Viscous Fluids as Models for Large Scale Strcuture

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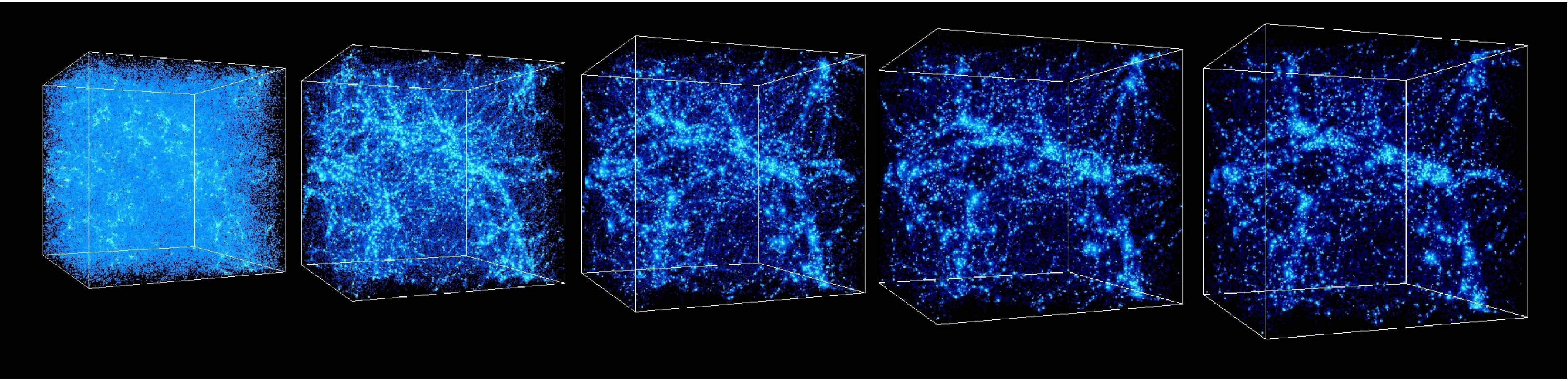
Based on
arXiv: 1509.03073
with Gerasimos Rigopoulos

Large Scale Structure



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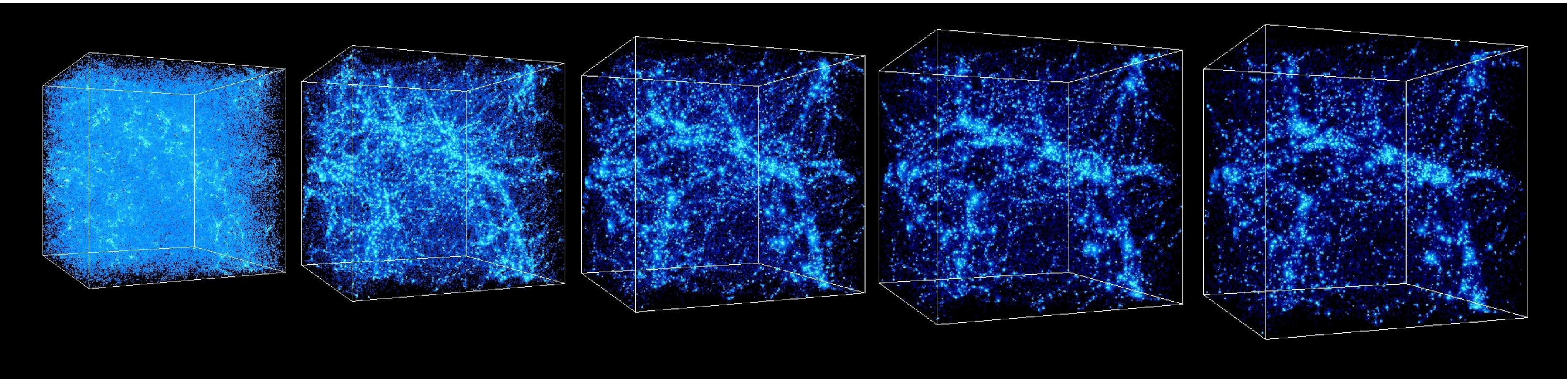
Initially small fluctuations —— Time evolution / Vlaslov equation ——> Complex Large Scale Structure

Large Scale Structure



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Initially small fluctuations

