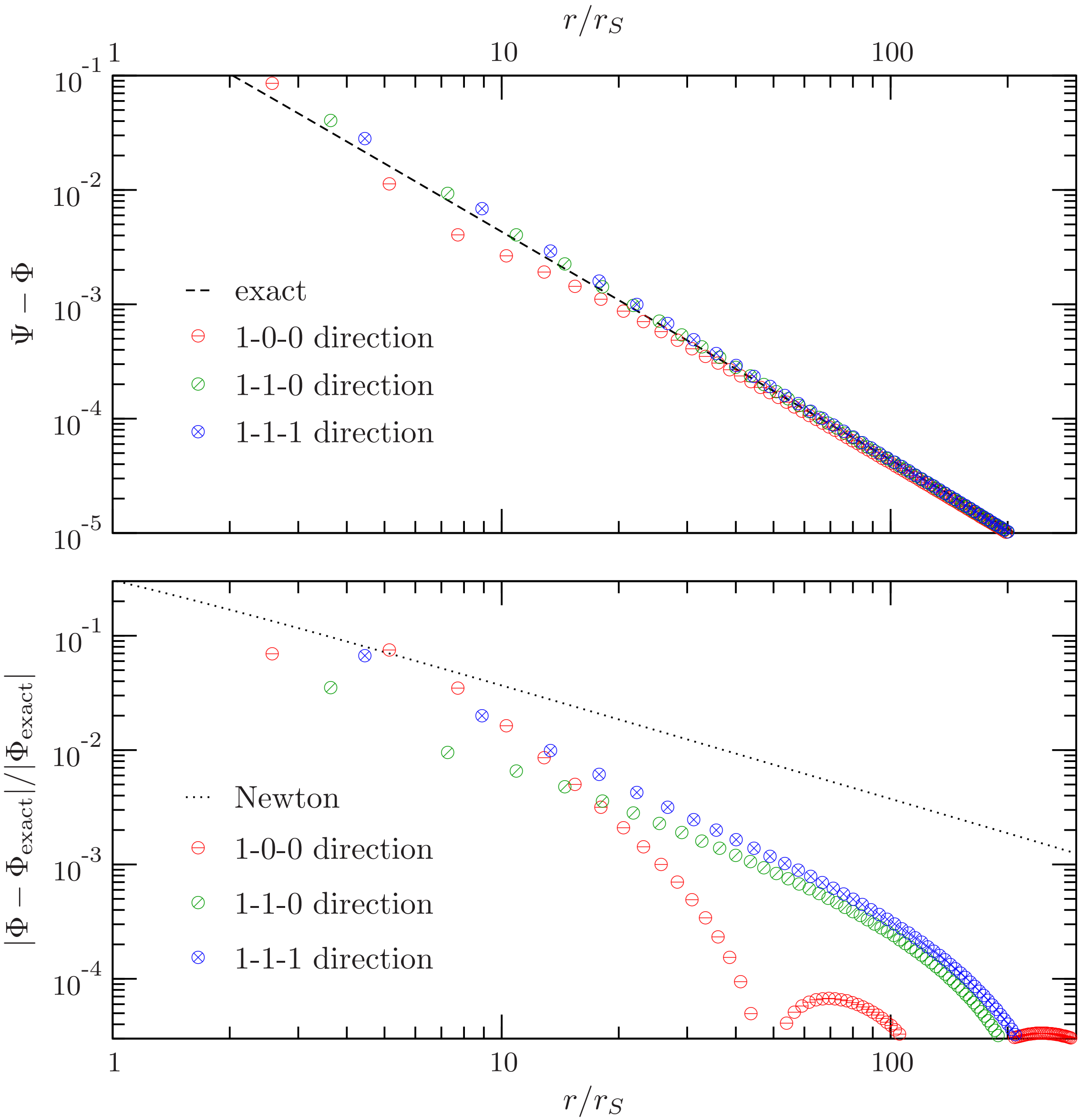
comparison $(4096)^3$ to $(1024)^3$ grid for a cosmic string simulation

pological defect simulations

- al defects, 400^3 grid [~1 Gflop/s…]
  - tor processor (NEC SX3)
- **ca 2005: cosmic strings, 512^3 grid**
  - MPI code w/ '1D' parallelisation (FFT issue)
- **ca 2009: co**
  - bigger
- **2012+: cos**
  - '2D' pa
  - huge in
  - could d

Schwarzschild test

(simulation uses 6144^3 lattice, so okay to $r \sim 1000 r_s$)



The metric around a point-mass should be close to Schwarzschild

expansion of metric:

Newtonian

$$\Psi(r) = -\frac{r_S}{2r} + \frac{r_S^2}{4r^2} - \frac{3r_S^3}{32r^3} + \ldots ,$$

$$\Phi(r) = -\frac{r_S}{2r} - \frac{3r_S^2}{16r^2} - \frac{r_S^3}{32r^3} - \frac{r_S^4}{512r^4}$$

