Evolution of linear perturbations in (Λ)LTB models

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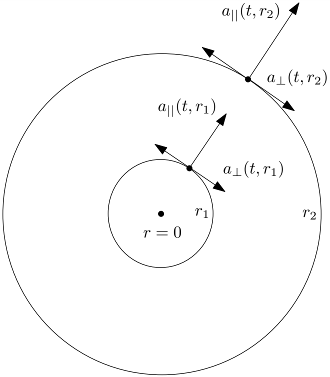
Heidelberg, October 26, 2015

Outline

1. (Λ)LTB models in general
2. Linear perturbations
3. Evolution equations and features
4. Results

Simplest inhomogeneous generalisation of the standard FLRW metric based on an exact solution of Einstein's field equations

1. radially inhomogeneous
2. spherically symmetric about single "central" worldline
3. dust solution
   $T_{\mu\nu} = \rho(t,r) u_\mu u_\nu$

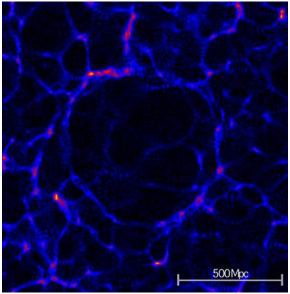


$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \frac{a_\parallel^2(t,r)}{1-\kappa(r)r^2}\mathrm{d}r^2 + r^2 a_\perp^2(t,r)\mathrm{d}\Omega^2$$

Motivation

Original idea:

- large Gpc scale spherical void
  (embedded into FLRW)
- accounts for DE effects on backward lightcone

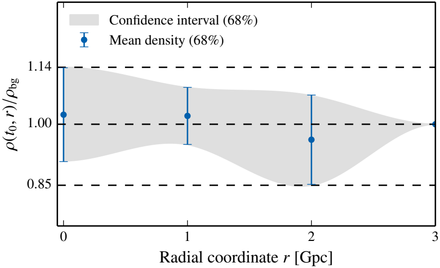


http://www.thphys.uni-heidelberg.de /∼cosmo/dokuwiki/doku.php/altmodels

Problems:

- global $h_{\rm CMB} < 0.4 \longleftrightarrow$ local $h_{\rm SNe} = 0.738$ *(Riess et al. (2011))*
- no evidence for strong anisotropy away from our position (kSZ effect)

- cosmological constant Λ
- Λ + radial inhom. $\rightarrow$ ΛLTB
- constrain deviations from homogeneity $\Rightarrow$ test the Copernican Principle



from Redlich et al. (2014)

Provide **more observables** to

1. definitely distinguish and ΛLTB model from ΛCDM
2. confirm and strengthen findings on spherical voids on a broad scientific basis

investigate linear structure growth in (Λ)LTB models

Linear structure growth in ΛLTB: structures evolve **position-dependent** and **anisotropic**

spherically symmetry at background level:

expansion into **spherical harmonics** $Y^{(\ell m)}(\theta, \phi)$ and cov. derivatives *(Clarkson et al. (2010), Gundlach & Martín-García (2000), Gerlach & Sengupta (1978))*

$$\phi = \sum_{(\ell m)} \phi^{(\ell m)} Y^{(\ell m)}$$

$$\phi_a = \sum_{(\ell m)} \phi^{(\ell m)} Y_a^{(\ell m)} + \bar{\phi}^{(\ell m)} \bar{Y}_a^{(\ell m)}$$

$$\phi_{ab} = \sum_{(\ell m)} \phi^{(\ell m)} Y_{ab}^{(\ell m)} + \bar{\phi}^{(\ell m)} \bar{Y}_{ab}^{(\ell m)}$$

$\Rightarrow$ Scalar-Vector-Tensor expressions on $\mathcal{S}^2$

Sets of gauge invariant quantities can be found

different from FLRW gauge invariant quantities $\{\Psi, \Phi, V_i, h_{ij}\}$!

metric perturbations - Clarkson et al. (2010)