Time evolution

→

~~Vlaslov equation~~

Fluid equations

Complex Large Scale Structure

Failure of Perturbation Theory

Solve fluid equation perturbatively

$$\partial_D \delta + \nabla \cdot \mathbf{w} = -\nabla \cdot (\delta \mathbf{w})$$

$$\partial_D \mathbf{w} + \frac{3}{2}\frac{\gamma}{D}\left(\mathbf{w} + \frac{\nabla \psi}{D}\right) = -\mathbf{w} \cdot \nabla \mathbf{w}$$

$$\nabla^2 \Psi = \frac{\delta}{D}$$

$$\mathbf{w} = \frac{d\mathbf{x}}{dD}$$

$$\gamma = \frac{\Omega_m}{f^2} \approx 1$$

$D =$ growth factor

Two possible sources of failure

Soft modes: $\sigma_d^2 = \frac{4\pi}{3}\int d^3q\, P_L(q)$ ⟶ Can be resumed (RPT)

Crocce, Scoccimaro 2005&2006

Gallilean invariance ⟶ Do not effect equal time correlators

Scoccimaro, Frieman 1995 Kehagias, Riotto 2013
Peloso, Pietroni 2013 Blas et. al. 2013

Hard modes: $\sigma_l^2 = 4\pi \int d^3q\, q^2 P_L(q)$ ⟶ Strong UV-dependence

Strong dependence on modes which are non-perturbative
not described by the fluid equations

Microscopic Stress

Infinite hierarchy of moment equations

$$\partial_D \delta + \nabla \cdot \mathbf{w} + \nabla \cdot (\delta \mathbf{w}) = 0$$

$$\partial_D \mathbf{w} + \frac{3}{2}\frac{\gamma}{D}\left(\mathbf{w} + \frac{\nabla\psi}{D}\right) + \mathbf{w} \cdot \nabla \mathbf{w} = \frac{\nabla\cdot\sigma}{1+\delta}$$

$$\partial_D \sigma + 3\frac{\gamma}{D}\sigma + \mathbf{w} \cdot \nabla\sigma + \sigma \cdot \nabla + (\nabla \mathbf{w})^T \cdot \sigma = \ldots$$

Single stream approximation $\sigma = 0$ and all higher moments

→ Closed set of equation → Good approximation during early stages

Only appears to be self-consistent → Caustics

Well defined if $\sigma \neq 0$ but, hierarchy can not be truncated

- Initial/background velocity dispersion
- Velocity dispersion is generated, when non-linear structures form

Effective Fluid for Warm Dark Matter

Shoji, Komatsu 2010
Boyanovsky et. al. 2008
Lesgourgues, Tram 2011

Stress suppresses structures at small scales $k \gg k_{\rm FS} \sim \frac{1}{\sqrt{\sigma}}$

Can construct an effective fluid description for $k \ll k_{\rm FS}$

Higher moments are of the order $\overline{w^n} \sim k_{\rm FS}^{-n}$ ⟶ Integrate out

Split into background and perturbation $\sigma = \bar{\sigma} + \delta\sigma$

To first order in $\mathbf{w}$ and $k_{\rm FS}^{-1}$ the stress reads

$$\sigma \approx \sigma_i \mathbf{1} \left(\frac{D_i}{D}\right)^3 + \delta\sigma_i \left(\frac{D_i}{D}\right)^3 + \sigma_i \mathbf{1} \left(\frac{D_i}{D}\right)^3 \int_{D_i}^{D} d\eta \, \left(\nabla^2 \mathbf{w} + \nabla\nabla \cdot \mathbf{w}\right)(\eta) + \ldots$$

Plug into Euler equation

$$\partial_D \mathbf{w} + \frac{3}{2}\frac{\gamma}{D}\left(\mathbf{w} + \frac{\nabla\psi}{D}\right) + \mathbf{w}\cdot\nabla\mathbf{w} =$$

$$= \nabla\cdot\delta\sigma_i \left(\frac{D_i}{D}\right)^3 + \sigma_i \left(\frac{D_i}{D}\right)^3 \nabla\delta + \sigma_i \left(\frac{D_i}{D}\right)^3 \int_{D_i}^{D} d\eta \, \left(\nabla^2 \mathbf{w} + \nabla\nabla \cdot \mathbf{w}\right)(\eta) + \ldots$$