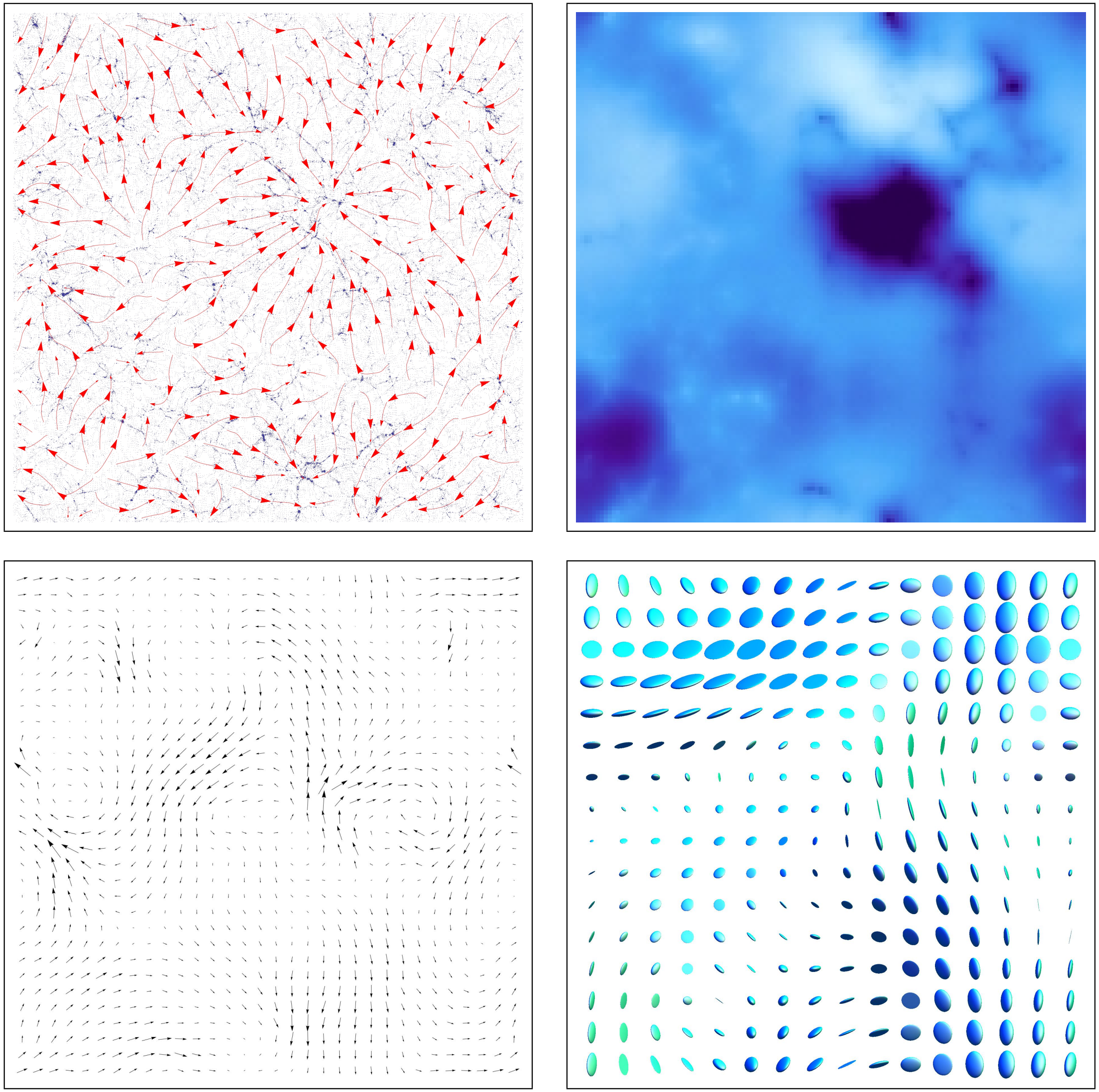
weak-field expansion

→ we should get perihelion precession of Mercury ☺

(& we are not limited to non-relativistic sources)

N-body particles & bulk flow

Φ-Ψ (same plane as particles)

