Slow-roll Inflation in the era of Euclid

Sébastien Clesse

S.C., C. Ringeval, *work in progress*

RWTH - Aachen University
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Gravity at the Largest Scales
2015, October 26-28, Heidelberg

Euclid Forecasts

Λ-CDM
$\Downarrow$
Extended models and parameterizations

Fisher Forecasts
$\Downarrow$
MCMC forecasts

Planck mock data
$\Downarrow$
Planck 2013, Planck 2015

In the context of inflation:

Scalar spectral index n_{s}, running α_s, tensor-to-scalar ratio r
$\Downarrow$
Hubble-flow parameters ϵ_1, ϵ_2, ϵ_3, slow-roll parameters, etc
$\Downarrow$
Single field models - `ASPIC` library

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Outline

Slow-roll single-field inflation

Inflaton field ϕ evolves slowly along its potential $V(\phi)$

Hubble-flow functions (slow-roll parameters)

$\epsilon_0 \equiv \frac{H_{\rm ini}}{H}$ and $\epsilon_{i+1} \equiv \frac{{\rm d} \ln |\epsilon_i|}{{\rm d}N}$ (N, e-fold number)

In slow-roll : $\epsilon_1 \simeq \frac{M_{\rm pl}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2$ $\epsilon_2 \simeq 2 M_{\rm pl}^2 \left[\left(\frac{V_{,\phi}}{V} \right)^2 - \frac{V_{,\phi\phi}}{V} \right]$

$$\epsilon_2 \epsilon_3 \simeq 2 M_{\rm pl}^4 \left[\frac{V_{,\phi\phi\phi} V_{\phi}}{V^2} - 3 \frac{V_{,\phi\phi}}{V} \left(\frac{V_{,\phi}}{V} \right)^2 + 2 \left(\frac{V_{,\phi}}{V} \right)^4 \right]$$

Observable predictions

- Scalar power spectrum amplitude: $A_{\rm s} \equiv \mathcal{P}_\zeta(k_{\rm p}) \simeq \frac{H^2}{8\pi^2 M_{\rm pl}^2 \epsilon_1}$
- Scalar spectral index *(at second order)*: $n_{\rm s} = 1 - 2\epsilon_1 - \epsilon_2 + (4C+2)\epsilon_1^2 + (2C-1)\epsilon_1\epsilon_2 + C\epsilon_2^2 - C\epsilon_2\epsilon_3$
- Running : $\alpha_s = 4\epsilon_1^2 + 2\epsilon_1\epsilon_2 + \epsilon_2^2 - \epsilon_2\epsilon_3$
- Tensor to scalar ratio: $r \simeq 16\epsilon_1$, Tensor spectral index: $n_{\rm t} \simeq -2\epsilon_1$

evaluated at t_* when the pivot scale $k_* = 0.05 {\rm Mpc}^{-1}$ exits the Hubble radius ($k_* = a(t_*)H(t_*)$) and $C \approx -0.73$

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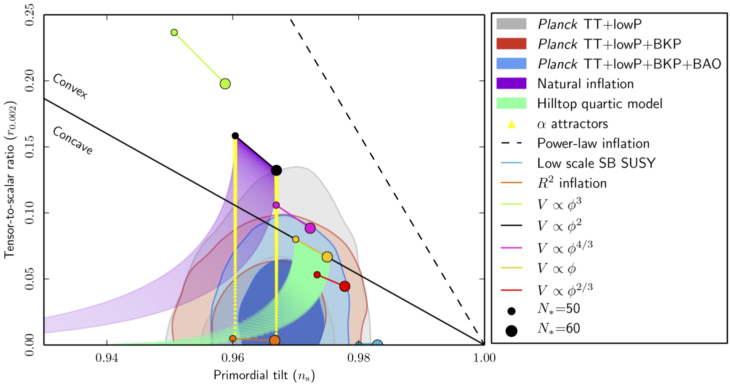
Contraints on $n_s, r, \alpha_s \ldots$ (Planck 2015, 1502.02111)



PRE-2020 experiments (Errard, Feeney, Peiris, Jaffe, 1509.0677):

- BICEP3 × Keck Array × Planck⇒ $\sigma(r) = 0.013$
- PIPER × Planck ⇒ $\sigma(r) = 8.3 \times 10^{-3}$
- SPT-3G × Planck ⇒ $\sigma(r) = 4.8 \times 10^{-3}$

After Planck...