"Noise" Sound-speed Non-local viscosity

Effective terms are in principle known ⟶ can be ressumed

FF, Yvonne Y.Y. Wong
arXiv 1412.2764

Averaged Equations of Motion

Average Fields over scale Λ^{-1}

$$\rho_\Lambda(\mathbf{x}) = \int d^3x' \, W_\Lambda\left(|\mathbf{x}-\mathbf{x}'|\right) \rho(\mathbf{x}')$$

$$\rho_\Lambda(\mathbf{x})\mathbf{w}_\Lambda(\mathbf{x}) = \int d^3x' \, W_\Lambda\left(|\mathbf{x}-\mathbf{x}'|\right) \rho(\mathbf{x}')\mathbf{w}(\mathbf{x}')$$

Averaged fluid equations (index Λ dropped):

$$\partial_D \delta + \nabla \cdot \mathbf{w} + \nabla \cdot (\delta \mathbf{w}) = 0$$

$$\partial_D \mathbf{w} + \frac{\gamma}{D}\left(\mathbf{w} + \frac{\nabla \psi}{D}\right) + \mathbf{w} \cdot \nabla \mathbf{w} = \frac{\nabla \cdot \sigma}{1+\delta}$$

Effective stress

Two possibilities

- Measure and use as a source   Pietroni et. al. 2011
- Parametrize in terms of long-wavelength fields

Baumann et. al. 2012

Stochastic Fluid Equations

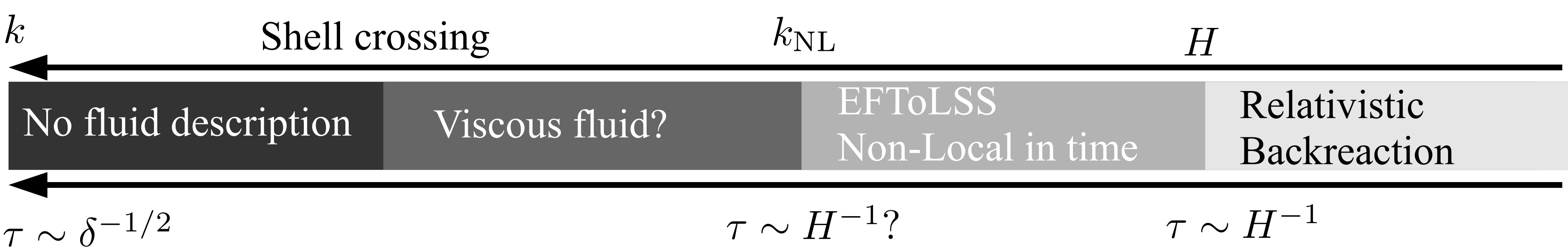
Effective viscous fluid with stochastic noise

$$\partial_D \mathbf{w} + \frac{3}{2}\frac{\gamma}{D}\left(\mathbf{w} + \frac{\nabla\psi}{D}\right) + \mathbf{w}\cdot\nabla\mathbf{w} = \mathbf{J} - \nu_1\nabla\delta + \nu_2\nabla^2\mathbf{w} + \nu_3\nabla\times\nabla\times\mathbf{w}$$

$$+\text{higher orders}$$

Interpreted as an effective field theory for long-wavelength perturbations

- Higher orders suppressed by powers of $\frac{\nabla}{k_*}$
- New terms constrained by symmetry
- Effective parameters a-priori unknown
  $\longrightarrow$ calibrate with simulations/observations
- Effective parameters ensure cutoff independence
  $\longrightarrow$ renormalization



Stochastic Adhesion Model

Rigopoulos 2014

Simplify:

Neglect Vorticity $\mathbf{w} = -\nabla h$

Zel'dovich approximation $\mathbf{w} = -\frac{\nabla \Psi}{D}$

Decoupled equation: $\partial_D h + \frac{1}{2} \left(\nabla h \right)^2 = J + \nu \nabla^2 h$

EFToLSS	Viscous Fluid
Cutoff at non-linear scale	Valid up virialization?
Viscosity is a perturbation	Viscosity at linear order
Non-local in time	Local in time

Carrasco et. al. 2012
Pajer, Zaldarriaga 2013
...