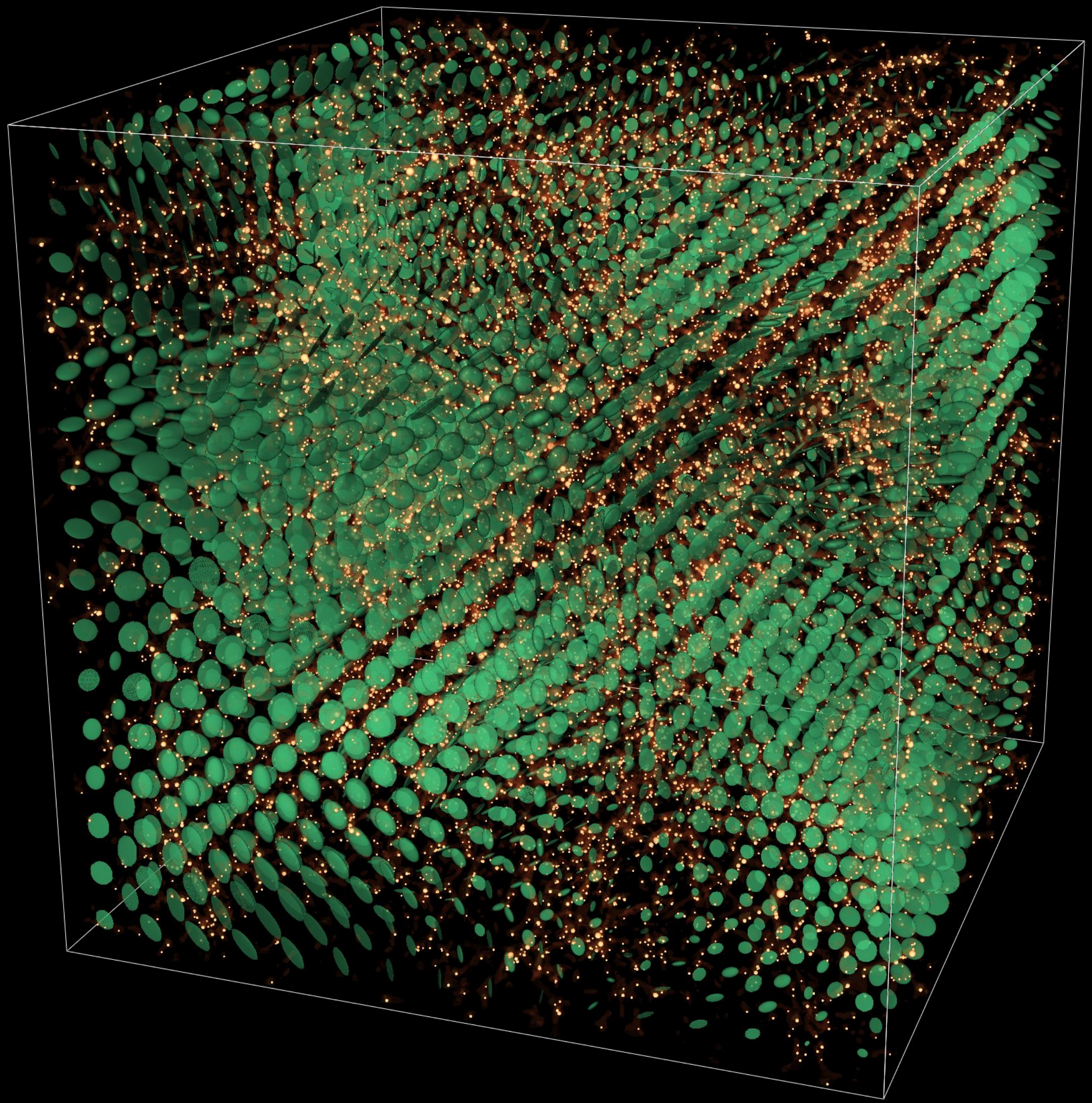


spin 1 perturbation B_i

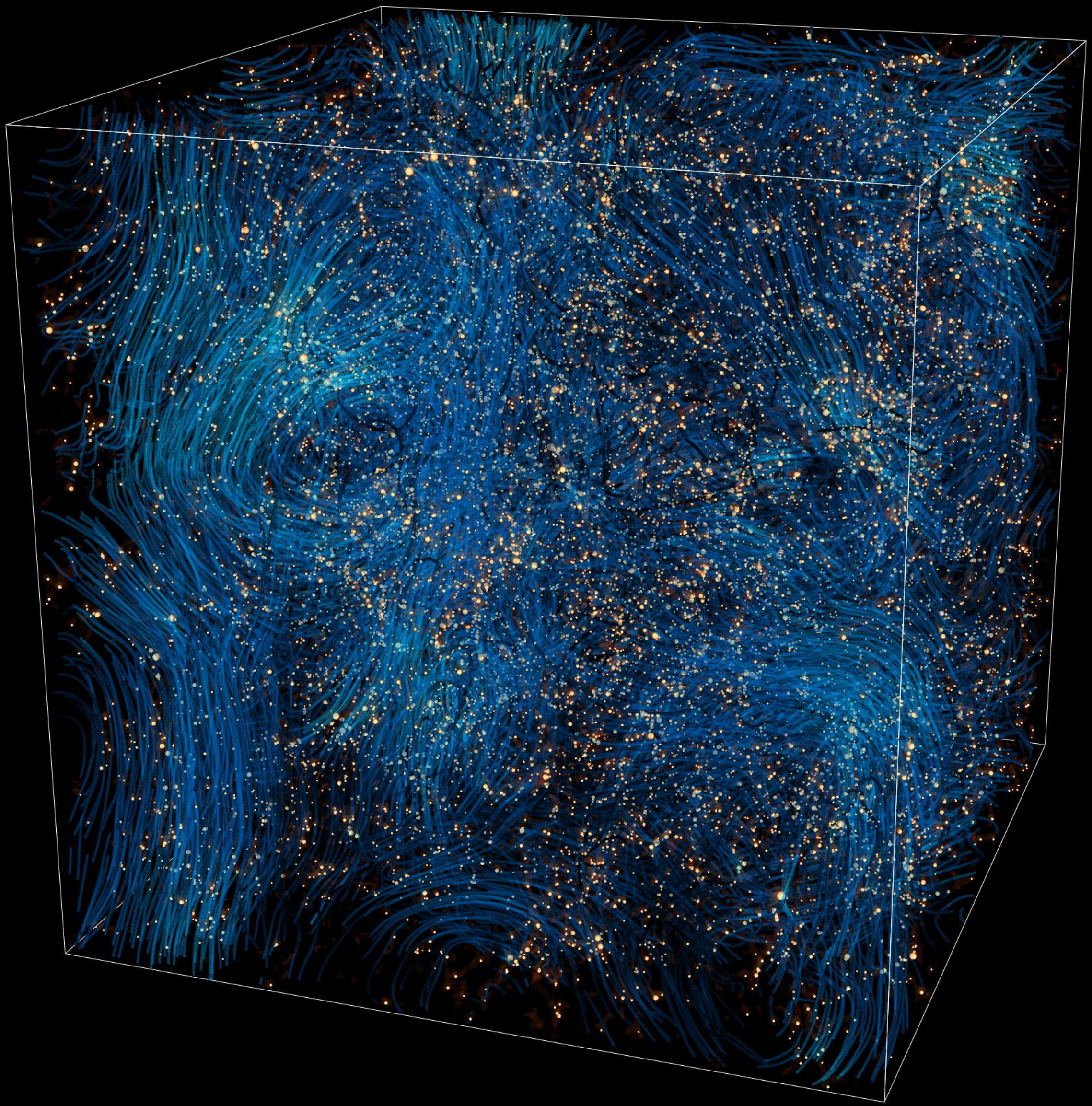
spin 2 field h_{ij} visualized through deformation of spheroid

(older figure with post-Newtonian