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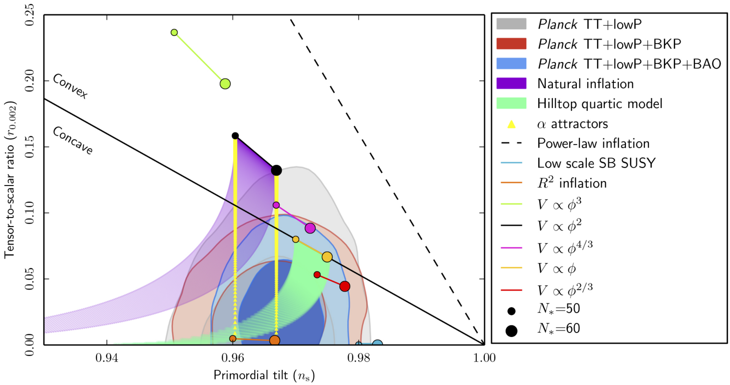

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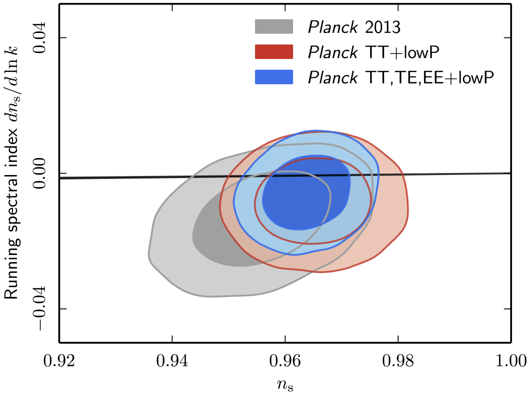
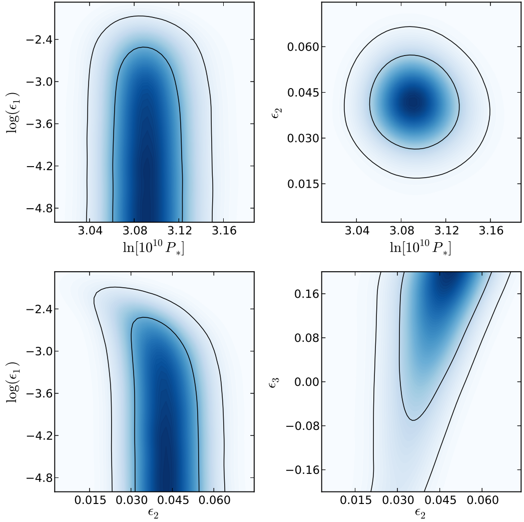


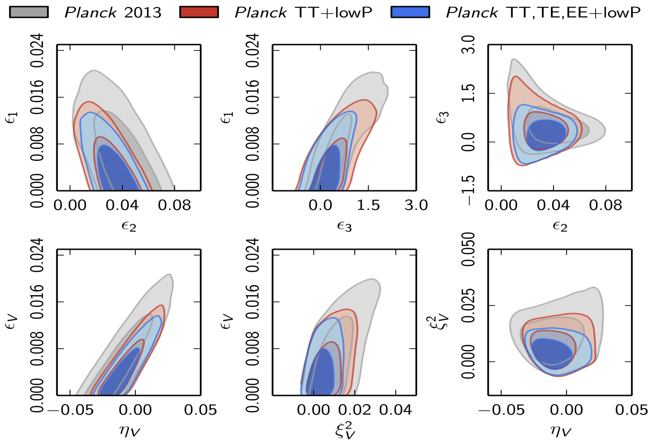
After Planck...