Blas et. al. 2015

Galilean Invariance

Galilean Transformation: $\mathbf{x} \to \mathbf{x} + \mathbf{T}(D)$ $\quad h(D) \to h(D) - \partial_D \mathbf{T}(D) \cdot \mathbf{x}$

Covariant derivative: $\partial_D + \frac{1}{2}(\nabla h) \cdot \nabla$ Original Fluid Equations Galilean Invariant

Gaussian Noise: $\langle J(D, \mathbf{k}) J(D', \mathbf{k}') \rangle = \Delta(D, D') \delta(\mathbf{k} + \mathbf{k}')$ → GI if local

From momentum conversation

Multiplicative noise

A non-local noise must be of the form $J(\mathbf{x}_{fl}) = J(\mathbf{x}) + \int_0^D d\eta \, (\nabla h)(\mathbf{x}) \cdot \nabla J(\mathbf{x}) + \ldots$

Non-local correlators

With $\mathbf{x}_{fl}(D) = \mathbf{x} + \int_{D_{in}}^D d\eta \, \nabla h(\mathbf{x}_{fl}(\eta), \eta)$

Viscosity: ∇h is not invariant → Build higher order operators from $\nabla^2 h$

$$\int_0^D dD' \, \nu(D, D') \nabla^2 h(D', \mathbf{x}_{fl}(D')) = \int_0^D dD' \, \nu(D, D') \nabla^2 h(D', \mathbf{x}(D'))$$

Non-local viscosity

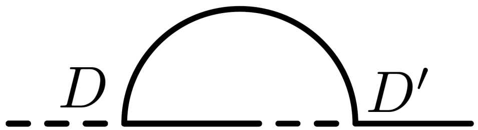
$$+ \int_0^D dD \int_0^{D'} d\eta \, \nu(D, D') \nabla^2 \left((\nabla h(x, \eta)) \cdot \nabla h(D', \mathbf{x}) \right) + \ldots$$

New vertex

Effective Filed Theory of Large Scale Structure

EFToLSS: SPT + perturbative corrections from ν and J

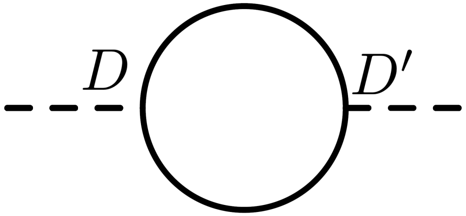
Propagator	Linear Power Spectrum	Vertex
$g(D, D') = \Theta(D - D')$	$P(k, D, D') = P_{in}(k)$	$\gamma\left(\mathbf{k}_1, \mathbf{k}_2\right) = \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{2}$