reconstruction from Newtonian simulation)

tensors



vectors



spectra

expectations to lowest order:

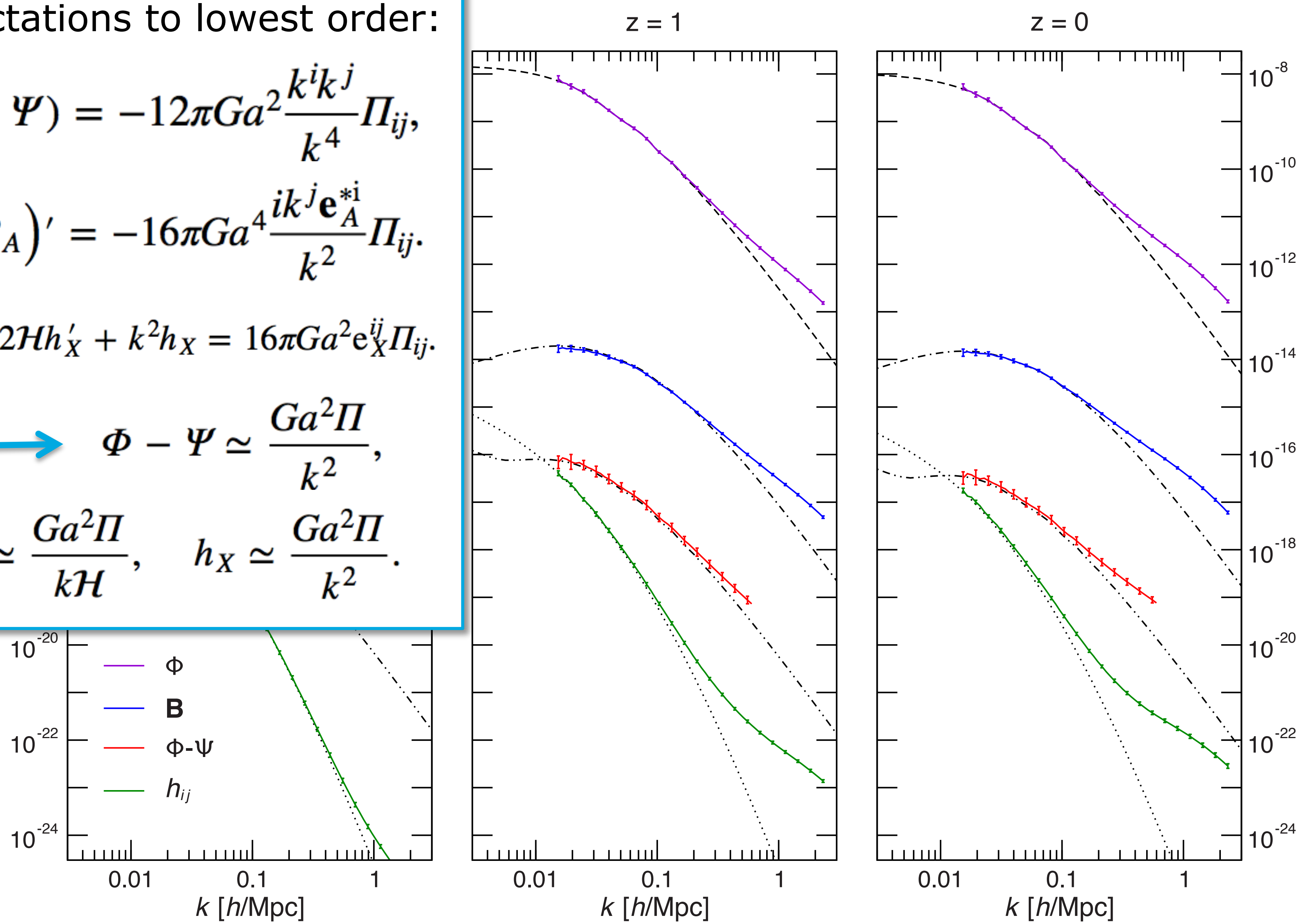
$$(\Phi - \Psi) = -12\pi G a^2 \frac{k^i k^j}{k^4} \Pi_{ij},$$

