

# Observational constraints in nonlocally modified gravity

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Gravity at the Largest Scales 2015

Heidelberg

## Introduction: Theory

## ● Inspiration

(Arkani-Hamed et al. 2002, Dvali 2006)

$$\mathcal{L}_{\text{proca}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu - A_\mu j^\mu \quad \Leftrightarrow \quad \mathcal{L}_{\text{nl}} = -\frac{1}{4}F_{\mu\nu}\left(1 - \frac{m^2}{\square}\right)F^{\mu\nu} - A_\mu j^\mu$$

where  $(\square^{-1}\phi)(x) = \int d^4y G(x,y)\phi(y)$

## ● Applying the same idea to Fierz-Pauli massive gravity

$$\mathcal{L}_{\text{nl}} = \frac{1}{2}h_{\mu\nu}\left(1 - \frac{m^2}{\square}\right)\mathcal{E}^{\mu\nu\rho\sigma}h_{\rho\sigma} - 2m^2\chi\frac{1}{\square}\partial_\mu\partial_\nu(h^{\mu\nu} - \eta^{\mu\nu}h)$$

→ Obstruction: covariantization  $\Rightarrow g^{\mu\nu}R_{\mu\nu} = 0$

"Covariant vDVZ discontinuity"

$$\left[\left(1 - \frac{m^2}{\square_g}\right)G_{\mu\nu}\right]^T = 8\pi GT_{\mu\nu} \quad (\text{Porrati 2002; Jaccard, Maggiore, Mitsou 2013})$$

▷ Unviable background cosmology

▷  $\square^{-1}R_{\mu\nu} \subset \square^{-1}G_{\mu\nu}$  generates instabilities

(Ferreira, Maroto 2013)

▷  $g_{\mu\nu}\square^{-1}R \subset \square^{-1}G_{\mu\nu}$  stable

(Foffa, Maggiore, Mitsou 2013)

## Introduction: Phenomenology

$$\text{Model RT :} \quad G_{\mu\nu} - m^2 (g_{\mu\nu} \square_{ret}^{-1} R)^T = 8\pi G T_{\mu\nu} \quad (\text{MM 2013})$$

where  $(\square^{-1}\phi)(x) = \int d^4y G(x,y)\phi(y)$

- Two models modifying General Relativity nonlocally – in the infrared
  - ▶  $m^2$  sets a new reference energy scale
    - Nonlocal terms contributes for  $\square_g \ll m^2$  and vice versa
- Phenomenological approach
- Interesting cosmology:
  - ▶ FRW background/linear perturbations
  - ▶ Observational constraints and model comparison

$$\text{Model RR :} \quad S_{RR} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} m^2 R \frac{1}{\square^2} R \right] \quad (\text{MM, Mancarella 2013})$$

# Application to Cosmology

## Model RT

$$G_{\mu\nu} - m^2(g_{\mu\nu}\square_{ret}^{-1}R)^T = 8\pi GT_{\mu\nu}$$

## Model RR

$$G_{\mu\nu} - m^2K_{\mu\nu}(\square_{ret}^{-1}R, \square_{ret}^{-2}R) = 8\pi GT_{\mu\nu}$$

- Resolution method: Localisation

$$\square V = R \quad \Rightarrow \quad V = \square^{-1}R + V^{(hom)}$$

- ▷ Auxiliary fields with *vanishing initial conditions*
- ▷ They are not genuine (*freely* propagating) degrees of freedom

$$G_{\mu\nu} + m^2 \left[ Ug_{\mu\nu} - \frac{1}{2}(\nabla_\mu S_\nu + \nabla_\nu S_\mu) \right] = 8\pi GT_{\mu\nu}$$

$$\square_g U = -R, \quad \partial_\mu U = \frac{1}{2}\nabla_\nu(\nabla_\mu S^\nu + \nabla^\nu S_\mu)$$

$$G_{\mu\nu} - m^2K_{\mu\nu}(V, S) = 8\pi GT_{\mu\nu}$$

$$\square_g V = R, \quad \square_g S = V$$

- Specialisation to flat FRW

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

# Background evolution

- Modified Friedmann equations :

