UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

A microscopic approach to cosmic structure formation

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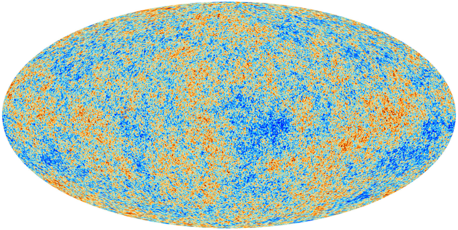
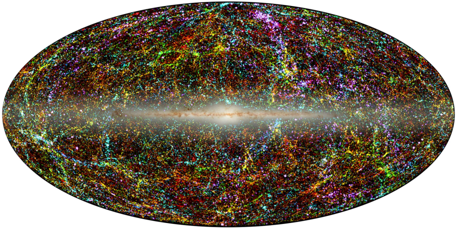
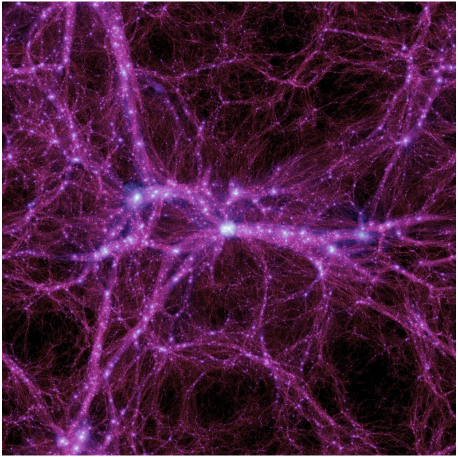
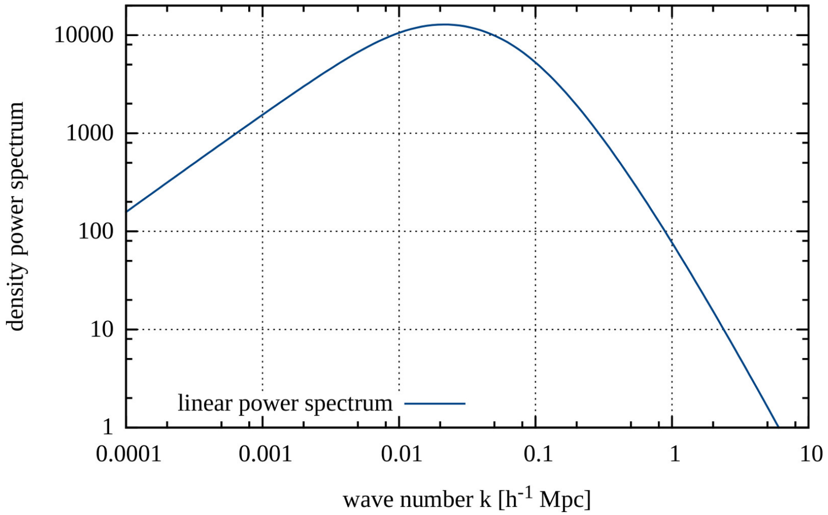
Problems in cosmic structure formation