$$\left(a^2 B_A\right)' = -16\pi G a^4 \frac{i k^j \mathbf{e}_A^{*i}}{k^2} \Pi_{ij}.$$

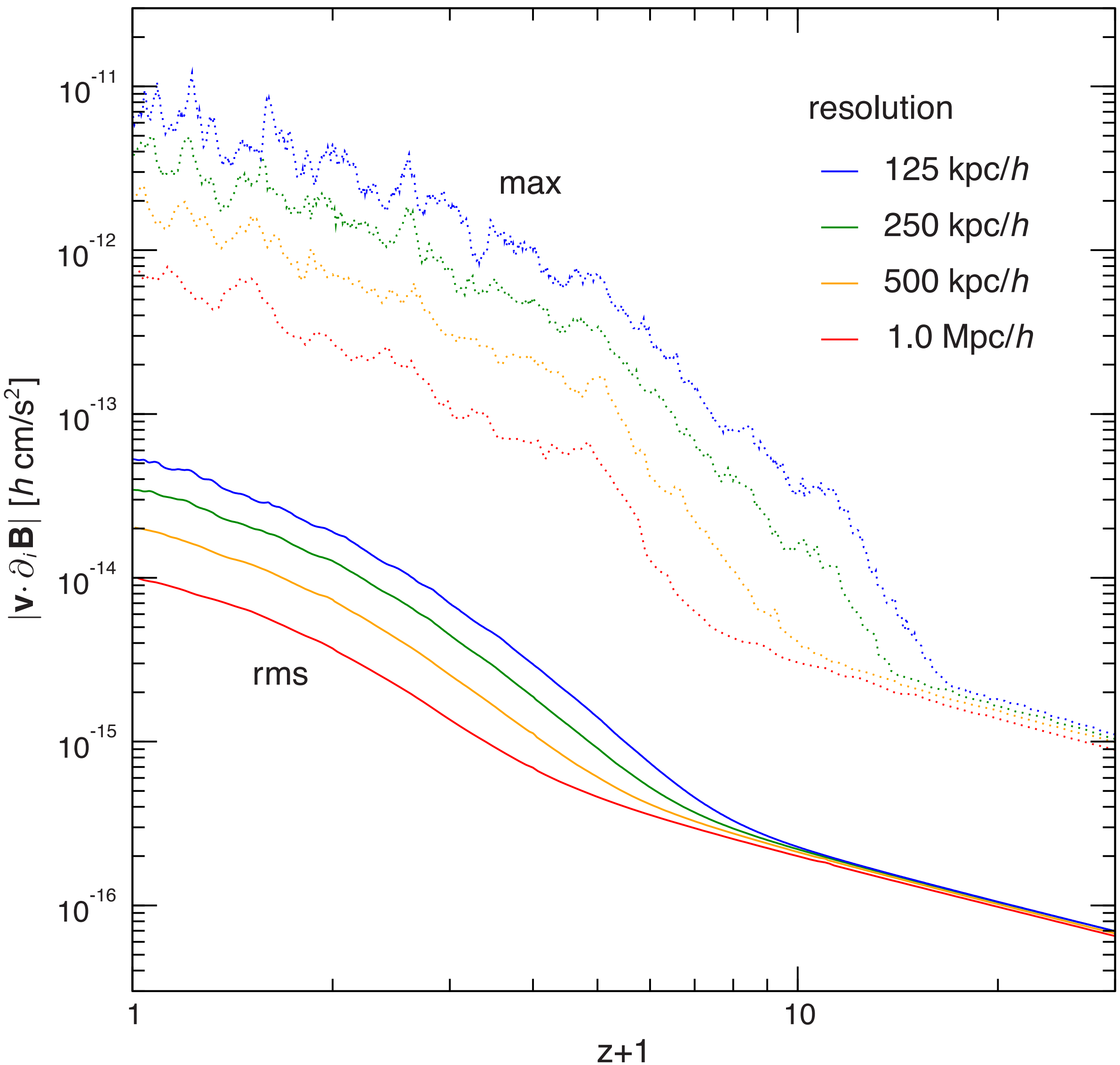
$$h_X'' + 2\mathcal{H} h_X' + k^2 h_X = 16\pi G a^2 e_X^{ij} \Pi_{ij}.$$