...or on $\epsilon_1, \epsilon_2, \epsilon_3$ (Martin, Ringeval, Trotta, Vennin, 1312.3529)



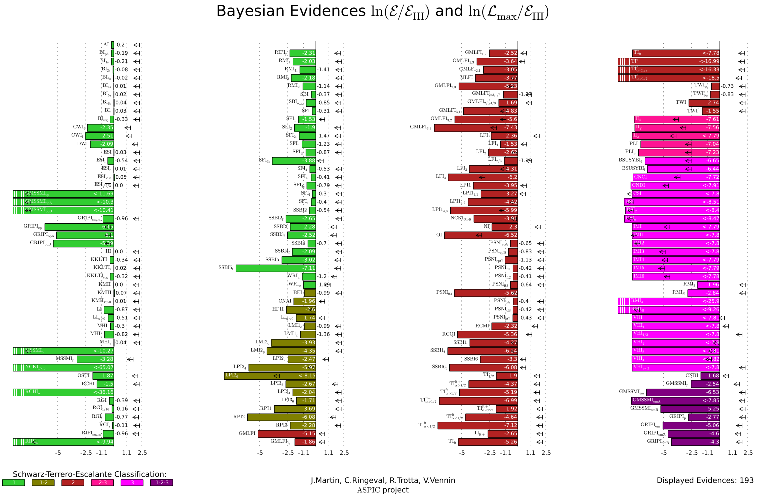
After Planck...

...or on $\epsilon_1, \epsilon_2, \epsilon_3$... or on slow-roll params. (Planck 2015, 1502.02111)



After Planck...

...and on single field models (Martin, Ringeval, Trotta, Vennin, 1312.3529)



Euclid Observables

Observables:

- **3D Galaxy Power spectrum** $P_\mathrm{g}(k,\mu,z)$ Probes the matter power spectrum $P_\delta(k)$ - biased
- **Galaxy Angular Power Spectrum** (cross-correlation between redshift bins i and j) : $C_{l,ij}^{(\mathrm{g})}$.
  Probes the matter power spectrum $P_\delta(k)$ - biased
- **Shear Angular Power Spectrum:** $C_{l,ij}^{(\mathrm{s})}$
  Probes the metric perturbation Ψ power spectrum
- **Cross-correlation galaxy-shear spectra:** $C_{l,ij}^{(\mathrm{g},s)}$
- **Cluster Mass Function** : N_{ij}, number of clusters in redshift bin i and mass bin j
  Probes the matter power spectrum $P_\delta(k)$

Euclid forecasts

- **Huang, Verde, Vernizzi, 1201.5955**
  MCMC, mock Planck, 3D galaxy power spectrum

$$\Delta n_{\rm s} = 0.002, \ \Delta \alpha_{\rm s} = 0.003$$

- **Audren, Lesgourgues, Bird, Haehnelt, Viel, 1210.2194**
  MCMC, mock Planck, galaxy P.S., shear angular P.S.

$$\text{Gal.: } \Delta n_{\rm s} = 2.8/0.37/3.4 \times 10^{-3}$$

$$(k_{max} = 0.1/0.6, \text{ no uncorr. err. / incl. uncorr. err.})$$

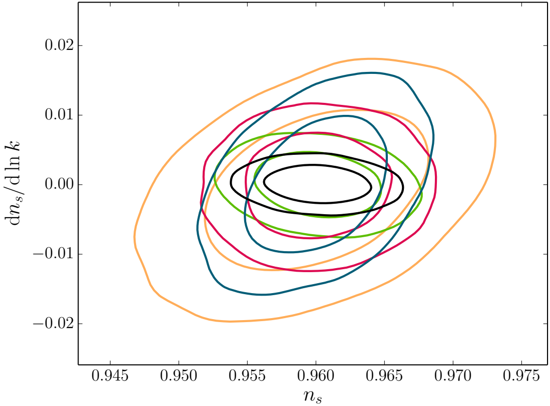
$$\text{Shear: } \Delta n_{\rm s} = 2.4/2.7 \times 10^{-3} \text{ (no uncorr. err. / incl. uncorr. err.)}$$