$$H^2(t) = 8\pi G \sum_i \bar{\rho}_i(t) + m^2 Y(\{\bar{V}_k\}, H(t))$$

+ auxiliary EoM for  $\{\bar{V}_k\}$

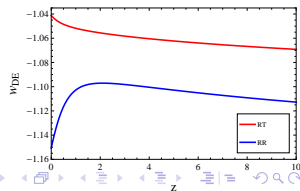
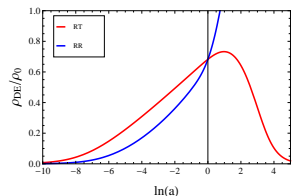
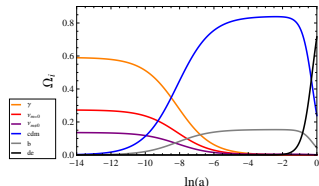
- $m^2 Y \equiv \bar{\rho}_{\text{DE}}(t)$ : Dynamical dark energy
- $\square^{-1} R|_{\text{RD}} = 0$  : Late-time effectiveness
- Flatness today:  $m_{\text{RT}} \simeq 0.67 H_0$ ,  $m_{\text{RR}} \simeq 0.28 H_0$
- From  $\dot{\bar{\rho}}_{\text{DE}} = -3H(1 + w_{\text{DE}})\bar{\rho}_{\text{DE}}$

Fit :  $w(t) = w_0 + (1 - a(t))w_a$

RT:  $w_0 \simeq -1.04$ ,  $w_a \simeq -0.02$

RR:  $w_0 \simeq -1.15$ ,  $w_a \simeq 0.08$

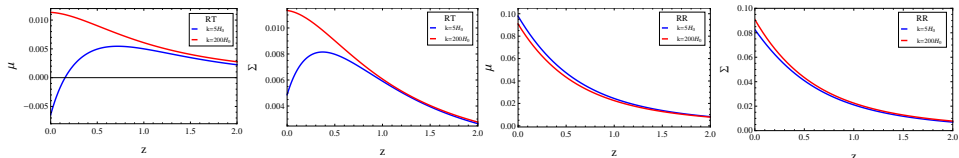
→ On the phantom side:  $w_{\text{DE}} < -1$



# Scalar perturbations and Structure Formation

- Gravitational  $\Psi$  and lensing potential  $(\Psi - \Phi)$  (YD, Foffa, Khosravi, Kunz, Maggiore 2014)

$$\Psi = [1 + \mu(z, k)] \Psi_{\Lambda\text{CDM}}, \quad (\Psi - \Phi) = [1 + \Sigma(z, k)] (\Psi - \Phi)_{\Lambda\text{CDM}}$$

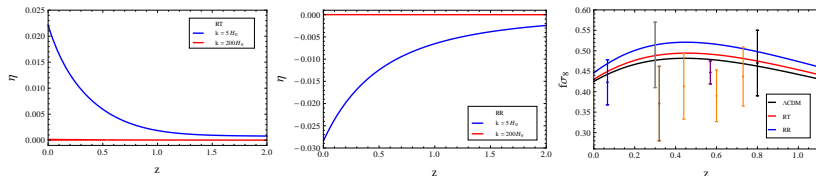


- Fit:  $\mu(t) = \mu_s a(t)^s$  RT:  $\mu_s = 0.01, s = 0.8$ , RR:  $\mu_s = 0.09, s = 2$  (EUCLID:  $\Delta\mu_s = 0.01$ )

- Gravitational slip and RSD (6dF, SDSS LRG, BOSS LOWZ+CMASS, WiggleZ, VIPERS)

$$\eta = (\Psi + \Phi)/\Phi,$$

$$f \equiv \frac{d \ln D}{d \ln a} \text{ with } D(a) \sim \delta_M(a)$$



- Consistency with structure formation

- Nonlinear structure formation for RR: N-body simulation

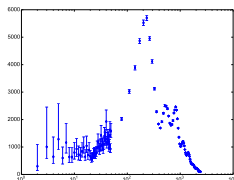
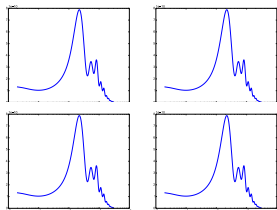
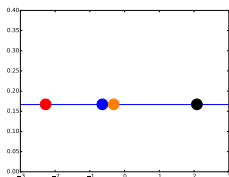
# Boltzmann Code and Parameter Inference