$$\rightarrow \quad \Phi - \Psi \simeq \frac{G a^2 \Pi}{k^2},$$

$$B_A \simeq \frac{G a^2 \Pi}{k\mathcal{H}}, \qquad h_X \simeq \frac{G a^2 \Pi}{k^2}.$$



frame-dragging contribution to acceleration



frame dragging is the largest non-Newtonian contribution to particle dynamics

it is more important on smaller scales

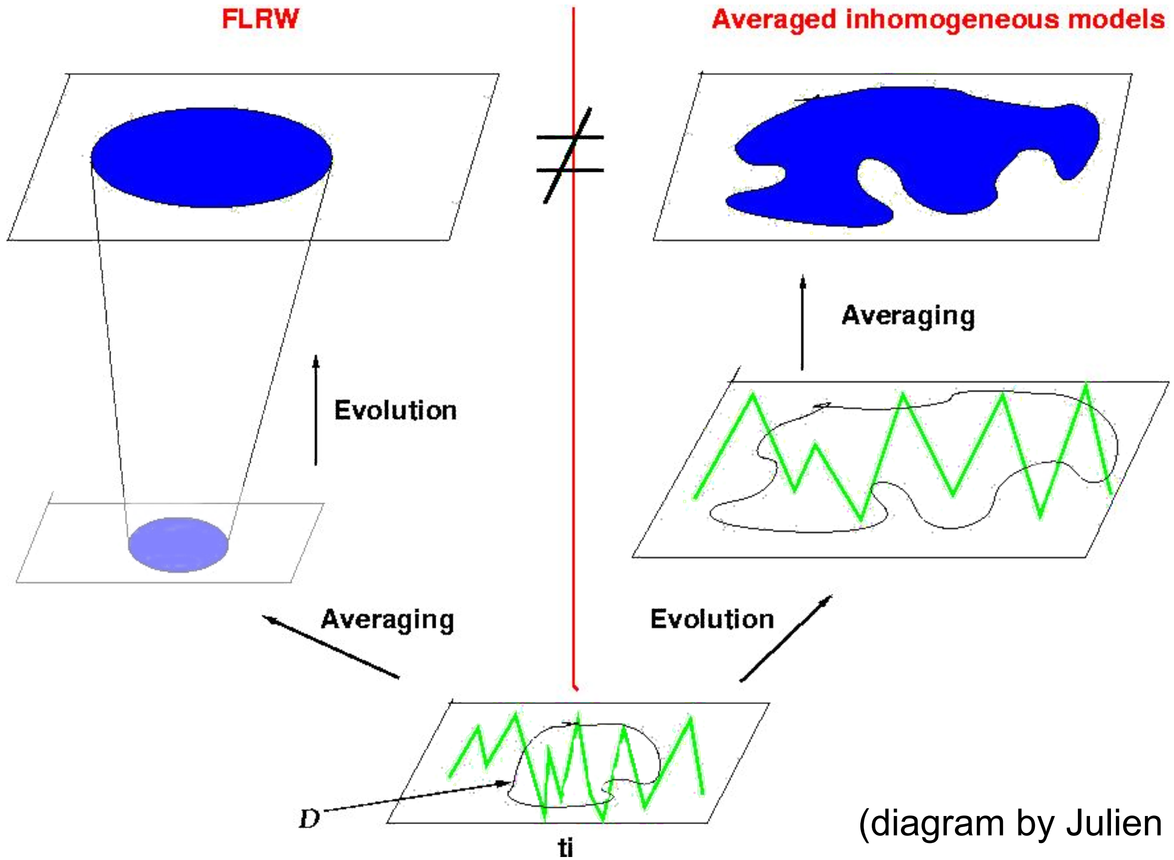
(but power spectra are not affected to scales shown there)

sub-dominant relative to scalar contribution at ~ 1:1000

but convergence needs more study

average and evolution

the average of the evolved universe is in general not the evolution of the averaged universe!