Abstract set of gauge invariant metric perturbations:
$\left\{\chi^{(\ell m)}, \varphi^{(\ell m)}, \varsigma^{(\ell m)}, \eta^{(\ell m)}\right\}$

$$\ddot{\chi} - \frac{\chi'' + C\chi'}{Z^2} + 3H_{\|}\dot{\chi} - \left[A - \frac{(\ell-1)(\ell+2)}{r^2 a_\perp^2}\right]\chi = \mathcal{S}_\chi(\varsigma, \varsigma', \varphi, \dot{\varphi})$$
$$\ddot{\varphi} + 4H_\perp\dot{\varphi} - \left(\frac{2\kappa}{a_\perp^2} - \Lambda\right)\varphi = \mathcal{S}_\varphi(\varsigma, \chi, \dot{\chi}, \chi', \ell)$$
$$\dot{\varsigma} + 2H_{\|}\varsigma = \mathcal{S}_\varsigma(\chi')$$
$$\eta = 0$$

Similar for matter perturbations $\left\{\Delta^{(\ell m)}, w^{(\ell m)}, v^{(\ell m)}\right\}$

Dynamical coupling of gauge invariants !

main differences compared to FLRW:

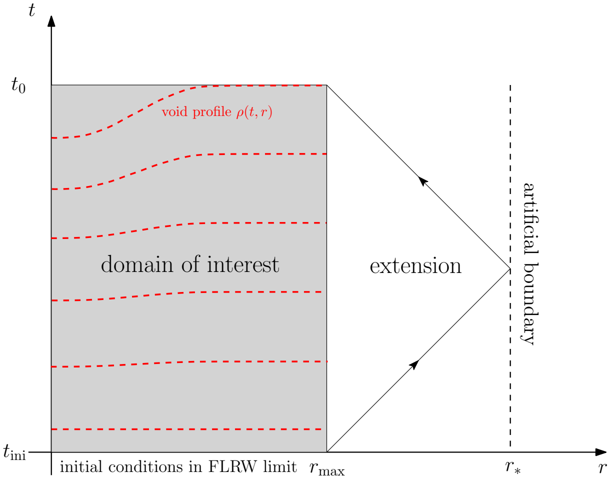
- **dynamical coupling** during spacetime evolution
- **physical interpretation**: LTB gauge invariants $\neq$ FLRW gauge invariants

Numerical solution

- initial conditions
- boundary conditions
- discretization (Finite Elements)

extract observables

- light propagation:
- sensitive to combination of gauge-invariants $\leftarrow$ Ricci- and Weyl focussing terms
- consequences: κ, γ



Dirichlet B.C.s

regularity at $r = 0$:

$$a^{(\ell m)}(t, 0) = 0$$

Gundlach et al. (2000)

artificial boundary at $r = r_*$:

$$a^{(\ell m)}(t, r_*) = 0$$

(adapted from February et al. (2014))

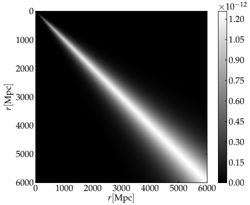
- initial universe at large redshift sufficiently homogeneous and isotropic
- start from initial scalar perturbations in spatially flat FLRW limit (Ψ)
- sample spherical harmonic coefficients from multivariate Gaussian distribution

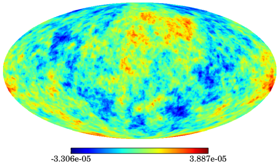
covariance matrices:

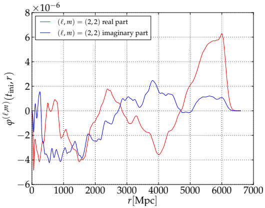
$$C_{\Psi}^{\ell}(r_i, r_j) = \frac{2}{\pi}\int_0^{\infty} \mathrm{d}k\; k^2 P_{\Psi}(k) j_{\ell}(kr_i) j_{\ell}(kr_j)$$

- obtain initial profiles

$$\varphi^{(\ell m)}(t_{\rm ini}, r) = -2\Psi^{(\ell m)}(t_{\rm ini}, r)$$

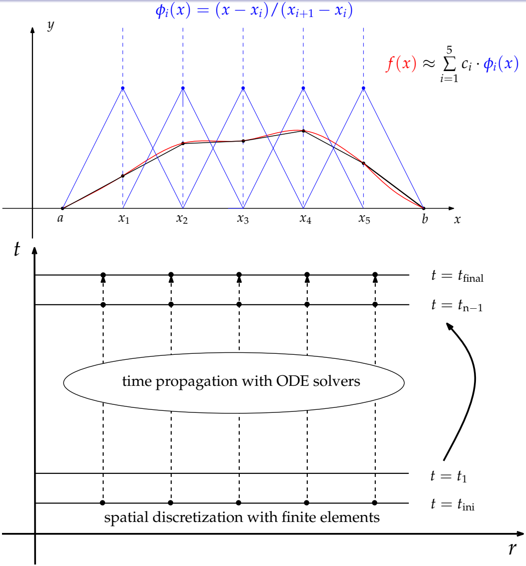






- discretize domain of interest in radius (**Finite Elements**)
- expand variables in basis polynomials
- time evolution of coefficients (**method of lines**)