- **Euclid Theory Group, Euclid Living Rev., 1206.1225**
  MCMC, mock Planck, Fisher matrix: Euclid only
- **Basse, Hamann, Hannestas, Wong, 1209.1043 and 1409.3469**
  MCMC, mock Planck, galaxy and shear angular P.S. + clusters

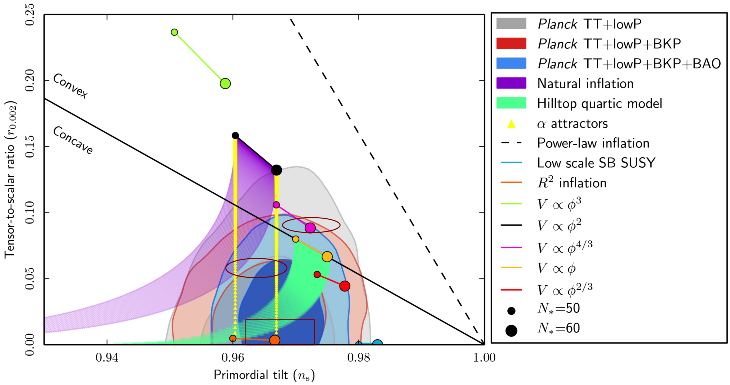
Data	$\sigma(\log A_{\rm s})$	$\sigma(n_{\rm s})$	$\sigma({\rm d}n_{\rm s}/{\rm d}\ln k)$	$\sigma(r)$
c	0.011	0.0052	0.0074	0.028
cs	0.0091	0.0030	0.0030	0.027
cg	0.0046	0.0035	0.0048	0.027
ccl	0.0068	0.0034	0.0064	0.026
cscl	0.0066	0.0028	0.0029	0.026
csgxcl	0.0032	0.0025	0.0017	0.026

Euclid forecasts

Forecasts on $n_{\rm s}, \alpha_s, r$ (Basse, Hamann, Hannestas, Wong, 1409.3469)
CMB, CMB+shear, CMB+Galaxy, CMB+Clusters,
CMB+shear+galaxy+clusters+cross-correlation



Adding clusters increases the lever arm to constrain the running


Convex
Concave
Tensor-to-scalar ratio ($r_{0.002}$)
Primordial tilt (n_s)
0.00
0.05
0.10
0.15
0.20
0.25
0.94
0.96
0.98
1.00
Planck TT+lowP
Planck TT+lowP+BKP
Planck TT+lowP+BKP+BAO
Natural inflation
Hilltop quartic model
α attractors
Power-law inflation
Low scale SB SUSY
R^2 inflation
$V \propto \phi^3$
$V \propto \phi^2$
$V \propto \phi^{4/3}$
$V \propto \phi$
$V \propto \phi^{2/3}$
N_*=50
N_*=60

Euclid forecasts

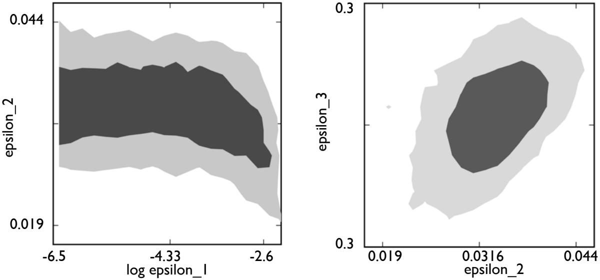
Our forecasts:

- Modified version of the modified `CAMB/COSMOMC` version from **Basse, Hamann, Hannestas, Wong, 1409.3469**
- Modified version `CLASS/MONTEPYTHON` from **Audren, Lesgourgues, Bird, Haehnelt, Viel, 1210.2194**
- REAL Planck data instead of mock CMB data
- Forecasts for Hubble-flow slow-roll parameters

Euclid forecasts

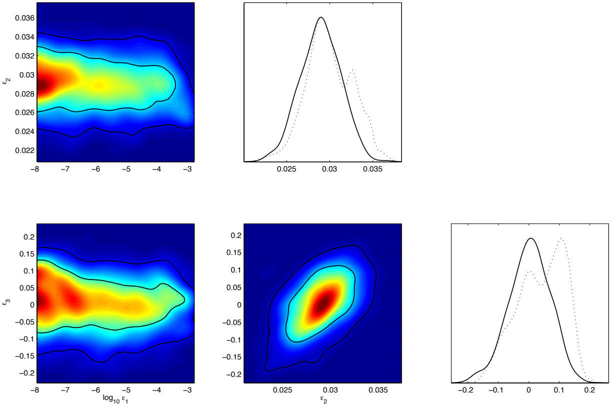
PRELIMINARY !