- Implementation in CLASS: Computation of CMB and LSS observables
- Observational constraints and model comparison with MONTEPYTHON  
(Lesgourgues, Audren et al.)
- Cosmological scenario: *Planck* baseline
  - ▷ 6 cosmo parameters varied:  $\{\omega_b, \omega_c, H_0, A_s, n_s, z_{\text{reio}}\}$
  - ▷ Neutrino: Two massless species  $N_{\text{eff}} = 2.03351$ , one massive  $m_\nu = 0.06\text{eV}$
- Datasets:
  - ▷ CMB: [Planck 2013](#), [Planck 2015](#)
  - ▷ Supernovae: SDSS-II/SNLS3 Joint Light-Curve Analysis (JLA 2014)
  - ▷ BAO: BOSS LOWZ+CMSS DR10&11 ([iso.](#), [aniso.](#)), 6dF and [SDSS MGS](#)
  - ▷  $H_0$ : HST ( $70.6 \pm 3.3$ ,  $73.0 \pm 2.4$ ,  $73.8 \pm 2.4$ )  
([YD, Foffa, Kunz, MM, Pettorino, 2014](#))  
([YD, Foffa, Kunz, MM, Pettorino, in prep.](#))

# Observational constraints and parameter inference

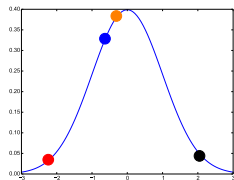
## ● Bayesian inference:

- ▷ Observed datasets: Planck 2013/2015, JLA, BAO, HST, etc
- ▷ Statistical models:  $\Lambda$ CDM, RT and RR with  $\{\omega_b, \omega_c, H_0, A_s, n_s, z_{\text{reio}}\}$
- ▷ Parameter estimation: Update our degree of belief through observations

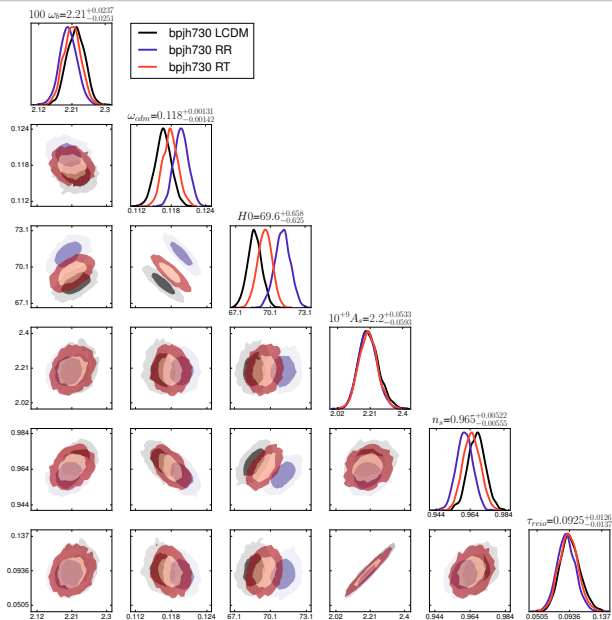


## ▷ Minimum $\chi^2$ estimation:

$$\chi^2 = \sum_{\text{dataset}} \chi_{\text{dataset}}^2 \quad \text{with} \quad \chi_{\text{dataset}}^2 = \sum_i \frac{(\theta_{\text{theo}}^i - \theta_{\text{obs}}^i)^2}{(\sigma_{\text{obs}}^i)^2}$$





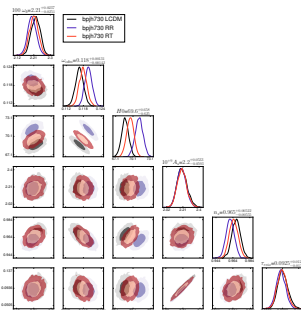


(YD, Foffa, Kunz, MM, Pettorino 2014)

## Observational constraints and parameter inference

| Param                 | Planck                       |                              |                              | BAO+Planck+JLA               |                              |                              |
|-----------------------|------------------------------|------------------------------|------------------------------|------------------------------|------------------------------|------------------------------|
|                       | $\Lambda$ CDM                | $g_{\mu\nu}\square^{-1}R$    | $R\square^{-2}R$             | $\Lambda$ CDM                | $g_{\mu\nu}\square^{-1}R$    | $R\square^{-2}R$             |
| $\omega_c$            | $0.1194^{+0.0027}_{-0.0026}$ | $0.1195^{+0.0026}_{-0.0028}$ | $0.1191^{+0.0027}_{-0.0028}$ | $0.1175^{+0.0015}_{-0.0014}$ | $0.1188^{+0.0014}_{-0.0014}$ | $0.1204^{+0.0014}_{-0.0013}$ |
| $H_0$                 | $67.56^{+1.2}_{-1.3}$        | $68.95^{+1.3}_{-1.3}$        | $71.67^{+1.5}_{-1.5}$        | $68.43^{+0.61}_{-0.69}$      | $69.3^{+0.68}_{-0.66}$       | $70.94^{+0.74}_{-0.7}$       |
| $\Delta\chi^2_{\min}$ | 9801.7                       | 9801.3                       | 9800.1                       | 10485.5                      | 10485.0                      | 10488.7                      |