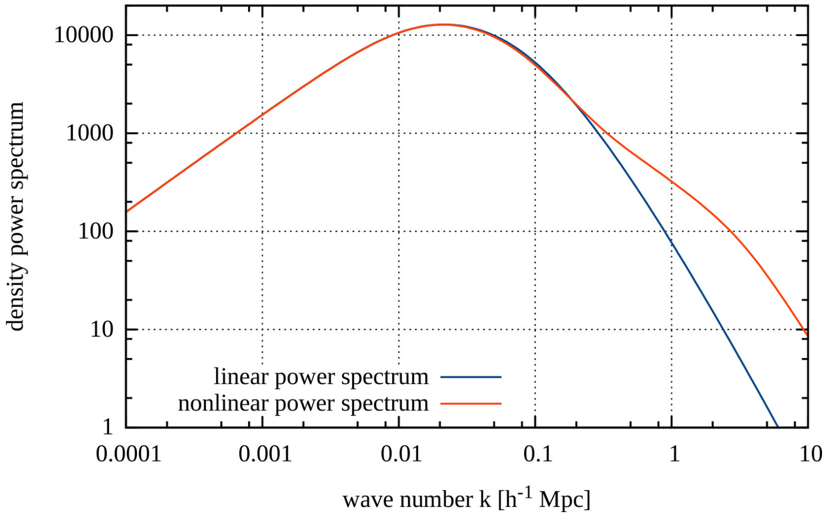




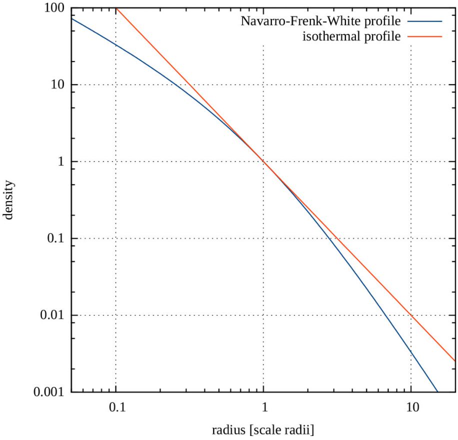
Problems in cosmic structure formation

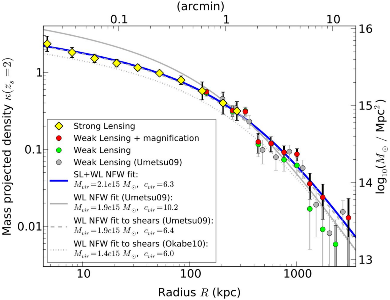


Problems in cosmic structure formation



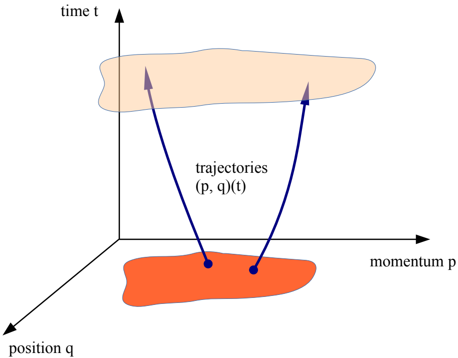
Problems in cosmic structure formation





Cosmological structure formation

- Non-equilibrium statistics of N classical particle trajectories
- Describe particle ensemble by partition sum (generating functional) Z
- Derive statistical properties by functional derivatives



Free generating functional

- Free functional:

$$Z_0[J,K] = \int \mathrm{d}\Gamma^{(\mathrm{i})} \exp\left(\mathrm{i} \int \langle J, \bar{x} \rangle \mathrm{d}t\right)$$

with phase-space trajectories

$$\bar{x}(t) = \begin{pmatrix} \bar{q} \\ \bar{p} \end{pmatrix} = \underbrace{G_{\mathrm{R}}(t,0)x^{(\mathrm{i})}}_{\text{free motion}} - \underbrace{\int_0^t G_{\mathrm{R}}(t,t')K(t')\mathrm{d}t'}_{\text{interaction}}$$

- Initial phase-space distribution

$$\mathrm{d}\Gamma^{(\mathrm{i})} = P(q,p)\mathrm{d}q\mathrm{d}p$$

Free generating functional with collective fields

- Include collective fields

$$Z_0[H,J,K] = \exp\left(\mathrm{i}H\hat{\Phi}\right)\int \mathrm{d}\Gamma^{(\mathrm{i})}\exp\left(\mathrm{i}\int\langle J,\bar{x}\rangle \mathrm{d}t\right)$$

with collective-field operator

$$\hat{\Phi} = \begin{pmatrix}\hat{\Phi}_\rho \\ \hat{\Phi}_B\end{pmatrix}, \quad \hat{\Phi}_\rho = \exp\left(-\vec{k}\cdot\frac{\delta}{\delta J}\right)$$

- Collective *response field* B quantifies the reaction of the ensemble to changes in particle positions,

$$\hat{\Phi}_B = \left(\vec{k}\cdot\frac{\delta}{\delta K}\right)\hat{\Phi}_\rho = \hat{b}\,\hat{\Phi}_\rho$$

