

Gravity at the Horizon:

from the cosmic dawn to ultra-large scales

Miguel Zumalacárregui

Instut für Theoretische Physik - University of Heidelberg → Nordita, Stockholm



UNIVERSITÄT
HEIDELBERG
Zukunft. Seit 1386.

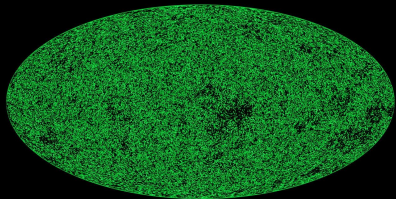


Gravity at the Largest Scales - Oct 2015

with L. Amendola, E. Bellini, J. Lesgourgues, V. Pettorino, J. Renk, I. Sawicki

(other work with D. Bettoni, F. König and M. Martinelli)

Horndeski in the Cosmic Linear Anisotropy Solving System



hi_class

WINTER 2016

- Scalar-tensor theories
- Early modified gravity^{*}
- Non-linear BAO evolution
- Ultra-large scales^{*}

^{*} at the horizon / beyond quasi-static

developed with [Emilio Bellini](#), [Julien Lesgourgues](#), [Iggy Sawicki](#)

Scalar-Tensor gravity

★ Old-School: $f(\phi)R + K(X, \phi)$ $X \equiv -(\partial\phi)^2/2$

▷ quintessence, $f(R)$, Brans-Dicke (Jordan '59, Brans & Dicke '61)

Scalar-Tensor gravity

★ Old-School: $f(\phi)R + K(X, \phi)$ $X \equiv -(\partial\phi)^2/2$

▷ quintessence, $f(R)$, Brans-Dicke (Jordan '59, Brans & Dicke '61)

★ Horndenski's Theory (1974)

$g_{\mu\nu} + \square\phi$ + Local + 4-D + Lorentz theory with 2^{nd} order Eqs.

$$G_2(X, \phi) - G_3(X, \phi)\square\phi + G_4(\phi, X)R + G_{4,X} [(\square\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}] \\ + G_5 G_{\mu\nu}\phi^{;\mu\nu} - \frac{G_{5,X}}{6} [(\square\phi)^3 - 3(\square\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu}{}^{;\nu}\phi_{;\nu}{}^{;\lambda}\phi_{;\lambda}{}^{;\mu}]$$

▷ all Old-school,

Scalar-Tensor gravity

★ Old-School: $f(\phi)R + K(X, \phi)$ $X \equiv -(\partial\phi)^2/2$

▷ quintessence, $f(R)$, Brans-Dicke (Jordan '59, Brans & Dicke '61)

★ Horndenski's Theory (1974)

$g_{\mu\nu} + \square\phi$ + Local + 4-D + Lorentz theory with 2^{nd} order Eqs.

$$G_2(X, \phi) - G_3(X, \phi)\square\phi + G_4(\phi, X)R + G_{4,X} [(\square\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}] \\ + G_5 G_{\mu\nu}\phi^{;\mu\nu} - \frac{G_{5,X}}{6} [(\square\phi)^3 - 3(\square\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu}{}^{;\nu}\phi_{;\nu}{}^{;\lambda}\phi_{;\lambda}{}^{;\mu}]$$

▷ all Old-school, kin. grav. braiding,

Scalar-Tensor gravity

- ★ Old-School: $f(\phi)R + K(X, \phi)$ $X \equiv -(\partial\phi)^2/2$
⊃ quintessence, $f(R)$, Brans-Dicke (Jordan '59, Brans & Dicke '61)

★ Horndenski's Theory (1974)

$g_{\mu\nu} + \square\phi$ + Local + 4-D + Lorentz theory with 2^{nd} order Eqs.

$$G_2(X, \phi) - G_3(X, \phi)\square\phi + G_4(\phi, X)R + G_{4,X} [(\square\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}] \\ + G_5 G_{\mu\nu}\phi^{;\mu\nu} - \frac{G_{5,X}}{6} [(\square\phi)^3 - 3(\square\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu}{}^{;\nu}\phi_{;\nu}{}^{;\lambda}\phi_{;\lambda}{}^{;\mu}]$$