$$\sim -\frac{k^2}{12}\Theta(D - D')I$$

Renormalizes $-k^2\nu(D, D')$

$$I = \int \frac{d^3q}{(2\pi)^3} q^2 P_{in}(q)$$

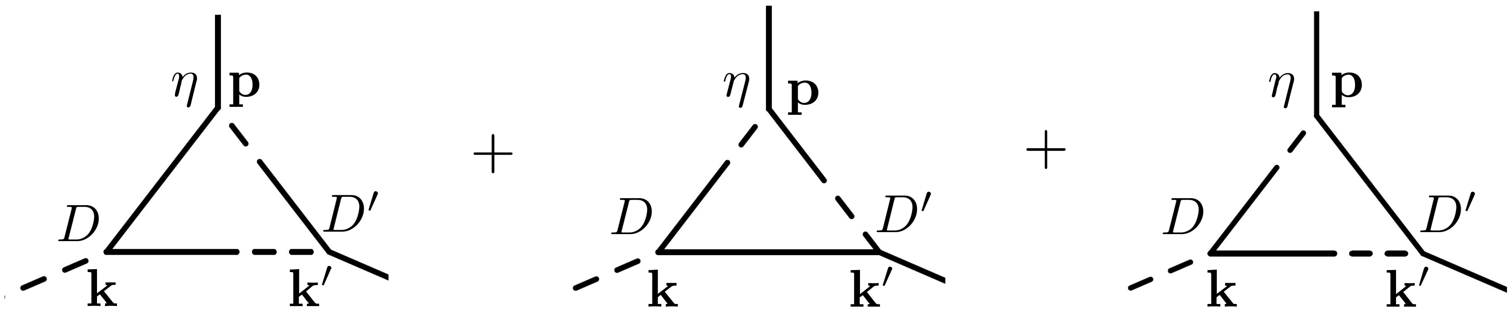


$$\sim \frac{1}{4}Y$$

Renormalizes $\Delta(D, D')$

$$Y = \int \frac{d^3q}{(2\pi)^3} q^4 P_{in}^2(q)$$

Effective Filed Theory of Large Scale Structure

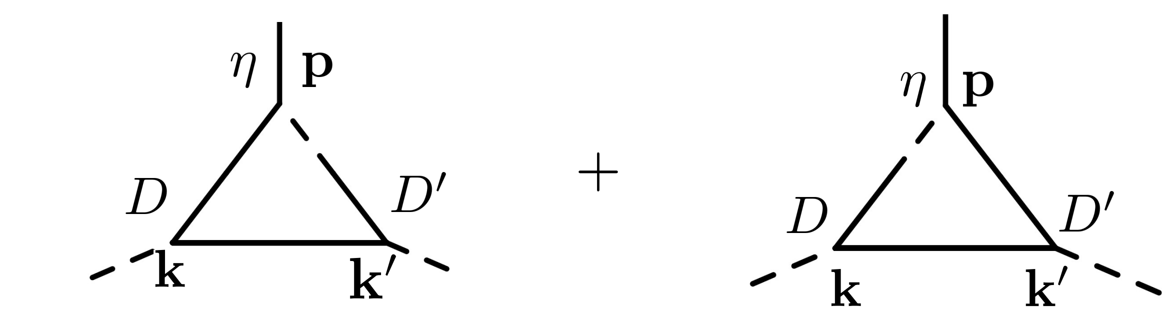


$$\sim 0\mathbf{k}'\cdot\mathbf{p} - \Theta(D-D')\Theta(D'-\eta)\frac{k^2\mathbf{k}'\cdot\mathbf{p}}{12}I + \Theta(D-D')\Theta(D-\eta)\frac{(\mathbf{k}'\cdot\mathbf{p})^2}{12}I + (D'\leftrightarrow\eta)$$

Original vertex protected

Corresponds to viscosity

Higher order vertex required



$$\sim \Theta(D'-\eta)\left(-\frac{\mathbf{k}\cdot\mathbf{p}}{16}Y - \frac{\mathbf{k}'\cdot\mathbf{p}}{16}Y + \frac{k^2}{12}Y\right) + (D'\leftrightarrow\eta)$$

Corresponds to noise

Multiplicative noise required

Viscous Fluid

Non-perturbative treatment of effective terms

Assume gaussian noise $\langle J(D,k)J(D',k')\rangle = \Delta\tilde{\nu}^3(D)\delta_D(D-D')\delta_D(\mathbf{k}+\mathbf{k}')$

Propagator: $g_k(D,D') = exp\left(-c_v^2 k^2 \int_{D'}^{D} d\eta\, \tilde{\nu}(\eta)\right)\Theta(D-D')$

Linear Power spectrum:

$$P(k,D,D') = P_{in}(k)g_k(D,0)g_k(D',0)$$
$$+\Delta\int_0^{\mathrm{min(D,D')}} d\eta\; g_k(D,\eta)g_k(D',\eta)\nu^3(\eta)$$

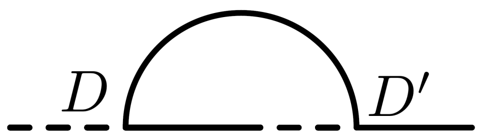
Asymptotic behavior:

$$P(k,D,D') = P_{in}(k)\left(1-\Theta(D-D')c_v^2k^2\int_0^{D'} d\eta\,\tilde{\nu}(\eta) + (D\leftrightarrow D')\right)\quad k\to 0$$

$$P(k,D,D') = \frac{\Delta\tilde{\nu}^2(D')}{2c_v^2k^2}g_k(D,D') + (D\leftrightarrow D')\quad k\to\infty$$