Forecasts on $\epsilon_1, \epsilon_2, \epsilon_3$ (S.C. Ringeval, *in preparation*), Shear P.S.



Euclid forecasts PRELIMINARY !

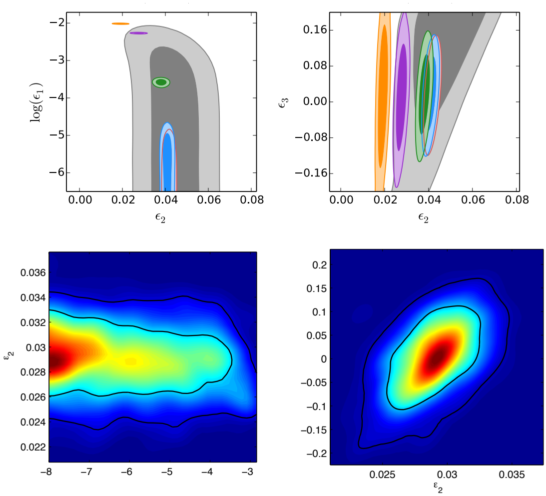
Forecasts on $\epsilon_1, \epsilon_2, \epsilon_3$ (S.C. Ringeval, *in preparation*)
Shear + galaxy + cross. corr. + clusters



4 times better reconstruction of ϵ_2 (!!! More statistics needed!!!)
Euclid could distinguish between concave vs. convex potentials i.e. large vs. small field models

Euclid vs future CMB experiments

Forecasts for PRISM-like experiment, Martin, Ringeval, Vennin, 1407.4034



For ϵ_2 and ϵ_3, Euclid is competitive with future CMB experiments

Summary and perspectives

Euclid: implications for single-field inflation

- Constraints on α_s optimally improved by a factor ~ 4
- Constraints on n_s and α_s optimally improved by a factor ~ 2
- Reconstruction of ϵ_2 (linked to $V_{,\phi\phi}$)
- Distinction between concave and convex potentials at $\gtrsim 2\sigma$

Some perspectives:

- **More statistics needed!**
- **Code validation!**
- *"GI alignement too difficult to be taken into account"*
- Bias: shape and marginalization
- Forecasts for ∼200 models of the ASPIC library
- Detecting features in the scalar PS - non-gaussianity
- Joint forecasts with 21cm experiments (SKA-2020)
- Joint forecasts with future CMB experiments (CoRE+)
- Degeneracies with DE/modified gravity parameters

A lot of work ⇒ please join the Euclid-TH WP4 (initial conditions) !

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Thank you for your attention

Euclid Specifications

Specifications: as in Basse, Hamann, Hannestas, Wong, 1409.3469

- 2 redshift bins for cosmic shear
- 11 redshift bins from $z = 0$ to $z = 3$, with $\Delta z = 0.15$ for galaxy
- 10 redshift bins / 10 mass bins for clusters from $z = 0$ to $z = 2$
- Cluster mass range: $[M_{\rm th}, 10^{16} M_\odot]$
- Field of view: $f_{\rm sky} = 0.37$
- 30 galaxies per arcmin^2
- Photometric uncertainty: simple Gaussian with $\sigma(z) = 0.03(1 + z)$
- Known bias $b = 1$ (optimistic!)
- Subdominant redshift space distortions
- Non-linearity cut-off: $l_{\rm max} = 2000$