⊃ all Old-school, kin. grav. braiding, covariant Galileon

Scalar-Tensor gravity

★ Old-School: $f(\phi)R + K(X, \phi)$ $X \equiv -(\partial\phi)^2/2$

▷ quintessence, $f(R)$, Brans-Dicke (Jordan '59, Brans & Dicke '61)

★ Horndenski's Theory (1974)

$g_{\mu\nu} + \square\phi$ + Local + 4-D + Lorentz theory with 2^{nd} order Eqs.

$$G_2(X, \phi) - G_3(X, \phi)\square\phi + G_4(\phi, X)R + G_{4,X} [(\square\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}] \\ + G_5 G_{\mu\nu}\phi^{;\mu\nu} - \frac{G_{5,X}}{6} [(\square\phi)^3 - 3(\square\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu}{}^{;\nu}\phi_{;\nu}{}^{;\lambda}\phi_{;\lambda}{}^{;\mu}]$$

▷ all Old-school, kin. grav. braiding, covariant Galileons

▷ proxy th. for massive gravity (de Rham & Heisenberg '11)

Scalar-Tensor gravity

- ★ Old-School: $f(\phi)R + K(X, \phi)$ $X \equiv -(\partial\phi)^2/2$
 - ▷ quintessence, $f(R)$, Brans-Dicke (Jordan '59, Brans & Dicke '61)

★ Horndenski's Theory (1974)

$g_{\mu\nu} + \boxed{\phi}$ + Local + 4-D + Lorentz theory with 2^{nd} order Eqs.

$$G_2(X, \phi) - G_3(X, \phi)\square\phi + G_4(\phi, X)R + G_{4,X} [(\square\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}] \\ + G_5 G_{\mu\nu}\phi^{;\mu\nu} - \frac{G_{5,X}}{6} [(\square\phi)^3 - 3(\square\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu}{}^{;\nu}\phi_{;\nu}{}^{;\lambda}\phi_{;\lambda}{}^{;\mu}]$$

- ▷ all Old-school, kin. grav. braiding, covariant Galileons
- ▷ proxy th. for massive gravity (de Rham & Heisenberg '11)
- ★ Beyond Horndeski (MZ & Garcia-Bellido '13)
 - ▷ General disformal coupling (Bekenstein '92)
 - ▷ "Covariantized" galileons (Gleyzes *et al.* '14)

Unifying approaches to DE:

Post Friedman	$15-22 \times f_i(t)$	Baker <i>et al.</i> '12
EFT for DE	$7-9 \times f_i(t)$	Gubitosi <i>et al.</i> , Bloomfield <i>et al.</i> '12 Gleyzes <i>et al.</i> '13 & '14, ...
EFT for \mathcal{L}_H	$5 \times f_i(t)$	Bellini & Sawicki '14

Parameterizations vs Lagrangian:

- Background \rightarrow often very constraining
- Non-linear effects
- Other regimes: Solar System, Lab., BH,...

2012: *"(I will) produce a computer implementation of the PPF formalism in the Boltzmann code CLASS"*

Unifying approaches to DE:

Post Friedman	$15-22 \times f_i(t)$	Baker <i>et al.</i> '12
EFT for DE	$7-9 \times f_i(t)$	Gubitosi <i>et al.</i> , Bloomfield <i>et al.</i> '12 Gleyzes <i>et al.</i> '13 & '14, ...
EFT for \mathcal{L}_H	$5 \times f_i(t)$	Bellini & Sawicki '14

Parameterizations vs Lagrangian:

- Background \rightarrow often very constraining
- Non-linear effects
- Other regimes: Solar System, Lab., BH,...