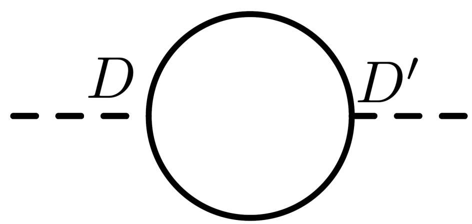
Viscous Fluid Renormalization

Viscosity



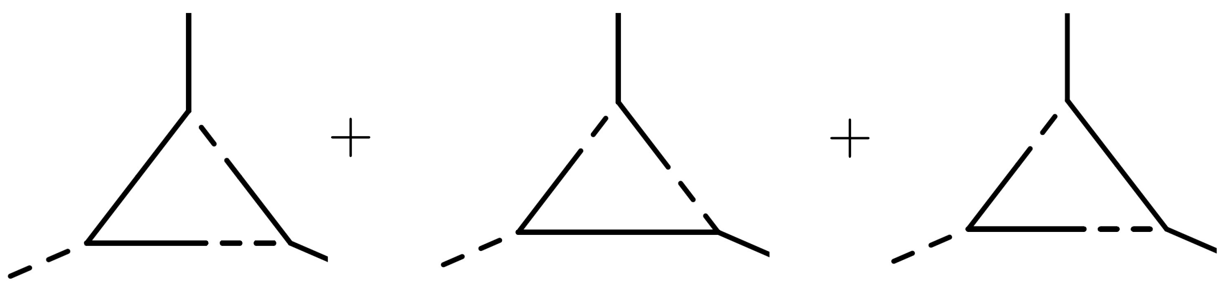
$$\sim -\tilde{\nu}(D)\frac{\Delta}{4c_v^4}\delta_D(D-D')\int\frac{d^3q}{(2\pi)^3}\frac{(q^2+\mathbf{q}\cdot\mathbf{k})\mathbf{q}\cdot\mathbf{k}}{q^4}$$

Noise



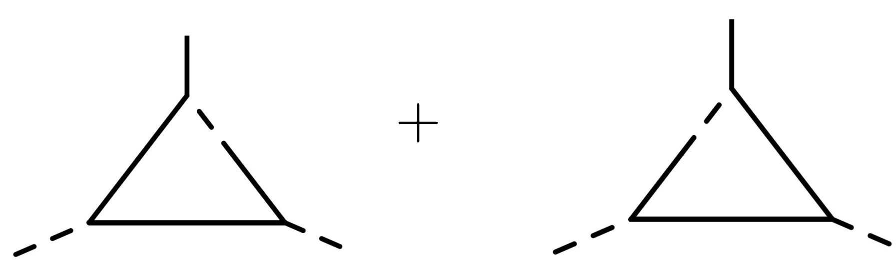
$$\sim \tilde{\nu}^3(D)\frac{\Delta^2}{8c_v^6}\delta_D(D-D')\int\frac{d^3q}{(2\pi)^3}\frac{1}{q^2}$$

Vertex



$= \text{finite}$

Multiplicative noise



$= \text{finite}$

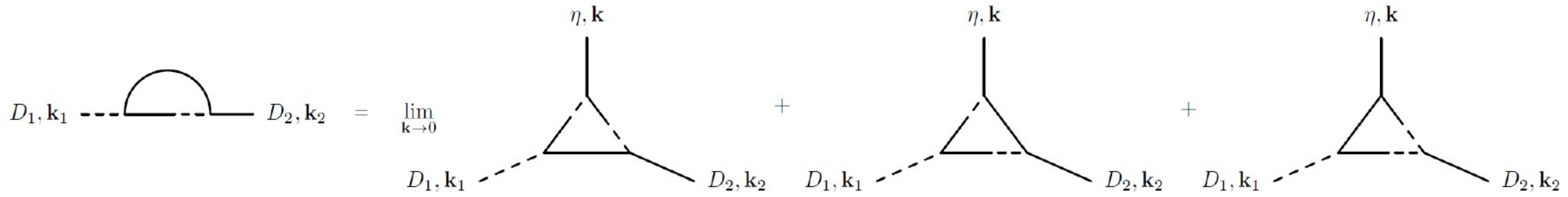
Ward-Identities

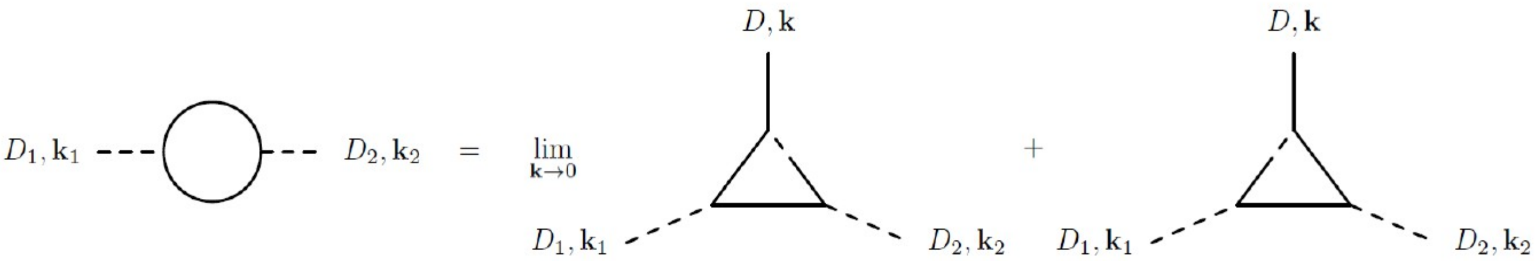
Written in terms of 1PI-vertices the Ward-Identities read, for $n \neq 1 \;\; m \neq 0$