Dune
Distributed and Unified Numerics Environment



evolve each spherical harmonic coefficients $(\ell, m) \leftarrow$ parallelised

Overview on algorithm & angular Powerspectra

background model

- void density profile
- asymptotically embedded into FLRW model

→

evolve backwards to initial time/redshift ($z_{\text{ini}} \sim 100$)

↓

- **initial perturbations** (FLRW limit)
- $\varphi^{(\ell m)} = -2\Psi^{(\ell m)}$
- $\chi^{(\ell m)} = \varsigma^{(\ell m)} = 0$

←

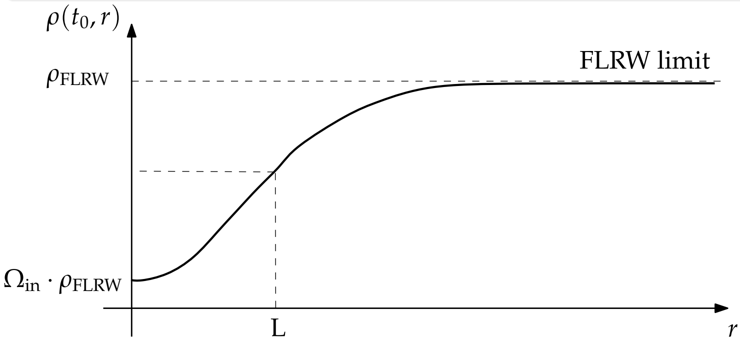
- Evolve linear PDEs for each (ℓ, m)-mode forward in time

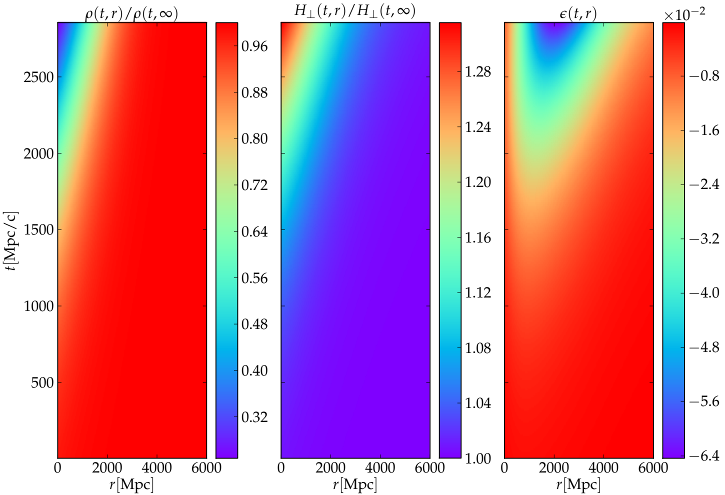
$$C^{\ell}(z) = \sum_{m=-\ell}^{\ell} \frac{\left|a^{(\ell m)}(t(z), r(z))\right|^2}{2\ell + 1}$$

LTB model embedded into EdS

Gaussian void profile: $\rho(t_0, r) = \rho_{\text{bg}} \cdot f(r, \Omega_{\text{in}}, L)$

$$f(r, \Omega_{\text{in}}, L) = 1 + (\Omega_{\text{in}} - 1)\exp\left(-\frac{r^2}{2L^2}\right)$$

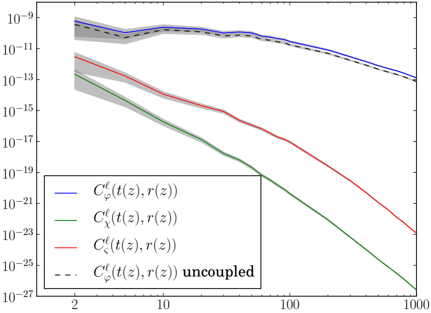




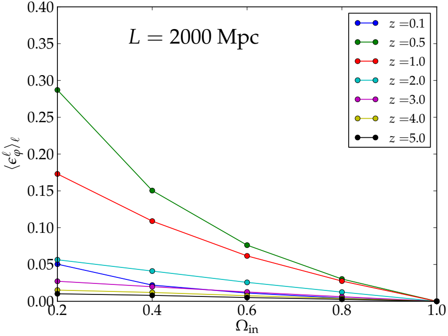
$$\epsilon(t,r) = \frac{H_\parallel - H_\perp}{H_\parallel + 2H_\perp}$$