2014: “(I will) produce a computer implementation of
~~the PPF formalism~~ Horndeski in the Boltzmann code CLASS”

Horndeski in four words

(Bellini & Sawicki JCAP '14)

Background expansion: $\longrightarrow H(t)$ (or Ω_{de} , or $w\dots$)

$$G_2 - G_3 \square \phi + G_4 R + G_{4,X} [\nabla \nabla \phi]^2 + G_5 G_{\mu\nu} \phi^{;\mu\nu} - \frac{G_{5,X}}{6} [\nabla \nabla \phi]^3$$

Horndeski in four words

(Bellini & Sawicki JCAP '14)

Background expansion: $\longrightarrow H(t)$ (or Ω_{de} , or $w\dots$)

$$G_2 - G_3 \square \phi + G_4 R + G_{4,X} [\nabla \nabla \phi]^2 + G_5 G_{\mu\nu} \phi^{;\mu\nu} - \frac{G_{5,X}}{6} [\nabla \nabla \phi]^3$$

Kineticity: α_K

Standard kinetic term $\rightarrow c_S^2$

Horndeski in four words

(Bellini & Sawicki JCAP '14)

Background expansion: $\longrightarrow H(t)$ (or Ω_{de} , or $w\dots$)

$$G_2 - G_3 \square \phi + G_4 R + G_{4,X} [\nabla \nabla \phi]^2 + G_5 G_{\mu\nu} \phi^{;\mu\nu} - \frac{G_{5,X}}{6} [\nabla \nabla \phi]^3$$

Kineticity: α_K

Standard kinetic term $\rightarrow c_S^2$

Braiding: α_B

Kinetic Mixing of $g_{\mu\nu}$ & ϕ

Horndeski in four words

(Bellini & Sawicki JCAP '14)

Background expansion: $\longrightarrow H(t)$ (or Ω_{de} , or $w\dots$)

$$G_2 - G_3 \square \phi + G_4 R + G_{4,X} [\nabla \nabla \phi]^2 + G_5 G_{\mu\nu} \phi^{;\mu\nu} - \frac{G_{5,X}}{6} [\nabla \nabla \phi]^3$$

Kineticity: α_K

Standard kinetic term $\rightarrow c_S^2$

Braiding: α_B

Kinetic Mixing of $g_{\mu\nu}$ & ϕ

M_p running: α_M

Variation rate of effective M_p

Horndeski in four words

(Bellini & Sawicki JCAP '14)

Background expansion: $\longrightarrow H(t)$ (or Ω_{de} , or $w\dots$)

$$G_2 - G_3 \square \phi + G_4 R + G_{4,X} [\nabla \nabla \phi]^2 + G_5 G_{\mu\nu} \phi^{;\mu\nu} - \frac{G_{5,X}}{6} [\nabla \nabla \phi]^3$$

Kineticity: α_K

Standard kinetic term $\rightarrow c_S^2$

Braiding: α_B

Kinetic Mixing of $g_{\mu\nu}$ & ϕ

M_p running: α_M

Variation rate of effective M_p

Tensor speed excess: α_T

Gravity waves $c_T^2 = 1 + \alpha_T$

Horndeski in four words

(Bellini & Sawicki JCAP '14)

Background expansion: $\longrightarrow H(t)$ (or Ω_{de} , or $w\dots$)

$$G_2 - G_3 \square \phi + G_4 R + G_{4,X} [\nabla \nabla \phi]^2 + G_5 G_{\mu\nu} \phi^{;\mu\nu} - \frac{G_{5,X}}{6} [\nabla \nabla \phi]^3$$

Kineticity: α_K

Standard kinetic term $\rightarrow c_S^2$

Braiding: α_B

Kinetic Mixing of $g_{\mu\nu}$ & ϕ

M_p running: α_M

Variation rate of effective M_p

Tensor speed excess: α_T

Gravity waves $c_T^2 = 1 + \alpha_T$

- $\alpha_M, \alpha_T \Rightarrow$ Mod. tensor eqs. (Saltas, Sawicki, Amendola, Kunz '14)
- $\alpha_K, \alpha_B \Rightarrow$ Kinetic terms