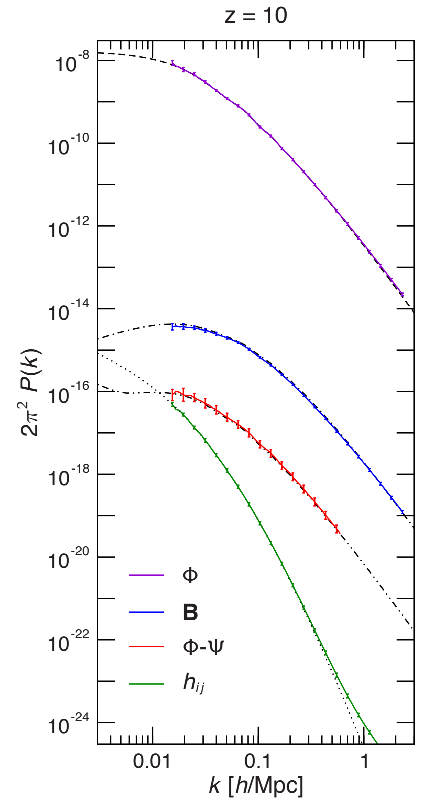
(diagram by Julien Larena)

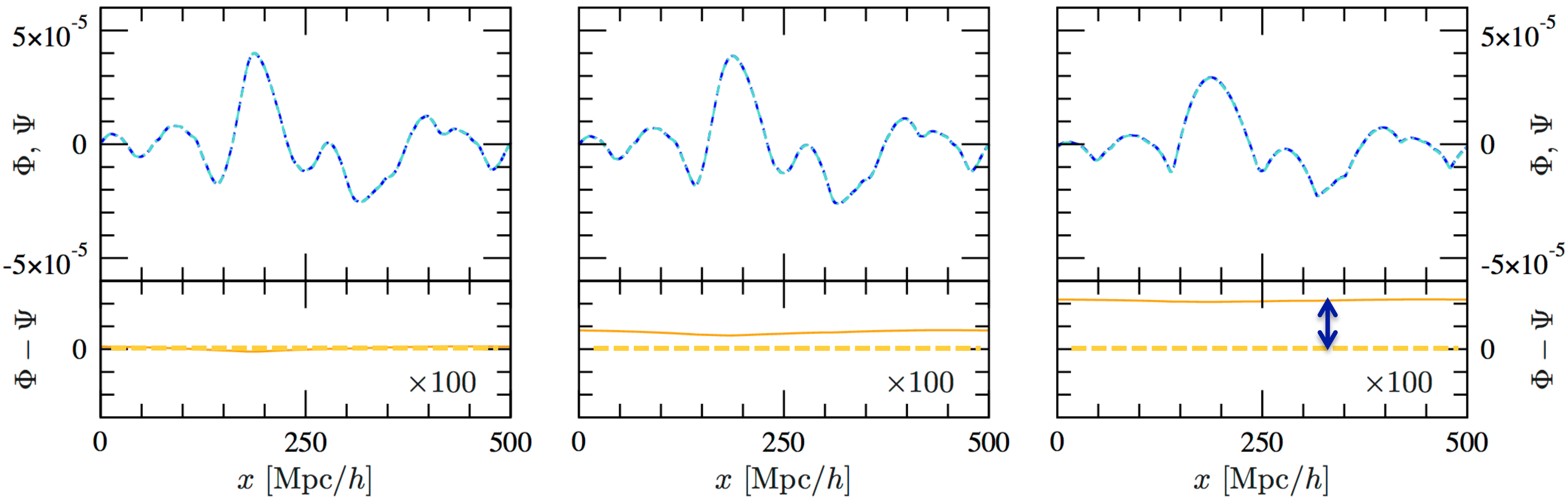
effect would become important around structure formation, same as DE