Generating functional with interactions

- Include interaction

$$Z[H,J,K] = \exp\left(\mathrm{i}\hat{S}_{\mathrm{I}}\right) Z_0[H,J,K]$$

with interaction operator

$$\hat{S}_{\mathrm{I}} = -\int \mathrm{d}1 \hat{\Phi}_B(-1) v(1) \hat{\Phi}_\rho(1)$$

where v is the Fourier-transformed potential

- Perturbation theory:

$$\mathrm{e}^{\mathrm{i}\hat{S}} = 1 + \mathrm{i}\hat{S} - \frac{1}{2}\hat{S}^2 + \dots$$

Correlations

- n-point density correlator

$$G_{\rho\rho\ldots\rho}(12\ldots n) = \hat{\Phi}_\rho\hat{\Phi}_\rho\ldots\hat{\Phi}_\rho\, Z_0[J,K]\Big|_{J,K=0} \\ = Z_0[L,0] \qquad (1)$$

with the shift L given by

$$L = -\sum_{i=1}^{n}\begin{pmatrix}\vec{k}_i \\ 0\end{pmatrix}\delta_\mathrm{D}(t-t_i)$$

- k-th order interaction terms are of the form

$$\delta G_{\rho\rho}(12) = \int \mathrm{d}1' b(-1')v(1')Z_0[L]$$

Specialisation to cosmology

1. Choose initial phase-space measure

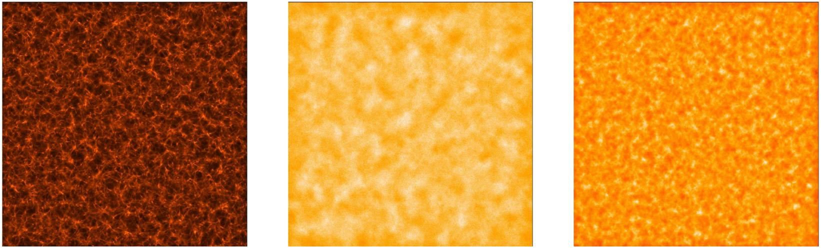
$$\mathrm{d}\Gamma^{(\mathrm{i})} = P(q,p)\,\mathrm{d}^{3N}q\,\mathrm{d}^{3N}p$$

   fully specified by initial power spectrum
2. Change time coordinate

$$t \to \tau = D_+(t) - D_+(t_\mathrm{i})$$

3. Adapt Green's function to expanding universe

Improved Zel'dovich trajectories



$$\vec{q}(\tau) = \vec{q}^{(\mathrm{i})} + \vec{p}^{(\mathrm{i})} \int_0^\tau \exp\left(h(\tau')\right) \mathrm{d}\tau'$$

$$h(\tau) = g^{-1}(\tau) - 1\,, \quad g(\tau) = a^2 D_+ f E(a)$$

$$\hat{v}(k) = -\frac{3}{2k^2}\frac{a}{g^2} \propto a^{-2} \text{ (EdS)}$$

Correlation terms

- In excellent approximation, the generating functional after application of arbitrary many density operators is

$$Z_0[L,0] = V^{-N} \int \mathrm{d}q \exp\left(-\frac{1}{2} L_p^\top C_{pp} L_p\right) \exp\left(\mathrm{i} \left\langle L_q, q \right\rangle\right)$$

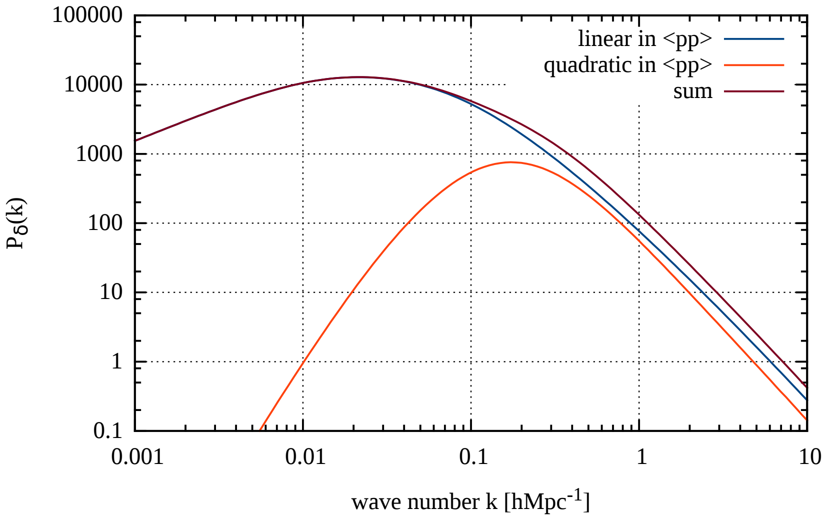
with the position and momentum shifts

$$L_q = -\sum_{i=1}^{n} \vec{k}_i \ , \quad L_p = -\sum_{i=1}^{n} g(t, t_i) \vec{k}_i$$