Theory specific relations:

- Quintessence: $\alpha_K \propto \Omega_{\text{DE}}$,
- JBD: $\alpha_K, \alpha_B = -\alpha_M$, Galileon-like: $\alpha_B + \alpha_M, \alpha_T$

Horndeski in four words

(Bellini & Sawicki JCAP '14)

Background expansion: $\longrightarrow H(t)$ (or Ω_{de} , or $w\dots$)

$$G_2 - G_3 \square \phi + G_4 R + G_{4,X} [\nabla \nabla \phi]^2 + G_5 G_{\mu\nu} \phi^{;\mu\nu} - \frac{G_{5,X}}{6} [\nabla \nabla \phi]^3$$

Kineticity: α_K

Standard kinetic term $\rightarrow c_S^2$

Braiding: α_B

Kinetic Mixing of $g_{\mu\nu}$ & ϕ

M_p running: α_M

Variation rate of effective M_p

Tensor speed excess: α_T

Gravity waves $c_T^2 = 1 + \alpha_T$

- $\alpha_M, \alpha_T \Rightarrow$ Mod. tensor eqs. (Saltas, Sawicki, Amendola, Kunz '14)
- $\alpha_K, \alpha_B \Rightarrow$ Kinetic terms

Theory specific relations:

- Quintessence: $\alpha_K \propto \Omega_{\text{DE}}$,
- JBD: $\alpha_K, \alpha_B = -\alpha_M$, Galileon-like: $\alpha_B + \alpha_M, \alpha_T$

BS is no BS

hi_class: How to use it...

Linear cosmology

$$\left. \begin{array}{l} G_2, G_3, G_4, G_5 \\ \phi(t_0), \dot{\phi}(t_0) \end{array} \right\} \longrightarrow \left\{ \begin{array}{l} \text{Kineticity } \alpha_K \\ \text{Braiding } \alpha_B \\ M_p \text{ running } \alpha_M \\ \text{Tensor excess } \alpha_T \end{array} \right\} \longrightarrow \left\{ \begin{array}{l} D_A(z) \\ C_\ell \\ P(k) \\ \dots \end{array} \right.$$

- Start with concrete model G 's + IC

- ★ covariant Galileons $G_2, G_3 \propto X$, $G_4, G_5 \propto X^2$

...

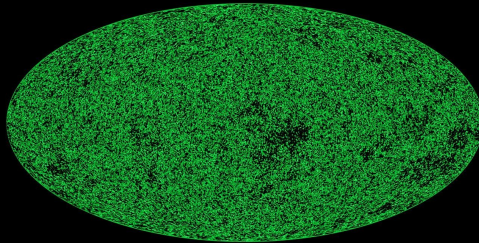
- Start with parameterization α 's + H

- ★ $\alpha_i = \text{constant}$

- ★ $\alpha_i \propto \Omega_{de}$

...

Early Modified Gravity



with Luca, Julien and Valeria

151x.xxxxx

Early Dark Energy

- Models that address coincidence problem
 - $\Rightarrow \Omega_{de} \neq 0$ at early times (e.g. growing ν quintessence)
- CMB $\Rightarrow \Omega_{de}(z_*) \lesssim 1\%$ (Pettorino, Amendola & Wetterich '13)

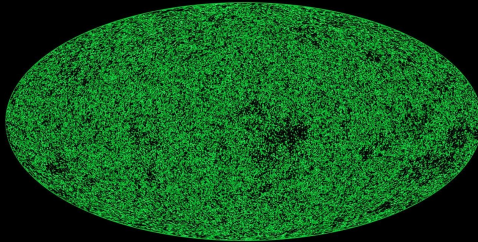
Early Modified Gravity

Soon in your favourite arxiv server...

How much early MG is allowed?