| Param                 | BAO+Planck+JLA+ $H_0 = 73.0 \pm 2.4$ |                              |                              |
|-----------------------|--------------------------------------|------------------------------|------------------------------|
|                       | $\Lambda$ CDM                        | $g_{\mu\nu}\square^{-1}R$    | $R\square^{-2}R$             |
| $\omega_c$            | $0.117^{+0.0014}_{-0.0014}$          | $0.1182^{+0.0013}_{-0.0014}$ | $0.1201^{+0.0013}_{-0.0013}$ |
| $H_0$                 | $68.72^{+0.61}_{-0.63}$              | $69.60^{+0.66}_{-0.63}$      | $71.14^{+0.72}_{-0.69}$      |
| $\Delta\chi^2_{\min}$ | 10488.9                              | 10487.3                      | 10489.3                      |

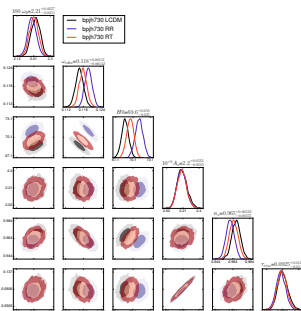


- Few parameters with  $\gtrsim 1\sigma$  deviation from  $\Lambda$ CDM  
→ Nonlocal models prefer a bigger  $H_0$
- Nonlocal vs  $\Lambda$ CDM: Overall  $|\Delta\chi^2| \lesssim 2$   
→ Mostly statistically equivalent to  $\Lambda$ CDM
- Planck: RR fits slightly better  $C_l^{TT}$  at low- $l$
- BAO+Planck+JLA: RR creates a Planck-JLA  $1\sigma$ -tension

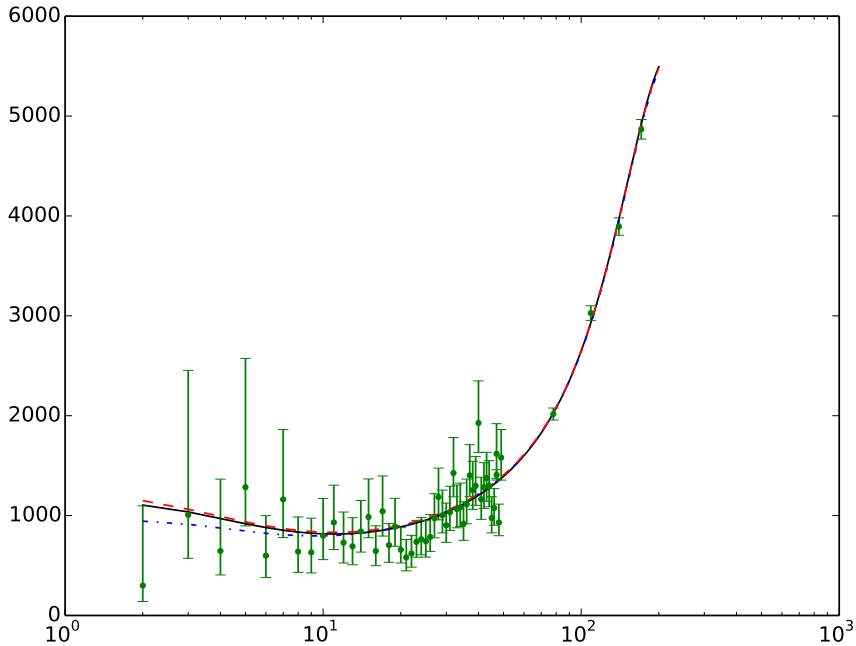
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| $\Delta\chi^2_{\min}$ | 1.6                          | 1.2                          | 0                            | 0.5                          | 0                            | 3.7                          |

| Param                 | BAO+Planck+JLA+ $H_0 = 73.0 \pm 2.4$ |                              |                              |
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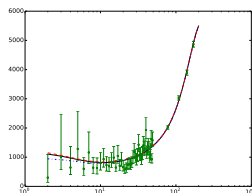
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## Observational constraints and parameter inference