deviation from FLRW background

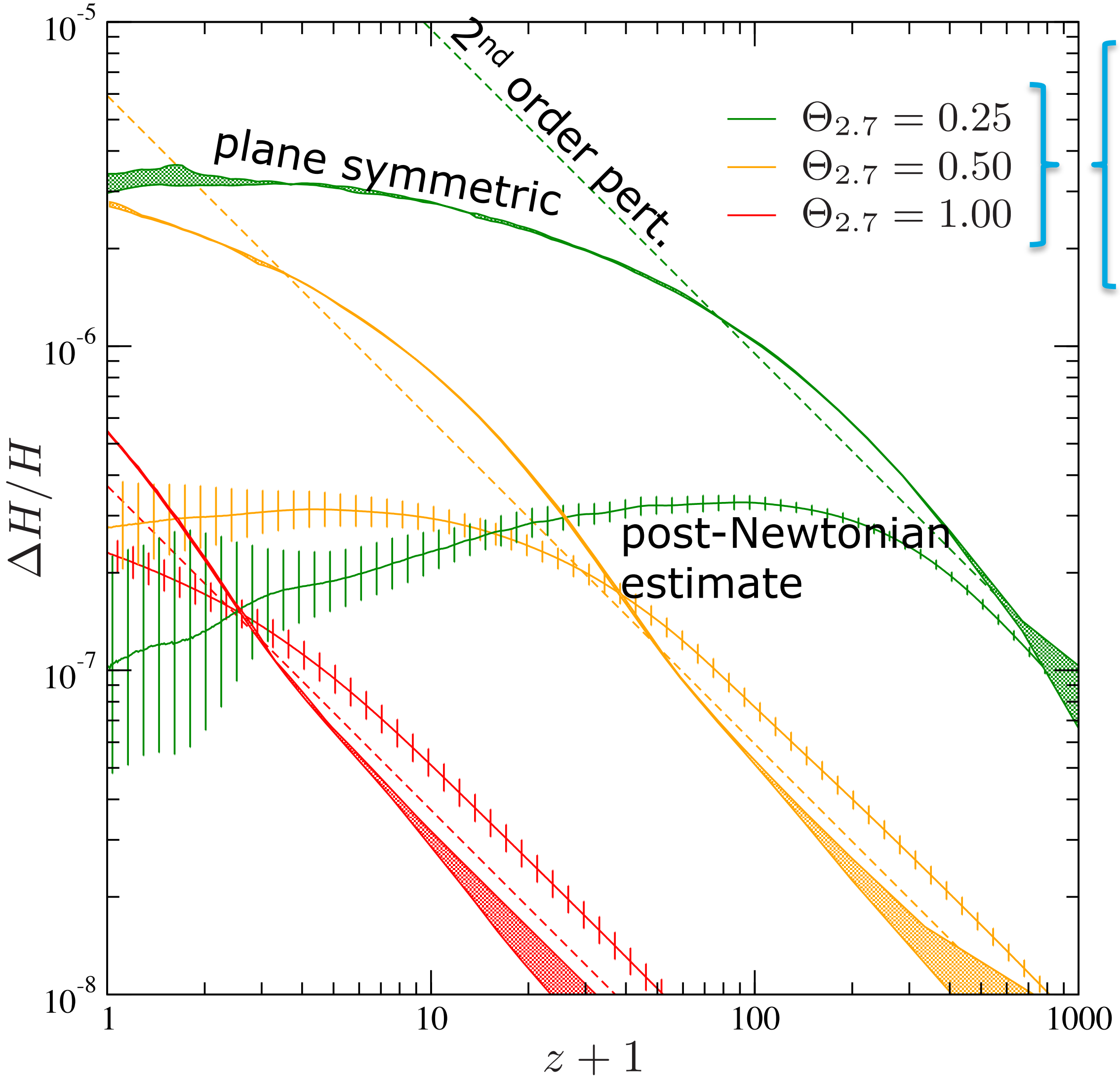
$$ds^2 = -(1+2\psi)dt^2 + a^2(1-2\phi)dx^2$$

- absorb Ψ zero mode into time redefinition
- interpret Φ zero mode as correction to chosen background evolution a(t)
- can check if background evolves differently than in FLRW → not possible in Newtonian simulations!





backreaction seems to stop!



Earlier k_{eq} should increase effect (→ Clarkson & Umeh arXiv: 1105.1886)

True at early times, but correction stops increasing when density perturbations go non-linear!

(Perturbation theory diverges there, can't predict what happens)

Is backreaction self-limiting? Can we understand this?

Layzer-Irvine equation & virialization

correction to expansion rate from zero mode: $\mathcal{H} \to \mathcal{H} - \Phi_0' = n^{\mu}_{;\mu}/3$

equation for evolution of zero mode:

$$2\Phi_0' + 3\mathcal{H}\Omega_m \Phi_0 = -\mathcal{H}\Omega_m \frac{T+U}{M}$$

(In a 'Newtonian interpretation', using $2T = \Sigma m_i v_i^2$ and $2U = \Sigma m_i \psi(x_i)$)

Newtonian gravity:

Layzer-Irvine equation $T' + U' + \mathcal{H}\left(2T + U\right) = 0$

virialization: 2T = -U

→ zero mode approaches a constant value $\Phi_0 \;\to\; -(T+U)/(3M)$

→ correction to expansion rate $\Delta\mathcal{H} \;=\; -\Phi_0'$
goes to zero in the virial limit!

conclusions

- Weak-field limit: cosmological GR N-body simulations are feasible → **gevolution**

  https://github.com/gevolution-code/gevolution-1.0.git
- 3D version working, based on **LATfield2** (latfield.org)
- Deviations from standard results small in ΛCDM:
  - Φ-Ψ, vectors & tensors subdominant also in non-linear regime (but can be taken into account now)
  - halo properties same as in Newtonian sims
  - backreaction appears to self-regulate (?)
- Approach allows for fully consistent treatment of relativistic 'stuff' (massive neutrinos, dark energy / modified gravity, cosmic strings, …)
- Missing: ray-tracing to obtain true observables
- (David plans to visit Heidelberg in January)