- Tensors $\Rightarrow \alpha_{M/T} \sim \mathcal{O}(1)$ (Amendola, Ballesteros & Pettorino '14)
- Scalars \Rightarrow BS + hi_class (see also Brax, Bruck, Clesse *et al.*)

Non-linear Effects



with Emilio Bellini

PRD 92 (2015) 6, 063522

Non-linear evolution of the BAO scale

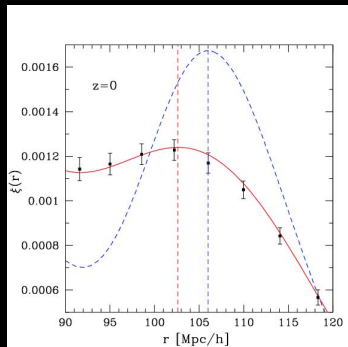
BAO scale in the galaxy distribution \rightarrow comoving standard ruler

$$r_{\text{phys}} = a(t) \cdot r_{\text{coord}}$$

Non-linear BAO evolution ($z=0$)

- Shift $\sim 0.3\%$ smaller
- Broadening $\sim 8 \text{ Mpc}/h$

(Prada *et al.* 1410.4684)



(from Crocce & Scoccimarro - PRD '08)

Non-linear evolution of the BAO scale

BAO scale in the galaxy distribution \rightarrow comoving standard ruler

$$r_{\text{phys}} = a(t) \cdot r_{\text{coord}}$$

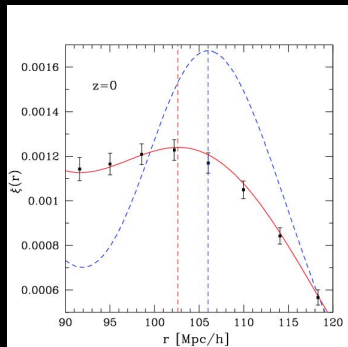
Non-linear BAO evolution ($z=0$)

- Shift $\sim 0.3\%$ smaller
- Broadening $\sim 8 \text{ Mpc}/h$

(Prada *et al.* 1410.4684)

In 2012:

"(My work will) deal with the dynamical evolution of the BAO scale in (MG), which will be measured with growing precision by current and forthcoming surveys."



(from Crocce & Scoccimarro - PRD '08)

in 2013:

E. Bellini \rightarrow Galileon bispectrum

Eulerian perturbation theory

Adjust to a template (Padmanabhan & White '08):

$$P(k) = P_{11}(k/\alpha) \approx P_{11}(k) - \boxed{(\alpha - 1)kP'_{11}(k)}$$

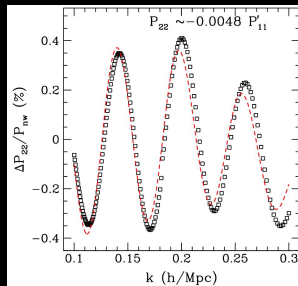
$$P(k) = \underbrace{P_{11}(k)}_{\text{linear}} + \underbrace{\sum_n P_{1n}(k)}_{\text{propagator}} + \underbrace{\sum_{n,m>1} P_{nm}(k)}_{\text{mode coupling}} \quad (P_{nm} \sim \langle \delta_n \delta_m \rangle)$$

$$\underbrace{\hspace{10em}}_{\propto P_{11}(k)}$$

- $P_{1n} \propto P_{11}$
- Mode coupling: $\supset (\dots)kP'_{11} \propto P_{22}$

Peak-background split (Sherwin & Zaldarriaga '12)

$$\alpha - 1 \approx \frac{47}{105} \sigma_{r_{BAO}}^2 \quad (\text{standard GR})$$



(from Padmanabhan & White - PRD'09)

Mode coupling & BAO shift in modified gravity

Non-linear gravitational interactions $\Psi = \mathcal{I}_1 \delta\rho + \mathcal{I}_2 \delta\rho^2 + \dots$

Modified mode coupling:

$$F_2(p, q, \mu) = C_0(t) + C_1(t)\mu \left(\frac{p}{q} + \frac{q}{p} \right) + C_2(t) \left[\mu^2 - \frac{1}{3} \right]$$