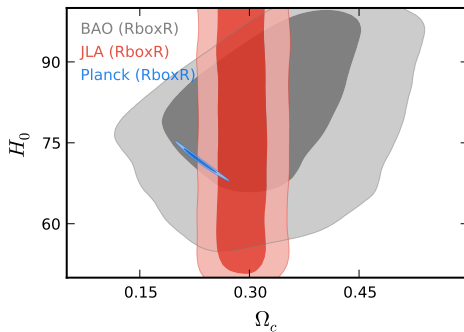
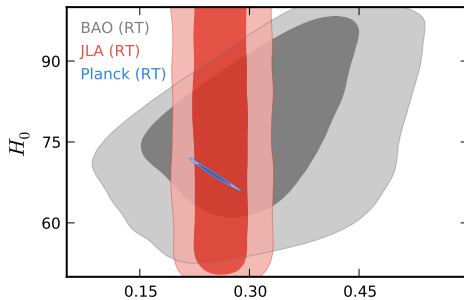
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| BAO+Planck+JLA+ $H_0 = 73.0 \pm 2.4$ |                             |                              |                              |
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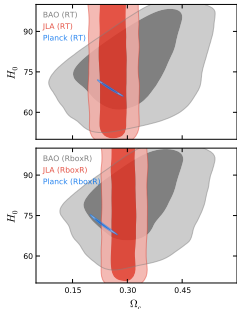
$$\Delta\omega_c = \Delta(\Omega_c h^2) \sim 1\%$$



## Observational constraints and parameter inference

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# Bayesian model selection

- Computation of the Bayes factor: done by considering the nested models

$$G_{\mu\nu} - m^2(g_{\mu\nu} \square_{\text{ret}}^{-1} R)^T - g_{\mu\nu} \Lambda = 8\pi G T_{\mu\nu}$$

$$\mathcal{L} = \frac{1}{16\pi G} [R - 2\Lambda - m^2 R \square^{-2} R] + \mathcal{L}_m$$

with cosmological parameter space  $\{\omega_b, H_0, A_s, n_s, z_{\text{reio}}, \Omega_\Lambda, \Omega_{de}\}$

→ Non-informative priors are flat on  $\Omega_\Lambda$  and  $\Omega_{de}$

- Three statistical models in each case:  $\mathcal{M}_{\Lambda+de}$ ,  $\mathcal{M}_\Lambda$ ,  $\mathcal{M}_{de}$
- Bayes theorem

$$P(\theta|d, \mathcal{M}) = \frac{P(d, \mathcal{M}|\theta)P(\theta|\mathcal{M})}{P(d, \mathcal{M})}$$

- Savage-Dickey density ratio:

$$B_{\Lambda/(\Lambda+de)} = \frac{P(d, \mathcal{M}_\Lambda)}{P(d, \mathcal{M}_{\Lambda+de})} = \frac{P(\Omega_{de}|d, \mathcal{M}_{\Lambda+de})}{P(\Omega_{de}|\mathcal{M}_{\Lambda+de})} \Big|_{\Omega_{de}=0}$$

→ Model  $\Lambda$  (dis)favored with betting odds  $B_{\Lambda/(\Lambda+de)} : 1$  wrt  $\Lambda + de$



# Conclusion

$$G_{\mu\nu} - m^2(g_{\mu\nu} \square_{\text{ret}}^{-1} R)^T = 8\pi G T_{\mu\nu}$$

$$\mathcal{L} = \frac{1}{16\pi G} [R - m^2 R \square^{-2} R] + \mathcal{L}_m$$

- Two observationally viable models of gravity (JCAP 1504 (2015) 04, 044, arXiv:1411.7692)
- Phenomenological side
  - ▶ Well behaved dynamical dark energy
  - ▶ Same number of free parameters than  $\Lambda$ CDM
  - ▶ Fit the data as well as  $\Lambda$ CDM

→ Provide observationally consistent alternatives to  $\Lambda$ CDM
- Theoretical side: Effective models/terms
  - ▶ Suggest effects/mechanisms for dynamical dark energy generation
  - ▶ Dimensional transmutation, conformal anomaly (Maggiore 2015)