average coupling strength

$$\epsilon_\varphi^\ell(z) = \left| \frac{\sqrt{C_\varphi^\ell(t(z), r(z))} - \sqrt{C_{\varphi,\mathrm{uc}}^\ell(t(z), r(z))}}{\sqrt{C_{\varphi,\mathrm{uc}}^\ell(t(z), r(z))}} \right|$$

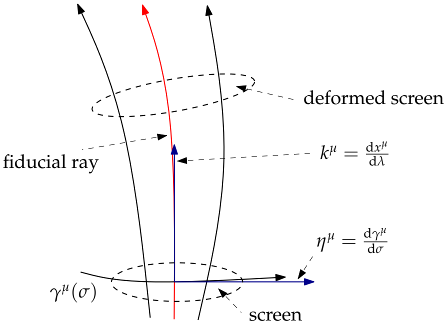


angular powerspectra at
$z = 0.5, \ \Omega_{\mathrm{in}} = 0.2, \ L = 2\mathrm{Gpc}$



coupling strength

Application to best fit ΛLTB nearly done !



Geodesic deviation:

$$\nabla_k^2 \eta^\mu = R^\mu{}_{\nu\alpha\beta} k^\alpha k^\beta \eta^\nu$$

decomposition into Sachs basis, affine parameter :

$$\eta^\mu = \eta_1 n_1^\mu + \eta_2 n_2^\mu$$

$$\frac{d}{d\lambda}(\dots) = k^\mu \nabla_\mu (\dots)$$

$$\eta_a(\lambda) = D_{ab}(\lambda) \left.\frac{d\eta_b}{d\lambda}\right|_{\lambda=0}$$

$$\frac{d^2 D_{ab}}{d\lambda^2} = \mathcal{T}_{ac} D_{cb}$$

$$\text{with } \mathcal{T}_{ab} = \underbrace{-\frac{1}{2} R_{\alpha\beta} k^\alpha k^\beta \delta_{ab}}_{\text{Ricci focussing}} + \underbrace{C_{\alpha\beta\gamma\delta} n_a^\alpha k^\beta k^\gamma n_b^\delta}_{\text{Weyl focussing}}$$

Express Ricci and Weyl focussing in terms of LTB gauge invariants

Jacobi equation: $D_{ab} = D_A^{(\mathrm{LTB})}\delta_{ab} + D_{ab}^{(1)}$ (central observer)

$$\frac{\mathrm{d}^2 D_{ab}^{(1)}}{\mathrm{d}\lambda^2} = -4\pi G\rho\,(t,r)\,(1+z)^2\,D_{ab}^{(1)} + ra_\perp\,(t,r)\,\mathcal{T}_{ab}^{(1)}\left[\{\eta^{(\ell m)},\chi^{(\ell m)},\varsigma^{(\ell m)},\varphi^{(\ell m)}\}\right]$$

Lensing amplification:

$$(A_{ab}) = \frac{(D_{ab})}{D_A^{(\mathrm{LTB})}} = \frac{(D_{ab})}{ra_\perp} = \begin{pmatrix} 1-\kappa & 0 \\ 0 & 1-\kappa \end{pmatrix} - \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix}.$$

Lensing quantities

$$\kappa = -\frac{1}{2}\frac{D_{11}^{(1)} + D_{22}^{(1)}}{ra_\perp}$$

$$\gamma_1 = -\frac{1}{2}\frac{D_{11}^{(1)} - D_{22}^{(1)}}{ra_\perp}$$

$$\gamma_2 = -\frac{1}{2}\frac{D_{12}^{(1)} + D_{21}^{(1)}}{ra_\perp}$$

- determine angular power spectra $C^\ell_{\kappa\kappa}$ and $C^\ell_{\gamma\gamma}$
- compare to observationally determined spectra

Conclusion

- **numerical solution** of evolution equations of gauge invariant perturbations in ΛLTB spacetimes (proof of concept)
- **coupling strength** significant in sufficiently deep and large voids (without Λ)
- application to best fit ΛLTB models nearly done
- observable predictions from **light propagation** (work in progress)