Kernel restrictions: $C_0 + \frac{2}{3}C_2 = 2C_1$, $C_1 = \frac{1}{2}$

(Takushima *et al.* '14, Bellini *et al.* '15)

Mode coupling & BAO shift in modified gravity

Non-linear gravitational interactions $\Psi = \mathcal{I}_1 \delta\rho + \mathcal{I}_2 \delta\rho^2 + \dots$

Modified mode coupling:

$$F_2(p, q, \mu) = C_0(t) + C_1(t)\mu \left(\frac{p}{q} + \frac{q}{p} \right) + C_2(t) \left[\mu^2 - \frac{1}{3} \right]$$

Kernel restrictions: $C_0 + \frac{2}{3}C_2 = 2C_1$, $C_1 = \frac{1}{2}$

(Takushima *et al.* '14, Bellini *et al.* '15)

Generalized shift formula (Bellini MZ '15)

$$\alpha_k - 1 = \frac{2}{5} \left(2C_0(t) - \frac{1}{2} \right) \langle \delta_L^2 \rangle \Rightarrow \begin{cases} \text{Linear growth: } \langle \delta_L^2 \rangle \approx \sigma_{r_{BAO}}^2 \\ \text{Non-linear gravity: } C_0 \neq \frac{17}{21} \end{cases}$$

BAO Shift for Galileons: linear growth

$$\alpha_k - 1 \approx \frac{2}{5} \left(2C_0(t) - \frac{1}{2} \right) \sigma_{r_{BAO}}^2 \Rightarrow \begin{cases} \text{Linear: } \sigma_{r_{BAO}}^2 \\ \text{Non-linear: } C_0 \end{cases}$$

- Implement Covariant Galileon in `hi_class` ✓
- Obtain $\delta_1(z)$, $P_{11}(k)$, & $\sigma_{r_{BAO}}$

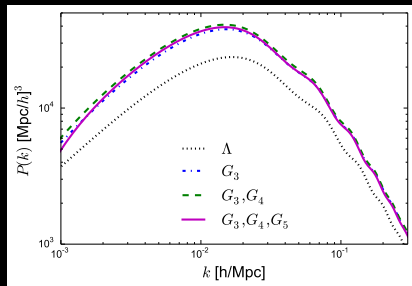
$$G_2 = -X$$

$$G_3 = c_3 X / M^3$$

$$G_4 = \frac{M_p^2}{2} + c_4 X^2 / M^6$$

$$G_5 = c_5 X^2 / M^9$$

Best fit models (Barreira *et al.* '14)



BAO Shift for Galileons: mode coupling

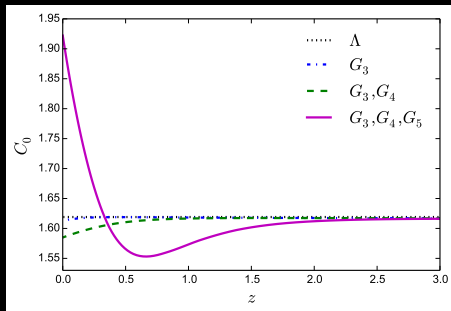
$$\alpha_k - 1 \approx \frac{2}{5} \left(2C_0(t) - \frac{1}{2} \right) \sigma_{r_{BAO}}^2 \Rightarrow \begin{cases} \text{Linear: } \sigma_{r_{BAO}}^2 \\ \text{Non-linear: } C_0 \end{cases}$$

- Expand \mathcal{L}_H over FRW:
scalar perturbations $\rightarrow \mathcal{O}(\delta^3)$
- Quasi-static + sub-horizon approx.
- Identify inhomogeneous sources:

$$\ddot{\delta}_2 + \dots = S_2 [\delta_1(p), \delta_1(q)]$$

- Integrate monopole component

$$S_2 \longrightarrow C_0(t)$$



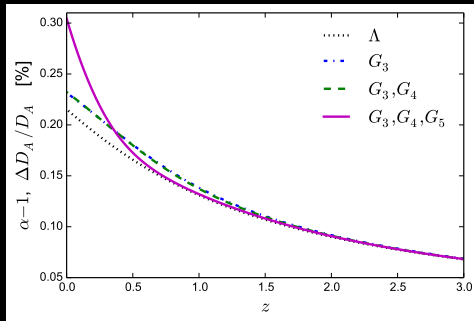
BAO Shift for Galileons

$$\alpha_k - 1 \approx \frac{2}{5} \left(2C_0(t) - \frac{1}{2} \right) \sigma_{r_{BAO}}^2 \Rightarrow \begin{cases} \text{Linear: } \sigma_{r_{BAO}}^2 \\ \text{Non-linear: } C_0 \end{cases}$$

- Can have significant enhancement at $z \sim 0$:)

In 2012:

"Further research will focus on the possibility of measuring such effects on LSS surveys."



BAO Shift for Galileons

$$\alpha_k - 1 \approx \frac{2}{5} \left(2C_0(t) - \frac{1}{2} \right) \sigma_{r_{BAO}}^2 \Rightarrow \begin{cases} \text{Linear: } \sigma_{r_{BAO}}^2 \\ \text{Non-linear: } C_0 \end{cases}$$

- Can have significant enhancement at $z \sim 0$:)
- Forecast \Rightarrow Utterly irrelevant... (Weinberg *et al.* '12)

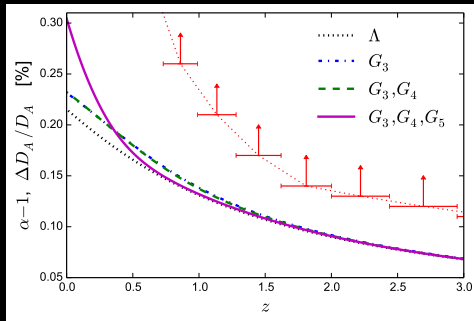
In 2012:

"Further research will focus on the possibility of measuring such effects on LSS surveys."

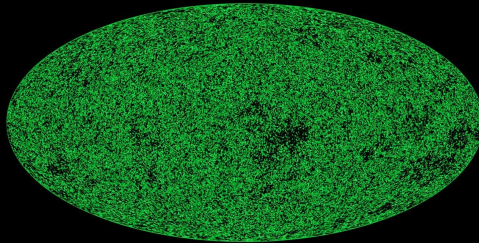
In 2015:

Nope...

but ask me again in 2016 ;)



Ultra-large Scales

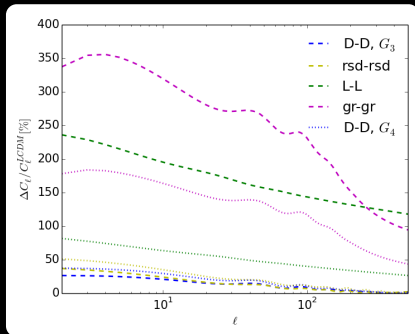
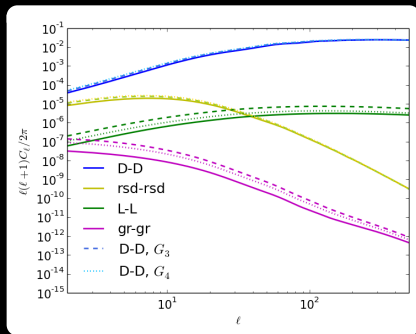


Janina Renk's Master Thesis

160x.xxxxx

Relativistic effects in \mathcal{L}_H

(J. Renk's Master thesis, cf. Ruth's talk)



- **Galileon** \Rightarrow modified $\Phi, \Psi \Rightarrow \uparrow$ Lens. & GR effects
- \uparrow correction for \downarrow dominant effects :(
 $\Delta\text{GR} > \Delta \text{lens} > \Delta\text{RSD} > \Delta \text{density}$)
- (maybe) detectable using multiple tracers (Alonso & Ferreira '15)