As in Audren, Lesgourgues, Bird, Haehnelt, Viel, 1210.2194

Euclid Observables 1

- **Galaxy Angular Power Spectrum** (cross-correlation between redshift bins i and j) :

$$C^{(\mathrm{g})}_{l,ij} = 4\pi \int \mathrm{d}\ln k \, S^{(\mathrm{g})}_{l,i}(k) \, S^{(\mathrm{g})}_{l,j}(k) \, \mathcal{P}_\zeta(k) \qquad (1)$$

Source function : $S^{(\mathrm{g})}_{l,i} = \int \mathrm{d}\chi j_l(k\chi) \mathcal{W}^{(\mathrm{g})}_i(\chi) T_\delta(k, \eta_0 - \chi)$

Window function: $\mathcal{W}^{(\mathrm{g})}_i(\chi) = b(k,\chi) \frac{H(z)\mathrm{d}n/\mathrm{d}z(z)}{n_i}$

with $n_i \equiv \int_{\Delta z_i} \mathrm{d}z' \mathrm{d}n/\mathrm{d}z'(z')$ and $\mathrm{d}n/\mathrm{d}z \propto z^2 \exp(-(z/z_0)^\beta)$

Parameters: $\beta = 1$, $z_0 = 0.3$

Noise : $C^{\mathrm{noise}}_{ij} = \delta_{ij} n_i^{-2}$

- **Shear Angular Power Spectrum:**

$$C^{(\mathrm{s})}_{l,ij} = 4\pi \int \mathrm{d}\ln k \, S^{(\mathrm{s})}_{l,i}(k) \, S^{(\mathrm{s})}_{l,j}(k) \, \mathcal{P}_\zeta(k) \qquad (2)$$

Source function :

$S^{(\mathrm{s})}_{l,i} = -2\sqrt{\frac{l(l^2-1)(l+2)}{4}} \int \mathrm{d}\chi \, j_l(k\chi) \mathcal{W}_i(\chi)^{(\mathrm{s})} T_\Psi(k, \eta_0 - \chi)$

Window function: $\mathcal{W}^{(\mathrm{s})}_i(\chi) = \int_\chi^\infty \mathrm{d}\chi' \, \frac{\chi - \chi'}{\chi\chi'} \, \frac{H(z)\mathrm{d}n/\mathrm{d}z(z)}{n_i}$

Noise : $C^{\mathrm{noise}}_{ij} = \delta_{ij} \langle \gamma^2 \rangle^{1/2} n_i^{-2}$ with $\langle \gamma^2 \rangle^{1/2} \sim 0.35$

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Euclid Observables 2

- **Cluster Mass Function** :

  N_{ij}, number of clusters in redshift bin i and mass bin j

$$N_{ij} = \Delta\Omega \int_0^\infty \mathrm{d}z \int_0^\infty \mathrm{d}M \, \mathrm{d}V \, \mathcal{W}^{c}_{ij}(M,z) \frac{\mathrm{d}n_{\mathrm{ST}}}{\mathrm{d}M}(M,z) \quad (3)$$

  $\Delta\Omega$ is the survey solid angle, $\mathrm{d}V$ is a comoving volume element
  $\mathcal{W}^c_{ij}$ is the window function for redshift and mass bins

  Sheth-Tormen mass function :

$$\frac{\mathrm{d}n_{\mathrm{ST}}}{\mathrm{d}M}(M,z) = \sqrt{\frac{1}{2\pi}} A \frac{\rho_m}{M^2} \left[1 + (a\nu)^{-p}\right] \sqrt{a\nu} \exp\left(-\frac{a\nu}{2}\right) \frac{\mathrm{d}\ln\nu}{\mathrm{d}\ln M} \quad (4)$$

  with $\nu(M,z) \equiv \delta_c^2(M,z)/\sigma_m^2$ and
  $\sigma_m^2(M,z) \equiv \int_0^\infty \mathrm{d}\ln k |W(k,R)|^2 \mathcal{P}_m(k)$

  $\delta_c(M,z)$ is the linear threshold density obtained from the model of spherical collapse

  $W(k,R)$ is the Fourier transform of the top-hat filter

  Parameters: $a = 0.707, A = 0.32, p = 0.3$

  Cosmology enters into the linear matter power spectrum $\mathcal{P}_m(k)$