- C_{pp} is the momentum correlation matrix

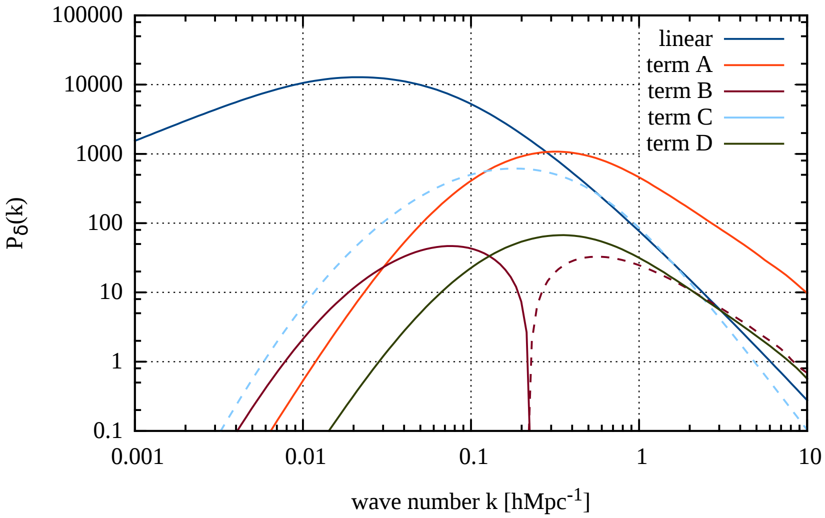
$$C_{pp} = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{\vec{k} \otimes \vec{k}}{k^4} P_\delta(k) \, \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}$$

Density power spectrum



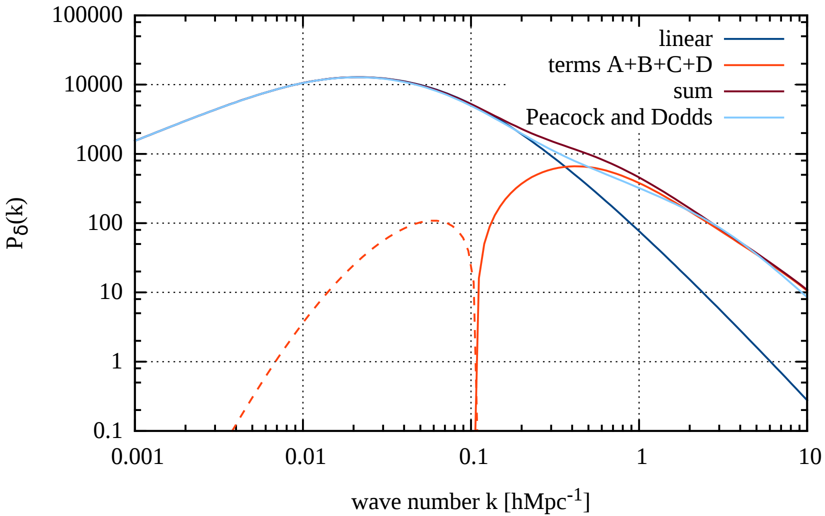
Improved Zel'dovich trajectories, no interaction

Density power spectrum



Improved Zel'dovich trajectories plus first-order interaction

Density power spectrum



Improved Zel'dovich trajectories plus first-order interaction

Summary

1. Non-equilibrium statistical field theory for dark-matter particles set up
2. Hamiltonian equations of motion, simple Green's function
3. Expansion parameter is deviation from unperturbed (improved Zel'dovich) trajectories
4. n-point statistics for collective fields obtained by functional derivatives
5. First-order perturbation theory reproduces numerical results already quite well

Further applications

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