Conclusions

- Contemporary scalar-tensor cosmology well understood
- Early MG strongly constrained:
 - ★ Stability priors
 - ★ effects on the CMB
- BAO: great standard ruler
 - ★ even for extreme gravity in future surveys
- Interesting Non-linear effects in MG (more to come...)
- Ultra-large scales
 - ★ rel. effects enhanced but hard to measure
- Early MG + Ultra-large \Rightarrow Horizon / beyond QS approx

Vielen Dank

Ω G_3 δ H $\phi_{\mu\nu}$ X ω V_X G_3 α_K
 $\sqrt{-g}$ Φ $R_{\mu\nu}$ ϕ α_M $\Gamma_{\mu\nu}^\lambda$ G_4 k^2 $\square\phi$ G_3 δ
 \mathcal{L}_H α_M ϕ Ψ H Π \mathcal{L}_H X $\square\phi$ \mathcal{L}_H
 α_N δ $\phi_{\mu\nu}$ h_ν α_K ϕ δ H G_3 Φ $\phi_{\mu\nu}^2$ α_M
 Ψ \mathcal{P} h_ν α_T ϕ^2 \mathcal{E} G_4 $\phi_{\mu\nu}$ R G_3 V_X θ X M_2^2
 ρ α_K H Ω \mathcal{P} $\Gamma_{\mu\nu}^\lambda$ X α_K $\square\phi$ $\phi_{\mu\nu}^2$ α_B
 δ H $\phi_{\mu\nu}$ Φ X ω V_X G_3 α_K
 $R_{\mu\nu}$ ϕ α_M $\Gamma_{\mu\nu}^\lambda$ G_4 k^2 G_3 $\phi_{\mu\nu}^2$ $\square G_3$ δ α_B
 ϕ Ψ H Π \mathcal{L}_H X $\square\phi$ \mathcal{L}_H $\phi_{\mu\nu}$ G_3 Φ G_3 h_ν ϕ α_T
 $\phi_{\mu\nu}$ h_ν α_H α_K ϕ δ H G_3 δ
 h_ν α_T ϕ^2 \mathcal{E} G_4 $\phi_{\mu\nu}$ R G_3 V_X θ X M_2^2
 α_H α_K ϕ δ H G_3 δ

hi_class

h_ν Ψ $R_{\mu\nu}$
 H $\phi_{\mu\nu}$ h_ν
 G_3 α_B $\square\phi$ k^2 $R_{\mu\nu}$
 \mathcal{L}_H G_3 G_3 α_B δ $\sqrt{-g}$ M_2^2
 H α_M V_X \mathcal{L}_H $\sqrt{-g}$ δ X Π Ψ Ψ α_H α_M G_4 Φ α_K α_M α_B α_I
 $\sqrt{-g}$ Ψ α_B δ $\phi_{\mu\nu}$ Ψ \mathcal{P} ρ α_K H Ω \mathcal{P} ϕ G_3 Φ α_K $\phi_{\mu\nu}^2$ α_I
 $R_{\mu\nu}$ ϕ^2 $\phi_{\mu\nu}$ h_ν α_T ϕ^2 θ $R_{\mu\nu}$ $\phi_{\mu\nu}$ X k^2 α_T α_M Ω \mathcal{P} ϕ α_H M_2^2 \mathcal{L}_H α_B α_I Ψ
 ϕ Ψ α_K ϕ^2 \mathcal{E} $\Gamma_{\mu\nu}^\lambda$ X ϕ Ψ H Π \mathcal{L}_H Π Φ \mathcal{E} α_B $\sqrt{-g}$ $\phi_{\mu\nu}$ δ Ψ θ
 $\Gamma_{\mu\nu}^\lambda$ G_3 \mathcal{L}_H X δ h_ν δ $\square\phi$ \mathcal{L}_H Π R \mathcal{E} α_B \mathcal{L}_H $\phi_{\mu\nu}$ δ Ψ $\sqrt{-g}$ \mathcal{L}_1
 G_3 k^2 X α_T G_4 \mathcal{L}_H

und auf Wiedersehen Heidelberg!