$$\sum_{i=1}^{n} \mathbf{p}_i \delta_D(D-\eta_i)\Gamma^{(n,m)} + \sum_{i=1}^{m} \mathbf{q}_i \delta_D(D-\lambda_i)\Gamma^{(n,m)} = \int d^3k \, \delta_D(\mathbf{k})\partial_D \partial_{\mathbf{k}} \Gamma^{(n,m+1)}$$

Kehagias, Riotto 2013
Peloso, Pietroni 2014
Frey, Täuber et. al 1994

At one loop





Ward-Identities

At one loop the WI can be written as

$$\Sigma\left(D_1, D_2; k_1\right) \mathbf{k}_1 \left(\theta\left(\eta - D_1\right) - \theta\left(\eta - D_2\right)\right) = \lim_{\mathbf{k}\to 0} \partial_{\mathbf{k}} \Pi\left(D_1, D_2, \eta; \mathbf{k}_1, \mathbf{k}\right)$$

$$\Phi\left(D_1, D_2; k_1\right) \mathbf{k}_1 \left(\theta\left(\eta - D_1\right) - \theta\left(\eta - D_2\right)\right) = \lim_{\mathbf{k}\to 0} \partial_{\mathbf{k}} \Psi\left(D_1, D_2, \eta; \mathbf{k}_1, \mathbf{k}\right)$$

In the limit $\mathbf{k} \to 0$ $\quad \Pi = \mathbf{k} \cdot \mathbf{k}_1 \tilde{\Pi}(k_1) \quad \Psi = \mathbf{k} \cdot \mathbf{k}_1 \tilde{\Psi}(k_1)$

Plugging in

$$\Pi\left(D_1, D_2, \eta; \mathbf{k}_1, \mathbf{k}\right) = \mathbf{k}_1 \cdot \mathbf{k} \Sigma\left(D_1, D_2; k_1\right) \left(\theta\left(\eta - D_1\right) - \theta\left(\eta - D_2\right)\right) + O(k^2)$$

$$\Psi\left(D_1, D_2, \eta; \mathbf{k}_1, \mathbf{k}\right) = \mathbf{k}_1 \cdot \mathbf{k} \Phi\left(D_1, D_2; k_1\right) \left(\theta\left(\eta - D_1\right) - \theta\left(\eta - D_2\right)\right) + O(k^2)$$

Non-Local: Divergences appears in higher vertices again

Local: Vertices protected

Comparison

Neglect the noise and assume a numerical small velocity

Choose $\nu(D) = c_v^2 D$

$$B_{EFT/Fluid} = B_{tree} + B_{1-loop} + g D_3^3 \frac{(\mathbf{k}_1 \cdot \mathbf{k}_2)^2}{6} P_{in}(k_1) P_{in}(k_2)$$

$$-c_v^2 \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{2} P_{in}(k_1) P_{in}(k_2) \left(\frac{k_1^2}{6}(D_3^2 + 3D_2^2 D_3) + \frac{k_2^2}{6}(D_3^2 + 3D_1^2 D_3) \right.$$

$$\left. + k_3^2 D_3^3 \left(\frac{1}{2} - \frac{1}{6} \right) \right) + permutations$$

c_v Fixed by 1-loop Power Spectrum → Difference from extra vertex and integrals $\int dD D D^n \neq \int dD \int dD D^n$

g Needs to be fixed by 2-loop Power Spectrum

EFT seems to prefer a local viscosity

Foreman, Senatore 2015
Carrasco et. al 2014

Conclusion and Outlook

- Counter terms in EFToLSS necessarily non-local
  - New vertices at higher order
  - WI guarantees that renormalization is possible
- Local Viscosity possible
- Local and Non-local viscosity differ on large scales
- Relation to the approach of Blas et. al.?
- Is the noise important?
  - Can the noise reproduce the PS at small scales
- Derive from UV-completion